Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

3-Logarithms/59-3.2.1-f+g-x-^m-A+B-log-e-a+b-x-over-c+d-x-^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [314]. This is test number [59].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (314)	0.00 (0)
Mathematica	95.86 (301)	4.14 (13)
Maxima	75.80 (238)	24.20 (76)
Maple	71.34 (224)	28.66 (90)
Fricas	66.88 (210)	33.12 (104)
Mupad	63.69 (200)	36.31 (114)
Giac	60.83 (191)	39.17 (123)
Sympy	36.31 (114)	63.69 (200)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

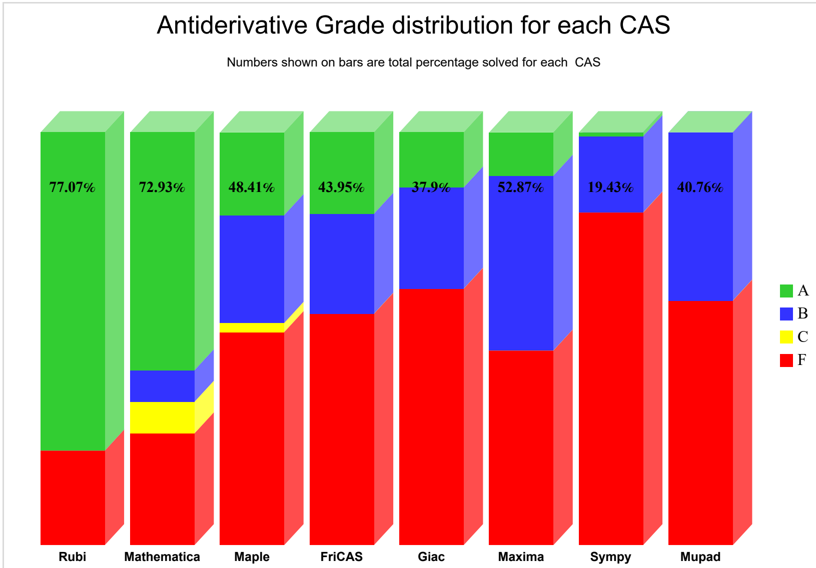
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

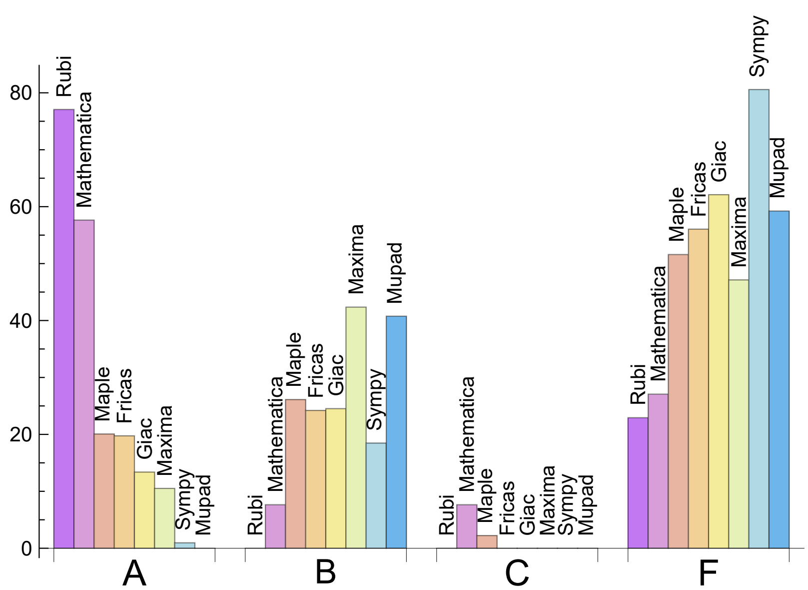
System	% A grade	% B grade	% C grade	% F grade
Rubi	77.070	0.000	0.000	22.930
Mathematica	57.643	7.643	7.643	27.070
Maple	20.064	26.115	2.229	51.592
Fricas	19.745	24.204	0.000	56.051
Giac	13.376	24.522	0.000	62.102
Maxima	10.510	42.357	0.000	47.134
Sympy	0.955	18.471	0.000	80.573
Mupad	0.000	40.764	0.000	59.236

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	13	100.00	0.00	0.00
Maxima	76	100.00	0.00	0.00
Maple	90	98.89	1.11	0.00
Fricas	104	92.31	7.69	0.00
Mupad	114	0.00	100.00	0.00
Giac	123	95.12	4.88	0.00
Sympy	200	19.00	69.00	12.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.29
Rubi	0.47
Mathematica	0.67
Fricas	1.25
Mupad	3.64
Giac	5.05
Maple	12.86
Sympy	17.70

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	213.77	1.01	138.00	1.00
Fricas	368.95	2.32	163.50	2.05
Mathematica	398.74	1.56	144.00	1.06
Mupad	429.54	2.05	160.00	1.54
Sympy	451.38	4.54	274.50	3.66
Maple	636.55	2.78	218.00	1.64
Maxima	692.84	4.16	428.00	3.08
Giac	980.03	5.14	224.00	1.65

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

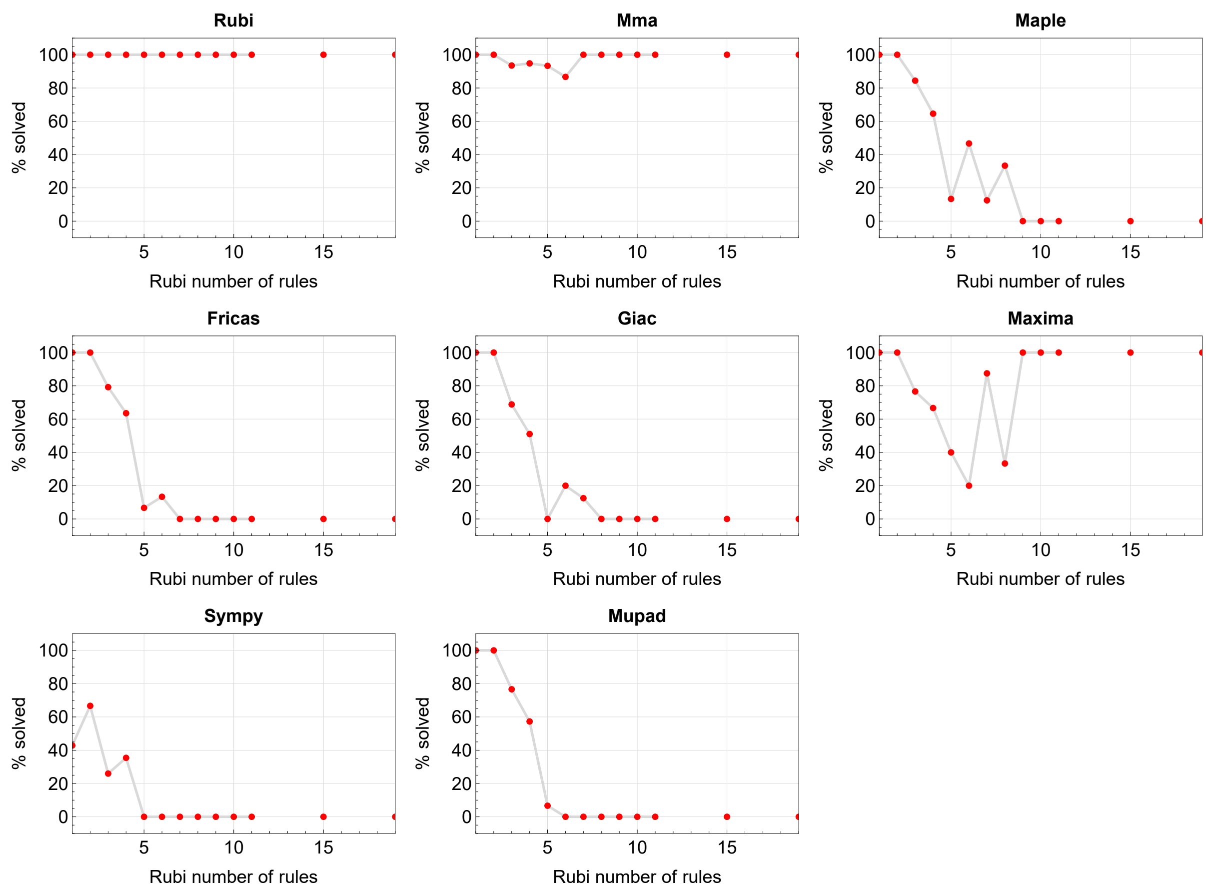


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

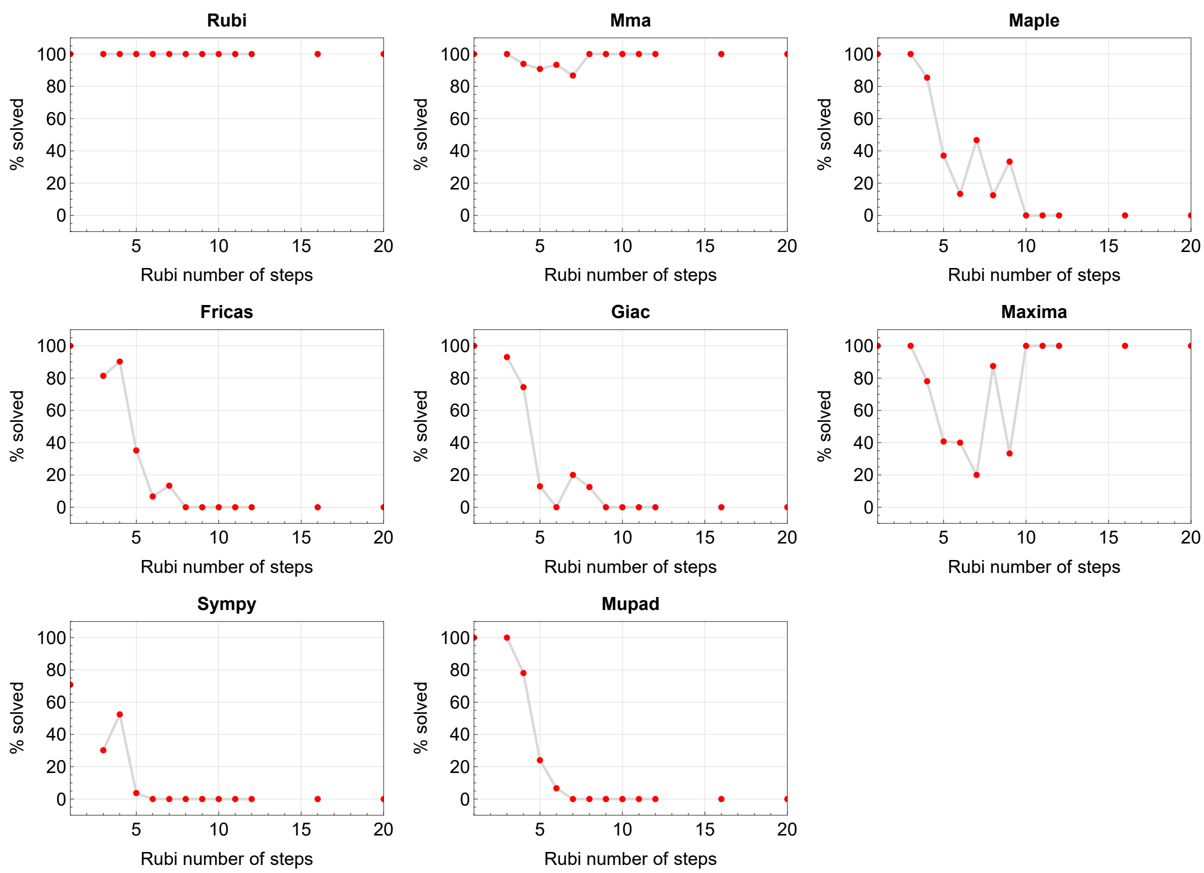


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

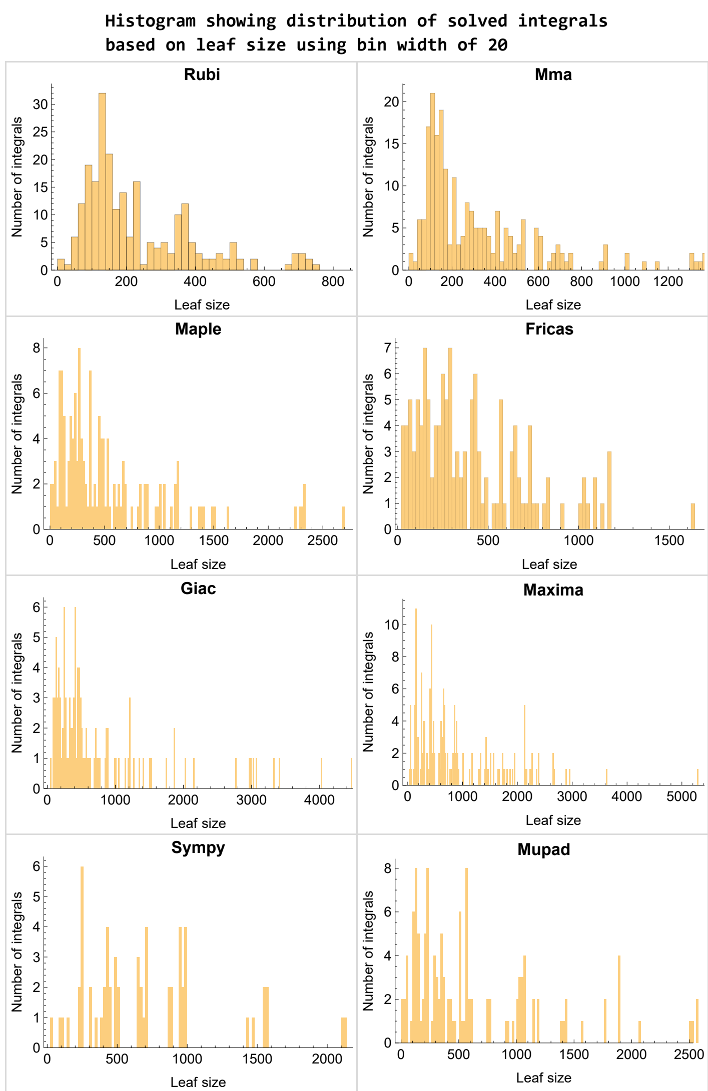


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

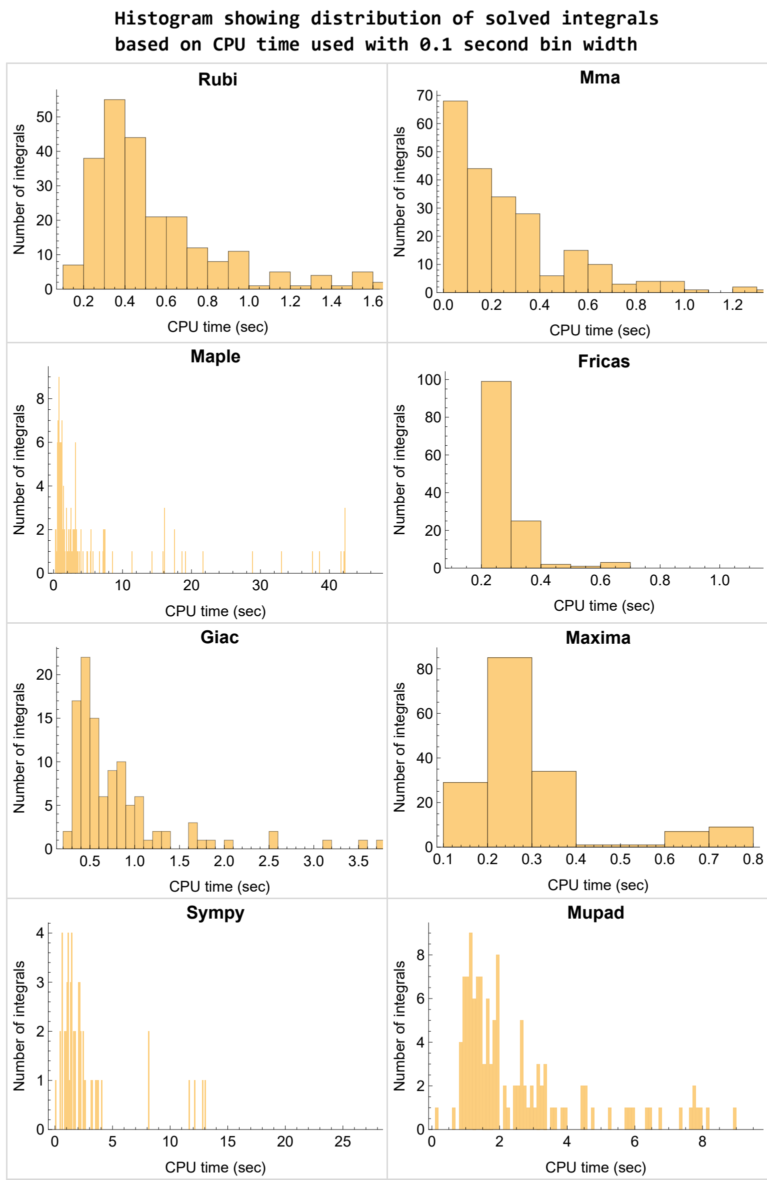


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

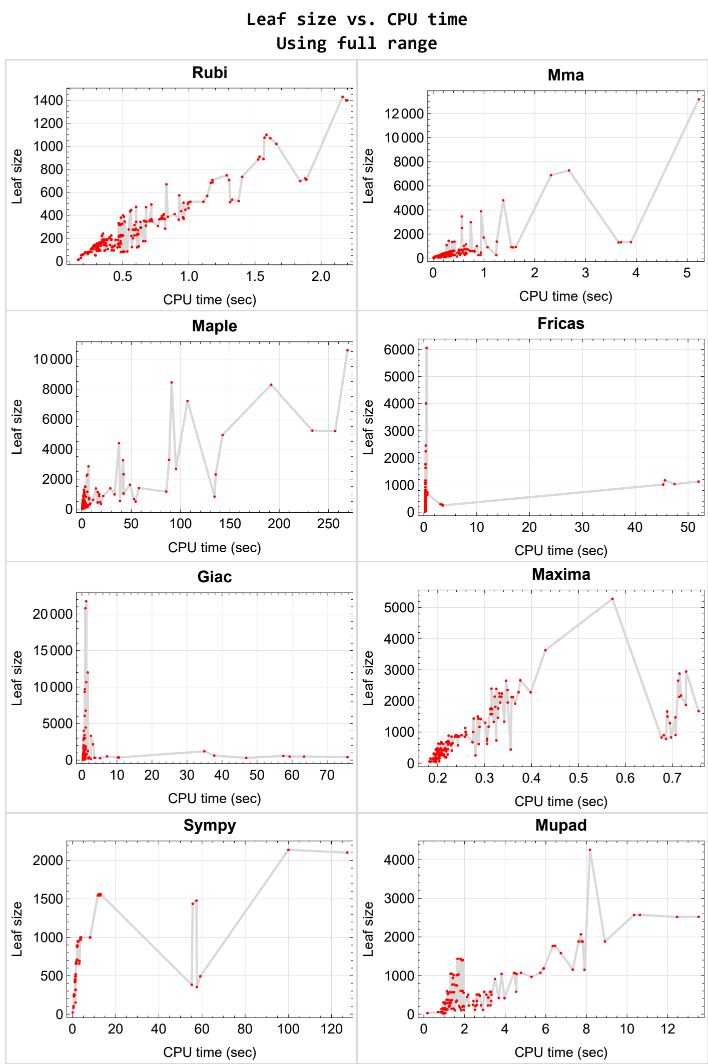


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 196, 197, 198, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 229, 303, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314}

Mathematica {}

Maple {151, 156, 157, 158, 298, 303, 305}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

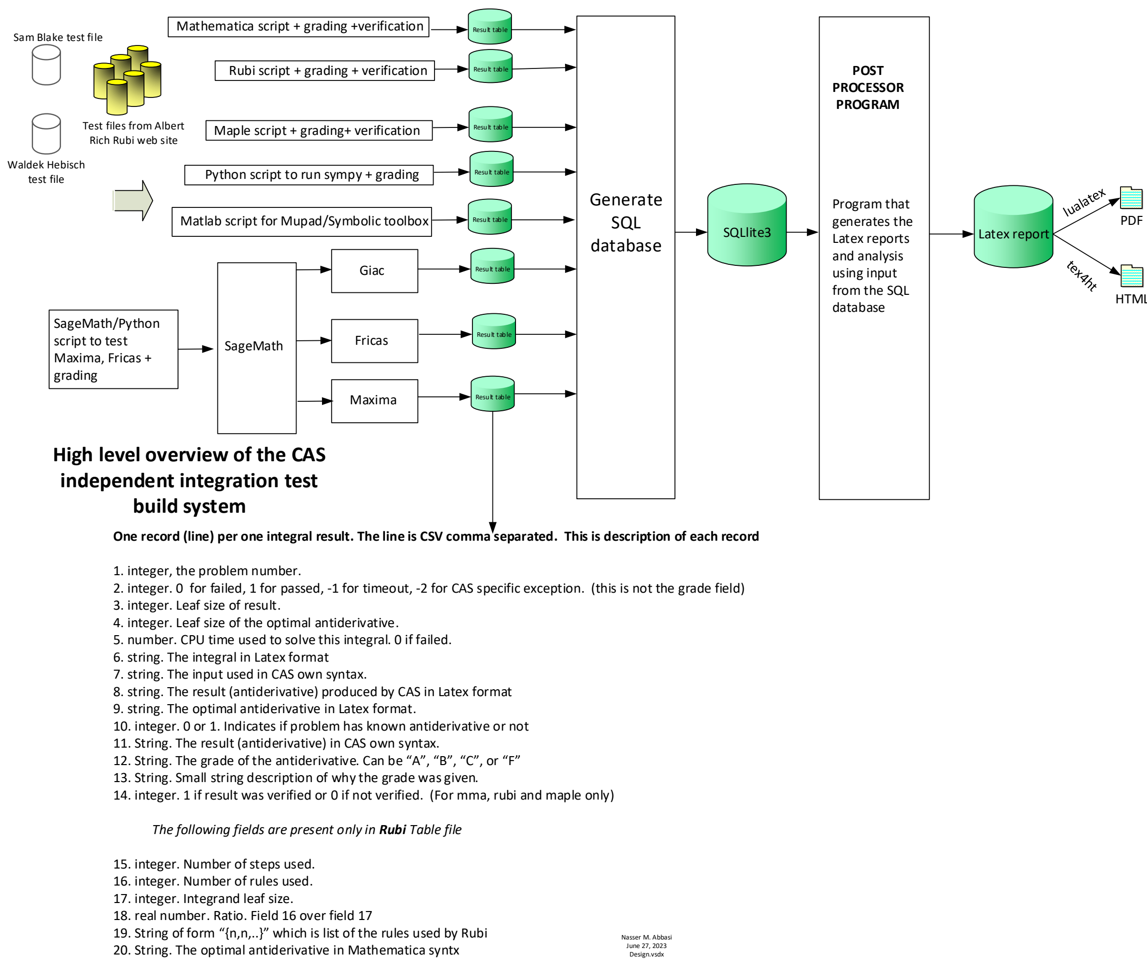
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 222, 223, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 311 }

B grade { 14, 71, 72, 101, 106, 147, 156, 157, 158, 159, 164, 165, 166, 167, 168, 244, 245, 276, 277, 306, 307, 308, 309, 310 }

C grade { 15, 16, 17, 18, 43, 44, 45, 46, 102, 103, 104, 105, 133, 134, 135, 136, 187, 188, 189, 190, 215, 216, 217, 218 }

F normal fail { 140, 141, 145, 146, 172, 222, 223, 227, 228, 229, 312, 313, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 6, 7, 34, 35, 43, 61, 90, 91, 93, 94, 102, 103, 106, 107, 108, 112, 113, 117, 118, 121, 122, 124, 125, 133, 134, 152, 175, 176, 177, 178, 179, 180, 187, 188, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 249, 262, 263, 264, 265, 266, 297 }

B grade { 1, 2, 3, 4, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 36, 37, 44, 45, 46, 57, 58, 59, 60, 63, 64, 65, 66, 88, 89, 92, 95, 96, 101, 104, 105, 119, 120, 123, 126, 127, 135, 136, 147, 148, 149, 150, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 181, 186, 189, 190, 235, 236, 237, 238, 239, 244, 267, 268, 269, 270, 271, 293, 294, 295, 296, 299, 300, 301, 302 }

C grade { 151, 156, 157, 158, 298, 303, 305 }

F normal fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 97, 98, 99, 100, 128, 129, 130, 131, 132, 140, 141, 145, 146, 159, 164, 165, 166, 167, 172, 182, 183, 184, 185, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 240, 241, 242, 243, 245, 246, 247, 248, 272, 273, 274, 275, 276, 277, 278, 279, 280, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { 304 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 4, 6, 7, 15, 22, 23, 27, 34, 35, 43, 50, 51, 55, 60, 61, 91, 93, 94, 102, 103, 104, 105, 106, 107, 108, 112, 113, 117, 122, 124, 125, 133, 134, 135, 136, 152, 172, 176, 178, 179, 187, 188, 189, 194, 195, 204, 206, 207, 215, 216, 217, 229, 230, 232, 233, 234, 249, 262, 265, 266, 296, 297 }

B grade { 1, 2, 3, 8, 9, 16, 17, 18, 28, 29, 30, 31, 32, 36, 37, 44, 45, 46, 56, 57, 58, 59, 63, 64, 88, 89, 90, 95, 96, 118, 119, 120, 121, 126, 127, 147, 148, 149, 150, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 180, 181, 190, 199, 200, 201, 202, 203, 208, 209, 218, 231, 236, 237, 263, 264, 268, 269, 293, 294, 295, 299, 300 }

C grade { }

F normal fail { 5, 10, 11, 12, 13, 14, 33, 38, 39, 40, 41, 42, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 177, 182, 183, 184, 185, 186, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { 65, 66, 238, 239, 270, 271, 301, 302 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 4, 7, 32, 34, 35, 57, 58, 59, 60, 61, 63, 64, 91, 94, 150, 152, 153, 176, 179, 206, 230, 231, 232, 233, 234, 236, 249, 266, 293, 295, 296, 297, 299 }

B grade { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 29, 30, 31, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 65, 66, 67, 68, 69, 88, 89, 90, 93, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 147, 148, 149, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 178, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 237, 238, 239, 240, 241, 242, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 294, 300, 301, 302, 303, 304 }

C grade { }

F normal fail { 5, 14, 22, 23, 27, 28, 33, 42, 50, 51, 55, 56, 62, 70, 71, 72, 73, 74, 75, 92, 101, 112, 113, 117, 118, 123, 132, 140, 141, 145, 146, 151, 159, 164, 165, 166, 167, 172, 177, 186, 194, 195, 199, 200, 205, 214, 222, 223, 227, 228, 229, 235, 243, 244, 245, 246, 247, 248, 267, 275, 276, 277, 278, 279, 280, 298, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 6, 7, 15, 16, 17, 34, 35, 43, 44, 45, 55, 56, 93, 94, 102, 103, 104, 105, 122, 125, 136, 150, 152, 153, 178, 179, 187, 188, 189, 199, 200, 204, 206, 207, 218, 263, 264, 265, 266, 296, 297, 299 }

B grade { 1, 2, 3, 4, 8, 9, 18, 29, 30, 31, 32, 33, 36, 37, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 95, 96, 106, 107, 108, 119, 120, 121, 124, 126, 127, 133, 147, 148, 149, 154, 155, 173, 174, 175, 176, 177, 180, 181, 190, 201, 202, 203, 208, 209, 215, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 268, 269, 270, 271, 295, 300, 301, 302 }

C grade { }

F normal fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 38, 39, 40, 41, 42, 50, 51, 62, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 134, 135, 140, 141, 145, 146, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 182, 183, 184, 185, 186, 194, 195, 205, 210, 211, 212, 213, 214, 216, 217, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314 }

F(-1) timedout fail { 67, 262, 293, 294, 303, 309 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 34, 35, 36, 37, 43, 44, 45, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 136, 147, 148, 149, 150, 152, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 190, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 262, 263, 264, 265, 266, 268, 269, 270, 271, 293, 294, 295, 296, 297, 299, 300, 301, 302 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 182, 183, 184, 185, 186, 194, 195, 199, 200, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 61, 234, 249 }

B grade { 4, 7, 32, 35, 60, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 230, 231, 232, 233, 262, 263, 264, 265, 266 }

C grade { }

F normal fail { 5, 14, 15, 16, 17, 22, 33, 42, 43, 44, 45, 50, 70, 92, 101, 106, 107, 108, 112, 113, 117, 123, 132, 140, 141, 145, 151, 159, 177, 186, 194, 195, 199, 205, 214, 222, 223, 227 }

F(-1) timedout fail { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 18, 23, 27, 28, 29, 30, 31, 34, 36, 37, 38, 39, 40, 41, 46, 51, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 79, 80, 81, 86, 87, 97, 98, 99, 100, 105, 114, 118, 128, 129, 130, 131, 136, 142, 146, 152, 153, 154, 155, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 182, 183, 184, 185, 190, 200, 210, 211, 212, 213, 218, 228, 229, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 255, 256, 259, 260, 261, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 290, 291, 292, 299, 300, 301, 302, 307, 308, 313, 314 }

F(-2) exception fail { 147, 148, 149, 150, 156, 157, 158, 164, 165, 166, 293, 294, 295, 296, 297, 298, 303, 304, 305, 306, 309, 310, 311, 312 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	158	146	1004	676	569	0	4462	1046
N.S.	1	0.84	0.78	5.34	3.60	3.03	0.00	23.73	5.56
time (sec)	N/A	0.310	0.079	17.529	0.207	0.337	0.000	1.060	1.378

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	134	124	755	479	426	0	3034	588
N.S.	1	0.86	0.79	4.84	3.07	2.73	0.00	19.45	3.77
time (sec)	N/A	0.294	0.072	7.215	0.215	0.317	0.000	0.813	1.142

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	110	103	528	309	296	0	1866	303
N.S.	1	0.89	0.83	4.26	2.49	2.39	0.00	15.05	2.44
time (sec)	N/A	0.272	0.041	2.894	0.197	0.281	0.000	0.810	0.962

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	82	73	276	156	160	352	880	134
N.S.	1	0.95	0.85	3.21	1.81	1.86	4.09	10.23	1.56
time (sec)	N/A	0.247	0.029	1.066	0.191	0.274	57.622	0.485	0.897

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	101	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.504	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	115	131	137	103	0	88	112
N.S.	1	1.00	1.72	1.96	2.04	1.54	0.00	1.31	1.67
time (sec)	N/A	0.228	0.043	3.125	0.189	0.280	0.000	0.758	2.516

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	142	114	271	259	265	2139	224	222
N.S.	1	0.94	0.75	1.79	1.72	1.75	14.17	1.48	1.47
time (sec)	N/A	0.322	0.101	7.393	0.205	0.268	100.059	0.782	1.292

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	167	145	440	432	482	0	381	349
N.S.	1	0.91	0.79	2.40	2.36	2.63	0.00	2.08	1.91
time (sec)	N/A	0.343	0.112	15.869	0.200	0.276	0.000	0.663	1.622

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	190	162	1043	651	733	0	541	603
N.S.	1	0.88	0.75	4.85	3.03	3.41	0.00	2.52	2.80
time (sec)	N/A	0.379	0.136	42.201	0.208	0.308	0.000	0.882	1.812

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	461	535	0	2945	0	0	0	0
N.S.	1	1.16	1.35	0.00	7.44	0.00	0.00	0.00	0.00
time (sec)	N/A	0.982	0.326	0.000	0.729	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	386	411	0	2175	0	0	0	0
N.S.	1	1.15	1.23	0.00	6.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.814	0.226	0.000	0.718	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	309	303	0	1501	0	0	0	0
N.S.	1	1.13	1.11	0.00	5.48	0.00	0.00	0.00	0.00
time (sec)	N/A	0.666	0.164	0.000	0.690	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	225	215	0	828	0	0	0	0
N.S.	1	1.15	1.10	0.00	4.22	0.00	0.00	0.00	0.00
time (sec)	N/A	0.478	0.136	0.000	0.697	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	127	537	0	0	0	0	0	0
N.S.	1	0.92	3.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.459	0.814	0.000	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	110	330	296	430	258	0	174	238
N.S.	1	0.81	2.43	2.18	3.16	1.90	0.00	1.28	1.75
time (sec)	N/A	0.299	0.331	3.136	0.216	0.272	0.000	0.989	2.608

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	224	463	672	861	651	0	478	506
N.S.	1	0.78	1.61	2.33	2.99	2.26	0.00	1.66	1.76
time (sec)	N/A	0.404	0.288	7.388	0.246	0.287	0.000	1.039	3.078

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	345	612	1146	1432	1164	0	840	1038
N.S.	1	0.77	1.37	2.56	3.20	2.60	0.00	1.88	2.32
time (sec)	N/A	0.505	0.415	16.000	0.287	0.317	0.000	1.321	4.562

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	615	473	700	2326	2136	1762	0	1206	1769
N.S.	1	0.77	1.14	3.78	3.47	2.87	0.00	1.96	2.88
time (sec)	N/A	0.580	0.556	42.228	0.336	0.332	0.000	1.694	6.460

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	53	95	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.51	2.71	1.06	1.06
time (sec)	N/A	0.194	0.408	0.053	0.431	0.253	34.232	27.573	0.758

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	56	35	35
N.S.	1	1.00	1.06	1.00	1.06	1.06	1.70	1.06	1.06
time (sec)	N/A	0.185	0.128	0.099	0.441	0.265	16.309	16.773	0.672

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	43	61	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.23	1.74	1.06	1.06
time (sec)	N/A	0.199	0.151	0.210	0.342	0.249	16.193	9.085	0.617

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	94	0	0	62	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.314	0.144	0.000	0.000	0.262	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	185	172	0	0	149	0	0	0
N.S.	1	0.94	0.87	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.456	0.282	0.000	0.000	0.280	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	329	81	187	37	37
N.S.	1	1.00	1.06	1.00	9.40	2.31	5.34	1.06	1.06
time (sec)	N/A	0.200	0.429	0.053	0.478	0.273	44.805	1.456	1.310

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	251	63	117	35	35
N.S.	1	1.00	1.06	1.00	7.61	1.91	3.55	1.06	1.06
time (sec)	N/A	0.188	0.463	0.018	0.488	0.264	58.784	0.789	0.835

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	186	85	128	37	37
N.S.	1	1.00	1.06	1.00	5.31	2.43	3.66	1.06	1.06
time (sec)	N/A	0.200	0.278	0.076	0.357	0.282	130.893	0.681	0.701

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	141	146	0	0	274	0	0	0
N.S.	1	0.92	0.95	0.00	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.380	0.161	0.000	0.000	0.273	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	314	276	254	0	0	755	0	0	0
N.S.	1	0.88	0.81	0.00	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.528	0.400	0.000	0.000	0.291	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	157	146	864	676	572	0	1876	1045
N.S.	1	0.84	0.78	4.60	3.60	3.04	0.00	9.98	5.56
time (sec)	N/A	0.321	0.072	17.540	0.207	0.327	0.000	1.090	1.314

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	133	124	652	479	429	0	1402	588
N.S.	1	0.85	0.79	4.18	3.07	2.75	0.00	8.99	3.77
time (sec)	N/A	0.291	0.060	7.138	0.199	0.297	0.000	0.836	1.141

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	109	101	463	309	297	0	990	303
N.S.	1	0.88	0.81	3.73	2.49	2.40	0.00	7.98	2.44
time (sec)	N/A	0.273	0.040	2.921	0.191	0.287	0.000	0.629	1.023

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	81	74	250	156	162	382	580	134
N.S.	1	0.94	0.86	2.91	1.81	1.88	4.44	6.74	1.56
time (sec)	N/A	0.250	0.026	1.069	0.195	0.276	55.240	0.441	0.871

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	85	101	0	0	0	0	566	0
N.S.	1	1.06	1.26	0.00	0.00	0.00	0.00	7.08	0.00
time (sec)	N/A	0.518	0.031	0.000	0.000	0.000	0.000	57.494	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	77	114	81	136	105	0	91	113
N.S.	1	0.75	1.12	0.79	1.33	1.03	0.00	0.89	1.11
time (sec)	N/A	0.211	0.040	3.108	0.194	0.271	0.000	0.598	1.064

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	142	115	272	259	266	2103	207	221
N.S.	1	0.94	0.76	1.80	1.72	1.76	13.93	1.37	1.46
time (sec)	N/A	0.317	0.089	7.432	0.194	0.276	127.349	0.927	1.265

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	166	146	440	433	483	0	405	349
N.S.	1	0.91	0.80	2.40	2.37	2.64	0.00	2.21	1.91
time (sec)	N/A	0.351	0.100	16.009	0.205	0.280	0.000	0.676	1.623

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	190	162	1043	652	735	0	684	603
N.S.	1	0.88	0.75	4.85	3.03	3.42	0.00	3.18	2.80
time (sec)	N/A	0.375	0.129	42.246	0.209	0.306	0.000	0.848	1.902

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	544	720	533	0	2880	0	0	0	0
N.S.	1	1.32	0.98	0.00	5.29	0.00	0.00	0.00	0.00
time (sec)	N/A	1.849	0.311	0.000	0.715	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	454	535	409	0	2129	0	0	0	0
N.S.	1	1.18	0.90	0.00	4.69	0.00	0.00	0.00	0.00
time (sec)	N/A	1.313	0.226	0.000	0.715	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	361	376	303	0	1473	0	0	0	0
N.S.	1	1.04	0.84	0.00	4.08	0.00	0.00	0.00	0.00
time (sec)	N/A	0.933	0.154	0.000	0.706	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	239	216	0	825	0	0	0	0
N.S.	1	1.09	0.98	0.00	3.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	0.124	0.000	0.677	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	128	268	0	0	0	0	0	0
N.S.	1	0.93	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.527	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	119	331	294	428	263	0	175	237
N.S.	1	0.73	2.03	1.80	2.63	1.61	0.00	1.07	1.45
time (sec)	N/A	0.265	0.257	3.121	0.200	0.277	0.000	0.987	2.422

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	240	464	672	861	654	0	407	505
N.S.	1	0.76	1.46	2.12	2.72	2.06	0.00	1.28	1.59
time (sec)	N/A	0.341	0.258	7.409	0.240	0.285	0.000	0.967	2.183

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	331	612	1146	1435	1167	0	776	1040
N.S.	1	0.77	1.43	2.67	3.34	2.72	0.00	1.81	2.42
time (sec)	N/A	0.464	0.382	16.052	0.277	0.294	0.000	1.308	3.823

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	399	700	2326	2138	1768	0	1265	1765
N.S.	1	0.74	1.31	4.34	3.99	3.30	0.00	2.36	3.29
time (sec)	N/A	0.499	0.510	42.183	0.332	0.317	0.000	1.685	6.344

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	53	95	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.51	2.71	1.06	1.06
time (sec)	N/A	0.196	0.165	0.053	0.437	0.257	24.070	27.328	0.703

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	56	35	35
N.S.	1	1.00	1.06	1.00	1.06	1.06	1.70	1.06	1.06
time (sec)	N/A	0.184	0.117	0.097	0.428	0.265	14.064	15.913	0.670

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	43	61	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.23	1.74	1.06	1.06
time (sec)	N/A	0.201	0.217	0.205	0.329	0.270	18.380	9.260	0.648

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	62	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.304	0.125	0.000	0.000	0.269	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	187	174	0	0	147	0	0	0
N.S.	1	0.94	0.87	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.398	0.260	0.000	0.000	0.261	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	329	81	187	37	37
N.S.	1	1.00	1.06	1.00	9.40	2.31	5.34	1.06	1.06
time (sec)	N/A	0.201	0.429	0.054	0.474	0.263	45.489	0.997	1.316

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	251	63	117	35	35
N.S.	1	1.00	1.06	1.00	7.61	1.91	3.55	1.06	1.06
time (sec)	N/A	0.184	0.470	0.019	0.484	0.261	58.587	0.691	0.894

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	186	85	128	37	37
N.S.	1	1.00	1.06	1.00	5.31	2.43	3.66	1.06	1.06
time (sec)	N/A	0.203	0.256	0.074	0.366	0.260	139.335	0.628	0.757

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	142	180	0	0	291	0	146	0
N.S.	1	0.92	1.17	0.00	0.00	1.89	0.00	0.95	0.00
time (sec)	N/A	0.354	0.145	0.000	0.000	0.278	0.000	0.362	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	342	288	0	0	770	0	325	0
N.S.	1	1.34	1.12	0.00	0.00	3.01	0.00	1.27	0.00
time (sec)	N/A	0.613	0.347	0.000	0.000	0.281	0.000	0.420	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	347	285	1160	631	736	0	11996	1433
N.S.	1	0.95	0.78	3.19	1.73	2.02	0.00	32.96	3.94
time (sec)	N/A	0.671	0.390	5.459	0.201	0.663	0.000	1.674	1.659

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	227	219	976	443	521	0	6772	766
N.S.	1	0.97	0.93	4.15	1.89	2.22	0.00	28.82	3.26
time (sec)	N/A	0.478	0.184	2.960	0.196	0.414	0.000	1.050	1.405

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	159	146	617	282	334	0	3408	371
N.S.	1	1.01	0.93	3.93	1.80	2.13	0.00	21.71	2.36
time (sec)	N/A	0.355	0.094	1.631	0.198	0.329	0.000	0.721	1.117

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	127	120	366	150	179	493	1215	153
N.S.	1	1.10	1.04	3.18	1.30	1.56	4.29	10.57	1.33
time (sec)	N/A	0.303	0.081	0.814	0.184	0.272	59.226	0.566	0.933

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	82	52	63	150	243	52
N.S.	1	1.00	1.00	1.46	0.93	1.12	2.68	4.34	0.93
time (sec)	N/A	0.188	0.008	0.218	0.182	0.292	1.388	0.332	0.694

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	154	122	0	0	0	0	0	0
N.S.	1	1.05	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	141	109	364	142	294	0	461	140
N.S.	1	1.55	1.20	4.00	1.56	3.23	0.00	5.07	1.54
time (sec)	N/A	0.288	0.089	3.514	0.191	3.144	0.000	0.546	1.365

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	173	1376	355	1175	0	2994	430
N.S.	1	1.00	0.91	7.24	1.87	6.18	0.00	15.76	2.26
time (sec)	N/A	0.421	0.296	14.207	0.202	45.636	0.000	0.865	3.156

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	274	264	3257	852	0	0	9692	1182
N.S.	1	0.97	0.93	11.51	3.01	0.00	0.00	34.25	4.18
time (sec)	N/A	0.575	0.424	41.680	0.237	0.000	0.000	0.852	5.886

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	371	359	5206	1761	0	0	21743	2569
N.S.	1	0.96	0.93	13.42	4.54	0.00	0.00	56.04	6.62
time (sec)	N/A	0.763	0.613	256.759	0.313	0.000	0.000	1.213	10.342

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	923	1100	757	0	2651	0	0	0	0
N.S.	1	1.19	0.82	0.00	2.87	0.00	0.00	0.00	0.00
time (sec)	N/A	1.615	0.662	0.000	0.712	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	565	706	506	0	1659	0	0	0	0
N.S.	1	1.25	0.90	0.00	2.94	0.00	0.00	0.00	0.00
time (sec)	N/A	1.159	0.361	0.000	0.689	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	290	410	362	0	899	0	0	0	0
N.S.	1	1.41	1.25	0.00	3.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.782	0.203	0.000	0.682	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	128	226	0	0	0	0	0	0
N.S.	1	0.95	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.604	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	367	1441	0	0	0	0	0	0
N.S.	1	1.24	4.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.716	0.304	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	220	418	0	0	0	0	0	0
N.S.	1	1.07	2.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	0.280	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	517	615	0	0	0	0	0	0
N.S.	1	1.33	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.991	0.809	0.000	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	747	909	918	0	0	0	0	0	0
N.S.	1	1.22	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.497	1.627	0.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1208	1429	1329	0	0	0	0	0	0
N.S.	1	1.18	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.178	3.893	0.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	43	31	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.34	0.97	1.06	1.06
time (sec)	N/A	0.188	0.170	0.055	0.434	0.279	28.897	26.492	0.723

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	32	29	32	32
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.97	1.07	1.07
time (sec)	N/A	0.182	0.126	0.095	0.430	0.254	16.379	16.024	0.667

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08
time (sec)	N/A	0.164	0.014	0.065	0.328	0.260	4.036	11.363	0.589

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	39	0	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.22	0.00	1.06	1.06
time (sec)	N/A	0.198	0.557	0.213	0.336	0.262	0.000	17.042	0.656

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	63	0	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.97	0.00	1.06	1.06
time (sec)	N/A	0.193	0.529	0.073	0.336	0.251	0.000	26.964	0.690

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	87	0	34	34
N.S.	1	1.00	1.06	1.00	1.06	2.72	0.00	1.06	1.06
time (sec)	N/A	0.195	5.458	0.075	0.356	0.263	0.000	35.221	0.687

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	342	71	32	34	34
N.S.	1	1.00	1.06	1.00	10.69	2.22	1.00	1.06	1.06
time (sec)	N/A	0.192	0.437	0.056	0.496	0.264	51.515	1.117	1.346

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	249	60	31	32	32
N.S.	1	1.00	1.07	1.00	8.30	2.00	1.03	1.07	1.07
time (sec)	N/A	0.181	0.325	0.017	0.514	0.263	60.304	0.777	0.823

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	195	54	22	26	26
N.S.	1	1.00	1.08	1.00	8.12	2.25	0.92	1.08	1.08
time (sec)	N/A	0.164	0.276	0.089	0.362	0.264	22.704	1.022	0.618

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	500	79	32	34	34
N.S.	1	1.00	1.06	1.00	15.62	2.47	1.00	1.06	1.06
time (sec)	N/A	0.194	0.587	0.074	0.378	0.258	146.187	1.176	0.783

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	752	120	0	34	34
N.S.	1	1.00	1.06	1.00	23.50	3.75	0.00	1.06	1.06
time (sec)	N/A	0.190	1.114	0.077	0.385	0.264	0.000	1.344	1.018

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	1001	161	0	34	34
N.S.	1	1.00	1.06	1.00	31.28	5.03	0.00	1.06	1.06
time (sec)	N/A	0.189	18.026	0.078	0.393	0.267	0.000	2.181	2.173

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	154	142	444	623	431	969	4036	1009
N.S.	1	0.86	0.79	2.47	3.46	2.39	5.38	22.42	5.61
time (sec)	N/A	0.312	0.072	1.115	0.206	0.305	3.516	0.531	1.470

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	130	120	315	439	318	706	2776	566
N.S.	1	0.87	0.81	2.11	2.95	2.13	4.74	18.63	3.80
time (sec)	N/A	0.288	0.064	0.883	0.204	0.283	2.043	0.447	1.254

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	106	99	207	280	222	491	1742	290
N.S.	1	0.90	0.84	1.75	2.37	1.88	4.16	14.76	2.46
time (sec)	N/A	0.267	0.040	0.700	0.213	0.263	1.314	0.422	1.127

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	78	69	109	144	125	253	869	126
N.S.	1	0.96	0.85	1.35	1.78	1.54	3.12	10.73	1.56
time (sec)	N/A	0.236	0.026	0.553	0.195	0.277	0.869	0.375	0.991

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	95	229	0	0	0	0	0
N.S.	1	1.00	1.19	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	0.035	1.149	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	105	87	132	83	233	125	104
N.S.	1	1.00	1.67	1.38	2.10	1.32	3.70	1.98	1.65
time (sec)	N/A	0.226	0.038	0.696	0.199	0.263	0.626	0.360	1.833

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	138	110	229	255	217	422	262	209
N.S.	1	0.96	0.76	1.59	1.77	1.51	2.93	1.82	1.45
time (sec)	N/A	0.318	0.079	0.986	0.209	0.255	1.057	0.404	1.734

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	163	141	361	428	406	656	416	339
N.S.	1	0.93	0.81	2.06	2.45	2.32	3.75	2.38	1.94
time (sec)	N/A	0.336	0.101	1.203	0.212	0.282	1.625	0.461	2.440

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	186	158	474	647	629	944	573	577
N.S.	1	0.90	0.77	2.30	3.14	3.05	4.58	2.78	2.80
time (sec)	N/A	0.362	0.121	2.155	0.223	0.264	2.408	0.494	2.974

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	365	436	511	0	2389	0	0	0	0
N.S.	1	1.19	1.40	0.00	6.55	0.00	0.00	0.00	0.00
time (sec)	N/A	0.915	0.306	0.000	0.325	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	309	365	391	0	1732	0	0	0	0
N.S.	1	1.18	1.27	0.00	5.61	0.00	0.00	0.00	0.00
time (sec)	N/A	0.766	0.214	0.000	0.316	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	292	287	0	1165	0	0	0	0
N.S.	1	1.15	1.13	0.00	4.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.635	0.142	0.000	0.289	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	212	203	0	611	0	0	0	0
N.S.	1	1.18	1.13	0.00	3.39	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	0.112	0.000	0.287	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	119	458	464	0	0	0	0	0
N.S.	1	0.93	3.58	3.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	0.939	1.184	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	102	314	176	416	150	434	192	222
N.S.	1	0.81	2.49	1.40	3.30	1.19	3.44	1.52	1.76
time (sec)	N/A	0.291	0.287	0.684	0.214	0.262	1.092	0.399	2.210

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	204	443	485	848	367	894	440	507
N.S.	1	0.76	1.65	1.81	3.16	1.37	3.34	1.64	1.89
time (sec)	N/A	0.387	0.285	1.053	0.243	0.263	2.110	0.426	2.603

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	315	582	892	1419	672	1544	725	1064
N.S.	1	0.75	1.39	2.13	3.39	1.61	3.69	1.73	2.55
time (sec)	N/A	0.475	0.405	1.339	0.291	0.274	11.684	0.514	4.432

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	575	433	666	1179	2123	1035	0	1014	1881
N.S.	1	0.75	1.16	2.05	3.69	1.80	0.00	1.76	3.27
time (sec)	N/A	0.560	0.589	2.322	0.357	0.283	0.000	0.561	7.614

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	114	30	158	30	0	1203	25
N.S.	1	1.04	4.07	1.07	5.64	1.07	0.00	42.96	0.89
time (sec)	N/A	0.177	0.044	0.844	0.204	0.251	0.000	34.962	1.058

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	61	22	0	320	15
N.S.	1	1.00	1.00	1.00	4.07	1.47	0.00	21.33	1.00
time (sec)	N/A	0.154	0.021	0.530	0.192	0.264	0.000	3.748	0.945

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	59	22	0	322	13
N.S.	1	1.00	1.00	1.31	4.54	1.69	0.00	24.77	1.00
time (sec)	N/A	0.157	0.018	0.532	0.203	0.266	0.000	3.572	0.979

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	52	95	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.62	2.97	1.06	1.06
time (sec)	N/A	0.192	0.562	0.866	0.221	0.257	8.352	13.898	1.855

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	34	56	32	32
N.S.	1	1.00	1.07	1.00	1.07	1.13	1.87	1.07	1.07
time (sec)	N/A	0.181	0.119	0.766	0.213	0.250	4.428	11.092	1.951

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	42	61	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.31	1.91	1.06	1.06
time (sec)	N/A	0.197	0.217	0.671	0.227	0.242	2.656	9.026	2.229

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	52	61	0	47	0	0	0
N.S.	1	1.00	1.04	1.22	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.292	0.137	1.850	0.000	0.249	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	95	89	117	0	130	0	0	0
N.S.	1	0.89	0.83	1.09	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.415	0.254	2.807	0.000	0.257	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	305	79	0	34	34
N.S.	1	1.00	1.06	1.00	9.53	2.47	0.00	1.06	1.06
time (sec)	N/A	0.200	0.930	1.062	0.225	0.254	0.000	0.888	4.301

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	227	61	274	32	32
N.S.	1	1.00	1.07	1.00	7.57	2.03	9.13	1.07	1.07
time (sec)	N/A	0.181	0.480	1.592	0.222	0.246	15.893	1.184	4.586

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	162	83	90	34	34
N.S.	1	1.00	1.06	1.00	5.06	2.59	2.81	1.06	1.06
time (sec)	N/A	0.200	0.401	0.691	0.230	0.278	1.582	0.662	5.580

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	91	87	113	0	199	0	0	0
N.S.	1	0.88	0.84	1.10	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	0.359	0.158	2.639	0.000	0.252	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	174	136	182	0	570	0	0	0
N.S.	1	0.82	0.64	0.86	0.00	2.69	0.00	0.00	0.00
time (sec)	N/A	0.477	0.468	3.918	0.000	0.268	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	156	144	446	885	454	998	490	1025
N.S.	1	0.86	0.79	2.45	4.86	2.49	5.48	2.69	5.63
time (sec)	N/A	0.314	0.065	1.201	0.223	0.330	3.610	59.292	1.707

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	132	122	318	647	341	707	355	567
N.S.	1	0.87	0.81	2.11	4.28	2.26	4.68	2.35	3.75
time (sec)	N/A	0.282	0.066	0.883	0.222	0.285	2.039	10.233	1.456

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	108	98	210	437	243	517	246	296
N.S.	1	0.90	0.82	1.75	3.64	2.02	4.31	2.05	2.47
time (sec)	N/A	0.262	0.038	0.704	0.356	0.276	1.390	1.813	1.303

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	108	250	148	250	129	120
N.S.	1	1.00	0.92	1.38	3.21	1.90	3.21	1.65	1.54
time (sec)	N/A	0.229	0.027	0.530	0.280	0.261	0.840	0.538	1.114

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	88	264	0	0	0	0	0
N.S.	1	1.00	1.06	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.487	0.031	0.704	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	111	93	187	110	255	187	108
N.S.	1	1.00	1.71	1.43	2.88	1.69	3.92	2.88	1.66
time (sec)	N/A	0.221	0.035	0.855	0.196	0.273	0.650	0.367	1.975

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	137	109	239	307	238	418	268	206
N.S.	1	0.99	0.79	1.73	2.22	1.72	3.03	1.94	1.49
time (sec)	N/A	0.303	0.080	1.214	0.199	0.270	1.028	0.330	1.945

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	165	140	379	480	430	677	477	341
N.S.	1	0.93	0.79	2.14	2.71	2.43	3.82	2.69	1.93
time (sec)	N/A	0.324	0.079	1.569	0.206	0.272	1.664	0.337	2.618

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	188	162	527	699	654	947	419	579
N.S.	1	0.90	0.78	2.53	3.36	3.14	4.55	2.01	2.78
time (sec)	N/A	0.343	0.118	2.708	0.221	0.287	2.416	0.663	3.313

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	377	448	523	0	2650	0	0	0	0
N.S.	1	1.19	1.39	0.00	7.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.951	0.318	0.000	0.346	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	373	402	0	1948	0	0	0	0
N.S.	1	1.17	1.26	0.00	6.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.822	0.225	0.000	0.333	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	292	298	0	1326	0	0	0	0
N.S.	1	1.15	1.17	0.00	5.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.644	0.154	0.000	0.319	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	220	207	0	727	0	0	0	0
N.S.	1	1.17	1.10	0.00	3.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	0.116	0.000	0.305	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	124	259	0	0	0	0	0	0
N.S.	1	0.94	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	0.931	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	106	321	188	574	200	454	380	228
N.S.	1	0.82	2.47	1.45	4.42	1.54	3.49	2.92	1.75
time (sec)	N/A	0.296	0.272	0.940	0.228	0.272	1.177	0.723	2.817

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	208	451	490	1001	410	879	0	503
N.S.	1	0.76	1.66	1.80	3.68	1.51	3.23	0.00	1.85
time (sec)	N/A	0.392	0.290	1.514	0.259	0.273	2.076	0.000	2.680

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	326	595	1019	1575	719	1561	0	1069
N.S.	1	0.76	1.39	2.38	3.67	1.68	3.64	0.00	2.49
time (sec)	N/A	0.485	0.399	2.112	0.313	0.261	12.152	0.000	4.783

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	445	680	1486	2279	1084	0	874	1883
N.S.	1	0.76	1.16	2.53	3.88	1.85	0.00	1.49	3.21
time (sec)	N/A	0.553	0.581	3.497	0.372	0.295	0.000	1.067	7.763

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	75	258	36	36
N.S.	1	1.00	1.06	1.00	1.06	2.21	7.59	1.06	1.06
time (sec)	N/A	0.199	0.103	0.718	0.217	0.246	14.055	0.526	2.267

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	57	165	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.78	5.16	1.06	1.06
time (sec)	N/A	0.190	0.081	0.628	0.220	0.290	4.835	0.577	2.450

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	65	170	36	36
N.S.	1	1.00	1.06	1.00	1.06	1.91	5.00	1.06	1.06
time (sec)	N/A	0.206	0.104	0.679	0.220	0.268	4.859	0.423	2.643

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	94	91	0	0	0	0	0	0	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	152	137	0	0	0	0	0	0	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	308	125	0	36	36
N.S.	1	1.00	1.06	1.00	9.06	3.68	0.00	1.06	1.06
time (sec)	N/A	0.196	0.321	0.663	0.227	0.269	0.000	0.643	6.234

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	230	107	558	34	34
N.S.	1	1.00	1.06	1.00	7.19	3.34	17.44	1.06	1.06
time (sec)	N/A	0.186	0.239	0.625	0.231	0.269	22.193	0.529	6.396

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	166	129	156	36	36
N.S.	1	1.00	1.06	1.00	4.88	3.79	4.59	1.06	1.06
time (sec)	N/A	0.199	0.151	0.615	0.223	0.246	2.862	0.473	7.564

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	150	135	0	0	0	0	0	0	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	266	225	0	0	0	0	0	0	0
N.S.	1	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	153	364	832	671	563	0	507	936
N.S.	1	0.89	2.13	4.87	3.92	3.29	0.00	2.96	5.47
time (sec)	N/A	0.298	0.540	134.474	0.220	0.263	0.000	7.178	1.428

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	129	273	664	467	417	0	363	520
N.S.	1	0.91	1.92	4.68	3.29	2.94	0.00	2.56	3.66
time (sec)	N/A	0.275	0.323	52.803	0.215	0.282	0.000	2.068	1.184

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	105	194	462	294	282	0	241	262
N.S.	1	0.93	1.72	4.09	2.60	2.50	0.00	2.13	2.32
time (sec)	N/A	0.248	0.193	18.612	0.203	0.275	0.000	0.830	1.033

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	81	126	265	154	163	0	131	127
N.S.	1	0.96	1.50	3.15	1.83	1.94	0.00	1.56	1.51
time (sec)	N/A	0.229	0.101	5.399	0.192	0.263	0.000	0.410	0.911

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	129	523	0	0	0	0	0
N.S.	1	1.00	1.63	6.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	0.071	2.591	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	107	89	130	116	107	0	111	97
N.S.	1	1.10	0.92	1.34	1.20	1.10	0.00	1.14	1.00
time (sec)	N/A	0.270	0.063	5.723	0.194	0.273	0.000	0.299	1.525

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	121	334	230	296	0	243	192
N.S.	1	1.00	0.88	2.44	1.68	2.16	0.00	1.77	1.40
time (sec)	N/A	0.295	0.203	19.185	0.202	0.273	0.000	0.325	1.239

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	162	143	504	400	540	0	454	317
N.S.	1	0.98	0.86	3.04	2.41	3.25	0.00	2.73	1.91
time (sec)	N/A	0.314	0.235	54.431	0.198	0.285	0.000	0.289	1.579

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	185	165	2309	618	820	0	718	555
N.S.	1	0.95	0.85	11.84	3.17	4.21	0.00	3.68	2.85
time (sec)	N/A	0.334	0.219	135.602	0.212	0.291	0.000	0.314	1.956

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	322	383	1709	10586	1871	0	0	0	0
N.S.	1	1.19	5.31	32.88	5.81	0.00	0.00	0.00	0.00
time (sec)	N/A	0.912	0.993	269.189	0.729	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	263	306	1149	7208	1284	0	0	0	0
N.S.	1	1.16	4.37	27.41	4.88	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	0.642	107.027	0.695	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	195	224	656	4394	779	0	0	0	0
N.S.	1	1.15	3.36	22.53	3.99	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	0.521	37.570	0.686	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	123	269	0	0	0	0	0	0
N.S.	1	0.94	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	107	236	299	449	339	0	0	200
N.S.	1	0.83	1.83	2.32	3.48	2.63	0.00	0.00	1.55
time (sec)	N/A	0.383	0.268	7.258	0.222	0.281	0.000	0.000	1.984

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	221	332	871	899	919	0	0	444
N.S.	1	0.81	1.21	3.18	3.28	3.35	0.00	0.00	1.62
time (sec)	N/A	0.487	0.353	21.602	0.238	0.303	0.000	0.000	2.122

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	342	432	1400	1500	1635	0	0	911
N.S.	1	0.80	1.01	3.28	3.51	3.83	0.00	0.00	2.13
time (sec)	N/A	0.577	0.475	57.746	0.285	0.329	0.000	0.000	3.506

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	470	1011	4946	2238	2458	0	0	1579
N.S.	1	0.80	1.72	8.43	3.81	4.19	0.00	0.00	2.69
time (sec)	N/A	0.657	0.604	142.586	0.337	0.371	0.000	0.000	6.745

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	809	747	6885	0	0	0	0	0	0
N.S.	1	0.92	8.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.212	2.318	0.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	614	573	4802	0	0	0	0	0	0
N.S.	1	0.93	7.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.912	1.379	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	376	352	2984	0	0	0	0	0	0
N.S.	1	0.94	7.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	0.740	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	186	174	2513	0	0	0	0	0	0
N.S.	1	0.94	13.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	0.567	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	150	524	543	1129	825	0	0	474
N.S.	1	0.82	2.85	2.95	6.14	4.48	0.00	0.00	2.58
time (sec)	N/A	0.444	0.505	38.576	0.259	0.287	0.000	0.000	2.761

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	315	693	1622	2246	2244	0	0	966
N.S.	1	0.81	1.78	4.16	5.76	5.75	0.00	0.00	2.48
time (sec)	N/A	0.550	0.692	48.694	0.331	0.356	0.000	0.000	5.298

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	493	1003	2687	3630	4008	0	0	2069
N.S.	1	0.81	1.64	4.40	5.94	6.56	0.00	0.00	3.39
time (sec)	N/A	0.678	0.856	95.365	0.429	0.410	0.000	0.000	7.731

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	830	669	1370	8292	5280	6057	0	0	4257
N.S.	1	0.81	1.65	9.99	6.36	7.30	0.00	0.00	5.13
time (sec)	N/A	0.788	1.251	191.847	0.572	0.528	0.000	0.000	8.175

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F(-1)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	94	0	0	0	62	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.409	0.000	0.000	0.000	0.285	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	154	142	442	619	433	969	2030	1008
N.S.	1	0.86	0.79	2.46	3.44	2.41	5.38	11.28	5.60
time (sec)	N/A	0.305	0.069	1.155	0.227	0.348	3.778	0.475	1.621

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	130	120	314	436	320	706	1506	566
N.S.	1	0.87	0.81	2.11	2.93	2.15	4.74	10.11	3.80
time (sec)	N/A	0.286	0.057	0.928	0.203	0.294	2.219	0.448	1.355

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	106	99	206	278	223	491	1056	290
N.S.	1	0.90	0.84	1.75	2.36	1.89	4.16	8.95	2.46
time (sec)	N/A	0.258	0.036	0.786	0.221	0.276	1.478	0.400	1.227

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	78	69	111	143	127	253	627	126
N.S.	1	0.96	0.85	1.37	1.77	1.57	3.12	7.74	1.56
time (sec)	N/A	0.232	0.027	0.645	0.198	0.278	0.935	0.456	1.077

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	95	149	0	0	0	617	0
N.S.	1	1.00	1.17	1.84	0.00	0.00	0.00	7.62	0.00
time (sec)	N/A	0.486	0.034	1.358	0.000	0.000	0.000	37.793	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	74	86	81	134	87	231	116	106
N.S.	1	1.16	1.34	1.27	2.09	1.36	3.61	1.81	1.66
time (sec)	N/A	0.206	0.035	0.733	0.198	0.270	0.652	0.583	1.849

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	138	128	225	255	221	422	234	208
N.S.	1	0.96	0.89	1.56	1.77	1.53	2.93	1.62	1.44
time (sec)	N/A	0.306	0.063	1.043	0.203	0.269	1.173	0.449	1.932

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	163	141	330	428	412	656	447	339
N.S.	1	0.93	0.81	1.89	2.45	2.35	3.75	2.55	1.94
time (sec)	N/A	0.345	0.096	1.269	0.208	0.276	1.770	0.437	2.610

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	186	166	437	647	637	944	751	578
N.S.	1	0.90	0.81	2.12	3.14	3.09	4.58	3.65	2.81
time (sec)	N/A	0.355	0.127	2.204	0.224	0.269	2.667	0.831	3.251

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	503	697	512	0	2395	0	0	0	0
N.S.	1	1.39	1.02	0.00	4.76	0.00	0.00	0.00	0.00
time (sec)	N/A	1.738	0.322	0.000	0.314	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	420	515	392	0	1735	0	0	0	0
N.S.	1	1.23	0.93	0.00	4.13	0.00	0.00	0.00	0.00
time (sec)	N/A	1.260	0.218	0.000	0.312	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	361	290	0	1172	0	0	0	0
N.S.	1	1.08	0.87	0.00	3.50	0.00	0.00	0.00	0.00
time (sec)	N/A	0.888	0.152	0.000	0.308	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	227	203	0	619	0	0	0	0
N.S.	1	1.12	1.00	0.00	3.06	0.00	0.00	0.00	0.00
time (sec)	N/A	0.589	0.114	0.000	0.305	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	120	252	346	0	0	0	0	0
N.S.	1	0.94	1.97	2.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	0.890	1.417	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	112	314	180	416	154	430	174	223
N.S.	1	0.73	2.05	1.18	2.72	1.01	2.81	1.14	1.46
time (sec)	N/A	0.260	0.270	0.730	0.217	0.269	1.208	0.395	3.152

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	219	444	485	847	373	892	369	507
N.S.	1	0.74	1.50	1.64	2.86	1.26	3.01	1.25	1.71
time (sec)	N/A	0.319	0.256	1.094	0.242	0.288	2.271	0.444	2.720

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	315	582	836	1420	680	1544	714	1064
N.S.	1	0.79	1.46	2.10	3.56	1.70	3.87	1.79	2.67
time (sec)	N/A	0.444	0.345	1.407	0.292	0.271	12.818	0.708	4.475

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	379	666	1112	2122	1045	0	1194	1880
N.S.	1	0.76	1.34	2.23	4.26	2.10	0.00	2.40	3.78
time (sec)	N/A	0.466	0.502	2.376	0.360	0.291	0.000	0.562	7.813

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	52	95	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.62	2.97	1.06	1.06
time (sec)	N/A	0.189	0.539	1.092	0.222	0.261	2.564	14.765	2.020

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	32	34	56	32	32
N.S.	1	1.00	1.07	1.00	1.07	1.13	1.87	1.07	1.07
time (sec)	N/A	0.179	0.115	0.909	0.224	0.252	2.042	11.642	2.312

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	42	61	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.31	1.91	1.06	1.06
time (sec)	N/A	0.194	0.324	0.780	0.223	0.255	2.920	9.754	3.064

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	55	0	50	0	0	0
N.S.	1	1.00	0.94	1.04	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.276	0.105	3.164	0.000	0.250	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	97	89	126	0	129	0	0	0
N.S.	1	0.89	0.82	1.16	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.355	0.216	3.355	0.000	0.246	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	309	79	400	34	34
N.S.	1	1.00	1.06	1.00	9.66	2.47	12.50	1.06	1.06
time (sec)	N/A	0.193	0.933	1.293	0.227	0.251	11.034	0.696	5.577

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	30	231	61	275	32	32
N.S.	1	1.00	1.07	1.00	7.70	2.03	9.17	1.07	1.07
time (sec)	N/A	0.179	0.458	1.158	0.227	0.252	6.975	1.196	6.859

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	166	83	92	34	34
N.S.	1	1.00	1.06	1.00	5.19	2.59	2.88	1.06	1.06
time (sec)	N/A	0.193	0.270	1.046	0.265	0.259	1.711	0.709	8.663

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	93	88	107	0	208	0	142	0
N.S.	1	0.89	0.85	1.03	0.00	2.00	0.00	1.37	0.00
time (sec)	N/A	0.318	0.141	3.787	0.000	0.283	0.000	0.586	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	199	135	268	0	584	0	291	0
N.S.	1	1.25	0.85	1.69	0.00	3.67	0.00	1.83	0.00
time (sec)	N/A	0.550	0.317	4.999	0.000	0.253	0.000	0.537	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	156	144	296	882	457	998	487	1024
N.S.	1	0.86	0.79	1.63	4.85	2.51	5.48	2.68	5.63
time (sec)	N/A	0.302	0.064	1.289	0.241	0.297	4.024	63.426	1.686

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	132	122	233	645	343	707	358	567
N.S.	1	0.87	0.81	1.54	4.27	2.27	4.68	2.37	3.75
time (sec)	N/A	0.287	0.051	0.990	0.231	0.287	2.190	10.537	1.488

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	108	98	185	436	245	517	243	296
N.S.	1	0.90	0.82	1.54	3.63	2.04	4.31	2.02	2.47
time (sec)	N/A	0.261	0.034	0.776	0.214	0.290	1.475	1.797	1.320

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	77	72	113	250	149	250	132	120
N.S.	1	0.99	0.92	1.45	3.21	1.91	3.21	1.69	1.54
time (sec)	N/A	0.232	0.027	0.650	0.213	0.256	0.916	0.702	1.164

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	87	162	0	0	0	0	0
N.S.	1	1.00	1.05	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	0.028	0.743	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	76	89	93	187	110	253	187	108
N.S.	1	0.75	0.87	0.91	1.83	1.08	2.48	1.83	1.06
time (sec)	N/A	0.208	0.033	0.836	0.218	0.265	0.696	0.417	2.908

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	138	128	162	306	240	418	268	206
N.S.	1	0.99	0.92	1.17	2.20	1.73	3.01	1.93	1.48
time (sec)	N/A	0.302	0.062	1.173	0.209	0.273	1.137	0.392	2.541

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	165	140	211	480	432	677	477	341
N.S.	1	0.93	0.79	1.19	2.71	2.44	3.82	2.69	1.93
time (sec)	N/A	0.325	0.073	1.410	0.215	0.283	1.783	0.380	3.315

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	188	162	232	699	658	947	424	579
N.S.	1	0.90	0.78	1.12	3.36	3.16	4.55	2.04	2.78
time (sec)	N/A	0.340	0.102	2.570	0.223	0.277	2.589	0.409	4.546

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	515	709	524	0	2660	0	0	0	0
N.S.	1	1.38	1.02	0.00	5.17	0.00	0.00	0.00	0.00
time (sec)	N/A	1.836	0.316	0.000	0.376	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	422	523	402	0	1950	0	0	0	0
N.S.	1	1.24	0.95	0.00	4.62	0.00	0.00	0.00	0.00
time (sec)	N/A	1.303	0.210	0.000	0.349	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	343	369	298	0	1333	0	0	0	0
N.S.	1	1.08	0.87	0.00	3.89	0.00	0.00	0.00	0.00
time (sec)	N/A	0.926	0.147	0.000	0.342	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	235	195	0	730	0	0	0	0
N.S.	1	1.11	0.92	0.00	3.46	0.00	0.00	0.00	0.00
time (sec)	N/A	0.599	0.111	0.000	0.325	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	125	259	0	0	0	0	0	0
N.S.	1	0.95	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	1.242	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	116	322	188	573	200	450	388	227
N.S.	1	0.74	2.05	1.20	3.65	1.27	2.87	2.47	1.45
time (sec)	N/A	0.260	0.270	0.962	0.243	0.273	1.198	0.653	3.279

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	222	452	490	1001	413	877	0	504
N.S.	1	0.74	1.51	1.64	3.35	1.38	2.93	0.00	1.69
time (sec)	N/A	0.325	0.277	1.418	0.276	0.280	2.186	0.000	3.328

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	325	595	694	1576	721	1561	0	1069
N.S.	1	0.80	1.46	1.71	3.87	1.77	3.84	0.00	2.63
time (sec)	N/A	0.445	0.357	1.847	0.316	0.277	13.024	0.000	5.733

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	388	680	886	2278	1088	0	883	1882
N.S.	1	0.77	1.36	1.77	4.55	2.17	0.00	1.76	3.76
time (sec)	N/A	0.468	0.515	3.063	0.398	0.291	0.000	1.095	8.911

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	75	258	36	36
N.S.	1	1.00	1.06	1.00	1.06	2.21	7.59	1.06	1.06
time (sec)	N/A	0.191	0.102	0.881	0.230	0.266	3.564	0.503	3.022

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	57	165	34	34
N.S.	1	1.00	1.06	1.00	1.06	1.78	5.16	1.06	1.06
time (sec)	N/A	0.180	0.079	0.734	0.226	0.253	2.774	0.515	3.536

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	65	170	36	36
N.S.	1	1.00	1.06	1.00	1.06	1.91	5.00	1.06	1.06
time (sec)	N/A	0.195	0.103	0.849	0.236	0.256	3.758	0.453	4.969

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	151	139	0	0	0	0	0	0	0
N.S.	1	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	312	125	792	36	36
N.S.	1	1.00	1.06	1.00	9.18	3.68	23.29	1.06	1.06
time (sec)	N/A	0.196	0.309	0.783	0.246	0.294	16.082	0.644	9.728

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	234	107	559	34	34
N.S.	1	1.00	1.06	1.00	7.31	3.34	17.47	1.06	1.06
time (sec)	N/A	0.183	0.236	0.737	0.233	0.263	10.733	0.562	11.962

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	170	129	158	36	36
N.S.	1	1.00	1.06	1.00	5.00	3.79	4.65	1.06	1.06
time (sec)	N/A	0.195	0.155	0.726	0.272	0.274	2.389	0.555	13.930

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	136	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	282	0	0	0	0	0	0	0
N.S.	1	1.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.589	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F(-1)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	94	0	0	0	62	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.411	0.000	0.000	0.000	0.283	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	343	279	594	593	636	1436	10664	1392
N.S.	1	0.97	0.79	1.67	1.67	1.79	4.05	30.04	3.92
time (sec)	N/A	0.650	0.383	1.207	0.222	0.637	55.682	1.233	1.865

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	223	215	412	415	445	998	6073	741
N.S.	1	0.98	0.95	1.81	1.83	1.96	4.40	26.75	3.26
time (sec)	N/A	0.472	0.172	1.030	0.211	0.384	8.132	0.793	1.587

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	155	142	265	262	280	658	3076	356
N.S.	1	1.03	0.95	1.77	1.75	1.87	4.39	20.51	2.37
time (sec)	N/A	0.347	0.088	0.931	0.205	0.308	3.120	0.607	1.289

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	123	114	122	140	150	318	1145	144
N.S.	1	1.13	1.05	1.12	1.28	1.38	2.92	10.50	1.32
time (sec)	N/A	0.293	0.070	0.768	0.201	0.263	1.421	0.495	1.081

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	51	54	56	83	406	47
N.S.	1	1.00	1.00	0.98	1.04	1.08	1.60	7.81	0.90
time (sec)	N/A	0.181	0.008	0.631	0.185	0.276	0.434	0.443	0.852

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	149	115	372	0	0	0	0	0
N.S.	1	1.06	0.82	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.042	3.980	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	137	105	242	138	255	0	511	166
N.S.	1	1.57	1.21	2.78	1.59	2.93	0.00	5.87	1.91
time (sec)	N/A	0.277	0.076	1.188	0.196	3.546	0.000	0.498	1.781

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	186	169	694	351	1017	0	2969	417
N.S.	1	1.02	0.92	3.79	1.92	5.56	0.00	16.22	2.28
time (sec)	N/A	0.405	0.291	1.894	0.208	45.301	0.000	0.523	3.689

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	270	260	1504	848	0	0	9339	1154
N.S.	1	0.98	0.95	5.47	3.08	0.00	0.00	33.96	4.20
time (sec)	N/A	0.551	0.393	3.046	0.258	0.000	0.000	0.784	7.324

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	367	355	2850	1757	0	0	20791	2518
N.S.	1	0.97	0.94	7.52	4.64	0.00	0.00	54.86	6.64
time (sec)	N/A	0.776	0.529	6.654	0.329	0.000	0.000	0.991	12.462

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	874	1069	733	0	2140	0	0	0	0
N.S.	1	1.22	0.84	0.00	2.45	0.00	0.00	0.00	0.00
time (sec)	N/A	1.537	0.628	0.000	0.321	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	532	682	486	0	1300	0	0	0	0
N.S.	1	1.28	0.91	0.00	2.44	0.00	0.00	0.00	0.00
time (sec)	N/A	1.161	0.334	0.000	0.301	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	270	392	346	0	673	0	0	0	0
N.S.	1	1.45	1.28	0.00	2.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.789	0.194	0.000	0.274	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	120	214	0	0	0	0	0	0
N.S.	1	0.96	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	0.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	347	1348	818	0	0	0	0	0
N.S.	1	1.25	4.87	2.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	0.370	4.263	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	212	402	0	0	0	0	0	0
N.S.	1	1.08	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.458	0.290	0.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	369	499	595	0	0	0	0	0	0
N.S.	1	1.35	1.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.985	0.767	0.000	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	714	885	894	0	0	0	0	0	0
N.S.	1	1.24	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.525	1.571	0.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1159	1398	1301	0	0	0	0	0	0
N.S.	1	1.21	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.135	3.650	0.000	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	61	30	29	32	29	20	103	28
N.S.	1	1.74	0.86	0.83	0.91	0.83	0.57	2.94	0.80
time (sec)	N/A	0.203	0.006	0.279	0.196	0.285	0.073	0.347	0.178

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	42	31	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.45	1.07	1.07	1.07
time (sec)	N/A	0.192	0.170	1.440	0.226	0.255	8.776	13.778	1.754

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	31	29	29	29
N.S.	1	1.00	1.07	1.00	1.07	1.15	1.07	1.07	1.07
time (sec)	N/A	0.178	0.116	2.559	0.228	0.256	4.695	10.803	1.884

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	25	19	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.19	0.90	1.10	1.10
time (sec)	N/A	0.154	0.013	1.006	0.211	0.280	0.848	11.526	0.986

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	38	31	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.31	1.07	1.07	1.07
time (sec)	N/A	0.180	0.644	1.079	0.237	0.260	2.742	17.436	1.889

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	62	32	31	31
N.S.	1	1.00	1.07	1.00	1.07	2.14	1.10	1.07	1.07
time (sec)	N/A	0.178	1.377	0.934	0.233	0.274	166.168	27.733	5.187

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	86	0	31	31
N.S.	1	1.00	1.07	1.00	1.07	2.97	0.00	1.07	1.07
time (sec)	N/A	0.177	21.204	1.453	0.226	0.270	0.000	38.331	7.908

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	318	69	0	31	31
N.S.	1	1.00	1.07	1.00	10.97	2.38	0.00	1.07	1.07
time (sec)	N/A	0.176	0.648	1.088	0.234	0.268	0.000	0.748	4.066

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	225	58	337	29	29
N.S.	1	1.00	1.07	1.00	8.33	2.15	12.48	1.07	1.07
time (sec)	N/A	0.167	0.410	0.799	0.246	0.260	22.021	1.202	4.513

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	171	52	158	23	23
N.S.	1	1.00	1.10	1.00	8.14	2.48	7.52	1.10	1.10
time (sec)	N/A	0.155	0.223	0.766	0.220	0.260	8.146	0.959	1.963

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	452	77	0	31	31
N.S.	1	1.00	1.07	1.00	15.59	2.66	0.00	1.07	1.07
time (sec)	N/A	0.179	1.435	0.609	0.247	0.266	0.000	0.936	5.479

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	688	118	0	31	31
N.S.	1	1.00	1.07	1.00	23.72	4.07	0.00	1.07	1.07
time (sec)	N/A	0.178	1.550	0.780	0.238	0.264	0.000	1.068	22.057

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	921	159	0	31	31
N.S.	1	1.00	1.07	1.00	31.76	5.48	0.00	1.07	1.07
time (sec)	N/A	0.178	48.734	2.677	0.256	0.284	0.000	1.733	31.878

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	345	282	599	855	660	1477	0	1403
N.S.	1	0.97	0.79	1.68	2.39	1.85	4.14	0.00	3.93
time (sec)	N/A	0.656	0.371	0.663	0.230	0.674	57.415	0.000	1.932

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	225	217	416	623	468	998	410	743
N.S.	1	0.98	0.95	1.82	2.72	2.04	4.36	1.79	3.24
time (sec)	N/A	0.460	0.166	0.573	0.231	0.388	8.165	75.813	1.628

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	157	142	270	419	301	692	257	362
N.S.	1	1.03	0.93	1.78	2.76	1.98	4.55	1.69	2.38
time (sec)	N/A	0.334	0.087	0.517	0.209	0.334	3.270	5.199	1.378

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	123	118	123	246	174	314	139	133
N.S.	1	1.18	1.13	1.18	2.37	1.67	3.02	1.34	1.28
time (sec)	N/A	0.283	0.070	0.391	0.204	0.285	1.408	0.702	1.110

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	54	57	80	104	82	50
N.S.	1	1.00	1.00	1.00	1.06	1.48	1.93	1.52	0.93
time (sec)	N/A	0.178	0.018	0.303	0.191	0.266	0.479	0.320	0.931

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	152	119	396	0	0	0	0	0
N.S.	1	1.06	0.83	2.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	0.041	3.368	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	140	108	245	192	279	0	454	191
N.S.	1	1.56	1.20	2.72	2.13	3.10	0.00	5.04	2.12
time (sec)	N/A	0.271	0.075	0.670	0.215	3.499	0.000	0.521	1.904

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	185	172	629	405	1036	0	486	412
N.S.	1	1.06	0.98	3.59	2.31	5.92	0.00	2.78	2.35
time (sec)	N/A	0.390	0.279	1.221	0.230	47.453	0.000	0.502	3.969

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	272	263	1293	900	0	0	1359	1147
N.S.	1	0.98	0.95	4.67	3.25	0.00	0.00	4.91	4.14
time (sec)	N/A	0.539	0.381	2.043	0.251	0.000	0.000	0.844	7.905

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	369	358	2299	1809	0	0	2159	2520
N.S.	1	0.97	0.94	6.03	4.75	0.00	0.00	5.67	6.61
time (sec)	N/A	0.751	0.514	5.498	0.323	0.000	0.000	3.176	13.522

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	869	1074	746	0	2351	0	0	0	0
N.S.	1	1.24	0.86	0.00	2.71	0.00	0.00	0.00	0.00
time (sec)	N/A	1.536	0.615	0.000	0.349	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	542	686	497	0	1458	0	0	0	0
N.S.	1	1.27	0.92	0.00	2.69	0.00	0.00	0.00	0.00
time (sec)	N/A	1.117	0.335	0.000	0.328	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	401	351	0	786	0	0	0	0
N.S.	1	1.43	1.25	0.00	2.80	0.00	0.00	0.00	0.00
time (sec)	N/A	0.806	0.181	0.000	0.305	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	124	220	0	0	0	0	0	0
N.S.	1	0.96	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.584	0.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	285	355	1370	0	0	0	0	0	0
N.S.	1	1.25	4.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.685	0.417	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	216	409	0	0	0	0	0	0
N.S.	1	1.08	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.451	0.285	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	381	510	603	0	0	0	0	0	0
N.S.	1	1.34	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.976	0.729	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	724	889	909	0	0	0	0	0	0
N.S.	1	1.23	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.485	1.538	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1154	1400	1317	0	0	0	0	0	0
N.S.	1	1.21	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.150	3.698	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	65	85	33	33
N.S.	1	1.00	1.06	1.00	1.06	2.10	2.74	1.06	1.06
time (sec)	N/A	0.184	0.111	0.747	0.237	0.333	21.419	0.565	2.262

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	54	83	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.86	2.86	1.07	1.07
time (sec)	N/A	0.174	0.080	0.635	0.223	0.328	6.770	0.561	2.350

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	48	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	2.09	0.96	1.09	1.09
time (sec)	N/A	0.158	0.023	0.533	0.220	0.323	1.636	0.476	1.109

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	61	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	1.97	0.00	1.06	1.06
time (sec)	N/A	0.188	0.062	0.611	0.233	0.314	0.000	0.513	2.406

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	85	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	2.74	0.00	1.06	1.06
time (sec)	N/A	0.184	0.062	0.650	0.237	0.311	0.000	1.197	7.127

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	109	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	3.52	0.00	1.06	1.06
time (sec)	N/A	0.185	0.067	0.702	0.249	0.283	0.000	0.511	12.412

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	321	115	0	33	33
N.S.	1	1.00	1.06	1.00	10.35	3.71	0.00	1.06	1.06
time (sec)	N/A	0.183	0.466	0.683	0.226	0.326	0.000	0.585	5.851

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	228	104	729	31	31
N.S.	1	1.00	1.07	1.00	7.86	3.59	25.14	1.07	1.07
time (sec)	N/A	0.174	0.245	0.579	0.227	0.329	26.300	0.578	6.344

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	174	98	333	25	25
N.S.	1	1.00	1.09	1.00	7.57	4.26	14.48	1.09	1.09
time (sec)	N/A	0.157	0.143	0.529	0.218	0.292	8.939	0.477	2.493

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	455	123	0	33	33
N.S.	1	1.00	1.06	1.00	14.68	3.97	0.00	1.06	1.06
time (sec)	N/A	0.184	0.283	0.540	0.235	0.313	0.000	0.761	8.003

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	690	164	0	33	33
N.S.	1	1.00	1.06	1.00	22.26	5.29	0.00	1.06	1.06
time (sec)	N/A	0.184	0.348	0.573	0.253	0.348	0.000	1.455	32.232

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	924	205	0	33	33
N.S.	1	1.00	1.06	1.00	29.81	6.61	0.00	1.06	1.06
time (sec)	N/A	0.185	0.356	0.757	0.262	0.333	0.000	1.091	48.710

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	348	463	1170	671	805	0	0	1434
N.S.	1	0.95	1.27	3.21	1.84	2.21	0.00	0.00	3.93
time (sec)	N/A	0.643	0.637	85.504	0.218	0.339	0.000	0.000	1.800

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	228	314	984	467	571	0	0	767
N.S.	1	0.97	1.33	4.17	1.98	2.42	0.00	0.00	3.25
time (sec)	N/A	0.464	0.386	33.088	0.217	0.318	0.000	0.000	1.491

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	160	204	623	294	365	0	304	372
N.S.	1	1.01	1.29	3.94	1.86	2.31	0.00	1.92	2.35
time (sec)	N/A	0.342	0.239	11.353	0.204	0.324	0.000	46.925	1.192

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	128	124	371	154	192	0	153	154
N.S.	1	1.10	1.07	3.20	1.33	1.66	0.00	1.32	1.33
time (sec)	N/A	0.287	0.108	3.158	0.198	0.323	0.000	2.533	1.031

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	83	59	59	0	58	53
N.S.	1	1.00	1.00	1.46	1.04	1.04	0.00	1.02	0.93
time (sec)	N/A	0.177	0.009	0.476	0.196	0.322	0.000	0.387	0.823

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	155	150	521	0	0	0	0	0
N.S.	1	1.05	1.01	3.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.443	0.068	2.521	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	136	117	368	151	250	0	169	141
N.S.	1	1.13	0.98	3.07	1.26	2.08	0.00	1.41	1.18
time (sec)	N/A	0.313	0.114	8.523	0.203	3.578	0.000	0.368	1.462

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	178	1385	382	1127	0	531	431
N.S.	1	1.00	0.93	7.25	2.00	5.90	0.00	2.78	2.26
time (sec)	N/A	0.390	0.348	28.812	0.218	52.004	0.000	0.520	3.177

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	275	273	3286	920	0	0	1530	1183
N.S.	1	0.97	0.96	11.57	3.24	0.00	0.00	5.39	4.17
time (sec)	N/A	0.545	0.620	88.437	0.261	0.000	0.000	1.138	5.901

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	372	366	5231	1912	0	0	3325	2570
N.S.	1	0.96	0.94	13.45	4.92	0.00	0.00	8.55	6.61
time (sec)	N/A	0.747	0.654	233.550	0.365	0.000	0.000	2.553	10.632

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F(-2)	F(-1)	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	570	706	906	8443	1671	0	0	0	0
N.S.	1	1.24	1.59	14.81	2.93	0.00	0.00	0.00	0.00
time (sec)	N/A	1.206	1.070	90.914	0.756	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	410	472	0	903	0	0	0	0
N.S.	1	1.39	1.61	0.00	3.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.863	0.580	180.000	0.707	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	130	217	2240	0	0	0	0	0
N.S.	1	0.95	1.58	16.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.591	0.098	4.852	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	367	1082	0	0	0	0	0	0
N.S.	1	1.22	3.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.781	0.264	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	220	3460	0	0	0	0	0	0
N.S.	1	1.06	16.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.540	0.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	393	517	13182	0	0	0	0	0	0
N.S.	1	1.32	33.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.038	5.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	875	1020	7279	0	0	0	0	0	0
N.S.	1	1.17	8.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.696	2.674	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	466	568	3890	0	0	0	0	0	0
N.S.	1	1.22	8.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.090	0.940	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	174	378	0	0	0	0	0	0
N.S.	1	0.86	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.616	0.190	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	425	509	0	0	0	0	0	0	0
N.S.	1	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.943	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	302	283	0	0	0	0	0	0	0
N.S.	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.787	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	629	734	0	0	0	0	0	0	0
N.S.	1	1.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.352	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [182] had the largest ratio of [.59375000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	0.84	33	0.121
2	A	4	4	0.86	33	0.121
3	A	4	4	0.89	33	0.121
4	A	4	4	0.95	31	0.129
5	A	7	6	1.00	33	0.182
6	A	3	2	1.00	33	0.061
7	A	4	4	0.94	33	0.121
8	A	4	4	0.91	33	0.121
9	A	4	4	0.88	33	0.121
10	A	10	9	1.16	35	0.257
11	A	8	7	1.15	35	0.200
12	A	7	6	1.13	35	0.171
13	A	6	5	1.15	33	0.152
14	A	5	4	0.92	35	0.114
15	A	4	3	0.81	35	0.086
16	A	4	3	0.78	35	0.086
17	A	4	3	0.77	35	0.086
18	A	4	3	0.77	35	0.086
19	N/A	1	0	1.00	35	0.000
20	N/A	1	0	1.00	33	0.000
21	N/A	1	0	1.00	35	0.000
22	A	4	3	1.00	35	0.086

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	3	0.94	35	0.086
24	N/A	1	0	1.00	35	0.000
25	N/A	1	0	1.00	33	0.000
26	N/A	1	0	1.00	35	0.000
27	A	5	4	0.92	35	0.114
28	A	4	3	0.88	35	0.086
29	A	4	4	0.84	33	0.121
30	A	4	4	0.85	33	0.121
31	A	4	4	0.88	33	0.121
32	A	4	4	0.94	31	0.129
33	A	8	7	1.06	33	0.212
34	A	3	2	0.75	33	0.061
35	A	4	4	0.94	33	0.121
36	A	4	4	0.91	33	0.121
37	A	4	4	0.88	33	0.121
38	A	20	19	1.32	35	0.543
39	A	16	15	1.18	35	0.429
40	A	12	11	1.04	35	0.314
41	A	8	7	1.09	33	0.212
42	A	5	4	0.93	35	0.114
43	A	4	3	0.73	35	0.086
44	A	4	3	0.76	35	0.086
45	A	5	4	0.77	35	0.114
46	A	5	4	0.74	35	0.114
47	N/A	1	0	1.00	35	0.000
48	N/A	1	0	1.00	33	0.000
49	N/A	1	0	1.00	35	0.000
50	A	4	3	1.00	35	0.086
51	A	4	3	0.94	35	0.086
52	N/A	1	0	1.00	35	0.000
53	N/A	1	0	1.00	33	0.000
54	N/A	1	0	1.00	35	0.000
55	A	5	4	0.92	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	7	6	1.34	35	0.171
57	A	3	3	0.95	30	0.100
58	A	3	3	0.97	30	0.100
59	A	3	3	1.01	30	0.100
60	A	3	3	1.10	28	0.107
61	A	1	1	1.00	22	0.045
62	A	5	4	1.05	30	0.133
63	A	4	3	1.55	30	0.100
64	A	3	3	1.00	30	0.100
65	A	3	3	0.97	30	0.100
66	A	3	3	0.96	30	0.100
67	A	5	4	1.19	32	0.125
68	A	5	4	1.25	32	0.125
69	A	5	4	1.41	30	0.133
70	A	9	8	0.95	24	0.333
71	A	4	3	1.24	32	0.094
72	A	5	4	1.07	32	0.125
73	A	5	4	1.33	32	0.125
74	A	5	4	1.22	32	0.125
75	A	5	4	1.18	32	0.125
76	N/A	1	0	1.00	32	0.000
77	N/A	1	0	1.00	30	0.000
78	N/A	1	0	1.00	24	0.000
79	N/A	1	0	1.00	32	0.000
80	N/A	1	0	1.00	32	0.000
81	N/A	1	0	1.00	32	0.000
82	N/A	1	0	1.00	32	0.000
83	N/A	1	0	1.00	30	0.000
84	N/A	1	0	1.00	24	0.000
85	N/A	1	0	1.00	32	0.000
86	N/A	1	0	1.00	32	0.000
87	N/A	1	0	1.00	32	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
88	A	4	4	0.86	30	0.133
89	A	4	4	0.87	30	0.133
90	A	4	4	0.90	30	0.133
91	A	4	4	0.96	28	0.143
92	A	7	6	1.00	30	0.200
93	A	3	2	1.00	30	0.067
94	A	4	4	0.96	30	0.133
95	A	4	4	0.93	30	0.133
96	A	4	4	0.90	30	0.133
97	A	10	9	1.19	32	0.281
98	A	8	7	1.18	32	0.219
99	A	7	6	1.15	32	0.188
100	A	6	5	1.18	30	0.167
101	A	5	4	0.93	32	0.125
102	A	4	3	0.81	32	0.094
103	A	4	3	0.76	32	0.094
104	A	4	3	0.75	32	0.094
105	A	4	3	0.75	32	0.094
106	A	1	1	1.04	29	0.034
107	A	1	1	1.00	18	0.056
108	A	1	1	1.00	20	0.050
109	N/A	1	0	1.00	32	0.000
110	N/A	1	0	1.00	30	0.000
111	N/A	1	0	1.00	32	0.000
112	A	4	3	1.00	32	0.094
113	A	4	3	0.89	32	0.094
114	N/A	1	0	1.00	32	0.000
115	N/A	1	0	1.00	30	0.000
116	N/A	1	0	1.00	32	0.000
117	A	5	4	0.88	32	0.125
118	A	4	3	0.82	32	0.094
119	A	4	4	0.86	32	0.125
120	A	4	4	0.87	32	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
121	A	4	4	0.90	32	0.125
122	A	4	4	1.00	30	0.133
123	A	7	6	1.00	32	0.188
124	A	3	2	1.00	32	0.062
125	A	4	4	0.99	32	0.125
126	A	4	4	0.93	32	0.125
127	A	4	4	0.90	32	0.125
128	A	11	10	1.19	34	0.294
129	A	9	8	1.17	34	0.235
130	A	8	7	1.15	34	0.206
131	A	6	5	1.17	32	0.156
132	A	5	4	0.94	34	0.118
133	A	4	3	0.82	34	0.088
134	A	4	3	0.76	34	0.088
135	A	4	3	0.76	34	0.088
136	A	4	3	0.76	34	0.088
137	N/A	1	0	1.00	34	0.000
138	N/A	1	0	1.00	32	0.000
139	N/A	1	0	1.00	34	0.000
140	A	4	3	0.97	34	0.088
141	A	4	3	0.90	34	0.088
142	N/A	1	0	1.00	34	0.000
143	N/A	1	0	1.00	32	0.000
144	N/A	1	0	1.00	34	0.000
145	A	5	4	0.90	34	0.118
146	A	4	3	0.85	34	0.088
147	A	3	3	0.89	31	0.097
148	A	3	3	0.91	31	0.097
149	A	3	3	0.93	31	0.097
150	A	3	3	0.96	29	0.103
151	A	7	6	1.00	31	0.194
152	A	3	3	1.10	31	0.097
153	A	3	3	1.00	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
154	A	3	3	0.98	31	0.097
155	A	3	3	0.95	31	0.097
156	A	9	8	1.19	33	0.242
157	A	8	7	1.16	33	0.212
158	A	7	6	1.15	31	0.194
159	A	6	5	0.94	33	0.152
160	A	5	4	0.83	33	0.121
161	A	5	4	0.81	33	0.121
162	A	5	4	0.80	33	0.121
163	A	5	4	0.80	33	0.121
164	A	6	5	0.92	33	0.152
165	A	6	5	0.93	33	0.152
166	A	6	5	0.94	31	0.161
167	A	7	6	0.94	33	0.182
168	A	6	5	0.82	33	0.152
169	A	5	4	0.81	33	0.121
170	A	5	4	0.81	33	0.121
171	A	5	4	0.81	33	0.121
172	A	5	4	0.98	36	0.111
173	A	4	4	0.86	30	0.133
174	A	4	4	0.87	30	0.133
175	A	4	4	0.90	30	0.133
176	A	4	4	0.96	28	0.143
177	A	7	6	1.00	30	0.200
178	A	3	2	1.16	30	0.067
179	A	4	4	0.96	30	0.133
180	A	4	4	0.93	30	0.133
181	A	4	4	0.90	30	0.133
182	A	20	19	1.39	32	0.594
183	A	16	15	1.23	32	0.469
184	A	12	11	1.08	32	0.344
185	A	8	7	1.12	30	0.233
186	A	5	4	0.94	32	0.125
187	A	4	3	0.73	32	0.094

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	4	3	0.74	32	0.094
189	A	5	4	0.79	32	0.125
190	A	5	4	0.76	32	0.125
191	N/A	1	0	1.00	32	0.000
192	N/A	1	0	1.00	30	0.000
193	N/A	1	0	1.00	32	0.000
194	A	4	3	1.00	32	0.094
195	A	4	3	0.89	32	0.094
196	N/A	1	0	1.00	32	0.000
197	N/A	1	0	1.00	30	0.000
198	N/A	1	0	1.00	32	0.000
199	A	5	4	0.89	32	0.125
200	A	7	6	1.25	32	0.188
201	A	4	4	0.86	32	0.125
202	A	4	4	0.87	32	0.125
203	A	4	4	0.90	32	0.125
204	A	4	4	0.99	30	0.133
205	A	7	6	1.00	32	0.188
206	A	3	2	0.75	32	0.062
207	A	4	4	0.99	32	0.125
208	A	4	4	0.93	32	0.125
209	A	4	4	0.90	32	0.125
210	A	20	19	1.38	34	0.559
211	A	16	15	1.24	34	0.441
212	A	12	11	1.08	34	0.324
213	A	8	7	1.11	32	0.219
214	A	5	4	0.95	34	0.118
215	A	4	3	0.74	34	0.088
216	A	4	3	0.74	34	0.088
217	A	5	4	0.80	34	0.118
218	A	5	4	0.77	34	0.118
219	N/A	1	0	1.00	34	0.000
220	N/A	1	0	1.00	32	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
221	N/A	1	0	1.00	34	0.000
222	A	4	3	1.00	34	0.088
223	A	4	3	0.92	34	0.088
224	N/A	1	0	1.00	34	0.000
225	N/A	1	0	1.00	32	0.000
226	N/A	1	0	1.00	34	0.000
227	A	5	4	0.93	34	0.118
228	A	7	6	1.37	34	0.176
229	A	5	4	0.98	36	0.111
230	A	3	3	0.97	27	0.111
231	A	3	3	0.98	27	0.111
232	A	3	3	1.03	27	0.111
233	A	3	3	1.13	25	0.120
234	A	1	1	1.00	19	0.053
235	A	5	4	1.06	27	0.148
236	A	4	3	1.57	27	0.111
237	A	3	3	1.02	27	0.111
238	A	3	3	0.98	27	0.111
239	A	3	3	0.97	27	0.111
240	A	5	4	1.22	29	0.138
241	A	5	4	1.28	29	0.138
242	A	5	4	1.45	27	0.148
243	A	9	8	0.96	21	0.381
244	A	4	3	1.25	29	0.103
245	A	5	4	1.08	29	0.138
246	A	5	4	1.35	29	0.138
247	A	5	4	1.24	29	0.138
248	A	5	4	1.21	29	0.138
249	A	4	3	1.74	14	0.214
250	N/A	1	0	1.00	29	0.000
251	N/A	1	0	1.00	27	0.000
252	N/A	1	0	1.00	21	0.000
253	N/A	1	0	1.00	29	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
254	N/A	1	0	1.00	29	0.000
255	N/A	1	0	1.00	29	0.000
256	N/A	1	0	1.00	29	0.000
257	N/A	1	0	1.00	27	0.000
258	N/A	1	0	1.00	21	0.000
259	N/A	1	0	1.00	29	0.000
260	N/A	1	0	1.00	29	0.000
261	N/A	1	0	1.00	29	0.000
262	A	3	3	0.97	29	0.103
263	A	3	3	0.98	29	0.103
264	A	3	3	1.03	29	0.103
265	A	3	3	1.18	27	0.111
266	A	1	1	1.00	21	0.048
267	A	5	4	1.06	29	0.138
268	A	4	3	1.56	29	0.103
269	A	3	3	1.06	29	0.103
270	A	3	3	0.98	29	0.103
271	A	3	3	0.97	29	0.103
272	A	5	4	1.24	31	0.129
273	A	5	4	1.27	31	0.129
274	A	5	4	1.43	29	0.138
275	A	9	8	0.96	23	0.348
276	A	4	3	1.25	31	0.097
277	A	5	4	1.08	31	0.129
278	A	5	4	1.34	31	0.129
279	A	5	4	1.23	31	0.129
280	A	5	4	1.21	31	0.129
281	N/A	1	0	1.00	31	0.000
282	N/A	1	0	1.00	29	0.000
283	N/A	1	0	1.00	23	0.000
284	N/A	1	0	1.00	31	0.000
285	N/A	1	0	1.00	31	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
286	N/A	1	0	1.00	31	0.000
287	N/A	1	0	1.00	31	0.000
288	N/A	1	0	1.00	29	0.000
289	N/A	1	0	1.00	23	0.000
290	N/A	1	0	1.00	31	0.000
291	N/A	1	0	1.00	31	0.000
292	N/A	1	0	1.00	31	0.000
293	A	3	3	0.95	31	0.097
294	A	3	3	0.97	31	0.097
295	A	3	3	1.01	31	0.097
296	A	3	3	1.10	29	0.103
297	A	1	1	1.00	23	0.043
298	A	5	4	1.05	31	0.129
299	A	3	3	1.13	31	0.097
300	A	3	3	1.00	31	0.097
301	A	3	3	0.97	31	0.097
302	A	3	3	0.96	31	0.097
303	A	6	5	1.24	33	0.152
304	A	6	5	1.39	31	0.161
305	A	9	8	0.95	25	0.320
306	A	5	4	1.22	33	0.121
307	A	6	5	1.06	33	0.152
308	A	6	5	1.32	33	0.152
309	A	6	5	1.17	33	0.152
310	A	6	5	1.22	31	0.161
311	A	7	6	0.86	25	0.240
312	A	5	4	1.20	33	0.121
313	A	7	6	0.94	33	0.182
314	A	6	5	1.17	33	0.152

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

3.1.1 Optimal result

Integrand size = 33, antiderivative size = 188

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{B(bc-ad)^4 g^4 n x}{5d^4} - \frac{B(bc-ad)^3 g^4 n (a+bx)^2}{10bd^3} \\
&\quad + \frac{B(bc-ad)^2 g^4 n (a+bx)^3}{15bd^2} - \frac{B(bc-ad) g^4 n (a+bx)^4}{20bd} \\
&\quad + \frac{g^4 (a+bx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{B(bc-ad)^5 g^4 n \log(c+dx)}{5bd^5}
\end{aligned}$$

output
```
1/5*B*(-a*d+b*c)^4*g^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*n*(b*x+a)^2/b/d^3+1
/15*B*(-a*d+b*c)^2*g^4*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*n*(b*x+a)^4
/b/d+1/5*g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/5*B*(-a*d+b*c)^
5*g^4*n*ln(d*x+c)/b/d^5
```

3.1.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{g^4\left((a+bx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{B(bc-ad)n\left(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4 \right.}{12d^5}\right.}{5b}
\end{aligned}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*
n*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*
c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x])
)/(12*d^5)))/(5*b)
```

3.1.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bn(bc-ad)\int \frac{g^5(a+bx)^4}{c+dx}dx}{5bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)\int \left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b) - (B*(b*c -
 a*d)*g^4*n*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d
^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)
^4*Log[c + d*x])/d^5))/(5*b)
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.1.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(176) = 352$.

Time = 17.53 (sec) , antiderivative size = 1004, normalized size of antiderivative = 5.34

method	result	size
parallelrisch	Expression too large to display	1004

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/60*(60*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*d^5*g^4*n+36*B*a^4*b*c*d^4*g^
4*n^2+60*B*a^3*b^2*c^2*d^3*g^4*n^2-90*B*a^2*b^3*c^3*d^2*g^4*n^2+54*B*a*b^4
*c^4*d*g^4*n^2-180*A*a^4*b*c*d^4*g^4*n+12*B*x*b^5*c^4*d*g^4*n^2+60*A*x*a^4
*b*d^5*g^4*n+12*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^4*n+3*B*x^4*a*b^
4*d^5*g^4*n^2-3*B*x^4*b^5*c*d^4*g^4*n^2+60*A*x^4*a*b^4*d^5*g^4*n+16*B*x^3*
a^2*b^3*d^5*g^4*n^2+4*B*x^3*b^5*c^2*d^3*g^4*n^2+120*A*x^3*a^2*b^3*d^5*g^4*
n+36*B*x^2*a^3*b^2*d^5*g^4*n^2-6*B*x^2*b^5*c^3*d^2*g^4*n^2+120*A*x^2*a^3*b
^2*d^5*g^4*n+48*B*x*a^4*b*d^5*g^4*n^2-120*B*x*a^3*b^2*c*d^4*g^4*n^2+120*B*
x*a^2*b^3*c^2*d^3*g^4*n^2-60*B*x*a*b^4*c^3*d^2*g^4*n^2+60*B*ln(e*((b*x+a)/
(d*x+c))^n)*a^4*b*c*d^4*g^4*n-120*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^2*
d^3*g^4*n+120*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^3*d^2*g^4*n-60*B*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^4*c^4*d*g^4*n-60*B*ln(b*x+a)*a^4*b*c*d^4*g^4*n^2+
120*B*ln(b*x+a)*a^3*b^2*c^2*d^3*g^4*n^2-120*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^
4*n^2+60*B*ln(b*x+a)*a*b^4*c^4*d*g^4*n^2+60*B*x^4*ln(e*((b*x+a)/(d*x+c))^n
)*a*b^4*d^5*g^4*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*d^5*g^4*n-20
*B*x^3*a*b^4*c*d^4*g^4*n^2+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*d^5
*g^4*n-60*B*x^2*a^2*b^3*c*d^4*g^4*n^2+30*B*x^2*a*b^4*c^2*d^3*g^4*n^2-48*B*
a^5*d^5*g^4*n^2-12*B*b^5*c^5*g^4*n^2-60*A*a^5*d^5*g^4*n+12*A*x^5*b^5*d^5*g
^4*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*g^4*n+12*B*ln(b*x+a)*a^5*d^5*g
^4*n^2-12*B*ln(b*x+a)*b^5*c^5*g^4*n^2)/d^5/n/b
```

3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(176) = 352$.

Time = 0.34 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.03

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5g^4x^5+12\,Ba^5d^5g^4n\log(bx+a)-12\,(Bb^5c^5-5\,Bab^4c^4d+10\,Ba^2b^3c^3d^2-10\,Ba^3b^2c^2d^3+5\,Ba\cdots}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*n*log(b*x + a) - 12*(B*b^5*c
^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a
^4*b*c*d^4)*g^4*n*log(d*x + c) + 3*(20*A*a*b^4*d^5*g^4 - (B*b^5*c*d^4 - B*
a*b^4*d^5)*g^4*n)*x^4 + 4*(30*A*a^2*b^3*d^5*g^4 + (B*b^5*c^2*d^3 - 5*B*a*b
^4*c*d^4 + 4*B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*A*a^3*b^2*d^5*g^4 - (B*b^5*
c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*g^4*n)
*x^2 + 12*(5*A*a^4*b*d^5*g^4 + (B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2
*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + 4*B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^
5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^
2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*
B*a*b^4*d^5*g^4*n*x^4 + 10*B*a^2*b^3*d^5*g^4*n*x^3 + 10*B*a^3*b^2*d^5*g^4*
n*x^2 + 5*B*a^4*b*d^5*g^4*n*x)*log((b*x + a)/(d*x + c)))/(b*d^5)
```

3.1.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(176) = 352$.

Time = 0.21 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.60

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{1}{5}\,Bb^4g^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{5}\,Ab^4g^4x^5 + Bab^3g^4x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Aab^3g^4x^4 + 2\,Ba^2b^2g^4x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+2\,Aa^2b^2g^4x^3 + 2\,Ba^3bg^4x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + 2\,Aa^3bg^4x^2$$
$$+\frac{1}{60}\,Bb^4g^4n\left(\frac{12\,a^5\log(bx+a)}{b^5}-\frac{12\,c^5\log(dx+c)}{d^5}-\frac{3\,(b^4cd^3-ab^3d^4)x^4-4\,(b^4c^2d^2-a^2b^2d^4)x^3+6\,(b}{b^4d^4}\right.$$
$$-\frac{1}{6}\,Bab^3g^4n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3}{b^3d^3}\right.$$
$$+Ba^2b^2g^4n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-2\,Ba^3bg^4n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Ba^4g^4n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Ba^4g^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^4g^4x$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/5*B*b^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b^4*g^4*x
^5 + B*a*b^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^3*g^4*
x^4 + 2*B*a^2*b^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*a^2
*b^2*g^4*x^3 + 2*B*a^3*b*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
2*A*a^3*b*g^4*x^2 + 1/60*B*b^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log
(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*
d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*
d^4)) - 1/6*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4
 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3
*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*a^2*b^2*g^4*n*(2*a^3*log(b*x + a)/b^3 -
2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*
x)/(b^2*d^2)) - 2*B*a^3*b*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d
^2 + (b*c - a*d)*x/(b*d)) + B*a^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)
/d) + B*a^4*g^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^4*g^4*x
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4462 vs. $2(176) = 352$.

Time = 1.06 (sec) , antiderivative size = 4462, normalized size of antiderivative = 23.73

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/60*(12*(B*b^10*c^6*g^4*n - 6*B*a*b^9*c^5*d*g^4*n - 5*(b*x + a)*B*b^9*c^6
*d*g^4*n/(d*x + c) + 15*B*a^2*b^8*c^4*d^2*g^4*n + 30*(b*x + a)*B*a*b^8*c^5
*d^2*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 - 20
*B*a^3*b^7*c^3*d^3*g^4*n - 75*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d*x + c)
- 60*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^7*
c^6*d^3*g^4*n/(d*x + c)^3 + 15*B*a^4*b^6*c^2*d^4*g^4*n + 100*(b*x + a)*B*a
^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/(
d*x + c)^2 + 60*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 5*(b*x + a
)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 6*B*a^5*b^5*c*d^5*g^4*n - 75*(b*x +
a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g
^4*n/(d*x + c)^2 - 150*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*x + c)^3 - 3
0*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + B*a^6*b^4*d^6*g^4*n + 30
*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^4*b^4*c^2
*d^6*g^4*n/(d*x + c)^2 + 200*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*n/(d*x + c)
^3 + 75*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 5*(b*x + a)*B*a^
6*b^3*d^7*g^4*n/(d*x + c) - 60*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4*n/(d*x + c)
^2 - 150*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 - 100*(b*x + a)^4
*B*a^3*b^3*c^3*d^7*g^4*n/(d*x + c)^4 + 10*(b*x + a)^2*B*a^6*b^2*d^8*g^4*n/
(d*x + c)^2 + 60*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3 + 75*(b*x +
 a)^4*B*a^4*b^2*c^2*d^8*g^4*n/(d*x + c)^4 - 10*(b*x + a)^3*B*a^6*b*d^9*...
```

3.1.9 Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 1046, normalized size of antiderivative = 5.56

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$=x^2\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c}{d}\right.}{10\,b\,d}\right.$$

$$\left.-\frac{a\,c\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{15\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,a^4\,g^4\,x+2\,B\,a^3\,b\,g^4\,x^2+2\,B\,a^2\,b^2\,g^4\,x^3+B\,a\,b^3\,g^4\,x^4+\frac{B\,b^4\,g^4\,x^5}{5}\right)+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+2\,B\,a\,d\,n-2\,B\,b\,c\,n)}{d}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B}{d}\right.}{5\,b\,d}\right.}{5\,b\,c}$$

$$a\,c\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right.$$

input
```
int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n)
)/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a
*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4)/d))
/(10*b*d) - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d)
 - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A
*b*c + B*a*d*n - B*b*c*n))/d) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*
d*n - B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c)
)/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) +
(A*a*b^3*c*g^4)/(3*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^4*g^4*x^5)/5
 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3
) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c + 2*B*a*d*n - 2*B*b*c*n))/d - ((5*a*d
+ 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b
*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d)
 - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4
)/d))/(5*b*d) - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(
5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d) + (2*a^2*b*g^4*(5*A*a*d +
 5*A*b*c + B*a*d*n - B*b*c*n))/d))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d +
5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(
5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c
*n))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c...
```

3.2 $\int (ag+bgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

3.2.1 Optimal result

Integrand size = 33, antiderivative size = 156

$$\int (ag+bgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = -\frac{B(bc-ad)^3g^3nx}{4d^3}+\frac{B(bc-ad)^2g^3n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3n(a+bx)^3}{12bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}+\frac{B(bc-ad)^4g^3n\log(c+dx)}{4bd^4}$$

output
```
-1/4*B*(-a*d+b*c)^3*g^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*n*(b*x+a)^2/b/d^2-1
/12*B*(-a*d+b*c)*g^3*n*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/b+1/4*B*(-a*d+b*c)^4*g^3*n*ln(d*x+c)/b/d^4
```

3.2.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = \frac{g^3\left((a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)n\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}\right)}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*
n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b
*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

3.2.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \, dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^3(a+bx)^4 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bn(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{4bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^3(a+bx)^4 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bg^3n(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{4b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^3(a+bx)^4 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bg^3n(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(a+bx)^4 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bg^3n(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b) - (B*(b*c -
 a*d)*g^3*n*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) +
 (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(4*b)
```

3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.2.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(146) = 292$.

Time = 7.22 (sec) , antiderivative size = 755, normalized size of antiderivative = 4.84

method	result
parallelrisch	$-36B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}b^{2}c^{2}d^{2}g^{3}n-6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c^{4}g^{3}n+6B\ln(bx+a)a^{4}d^{4}g^{3}n^{2}+6B\ln(bx+a)b^{4}c^{4}g^{3}n^{2}+9B\,a^{3}bc\,d^{3}g$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-36*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^2*d^2*g^3*n-6*B*ln(e*((b*x
+a)/(d*x+c))^n)*b^4*c^4*g^3*n+6*B*ln(b*x+a)*a^4*d^4*g^3*n^2+6*B*ln(b*x+a)*
b^4*c^4*g^3*n^2+9*B*a^3*b*c*d^3*g^3*n^2+24*B*a^2*b^2*c^2*d^2*g^3*n^2-21*B*
a*b^3*c^3*d*g^3*n^2-60*A*a^3*b*c*d^3*g^3*n+6*B*x^4*ln(e*((b*x+a)/(d*x+c))^
n)*b^4*d^4*g^3*n+2*B*x^3*a*b^3*d^4*g^3*n^2-2*B*x^3*b^4*c*d^3*g^3*n^2+24*A*
x^3*a*b^3*d^4*g^3*n+9*B*x^2*a^2*b^2*d^4*g^3*n^2+3*B*x^2*b^4*c^2*d^2*g^3*n^
2+36*A*x^2*a^2*b^2*d^4*g^3*n+18*B*x*a^3*b*d^4*g^3*n^2-6*B*x*b^4*c^3*d*g^3*
n^2+24*A*x*a^3*b*d^4*g^3*n+24*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^3*d*g^3*
n-24*B*ln(b*x+a)*a^3*b*c*d^3*g^3*n^2+36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g^3*n^
2-24*B*ln(b*x+a)*a*b^3*c^3*d*g^3*n^2+24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^3*d^4*g^3*n+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*d^4*g^3*n-12*B*x^
2*a*b^3*c*d^3*g^3*n^2+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*d^4*g^3*n-36*
B*x*a^2*b^2*c*d^3*g^3*n^2+24*B*x*a*b^3*c^2*d^2*g^3*n^2+24*B*ln(e*((b*x+a)/
(d*x+c))^n)*a^3*b*c*d^3*g^3*n-18*B*a^4*d^4*g^3*n^2+6*B*b^4*c^4*g^3*n^2-24*
A*a^4*d^4*g^3*n+6*A*x^4*b^4*d^4*g^3*n)/d^4/n/b
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(146) = 292$.

Time = 0.32 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.73

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{6\,Ab^4d^4g^3x^4+6\,Ba^4d^4g^3n\log(bx+a)+6\,(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3)g^3n\log(dx+\cdots}{\cdots}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*n*log(b*x + a) + 6*(B*b^4*c^4
- 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*n*log(d*x +
 c) + 2*(12*A*a*b^3*d^4*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(
12*A*a^2*b^2*d^4*g^3 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^2*d^4)
*g^3*n)*x^2 + 6*(4*A*a^3*b*d^4*g^3 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*
B*a^2*b^2*c*d^3 - 3*B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*a*b
^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x)*log(e) + 6
*(B*b^4*d^4*g^3*n*x^4 + 4*B*a*b^3*d^4*g^3*n*x^3 + 6*B*a^2*b^2*d^4*g^3*n*x^
2 + 4*B*a^3*b*d^4*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b*d^4)
```

3.2.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.21 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\begin{aligned}\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= \frac{1}{4}Bb^3g^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&+\frac{1}{4}Ab^3g^3x^4+Bab^2g^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&+Aab^2g^3x^3+\frac{3}{2}Ba^2bg^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}Aa^2bg^3x^2 \\
&-\frac{1}{24}Bb^3g^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3\right. \\
&+\frac{1}{2}Bab^2g^3n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&-\frac{3}{2}Ba^2bg^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&+Ba^3g^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&+Ba^3g^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^3g^3x\end{aligned}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/4*B*b^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*g^3*x
^4 + B*a*b^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^2*g^3*
x^3 + 3/2*B*a^2*b*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a
^2*b*g^3*x^2 - 1/24*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x +
c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 +
6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*g^3*n*(2*a^3*log(b*x + a
)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2)) - 3/2*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(
d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*g^3*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*a^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^3
*g^3*x
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3034 vs. $2(146) = 292$.

Time = 0.81 (sec) , antiderivative size = 3034, normalized size of antiderivative = 19.45

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
-1/24*(6*(B*b^8*c^5*g^3*n - 5*B*a*b^7*c^4*d*g^3*n - 4*(b*x + a)*B*b^7*c^5*
d*g^3*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^3*n + 20*(b*x + a)*B*a*b^6*c^4*
d^2*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 10*B
*a^3*b^5*c^2*d^3*g^3*n - 40*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) -
30*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^5*c^5
*d^3*g^3*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^3*n + 40*(b*x + a)*B*a^3*b^4*
c^2*d^4*g^3*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)
^2 + 20*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^3*
n - 20*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^
3*c^2*d^5*g^3*n/(d*x + c)^2 - 40*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x
+ c)^3 + 4*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*
b^2*c*d^6*g^3*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x
+ c)^3 - 6*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 20*(b*x + a)^3*B*a^
4*b*c*d^7*g^3*n/(d*x + c)^3 + 4*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x + c)^3)*l
og((b*x + a)/(d*x + c))/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x + c) + 6*(b*x
+ a)^2*b^2*d^6/(d*x + c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3 + (b*x + a)^4
*d^8/(d*x + c)^4) + (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 38*(b*x
 + a)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 190*(b*x
 + a)*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n
/(d*x + c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 380*(b*x + a)*B*a^2*b^5*c^...
```

3.2. $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.2.9 Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$
$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$
$$+x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c}{d}\right)}{4\,b\,d}+\frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n)}{2\,d}-\frac{a\,c\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$
$$+\frac{\ln(c+d\,x)\,\left(-4\,B\,n\,a^3\,c\,d^3\,g^3+6\,B\,n\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,n\,a\,b^2\,c^3\,d\,g^3+B\,n\,b^3\,c^4\,g^3\right)}{4\,d^4}$$
$$+\frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,n\,\ln(a+b\,x)}{4\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*g^
3*(4*a*d + 4*b*c))/(12*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n
 - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(
8*b*d) - (a*b*g^3*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*a*b^
2*c*g^3)/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^3*g^3*x^4)/4 + B*a^
3*g^3*x + (3*B*a^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*(
(((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4
*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(6*A*a*d + 4*A*b
*c + B*a*d*n - B*b*c*n))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*g^3*(8*A*a
*d + 12*A*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*d) - (a*c*((b^2*g^3*(16*A*a*d +
 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))
/(b*d)) + (log(c + d*x)*(B*b^3*c^4*g^3*n - 4*B*a^3*c*d^3*g^3*n - 4*B*a*b^2
*c^3*d*g^3*n + 6*B*a^2*b*c^2*d^2*g^3*n))/(4*d^4) + (A*b^3*g^3*x^4)/4 + (B*
a^4*g^3*n*log(a + b*x))/(4*b)
```

3.2. $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.3 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

3.3.1 Optimal result

Integrand size = 33, antiderivative size = 124

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{B(bc-ad)^2g^2nx}{3d^2} - \frac{B(bc-ad)g^2n(a+bx)^2}{6bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} - \frac{B(bc-ad)^3g^2n\log(c+dx)}{3bd^3}$$

output
```
1/3*B*(-a*d+b*c)^2*g^2*n*x/d^2-1/6*B*(-a*d+b*c)*g^2*n*(b*x+a)^2/b/d+1/3*g^
2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/3*B*(-a*d+b*c)^3*g^2*n*ln(
d*x+c)/b/d^3
```

3.3.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.83

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{g^2\left((a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{B(-bc+ad)n\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}\right)}{3b}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*
d)*n*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c +
 d*x]))/(2*d^3)))/(3*b)
```

3.3.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bn(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bg^2n(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bg^2n(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bg^2n(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) - (B*(b*c -
 a*d)*g^2*n*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2
*Log[c + d*x])/d^3))/(3*b)
```

3.3.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.3.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(116) = 232$.

Time = 2.89 (sec) , antiderivative size = 528, normalized size of antiderivative = 4.26

method	result
parallelrisch	$\frac{-6B\ln(bx+a)a^2bc\,d^2g^2n^2+6B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^2d^3g^2n+6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b\,d^3g^2n-6Bxa\,b^2c\,d^2g^2n^2+6B\ln\left(e\left(\frac{bx+\ldots}{dx\ldots}\right.\right.\ldots}{\ldots}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-6*B*ln(b*x+a)*a^2*b*c*d^2*g^2*n^2+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^2*d^3*g^2*n+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^3*g^2*n-6*B*x*a*b^
2*c*d^2*g^2*n^2+6*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*c*d^2*g^2*n-6*B*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b^2*c^2*d*g^2*n+6*B*ln(b*x+a)*a*b^2*c^2*d*g^2*n^2-4*
B*a^3*d^3*g^2*n^2-2*B*b^3*c^3*g^2*n^2-6*A*a^3*d^3*g^2*n+2*B*x^3*ln(e*((b*x
+a)/(d*x+c))^n)*b^3*d^3*g^2*n+B*x^2*a*b^2*d^3*g^2*n^2-B*x^2*b^3*c*d^2*g^2*
n^2+6*A*x^2*a*b^2*d^3*g^2*n+4*B*x*a^2*b*d^3*g^2*n^2+2*B*x*b^3*c^2*d*g^2*n^
2+6*A*x*a^2*b*d^3*g^2*n+B*a^2*b*c*d^2*g^2*n^2+5*B*a*b^2*c^2*d*g^2*n^2-12*A
*a^2*b*c*d^2*g^2*n+2*A*x^3*b^3*d^3*g^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3
*c^3*g^2*n+2*B*ln(b*x+a)*a^3*d^3*g^2*n^2-2*B*ln(b*x+a)*b^3*c^3*g^2*n^2)/b/
d^3/n
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.39

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3g^2x^3+2\,Ba^3d^3g^2n\log(bx+a)-2\,(Bb^3c^3-3\,Bab^2c^2d+3\,Ba^2bcd^2)g^2n\log(dx+c)+(6\,Aab^2d^3g\ldots}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*n*log(b*x + a) - 2*(B*b^3*c^3 -
 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*n*log(d*x + c) + (6*A*a*b^2*d^3*g^
2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*A*a^2*b*d^3*g^2 + (B*b^3
*c^2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3
+ 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n
*x^3 + 3*B*a*b^2*d^3*g^2*n*x^2 + 3*B*a^2*b*d^3*g^2*n*x)*log((b*x + a)/(d*x
 + c)))/(b*d^3)
```

3.3.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time = 0.20 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\begin{aligned}
&\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\
&=\frac{1}{3}Bb^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Ab^2g^2x^3\\
&\quad+Babg^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aabg^2x^2\\
&\quad+\frac{1}{6}Bb^2g^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)\\
&\quad-Babg^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)\\
&\quad+Ba^2g^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)\\
&\quad+Ba^2g^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^2g^2x
\end{aligned}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/3*B*b^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*g^2*x
^3 + B*a*b*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b*g^2*x^2
+ 1/6*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*a*b*g^2*n*(a
^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^2*
g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^2*g^2*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*a^2*g^2*x
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1866 vs. $2(116) = 232$.

Time = 0.81 (sec) , antiderivative size = 1866, normalized size of antiderivative = 15.05

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/6*(2*(B*b^6*c^4*g^2*n - 4*B*a*b^5*c^3*d*g^2*n - 3*(b*x + a)*B*b^5*c^4*d*
g^2*n/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g^2*n + 12*(b*x + a)*B*a*b^4*c^3*d^2
*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 4*B*a^3
*b^3*c*d^3*g^2*n - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 12*(b*
x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*b^2*d^4*g^2*n + 12*(b*x
 + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g
^2*n/(d*x + c)^2 - 3*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 12*(b*x + a)^
2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c
)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3 - 3*(b*x + a)*b^2*d^4/(d*x + c) + 3
*(b*x + a)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3*d^6/(d*x + c)^3) + (3*B*b^6*c
^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n - 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c
) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 28*(b*x + a)*B*a*b^4*c^3*d^2*g^2*n/(d*x +
 c) + 4*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*g
^2*n - 42*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a
*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^2*n + 28*(b*x + a)*B*a^
3*b^2*c*d^4*g^2*n/(d*x + c) + 24*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(d*x
+ c)^2 - 7*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a^3*b*
c*d^5*g^2*n/(d*x + c)^2 + 4*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2 + 2*B*
b^6*c^4*g^2*log(e) - 8*B*a*b^5*c^3*d*g^2*log(e) - 6*(b*x + a)*B*b^5*c^4*d*
g^2*log(e)/(d*x + c) + 12*B*a^2*b^4*c^2*d^2*g^2*log(e) + 24*(b*x + a)*B...
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-\frac{\ln(c+d\,x)\,(3\,B\,n\,a^2\,c\,d^2\,g^2-3\,B\,n\,a\,b\,c^2\,d\,g^2+B\,n\,b^2\,c^3\,g^2)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}+\frac{B\,a^3\,g^2\,n\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^
2*x^2) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*
n))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d +
3*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b*c*g^2)/d) + x^2*((b*g^2*(9*A*a*d
+ 3*A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*g^2*(3*a*d + 3*b*c))/(6*d)) -
 (log(c + d*x)*(B*b^2*c^3*g^2*n + 3*B*a^2*c*d^2*g^2*n - 3*B*a*b*c^2*d*g^2*
n))/(3*d^3) + (A*b^2*g^2*x^3)/3 + (B*a^3*g^2*n*log(a + b*x))/(3*b)
```

3.4 $\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.4.1 Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)gnx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}+\frac{B(bc-ad)^2gn\log(c+dx)}{2bd^2}$$

output
```
-1/2*B*(-a*d+b*c)*g*n*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
b+1/2*B*(-a*d+b*c)^2*g*n*ln(d*x+c)/b/d^2
```

3.4.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g\left((a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{B(-bc+ad)n(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*d)
*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

3.4.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}-\frac{Bn(bc-ad)\int \frac{g^2(a+bx)}{c+dx}dx}{2bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}-\frac{Bgn(bc-ad)\int \frac{a+bx}{c+dx}dx}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}-\frac{Bgn(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}-\frac{Bgn(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b) - (B*(b*c - a
*d)*g*n*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(2*b)
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.4.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(80) = 160$.

Time = 1.07 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.21

method	result
parallelrisch	$\frac{B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}d^{2}gn+A\,x^{2}b^{2}d^{2}gn+B\ln(bx+a)a^{2}d^{2}g\,n^{2}-2B\ln(bx+a)abcdg\,n^{2}+B\ln(bx+a)b^{2}c^{2}g\,n^{2}+2Bx\ln\left(e\left(\frac{bx+c}{dx+c}\right.\right.}{}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*g*n+A*x^2*b^2*d^2*g*n+B*ln(b*
x+a)*a^2*d^2*g*n^2-2*B*ln(b*x+a)*a*b*c*d*g*n^2+B*ln(b*x+a)*b^2*c^2*g*n^2+2
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*g*n+B*x*a*b*d^2*g*n^2-B*x*b^2*c*d*g
*n^2+2*A*x*a*b*d^2*g*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*g*n-B*ln(e*((
b*x+a)/(d*x+c))^n)*b^2*c^2*g*n-B*a^2*d^2*g*n^2+B*b^2*c^2*g*n^2-2*A*a^2*d^2
*g*n-3*A*a*b*c*d*g*n)/b/d^2/n
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.86

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+Ba^2d^2gn\log(bx+a)+(Bb^2c^2-2\,Babcd)gn\log(dx+c)+(2\,Aabd^2g-(Bb^2cd-Babd^2)gn}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*n*log(b*x + a) + (B*b^2*c^2 - 2*B*a*b*c
*d)*g*n*log(d*x + c) + (2*A*a*b*d^2*g - (B*b^2*c*d - B*a*b*d^2)*g*n)*x + (
B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*log(e) + (B*b^2*d^2*g*n*x^2 + 2*B*a*b*d
^2*g*n*x)*log((b*x + a)/(d*x + c)))/(b*d^2)
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(73) = 146.

Time = 57.62 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.09

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\begin{cases} agx\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ ag\left(Ax+\frac{Bc\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}+Bnx+Bx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)\right) \\ Aagx+\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b}-\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bbgnx^2}{4}+\frac{Bbgx^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2} \\ Aagx+\frac{Abgx^2}{2}+\frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b}+\frac{Ba^2g\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b}-\frac{Bacgn\log\left(\frac{c}{d}+x\right)}{d}+\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c+dx}+\right.\right. \end{cases}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((a*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (a*g*(A*x
+ B*c*log(e*(a/(c + d*x))**n)/d + B*n*x + B*x*log(e*(a/(c + d*x))**n)), Eq
(b, 0)), (A*a*g*x + A*b*g*x**2/2 + B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b)
- B*a*g*n*x/2 + B*a*g*x*log(e*(a/c + b*x/c)**n) - B*b*g*n*x**2/4 + B*b*g*x
**2*log(e*(a/c + b*x/c)**n)/2, Eq(d, 0)), (A*a*g*x + A*b*g*x**2/2 + B*a**2
*g*n*log(c/d + x)/(2*b) + B*a**2*g*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)
/(2*b) - B*a*c*g*n*log(c/d + x)/d + B*a*g*n*x/2 + B*a*g*x*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n) + B*b*c**2*g*n*log(c/d + x)/(2*d**2) - B*b*c*g*n*x
/(2*d) + B*b*g*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\begin{aligned}
\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx
&= \frac{1}{2}\,Bbgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}\,Abgx^2 \\
&\quad -\frac{1}{2}\,Bbgn\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad +Bagn\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)+Bagx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aagx
\end{aligned}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
1/2*B*b*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*g*x^2 - 1/2
*B*b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*g*x*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + A*a*g*x
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(80) = 160$.

Time = 0.48 (sec) , antiderivative size = 880, normalized size of antiderivative = 10.23

$$\int (ag + bgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx =$$

$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{2\,(bx+a)Bb^3c^3dgn}{dx+c} + 3\,Ba^2b^2cd^2gn + \frac{6\,(bx+a)Bab^2c^2d^2gn}{dx+c} - Ba^3bd^3gn - \frac{6\,(bx+}{}\right.}{b^2d^2 - \frac{2\,(bx+a)bd^3}{dx+c} + \frac{(bx+a)^2d^4}{(dx+c)^2}}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
-1/2*((B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - 2*(b*x + a)*B*b^3*c^3*d*g*n/(
d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x +
c) - B*a^3*b*d^3*g*n - 6*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + 2*(b*x +
a)*B*a^3*d^4*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a
)*b*d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^4*c^3*g*n - 3*B*a*
b^3*c^2*d*g*n - (b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*
n + 3*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x +
 a)*B*a^2*b*c*d^3*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) + B*b^
4*c^3*g*log(e) - 3*B*a*b^3*c^2*d*g*log(e) - 2*(b*x + a)*B*b^3*c^3*d*g*log(
e)/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*log(e) + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*
log(e)/(d*x + c) - B*a^3*b*d^3*g*log(e) - 6*(b*x + a)*B*a^2*b*c*d^3*g*log(
e)/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*g*log(e)/(d*x + c) + A*b^4*c^3*g - 3*
A*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) + 3*A*a^2*b^2*c*d^2*
g + 6*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + c) - A*a^3*b*d^3*g - 6*(b*x + a)*
A*a^2*b*c*d^3*g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*g/(d*x + c))/(b^2*d^2 -
2*(b*x + a)*b*d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^3*c^3*g*
n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b + (b
*x + a)*d/(d*x + c))/(b*d^2) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*
a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b*d^2))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)$$
$$+\frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,g\,n-2\,B\,a\,c\,d\,g\,n\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,n\,\ln(a+b\,x)}{2\,b}$$

input
```
int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((g*(4*A*a*d + 2*A*b*c + B*a*d*n - B*b*c*n))/(2*d) - (A*g*(2*a*d + 2*b*c
))/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b*g*x^2)/2 + B*a*g*x) + (lo
g(c + d*x)*(B*b*c^2*g*n - 2*B*a*c*d*g*n))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2
*g*n*log(a + b*x))/(2*b)
```

3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx$

3.5.1 Optimal result

Integrand size = 33, antiderivative size = 84

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx=-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg}+\frac{Bn\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g+B*n*polylog(2
,1+(-a*d+b*c)/d/(b*x+a))/b/g
```

3.5.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx =\frac{\log(g(a+bx))\left(-Bn\log(g(a+bx))+2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+2Bn\,\mathrm{PolyLog}\left(2\right.}{2bg}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

output
```
(Log[g*(a + b*x)]*(-(B*n*Log[g*(a + b*x)]) + 2*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*n*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(2*b*g)
```

3.5.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2941, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{ag+bgx}dx$$

$$\Big\downarrow \text{2941}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{Bn(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$-\frac{Bn(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$-\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2752}$$

$$\frac{Bn\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*g)) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

3.5.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2941
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a + b*
x))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d
)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f -
 a*g, 0]
```

3.5.4 Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{bgx+ag}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

3.5.5 Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="frica
s")
```

output
```
integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g), x)
```

3.5.6 Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n)/(a + b*x), x))/g
```

3.5.7 Maxima [F]

$$\int \frac{A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag + bgx} \, dx = \int \frac{B \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{bgx + ag} \, dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="maxim
a")
```

output
```
B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/(b*g) +
 integrate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2
*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g
)
```

3.5.8 Giac [F]

$$\int \frac{A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag + bgx} \, dx = \int \frac{B \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{bgx + ag} \, dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g), x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag + bgx} \, dx = \int \frac{A + B \ln \left(e \left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{a\,g + b\,g\,x} \, dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x), x)
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx$

3.6.1 Optimal result

Integrand size = 33, antiderivative size = 67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{Bn}{bg^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-B*n/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^2/
(b*x+a)
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg(ag+bgx)} + \frac{B(bc-ad)n\left(-\frac{1}{(bc-ad)(a+bx)} - \frac{d\log(a+bx)}{(bc-ad)^2} + \frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

output
```
-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g*(a*g + b*g*x))) + (B*(b*c -
a*d)*n*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d
*Log[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2949, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^2} \, dx$$
$$\Big\downarrow \text{2949}$$
$$\frac{\int \frac{(c+dx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2 (bc-ad)}$$
$$\Big\downarrow \text{2741}$$
$$\frac{-\frac{(c+dx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{g^2 (bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

output
```
(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(a + b*x))/((b*c - a*d)*g^2)
```

3.6.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.6.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.96

method	result	size
parallelrisch	$-\frac{Ba\,b^{2}d^{2}n^{2}-B\,b^{3}cd\,n^{2}+Aa\,b^{2}d^{2}n-A\,b^{3}cdn-Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{2}n-B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}cdn}{g^{2}(bx+a)b^{3}dn(ad-cb)}$	131

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
-(B*a*b^2*d^2*n^2-B*b^3*c*d*n^2+A*a*b^2*d^2*n-A*b^3*c*d*n-B*x*ln(e*((b*x+a
)/(d*x+c))^n)*b^3*d^2*n-B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d*n)/g^2/(b*x+a)
/b^3/d/n/(a*d-b*c)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(ag+bgx)^{2}}\,dx = -\frac{Abc-Aad+(Bbc-Bad)n+(Bbc-Bad)\log(e)+(Bbdnx+Bbcn)\log\left(\frac{bx+a}{dx+c}\right)}{(b^{3}c-ab^{2}d)g^{2}x+(ab^{2}c-a^{2}bd)g^{2}}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="fri
cas")
```

output
```
-(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) + (B*b*d*n*x
+ B*b*c*n)*log((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c -
 a^2*b*d)*g^2)
```

3.6.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

3.6.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(67) = 134.

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.04

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -Bn\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{b^2g^2x+abg^2} - \frac{A}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="max
ima")
```

output
```
-B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*l
og(d*x + c)/((b^2*c - a*b*d)*g^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n)/(b^2*g^2*x + a*b*g^2) - A/(b^2*g^2*x + a*b*g^2)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\left(\frac{(dx+c)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2}+\frac{(Bn+B\log(e)+A)(dx+c)}{(bx+a)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="gia
c")
```

output
```
-((d*x + c)*B*n*log((b*x + a)/(d*x + c))/((b*x + a)*g^2) + (B*n + B*log(e)
 + A)*(d*x + c)/((b*x + a)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\,n}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{b\,\left(a\,g^2+b\,g^2\,x\right)} - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^2,x)
```

output
```
- (A + B*n)/(b^2*g^2*x + a*b*g^2) - (B*log(e*((a + b*x)/(c + d*x))^n))/(b*
(a*g^2 + b*g^2*x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)
/(b*g^2*(a*d - b*c))
```

3.7 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$

3.7.1 Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{Bn}{4bg^3(a+bx)^2}+\frac{Bdn}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2g^3}$$

output
```
-1/4*B*n/b/g^3/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*n*ln
(b*x+a)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x
+a)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

3.7.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(-3*a*d
 + b*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log
[c + d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

3.7.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^3} \, dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{Bn(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{2bg}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{2bg^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{2bg^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{2bg^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g^3*(a + b*x)^2) + (B*(b*c
- a*d)*n*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) +
 (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3))/(2*
b*g^3)
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.7.4 Maple [A] (verified)

Time = 7.39 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.79

method	result
parallelrisch	$-\frac{3B\,a^2b^3d^3n^2+B\,b^5c^2d\,n^2+2A\,a^2b^3d^3n+2A\,b^5c^2dn-4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^4d^3n-4B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^4c\,d^2n-4Ba\,b^4c\,d^2}{4g^3(bx+a)^2(a^2d^2-2abcd+b}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(3*B*a^2*b^3*d^3*n^2+B*b^5*c^2*d*n^2+2*A*a^2*b^3*d^3*n+2*A*b^5*c^2*d*
n-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^3*n-4*B*ln(e*((b*x+a)/(d*x+c))^n
)*a*b^4*c*d^2*n-4*B*a*b^4*c*d^2*n^2-4*A*a*b^4*c*d^2*n-2*B*x^2*ln(e*((b*x+a
)/(d*x+c))^n)*b^5*d^3*n+2*B*x*a*b^4*d^3*n^2-2*B*x*b^5*c*d^2*n^2+2*B*ln(e*(
(b*x+a)/(d*x+c))^n)*b^5*c^2*d*n)/g^3/(b*x+a)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)
/b^4/d/n
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\,(Bb^2cd-Babd^2)nx+(Bb^2c^2-4\,Babcd+3\,Ba^2d^2)n+2\,(Bb^2c^2-2}{4\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="fri
cas")
```

output
```
-1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*
n*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c
*d + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2
 - 2*B*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2
*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a
^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2139 vs. 2(133) = 266.

Time = 100.06 (sec) , antiderivative size = 2139, normalized size of antiderivative = 14.17

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

output
```
Piecewise((zoo*(A + B*log(0**n*e))/(g**3*x**2), Eq(a, 0) & Eq(b, 0)), (-A*
d**2/(2*b**3*c**2*g**3 + 4*b**3*c*d*g**3*x + 2*b**3*d**2*g**3*x**2) - B*d*
*2*log(e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n)/(2*b**3*c**2*g**3 + 4*b*
*3*c*d*g**3*x + 2*b**3*d**2*g**3*x**2), Eq(a, b*c/d)), ((A*x + B*c*log(e*(
a/(c + d*x))**n)/d + B*n*x + B*x*log(e*(a/(c + d*x))**n))/(a**3*g**3), Eq(
b, 0)), (-2*A*a**2*d**2/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*d*g**3 + 8*a**
3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d*g**3*x + 4*a
**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*d*g**3*x**2 +
4*b**5*c**2*g**3*x**2) + 4*A*a*b*c*d/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*d
*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d
*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x - 8*a*b**4*c*d
*g**3*x**2 + 4*b**5*c**2*g**3*x**2) - 2*A*b**2*c**2/(4*a**4*b*d**2*g**3 -
8*a**3*b**2*c*d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b**3*c**2*g**3 - 1
6*a**2*b**3*c*d*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a*b**4*c**2*g**3*x
 - 8*a*b**4*c*d*g**3*x**2 + 4*b**5*c**2*g**3*x**2) - 3*B*a**2*d**2*n/(4*a*
*4*b*d**2*g**3 - 8*a**3*b**2*c*d*g**3 + 8*a**3*b**2*d**2*g**3*x + 4*a**2*b
**3*c**2*g**3 - 16*a**2*b**3*c*d*g**3*x + 4*a**2*b**3*d**2*g**3*x**2 + 8*a
*b**4*c**2*g**3*x - 8*a*b**4*c*d*g**3*x**2 + 4*b**5*c**2*g**3*x**2) + 4*B*
a*b*c*d*n/(4*a**4*b*d**2*g**3 - 8*a**3*b**2*c*d*g**3 + 8*a**3*b**2*d**2*g*
*3*x + 4*a**2*b**3*c**2*g**3 - 16*a**2*b**3*c*d*g**3*x + 4*a**2*b**3*d*...
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{1}{4}Bn\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{2}{(b^3c^2 -}\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)} - \frac{A}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="max
ima")
```

output
```
1/4*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
 a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(
b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^
3*x + a^2*b*g^3)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.48

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\left(Bbn-\frac{2(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{Bbn-\frac{4(bx+a)Bdn}{dx+c}+2Bb\log(e)-\frac{4(bx+a)Bd\log(e)}{dx+c}+2Ab-\frac{4(bx+}{dx}}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="gia
c")
```

output
```
-1/4*(2*(B*b*n - 2*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b
*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2) + (B*b*n
- 4*(b*x + a)*B*d*n/(d*x + c) + 2*B*b*log(e) - 4*(b*x + a)*B*d*log(e)/(d*x
 + c) + 2*A*b - 4*(b*x + a)*A*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^
2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)
^2)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.47

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2} - \frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)} - \frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^3,x)
```

output
```
- ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)
/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log(e*((a
 + b*x)/(c + d*x))^n))/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*b*g^3*x)) - (B*d^
2*n*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b
*d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2)
```

3.7. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$

3.8 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx$

3.8.1 Optimal result

Integrand size = 33, antiderivative size = 183

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = &-\frac{Bn}{9bg^4(a+bx)^3}+\frac{Bdn}{6b(bc-ad)g^4(a+bx)^2}\\ &-\frac{Bd^2n}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3g^4}\\ &-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3g^4}\end{aligned}$$

output
```
-1/9*B*n/b/g^4/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2*n/
b/(-a*d+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-
A-B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^4/(b*x+a)^3+1/3*B*d^3*n*ln(d*x+c)/b/(-a
*d+b*c)^3/g^4
```

3.8.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3 \cdots}{(bc-ad)^3}}{18bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

output
```
-1/18*(6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(11*a^
2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3
*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3
)/(b*g^4*(a + b*x)^3)
```

3.8.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^4} \, dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{Bn(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

output
```
-1/3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g^4*(a + b*x)^3) + (B*(b*c
- a*d)*n*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^
2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d
^3*Log[c + d*x])/(b*c - a*d)^4))/(3*b*g^4)
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.8.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(174) = 348$.

Time = 15.87 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.40

method	result
parallelrisch	$-\frac{11B\,a^3b^4d^4n^2-2B\,b^7c^3d\,n^2+6A\,a^3b^4d^4n-6A\,b^7c^3dn-18B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^6d^4n-18Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b^5d^4n-18B}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(11*B*a^3*b^4*d^4*n^2-2*B*b^7*c^3*d*n^2+6*A*a^3*b^4*d^4*n-6*A*b^7*c^
3*d*n-18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^4*n-18*B*x*ln(e*((b*x+a)/
(d*x+c))^n)*a^2*b^5*d^4*n-18*B*x*a*b^6*c*d^3*n^2-18*B*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b^5*c*d^3*n+18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^2*n+6*B*x
^2*a*b^6*d^4*n^2-6*B*x^2*b^7*c*d^3*n^2+15*B*x*a^2*b^5*d^4*n^2+3*B*x*b^7*c^
2*d^2*n^2-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d*n-18*B*a^2*b^5*c*d^3*n^2
+9*B*a*b^6*c^2*d^2*n^2-18*A*a^2*b^5*c*d^3*n+18*A*a*b^6*c^2*d^2*n-6*B*x^3*l
n(e*((b*x+a)/(d*x+c))^n)*b^7*d^4*n)/g^4/(b*x+a)^3/(a^3*d^3-3*a^2*b*c*d^2+3
*a*b^2*c^2*d-b^3*c^3)/n/b^5/d
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(171) = 342$.

Time = 0.28 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.63

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{6\,Ab^3c^3-18\,Aab^2c^2d+18\,Aa^2bcd^2-6\,Aa^3d^3+6\,(Bb^3cd^2-Bab^2d^3)nx^2-3\,(Bb^3c^2d-6\,Bab^2cd^2+5}{18\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="fri
cas")
```

output
```
-1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6
*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*
B*a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*
B*a^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^
3)*log(e) + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x +
 (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log((b*x + a)/(d*x + c
)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3
*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3
*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (
a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
```

output
```
Timed out
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(171) = 342$.

Time = 0.20 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.36

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}\,Bn\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b}\right. - \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)} - \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="max
ima")
```

output
```
-1/18*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*
d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^
5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*
c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4)
+ 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d
^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
- a^3*b*d^3)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4
*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A/(b^4*g^4*x^3
 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(171) = 342$.

Time = 0.66 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.08

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4}\,dx =$$

$$-\frac{1}{18}\left(\frac{6\left(Bb^2n - \frac{3\,(bx+a)Bbdn}{dx+c} + \frac{3\,(bx+a)^2Bd^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3} - \frac{2\,(bx+a)^3abcdg^4}{(dx+c)^3} + \frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}} + \frac{2\,Bb^2n - \frac{9\,(bx+a)Bbdn}{dx+c} + \frac{18\,(bx+a)^2Bd^2n}{(dx+c)^2} + 6\,Bb^2}{(bx+\ldots}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="gia
c")
```

output
```
-1/18*(6*(B*b^2*n - 3*(b*x + a)*B*b*d*n/(d*x + c) + 3*(b*x + a)^2*B*d^2*n/
(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3
 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x +
c)^3) + (2*B*b^2*n - 9*(b*x + a)*B*b*d*n/(d*x + c) + 18*(b*x + a)^2*B*d^2*
n/(d*x + c)^2 + 6*B*b^2*log(e) - 18*(b*x + a)*B*b*d*log(e)/(d*x + c) + 18*
(b*x + a)^2*B*d^2*log(e)/(d*x + c)^2 + 6*A*b^2 - 18*(b*x + a)*A*b*d/(d*x +
 c) + 18*(b*x + a)^2*A*d^2/(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)
^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x
+ c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,b\,c^2\,n}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{3\,b\,g^4\,(a+b\,x)^3} - \frac{B\,b\,d^2\,n\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{7\,B\,a\,c\,d\,n}{18\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{11\,B\,a^2\,d^2\,n}{18\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{5\,B\,a\,d^2\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,b\,c\,d\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^4,x)
```

output
```
(2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*g^4*(a*d - b*
c)^2*(a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*b
*c^2*n)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i
 + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (B*log(e*((a + b*x
)/(c + d*x))^n))/(3*b*g^4*(a + b*x)^3) - (B*b*d^2*n*x^2)/(3*g^4*(a*d - b*c
)^2*(a + b*x)^3) + (7*B*a*c*d*n)/(18*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*
B*a^2*d^2*n)/(18*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*n*x)/(6*g^4
*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c*d*n*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)
^3)
```

3.9 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$

3.9.1 Optimal result

Integrand size = 33, antiderivative size = 215

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = & -\frac{Bn}{16bg^5(a+bx)^4}+\frac{Bdn}{12b(bc-ad)g^5(a+bx)^3} \\ & -\frac{Bd^2n}{8b(bc-ad)^2g^5(a+bx)^2} \\ & +\frac{Bd^3n}{4b(bc-ad)^3g^5(a+bx)}+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4g^5} \\ & -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4g^5}\end{aligned}$$

output
```
-1/16*B*n/b/g^5/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2*
n/b/(-a*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*
B*d^4*n*ln(b*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/
b/g^5/(b*x+a)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

3.9.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4}+\frac{Bn\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}}{4bg^5}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^4) + (B*n*((-3*(b*c -
a*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)
^2)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 1
2*d^4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

3.9.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{Bn(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{4bg}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{4bg^5}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int\left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+bx}\right.}{4bg^5}$$
$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{4bg^5}-$$
$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

output
```
-1/4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g^5*(a + b*x)^4) + (B*(b*c
- a*d)*n*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^
3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (
d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5))/(4*b*
g^5)
```

3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(204) = 408$.

Time = 42.20 (sec) , antiderivative size = 1043, normalized size of antiderivative = 4.85

method	result	size
parallelrisch	Expression too large to display	1043

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(25*B*x^4*a^6*b^3*c*d^4*n^2-48*B*x^4*a^5*b^4*c^2*d^3*n^2+36*B*x^4*a^4
*b^5*c^3*d^2*n^2-16*B*x^4*a^3*b^6*c^4*d*n^2+12*A*x^4*a^6*b^3*c*d^4*n-48*A*
x^4*a^5*b^4*c^2*d^3*n+72*A*x^4*a^4*b^5*c^3*d^2*n-48*A*x^4*a^3*b^6*c^4*d*n+
88*B*x^3*a^7*b^2*c*d^4*n^2-180*B*x^3*a^6*b^3*c^2*d^3*n^2+144*B*x^3*a^5*b^4
*c^3*d^2*n^2-64*B*x^3*a^4*b^5*c^4*d*n^2+48*A*x^3*a^7*b^2*c*d^4*n-192*A*x^3
*a^6*b^3*c^2*d^3*n+288*A*x^3*a^5*b^4*c^3*d^2*n-192*A*x^3*a^4*b^5*c^4*d*n+1
08*B*x^2*a^8*b*c*d^4*n^2-240*B*x^2*a^7*b^2*c^2*d^3*n^2+210*B*x^2*a^6*b^3*c
^3*d^2*n^2-96*B*x^2*a^5*b^4*c^4*d*n^2-192*A*x*a^8*b*c^2*d^3*n+48*B*x^3*ln(
e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c*d^4*n+72*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
a^8*b*c*d^4*n+3*B*x^4*a^2*b^7*c^5*n^2+12*A*x^4*a^2*b^7*c^5*n+12*B*x^3*a^3*
b^6*c^5*n^2+48*A*x^3*a^3*b^6*c^5*n+18*B*x^2*a^4*b^5*c^5*n^2+72*A*x^2*a^4*b
^5*c^5*n+48*B*x*a^9*c*d^4*n^2+12*B*x*a^5*b^4*c^5*n^2+48*A*x*a^9*c*d^4*n+48
*A*x*a^5*b^4*c^5*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^9*c^2*d^3*n-12*B*ln(e*
((b*x+a)/(d*x+c))^n)*a^6*b^3*c^5*n+72*A*x^2*a^8*b*c*d^4*n-288*A*x^2*a^7*b^
2*c^2*d^3*n+432*A*x^2*a^6*b^3*c^3*d^2*n-288*A*x^2*a^5*b^4*c^4*d*n+48*B*x*l
n(e*((b*x+a)/(d*x+c))^n)*a^9*c*d^4*n-120*B*x*a^8*b*c^2*d^3*n^2+120*B*x*a^7
*b^2*c^3*d^2*n^2-60*B*x*a^6*b^3*c^4*d*n^2+288*A*x*a^7*b^2*c^3*d^2*n-192*A*
x*a^6*b^3*c^4*d*n-72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3*d^2*n+48*B*ln(e
*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^4*d*n+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a
^6*b^3*c*d^4*n)/g^5/(b*x+a)^4/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-...
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. 2(201) = 402.

Time = 0.31 (sec) , antiderivative size = 733, normalized size of antiderivative = 3.41

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\,(Bb^4cd^3 - Bab^3d^4)nx^3 + 6\,(Bb^4c}{48\,((b^9c^4 - 4\,ab^8c^3d + 6\,a}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="fri
cas")
```

output
```
-1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b
*c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^
2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*
b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16
*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n
 + 12*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3
 + B*a^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2
*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^
2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log((b*x + a)/(d*x + c)))/((b^9*c^4 -
4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4
 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 +
a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^
2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^
3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*
c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g
^5)
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(201) = 402$.

Time = 0.21 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.03

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{1}{48}\,Bn\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - 2}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\,(}\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$
$$- \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="max
ima")
```

output
```
1/48*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5
*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4
*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b
^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6
*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b
*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2
*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 -
 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) -
1/4*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^
3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A/(b^5*g^5*x^4
+ 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(201) = 402$.

Time = 0.88 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.52

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{1}{48}\left(\frac{12\left(Bb^3n - \frac{4\,(bx+a)Bb^2dn}{dx+c} + \frac{6\,(bx+a)^2Bbd^2n}{(dx+c)^2} - \frac{4\,(bx+a)^3Bd^3n}{(dx+c)^3}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4b^3c^3g^5}{(dx+c)^4} - \frac{3\,(bx+a)^4ab^2c^2dg^5}{(dx+c)^4} + \frac{3\,(bx+a)^4a^2bcd^2g^5}{(dx+c)^4} - \frac{(bx+a)^4a^3d^3g^5}{(dx+c)^4}} + \frac{3\,Bb^3n - \frac{16\,(bx+a)Bb^2dn}{dx+c} + 36}{}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="gia
c")
```

output
```
-1/48*(12*(B*b^3*n - 4*(b*x + a)*B*b^2*d*n/(d*x + c) + 6*(b*x + a)^2*B*b*d
^2*n/(d*x + c)^2 - 4*(b*x + a)^3*B*d^3*n/(d*x + c)^3)*log((b*x + a)/(d*x +
 c))/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/
(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*
d^3*g^5/(d*x + c)^4) + (3*B*b^3*n - 16*(b*x + a)*B*b^2*d*n/(d*x + c) + 36*
(b*x + a)^2*B*b*d^2*n/(d*x + c)^2 - 48*(b*x + a)^3*B*d^3*n/(d*x + c)^3 + 1
2*B*b^3*log(e) - 48*(b*x + a)*B*b^2*d*log(e)/(d*x + c) + 72*(b*x + a)^2*B*
b*d^2*log(e)/(d*x + c)^2 - 48*(b*x + a)^3*B*d^3*log(e)/(d*x + c)^3 + 12*A*
b^3 - 48*(b*x + a)*A*b^2*d/(d*x + c) + 72*(b*x + a)^2*A*b*d^2/(d*x + c)^2
- 48*(b*x + a)^3*A*d^3/(d*x + c)^3)/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 -
 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5
/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 -
a*d/(b*c - a*d)^2)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\,c}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,c}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4}$$

$$-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{4\,b\,\left(a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4\right)}$$

$$-\frac{B\,d^4\,n\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,\left(a\,d-b\,c\right)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{\left(a\,d-b\,c\right)^4}\right)}{2\,b\,g^5\,\left(a\,d-b\,c\right)^4}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^5,x)
```

output
```
- ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*
b^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(1
2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n
 + 13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 +
16*a^3*b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log(e*((a +
 b*x)/(c + d*x))^n))/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2
*b^2*g^5*x^2 + 4*a^3*b*g^5*x)) - (B*d^4*n*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d
^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4)
- (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c
)^4))/(2*b*g^5*(a*d - b*c)^4)
```

3.9. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$

3.10 $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

3.10.1 Optimal result

Integrand size = 35, antiderivative size = 396

$$
\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= -\frac{B(bc-ad)g^4n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} \\
&\quad+\frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad+\frac{B(bc-ad)^2g^4n(a+bx)^3\left(4A+Bn+4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&\quad-\frac{B(bc-ad)^3g^4n(a+bx)^2\left(12A+7Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60bd^3} \\
&\quad+\frac{B(bc-ad)^4g^4n(a+bx)\left(12A+13Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^4} \\
&\quad+\frac{B(bc-ad)^5g^4n\left(12A+25Bn+12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30bd^5} \\
&\quad+\frac{2B^2(bc-ad)^5g^4n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}
$$

output
```
-1/10*B*(-a*d+b*c)*g^4*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5
*g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/30*B*(-a*d+b*c)^2*g^4
*n*(b*x+a)^3*(4*A+B*n+4*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/60*B*(-a*d+b*
c)^3*g^4*n*(b*x+a)^2*(12*A+7*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+1/3
0*B*(-a*d+b*c)^4*g^4*n*(b*x+a)*(12*A+13*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)
)/b/d^4+1/30*B*(-a*d+b*c)^5*g^4*n*(12*A+25*B*n+12*B*ln(e*((b*x+a)/(d*x+c))
^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*n^2*polylog(2
,d*(b*x+a)/b/(d*x+c))/b/d^5
```

3.10.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.35

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{B(bc-ad)n\left(24Abd(bc-ad)^3x+24Bd(bc-ad)^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-12d^2(bc-ad)^2(\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d
)*n*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) - 6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
) - 24*B*(b*c - a*d)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d
)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6
*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^
3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c)
+ a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*n*((2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])))/(12*d^5)))/(5*b)
```

3.10.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {2949, 2781, 2784, 2784, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 \, dx$$

$$\Big\downarrow \text{2949}$$

$$g^4 (bc-ad)^5 \int \frac{(a+bx)^4 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g^4 (bc-ad)^5 \left(\frac{(a+bx)^5 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn \int \frac{(a+bx)^4 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}\right)$$

$$\Big\downarrow \text{2784}$$

$$g^4 (bc-ad)^5 \left(\frac{(a+bx)^5 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn \left(\frac{(a+bx)^4 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\int \frac{(a+bx)^3 \left(4A+Bn+4B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}\cdots}{4d}\cdots\right)}{5b}\cdots\right)$$

$$\Big\downarrow \text{2784}$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-}{}\right.}{5b}\right.$$

$\downarrow$ 2784

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-}{}\right.}{}\right.$$

$\downarrow$ 27

3.10. $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{g^4(bc- \quad 2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} -}{}\right.}{}\right.$$

$\downarrow$ 2784

$$ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{g^4(bc- \quad 2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} -}{}\right.}{}\right.$$

$\downarrow$ 2754

3.10. $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-}{}\right.}{}\right.$$

$\downarrow$ 2838

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-}{}\right.}{}\right.$$

3.10. $\int(ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^5*g^4*(((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(5*b*(c + d*x)^5*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*(((a + b*x)^4*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d*(c + d*x)^4*(b - (d*(a + b*x))
/(c + d*x))^4) - (((a + b*x)^3*(4*A + B*n + 4*B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(
12*A + 7*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b -
 (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(12*A + 13*B*n + 12*B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-((
(12*A + 25*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x)
)/(b*(c + d*x))])/d) - (12*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)
/d)/d)/(3*d))/(4*d)))/(5*b))
```

3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.10.4 Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.10.5 Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log(e*((b*x + a)/
(d*x + c))^n)^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2
*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log(e*((b*x + a)/(d*x + c))^n)
, x)
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2945 vs. $2(381) = 762$.

Time = 0.73 (sec) , antiderivative size = 2945, normalized size of antiderivative = 7.44

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
2/5*A*B*b^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b^4*g
^4*x^5 + 2*A*B*a*b^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
a*b^3*g^4*x^4 + 4*A*B*a^2*b^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + 2*A^2*a^2*b^2*g^4*x^3 + 4*A*B*a^3*b*g^4*x^2*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + 2*A^2*a^3*b*g^4*x^2 + 1/30*A*B*b^4*g^4*n*(12*a^5*log(b*x +
 a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^
4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4
 - a^4*d^4)*x)/(b^4*d^4)) - 1/3*A*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 -
6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a
^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*a^2*b^2*g^4*n*
(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^
2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*a^3*b*g^4*n*(a^2*log(b*x +
 a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^4*g^4*n*(a
*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^4*g^4*x*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n) + A^2*a^4*g^4*x - 1/30*((25*g^4*n^2 + 12*g^4*n*log(e))*
b^4*c^5 - (113*g^4*n^2 + 60*g^4*n*log(e))*a*b^3*c^4*d + 4*(49*g^4*n^2 + 30
*g^4*n*log(e))*a^2*b^2*c^3*d^2 - 12*(13*g^4*n^2 + 10*g^4*n*log(e))*a^3*b*c
^2*d^3 + 12*(4*g^4*n^2 + 5*g^4*n*log(e))*a^4*c*d^4)*B^2*log(d*x + c)/d^5 -
 2/5*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2
 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2...
```

3.10.8 Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^4 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^4*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^4 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.11 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

3.11.1 Optimal result

Integrand size = 35, antiderivative size = 335

$$\begin{aligned}\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx &= -\frac{B(bc-ad)g^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\ &+ \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\ &+ \frac{B(bc-ad)^2g^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^2} \\ &- \frac{B(bc-ad)^3g^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12bd^3} \\ &- \frac{B(bc-ad)^4g^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12bd^4} \\ &- \frac{B^2(bc-ad)^4g^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)*g^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*
g^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/12*B*(-a*d+b*c)^2*g^3*
n*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/12*B*(-a*d+b*c
)^3*g^3*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/12*B*(
-a*d+b*c)^4*g^3*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)
/b/(d*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+
c))/b/d^4
```

3.11.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.23

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$=\frac{g^3\left((a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3d^2(-bc+ad)(a+\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x -
 d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a + b
*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)]))/(3*d^4)))/(4*b)
```

3.11.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2949, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{ 2949}$$

$$g^3(bc-ad)^4\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2781}$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}\right)$$

$\downarrow$ 2784

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots}{\cdots}\right.}{2b}\right.$$

$\downarrow$ 2754

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2838

3.11. $\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots}{\cdots}\right)}{\cdots}\right)$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^4*g^3*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(
c + d*x))^3) - (((a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^
n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A
+ 5*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b
*x))/(c + d*x))) - (-(((6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])
*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6*B*n*PolyLog[2, (d*(a + b*x)
)/(b*(c + d*x))])/d)/d)/(2*d))/(3*d)))/(2*b))
```

3.11.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.11.4 Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.11.5 Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b
^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^
n), x)
```

3.11.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2175 vs. $2(322) = 644$.

Time = 0.72 (sec) , antiderivative size = 2175, normalized size of antiderivative = 6.49

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
1/2*A*B*b^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b^3*g
^3*x^4 + 2*A*B*a*b^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
a*b^2*g^3*x^3 + 3*A*B*a^2*b*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + 3/2*A^2*a^2*b*g^3*x^2 - 1/12*A*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*
c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*a*b^2*g^3*n*(2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^
2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^3*g^3*n*(a*log(b
*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^3*g^3*x*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n) + A^2*a^3*g^3*x + 1/12*((11*g^3*n^2 + 6*g^3*n*log(e))*b^3*c^4
 - 2*(19*g^3*n^2 + 12*g^3*n*log(e))*a*b^2*c^3*d + 9*(5*g^3*n^2 + 4*g^3*n*l
og(e))*a^2*b*c^2*d^2 - 6*(3*g^3*n^2 + 4*g^3*n*log(e))*a^3*c*d^3)*B^2*log(d
*x + c)/d^4 + 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2
*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(log(b*x + a)*log(
(b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b
*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 - 3*B^2*a^4*d^4*g^3*n^2*log(b
*x + a)^2 - 2*(b^4*c*d^3*g^3*n*log(e) - (g^3*n*log(e) + 6*g^3*log(e)^2)*a*
b^3*d^4)*B^2*x^3 + ((g^3*n^2 + 3*g^3*n*log(e))*b^4*c^2*d^2 - 2*(g^3*n^2 +
6*g^3*n*log(e))*a*b^3*c*d^3 + (g^3*n^2 + 9*g^3*n*log(e) + 18*g^3*log(e)...
```

3.11.8 Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.12 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

3.12.1 Optimal result

Integrand size = 35, antiderivative size = 274

$$\begin{aligned}
&\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&\quad + \frac{B(bc-ad)^2g^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^2} \\
&\quad + \frac{B(bc-ad)^3g^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} \\
&\quad + \frac{2B^2(bc-ad)^3g^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*g^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/3*
g^2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/3*B*(-a*d+b*c)^2*g^2*n
*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2+1/3*B*(-a*d+b*c)^3*
g^2*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b
/d^3+2/3*B^2*(-a*d+b*c)^3*g^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

3.12.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.11

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g^2\left((a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-d^2(a+bx)^2\left(A+B\,\mathrm{lo}\right.\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d
)*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/
(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2
*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)*Log[
c + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/
(3*b)
```

3.12.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2949, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2949}$$

$$g^2(bc-ad)^3\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)$$

$\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d}{2d}\right.}{3b}\right.$$

$\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\int}{2d}\right.}{3b}\right.$$

$\downarrow$ 2754

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\right.}{3}\right.$$

3.12. $\int(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$\Big\downarrow \text{2838}$$

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-}{}\right.}{3b}\right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^3*g^2*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))
/(c + d*x))^2) - (((a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^
n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B*n + 2*B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) -
 (2*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))
```

3.12.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.12.4 Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.12.5 Fricas [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^2 dx = \int (bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A
*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c)
)^n), x)
```

3.12.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. 2(263) = 526.

Time = 0.69 (sec) , antiderivative size = 1501, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
2/3*A*B*b^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b^2*g
^2*x^3 + 2*A*B*a*b*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*
b*g^2*x^2 + 1/3*A*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*
A*B*a*b*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x
/(b*d)) + 2*A*B*a^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^
2*g^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a^2*g^2*x - 1/3*((3*g
^2*n^2 + 2*g^2*n*log(e))*b^2*c^3 - (7*g^2*n^2 + 6*g^2*n*log(e))*a*b*c^2*d
+ 2*(2*g^2*n^2 + 3*g^2*n*log(e))*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^
3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2 - a^3*d^3*g^
2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - B^2*a
^3*d^3*g^2*n^2*log(b*x + a)^2 - (b^3*c*d^2*g^2*n*log(e) - (g^2*n*log(e) +
3*g^2*log(e)^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^
2*n^2 + 3*a^2*b*c*d^2*g^2*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^3*c^3*g^
2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2)*B^2*log(d*x + c)^2
+ ((g^2*n^2 + 2*g^2*n*log(e))*b^3*c^2*d - 2*(g^2*n^2 + 3*g^2*n*log(e))*a*b
^2*c*d^2 + (g^2*n^2 + 4*g^2*n*log(e) + 3*g^2*log(e)^2)*a^2*b*d^3)*B^2*x +
(2*a*b^2*c^2*d*g^2*n^2 - 5*a^2*b*c*d^2*g^2*n^2 + (3*g^2*n^2 + 2*g^2*n*log(
e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*...
```

3.12.8 Giac [F]

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.13 $\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

3.13.1 Optimal result

Integrand size = 33, antiderivative size = 196

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{B(bc-ad)^2gn\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2} - \frac{B^2(bc-ad)^2gn^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

output
```
-B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/2*g*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-B*(-a*d+b*c)^2*g*n*(A+B*n+B*ln(e*(
(b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-B^2*(-a*d+b*c)^2*g*n^2
*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

3.13.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{g\left((a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(2Abdx+2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2B(bc-ad)n\log(c+dx)-2(bc-ad)\left(A\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)*
n*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c -
 a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - L
og[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2
))/(2*b)
```

3.13.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2949, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$
$$\Big\downarrow \text{ 2949}$$
$$g(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{ 2781}$$

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)$$

$\Big\downarrow$ 2784

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)$$

$\Big\downarrow$ 2754

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(\cdots\right.}{\cdots}}{d}\right.}{b}\right.$$

$\Big\downarrow$ 2838

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+\cdots\right.}{d}}{d}\right.}{b}\right.$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

3.13. $\int(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)^2*g*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2
*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(((a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) - (-(((A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x)
)/(b*(c + d*x))])/d) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)
)/b)
```

3.13.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.13.4 Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.13.5 Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}dx$$
$$=\int (bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n), x)
```

3.13.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(193) = 386$.

Time = 0.70 (sec) , antiderivative size = 828, normalized size of antiderivative = 4.22

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = ABbgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + \frac{1}{2}A^2bgx^2 - ABbgn\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) + 2ABagn\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right) + 2ABagx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + A^2agx + \frac{((gn^2+gn\log(e))bc^2 - (gn^2+2gn\log(e))acd)B^2\log(dx+c)}{d^2} + \frac{(b^2c^2gn^2 - 2abcdgn^2 + a^2d^2gn^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2} - \frac{B^2a^2d^2gn^2\log(bx+a)^2 - B^2b^2d^2gx^2\log(e)^2 + 2(b^2c^2gn^2 - 2abcdgn^2)B^2\log(bx+a)\log(dx+c) - (}{}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
A*B*b*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*g*x^2 - A*B
*b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + 2*A*B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a*g*x*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*g*x + ((g*n^2 + g*n*log(e))*b*c^2
- (g*n^2 + 2*g*n*log(e))*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g*n^2 - 2*
a*b*c*d*g*n^2 + a^2*d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d)
 + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*
g*n^2*log(b*x + a)^2 - B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c^2*g*n^2 - 2*a
*b*c*d*g*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g
*n^2)*B^2*log(d*x + c)^2 + 2*(b^2*c*d*g*n*log(e) - (g*n*log(e) + g*log(e)^
2)*a*b*d^2)*B^2*x + 2*(a*b*c*d*g*n^2 - (g*n^2 + g*n*log(e))*a^2*d^2)*B^2*l
og(b*x + a) - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((b*x + a)^n)^2 -
 (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((d*x + c)^n)^2 - 2*(B^2*b^2*d
^2*g*x^2*log(e) + B^2*a^2*d^2*g*n*log(b*x + a) - (b^2*c*d*g*n - (g*n + 2*g
*log(e))*a*b*d^2)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*log(d*x + c))*
log((b*x + a)^n) + 2*(B^2*b^2*d^2*g*x^2*log(e) + B^2*a^2*d^2*g*n*log(b*x +
 a) - (b^2*c*d*g*n - (g*n + 2*g*log(e))*a*b*d^2)*B^2*x + (b^2*c^2*g*n - 2*
a*b*c*d*g*n)*B^2*log(d*x + c) + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*lo
g((b*x + a)^n))*log((d*x + c)^n))/(b*d^2)
```

3.13.8 Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.14 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

3.14.1 Optimal result

Integrand size = 35, antiderivative size = 138

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output

```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*n*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*n^2*
polylog(3,b*(d*x+c)/d/(b*x+a))/b/g
```

3.14.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 537 vs. 2(138) = 276.

Time = 0.81 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.89

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$

$$= \frac{3\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2+3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)\left(\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

output
```
(3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/
(c + d*x)])^2 + 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a +
 b*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d
 + x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b
*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*n^2*(Log[a/b +
 x]^3 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a + b*x
]*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*Log[a/b
+ x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/b + x]*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c +
 d*x)])*(Log[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)
] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[3, (d*(a + b*x))/(
-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(3*b*g)
```

3.14.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2949, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ag+bgx} \, dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{ 2779}$$

$$\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \; 7143$$

$$\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))
/b)/g
```

3.14.3.1 Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.14.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}\,dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

3.14.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="fri
cas")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(b*g*x + a*g), x)
```

3.14.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)**2/(a + b*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)/(a + b*x), x))/g
```

3.14.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="max
ima")
```

output
```
B^2*log(b*x + a)*log((d*x + c)^n)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - i
ntegrate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log
((b*x + a)^n)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log
(e) + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log((b*x + a)^n) - 2*(B^2*b*
c*log(e) + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d
*n)*log(b*x + a) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n
))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

3.14.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="gia
c")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x + a*g), x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x), x)
```

3.15 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

3.15.1 Optimal result

Integrand size = 35, antiderivative size = 136

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{2B^2n^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
-2*B^2*n^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))^2/(-a*d+b*c)/g^2/(b*x+a)
```

3.15.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.43

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(a+bx)\left(A+\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a +
b*x)*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
 B*d*n*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2
*(a + b*x)))
```

3.15.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2949, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2} \, dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2 (bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{2 B n \int \frac{(c+dx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{g^2 (bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{2 B n \left(-\frac{(c+dx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{B n (c+dx)}{a+bx}\right)-\frac{(c+dx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{g^2 (bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2*B*n
*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(a + b*x)))/((b*c - a*d)*g^2)
```

3.15.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(136) = 272$.

Time = 3.14 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.18

method	result
parallelrisch	$-\frac{-A^{2} b^{3} c d n+2 B^{2} a b^{2} d^{2} n^{3}-2 B^{2} b^{3} c d n^{3}+A^{2} a b^{2} d^{2} n+2 A B a b^{2} d^{2} n^{2}-2 A B b^{3} c d n^{2}-B^{2} x \ln \left(e \left(\frac{b x+a}{d x+c}\right)^{n}\right)^{2} b^{3} d^{2} n-2 B^{2} x \ln \left(e \right.}{g^{2}(b x-}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOS
E)
```

output
```
-(-A^2*b^3*c*d*n+2*B^2*a*b^2*d^2*n^3-2*B^2*b^3*c*d*n^3+A^2*a*b^2*d^2*n+2*A
*B*a*b^2*d^2*n^2-2*A*B*b^3*c*d*n^2-B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d
^2*n-2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^2*n^2-B^2*ln(e*((b*x+a)/(d*x+
c))^n)^2*b^3*c*d*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d*n^2-2*A*B*x*ln(
e*((b*x+a)/(d*x+c))^n)*b^3*d^2*n-2*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d*n
)/g^2/(b*x+a)/b^3/d/n/(a*d-b*c)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.90

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\frac{A^2bc-A^2ad+2\left(B^2bc-B^2ad\right)n^2+\left(B^2bc-B^2ad\right)\log\left(e\right)^2+\left(B^2bdn^2x+B^2bcn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\,(A$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="f
ricas")
```

output
```
-(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(
e)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b
*c - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n + (B^2*b*d*
n*x + B^2*b*c*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*b*c*n^2 + A*B*b
*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b^3*c - a*
b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

3.15.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$=\frac{\int\frac{A^2}{a^2+2abx+b^2x^2}\,dx+\int\frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^2+2abx+b^2x^2}\,dx+\int\frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^2+2abx+b^2x^2}\,dx}{g^2}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2,x)
```

output
```
(Integral(A**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integral(B**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integr
al(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**2 + 2*a*b*x + b**2*x*
*2), x))/g**2
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(136) = 272$.

Time = 0.22 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.16

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -2\,ABn\left(\frac{1}{b^2g^2x+abg^2} + \frac{d\log(bx+a)}{(b^2c-abd)g^2} - \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \left(2\,n\left(\frac{1}{b^2g^2x+abg^2} + \frac{d\log(bx+a)}{(b^2c-abd)g^2} - \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) - \frac{((bdx+ad)\log}{} \right. - \frac{B^2\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)^2}{b^2g^2x+abg^2} - \frac{2\,AB\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{b^2g^2x+abg^2} - \frac{A^2}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="m
axima")
```

output
```
-2*A*B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) -
 d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - (2*n*(1/(b^2*g^2*x + a*b*g^2) + d
*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)
)*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + a*d)*log(b*x + a)^2 +
 (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x +
a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n^2/(a*b^2
*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B^2 - B^2*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)^2/(b^2*g^2*x + a*b*g^2) - 2*A*B*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.28

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\left(\frac{(dx+c)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)g^2} + \frac{2\,(B^2n^2+B^2n\log(e)+ABn)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2} + \frac{(2\,B^2n^2+2\,B^2n\log\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="g
iac")
```

output
```
-((d*x + c)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)*g^2) + 2*(B^2*n^
2 + B^2*n*log(e) + A*B*n)*(d*x + c)*log((b*x + a)/(d*x + c))/((b*x + a)*g^
2) + (2*B^2*n^2 + 2*B^2*n*log(e) + B^2*log(e)^2 + 2*A*B*n + 2*A*B*log(e) +
 A^2)*(d*x + c)/((b*x + a)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.75

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{b\,(a\,g^2+b\,g^2\,x)} - \frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,B^2\,n}{x\,b^2\,g^2+a\,b\,g^2} + \frac{2\,A\,B}{x\,b^2\,g^2+a\,b\,g^2}\right) - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)1\mathrm{i}}{a\,d-b\,c}\right)(A+B\,n)\,4\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^2,x)
```

output
```
- (A^2 + 2*B^2*n^2 + 2*A*B*n)/(b^2*g^2*x + a*b*g^2) - log(e*((a + b*x)/(c
+ d*x))^n)^2*(B^2/(b*(a*g^2 + b*g^2*x)) - (B^2*d)/(b*g^2*(a*d - b*c))) - l
og(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(b^2*g^2*x + a*b*g^2) + (2*A*B)/(
b^2*g^2*x + a*b*g^2)) - (B*d*n*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b
*g^2))*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*g^2*(a*d - b*c))
```

3.15. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

3.16 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

3.16.1 Optimal result

Integrand size = 35, antiderivative size = 288

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{2B^2dn^2(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} + \frac{2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bBn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output
```
2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d
+b*c)^2/g^3/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*
d+b*c)^2/g^3/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(
-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*
d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a
*d+b*c)^2/g^3/(b*x+a)^2
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-4d^2(a+bx)^2}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)
*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*
c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*L
og[c + d*x]) + 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) -
 2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x
])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2
)/(b*g^3*(a + b*x)^2)
```

3.16.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3}dx$$

$$\Big\downarrow \; 2949$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2795$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{2Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{b(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(e\left(\cdots\right.\right.\right)}{a+b\cdots}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

output
```
((2*B^2*d*n^2*(c + d*x))/(a + b*x) - (b*B^2*n^2*(c + d*x)^2)/(4*(a + b*x)^
2) + (2*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)
- (b*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^
2) + (d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (b
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2))/((
b*c - a*d)^2*g^3)
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.16.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(282) = 564$.

Time = 7.39 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.33

method	result
parallelrisch	$-\frac{7B^2a^2b^3d^3n^3+B^2b^5c^2d\,n^3+2A^2a^2b^3d^3n+2A^2b^5c^2dn-8AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^4c\,d^2n-8ABa\,b^4c\,d^2n^2+4ABxa\,b^4d^3n^2-4A}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/4*(7*B^2*a^2*b^3*d^3*n^3+B^2*b^5*c^2*d*n^3+2*A^2*a^2*b^3*d^3*n+2*A^2*b^
5*c^2*d*n-8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c*d^2*n-8*A*B*a*b^4*c*d^2*
n^2+4*A*B*x*a*b^4*d^3*n^2-4*A*B*x*b^5*c*d^2*n^2-4*B^2*ln(e*((b*x+a)/(d*x+c
))^n)^2*a*b^4*c*d^2*n-8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c*d^2*n^2+4*A*
B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d*n-4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n
)*b^5*d^3*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^4*d^3*n-8*B^2*x*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b^4*d^3*n^2-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*
d^2*n^2-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^3*n-2*B^2*x^2*ln(e*((b*x
+a)/(d*x+c))^n)^2*b^5*d^3*n-6*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^3*n^
2+6*B^2*x*a*b^4*d^3*n^3-6*B^2*x*b^5*c*d^2*n^3+2*B^2*ln(e*((b*x+a)/(d*x+c))
^n)^2*b^5*c^2*d*n+2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d*n^2-8*B^2*a*b^
4*c*d^2*n^3+6*A*B*a^2*b^3*d^3*n^2+2*A*B*b^5*c^2*d*n^2-4*A^2*a*b^4*c*d^2*n)
/g^3/(b*x+a)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/d/n
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. 2(282) = 564.

Time = 0.29 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.26

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{2\,A^2b^2c^2-4\,A^2abcd+2\,A^2a^2d^2+(B^2b^2c^2-8\,B^2abcd+7\,B^2a^2d^2)n^2+2\,(B^2b^2c^2-2\,B^2abcd+B^2a^2d}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="f
ricas")
```

output
```
-1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2
*a*b*c*d + 7*B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d
^2)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2
 - 2*B^2*a*b*c*d)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b^2*c^2 - 4*A*B
*a*b*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 + 2*(A*
B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a
^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d
+ 3*B^2*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c
^2 - 2*B^2*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((B^2*b^2*c^2
- 4*B^2*a*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*
b^2*c^2 - 2*A*B*a*b*c*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b
*d^2)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.16.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{\int \frac{A^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx}{g^3}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3,x)
```

output
```
(Integral(A**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integ
ral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x + 3
*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(282) = 564$.

Time = 0.25 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.99

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{1}{2}ABn\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{}{(b^3c^2}\right.$$
$$+\frac{1}{4}\left(2\,n\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{}{(b^3}\right.\right.$$
$$-\frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}$$
$$-\frac{A^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="m
axima")
```

output
```
1/2*A*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c
 - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3)) + 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c -
a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g
^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2
*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2
+ 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a
^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*
d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2
 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*n^2
/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a
*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3
 + a^3*b^2*d^2*g^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3
*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

3.16.8 Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.66

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\left(\frac{2\left(B^2bn^2-\frac{2\,(bx+a)B^2dn^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{2\left(B^2bn^2-\frac{4\,(bx+a)B^2dn^2}{dx+c}+2\,B^2bn\log\left(e\right)-\frac{4\,(bx+a)B^2dn\log(e}{dx+c}\right.}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg}{(dx+c)^2}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="g
iac")
```

output
```
-1/4*(2*(B^2*b*n^2 - 2*(b*x + a)*B^2*d*n^2/(d*x + c))*log((b*x + a)/(d*x +
 c))^2/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2)
 + 2*(B^2*b*n^2 - 4*(b*x + a)*B^2*d*n^2/(d*x + c) + 2*B^2*b*n*log(e) - 4*(
b*x + a)*B^2*d*n*log(e)/(d*x + c) + 2*A*B*b*n - 4*(b*x + a)*A*B*d*n/(d*x +
 c))*log((b*x + a)/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)
^2*a*d*g^3/(d*x + c)^2) + (B^2*b*n^2 - 8*(b*x + a)*B^2*d*n^2/(d*x + c) + 2
*B^2*b*n*log(e) - 8*(b*x + a)*B^2*d*n*log(e)/(d*x + c) + 2*B^2*b*log(e)^2
- 4*(b*x + a)*B^2*d*log(e)^2/(d*x + c) + 2*A*B*b*n - 8*(b*x + a)*A*B*d*n/(
d*x + c) + 4*A*B*b*log(e) - 8*(b*x + a)*A*B*d*log(e)/(d*x + c) + 2*A^2*b -
 4*(b*x + a)*A^2*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)
^2*a*d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$

$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)} - \frac{B^2\,d^2}{2\,b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right) - \frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+6\,A\,B\,a\,d\,n-2\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,\left(3\,b\,B^2\,n^2+2\,A\,b\,B\,n\right)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2} - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2} + \frac{B^2\,d^2\,\left(\frac{b\,g^3\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{b^2\,g^3\,n\,x\,(a\,d-b\,c)}{d}+\frac{a\,b\,g^3\,n\,(a\,d-b\,c)}{2\,d}\right)}{b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2\right)}\right) - \frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x-\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(2\,A+3\,B\,n\right)\,1\mathrm{i}}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*
b*g^3*x)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2
*a*d - 2*A^2*b*c + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n
)/(2*(a*d - b*c)) + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b*
g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*
B)/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) + (B^2*d^2*((b*g^3*n*(a*d - b
*c)*(2*a*d - b*c))/(2*d^2) + (b^2*g^3*n*x*(a*d - b*c))/d + (a*b*g^3*n*(a*d
 - b*c))/(2*d)))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*b*g^3 + b^3*g
^3*x^2 + 2*a*b^2*g^3*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2*g^3 - 2*a
^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b
*g^3*(a*d - b*c)^2)
```

3.16. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

3.17 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

3.17.1 Optimal result

Integrand size = 35, antiderivative size = 448

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} - \frac{2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)} + \frac{bBdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}$$

output
```
-2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/
(-a*d+b*c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*
x+a)^3-2*B*d^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/
(b*x+a)+b*B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4
/(b*x+a)^2-2/9*b^2*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c
)^3/g^4/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)
^3/g^4/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^
3/g^4/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+
b*c)^3/g^4/(b*x+a)^3
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.37

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{18\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(12A(bc-ad)^3+4B(bc-ad)^3n-18Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2n(a+bx)+36Ad^2(bc-ad)(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

output
```
-1/54*(18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*A*(b*c - a*d
)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c -
 a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a
*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*n*(a + b*
x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*
d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[e
*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[e*((a + b
*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c
+ d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*n*(a + b*x)^3*Lo
g[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[
c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x
] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log[a +
 b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (
d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)]))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

3.17.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}-\frac{d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}-\frac{2Bd^2n(c+dx)\left(B\right.}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

output
```
((-2*B^2*d^2*n^2*(c + d*x))/(a + b*x) + (b*B^2*d*n^2*(c + d*x)^2)/(2*(a +
b*x)^2) - (2*b^2*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B*d^2*n*(c + d
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (b*B*d*n*(c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (2*b^2*B*n*(c + d
*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^3) - (d^2*(c +
d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) + (b*d*(c + d*x)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^2 - (b^2*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(a + b*x)^3))/((b*c - a*d)^3
*g^4)
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(440) = 880.

Time = 16.00 (sec) , antiderivative size = 1146, normalized size of antiderivative = 2.56

method	result	size
parallelrisch	Expression too large to display	1146

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/54*(-108*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^4*n-108*B^2*x*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b^6*c*d^3*n^2-108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^
2*b^5*d^4*n-108*A*B*x*a*b^6*c*d^3*n^2-108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^
2*b^5*c*d^3*n+108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^2*n-108*A*B*a^
2*b^5*c*d^3*n^2+54*A*B*a*b^6*c^2*d^2*n^2-36*A*B*x^3*ln(e*((b*x+a)/(d*x+c))
^n)*b^7*d^4*n-54*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^6*d^4*n-162*B^2*x
^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^4*n^2-36*B^2*x^2*ln(e*((b*x+a)/(d*x+c
))^n)*b^7*c*d^3*n^2+36*A*B*x^2*a*b^6*d^4*n^2-36*A*B*x^2*b^7*c*d^3*n^2-54*B
^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*d^4*n-108*B^2*x*ln(e*((b*x+a)/(d*
x+c))^n)*a^2*b^5*d^4*n^2+18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^2*n^
2-162*B^2*x*a*b^6*c*d^3*n^3+90*A*B*x*a^2*b^5*d^4*n^2+18*A*B*x*b^7*c^2*d^2*
n^2-54*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*c*d^3*n+54*B^2*ln(e*((b*x+a
)/(d*x+c))^n)^2*a*b^6*c^2*d^2*n-108*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*
c*d^3*n^2+54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^2*n^2-36*A*B*ln(e*(
(b*x+a)/(d*x+c))^n)*b^7*c^3*d*n-108*B^2*a^2*b^5*c*d^3*n^3+27*B^2*a*b^6*c^2
*d^2*n^3+66*A*B*a^3*b^4*d^4*n^2-12*A*B*b^7*c^3*d*n^2-54*A^2*a^2*b^5*c*d^3*
n+54*A^2*a*b^6*c^2*d^2*n+85*B^2*a^3*b^4*d^4*n^3-4*B^2*b^7*c^3*d*n^3+18*A^2
*a^3*b^4*d^4*n-18*A^2*b^7*c^3*d*n-18*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b
^7*d^4*n-66*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^4*n^2+66*B^2*x^2*a*b^6
*d^4*n^3-66*B^2*x^2*b^7*c*d^3*n^3+147*B^2*x*a^2*b^5*d^4*n^3+15*B^2*x*b^...
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1164 vs. $2(440) = 880$.

Time = 0.32 (sec) , antiderivative size = 1164, normalized size of antiderivative = 2.60

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{18\,A^2b^3c^3 - 54\,A^2ab^2c^2d + 54\,A^2a^2bcd^2 - 18\,A^2a^3d^3 + (4\,B^2b^3c^3 - 27\,B^2ab^2c^2d + 108\,B^2a^2bcd^2 - 85\,B}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="f
ricas")
```

output
```
-1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a
^3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^
2*a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*(A*B*b^3*c*
d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*
a^2*b*c*d^2 - B^2*a^3*d^3)*log(e)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^
2*d^3*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d +
 3*B^2*a^2*b*c*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 + 6*(2*A*B*b^3*c^3 - 9
*A*B*a*b^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3)*n - 3*((5*B^2*b^3*
c^2*d - 54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*(A*B*b^3*c^2*d - 6*
A*B*a*b^2*c*d^2 + 5*A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*
c^2*d + 18*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 + 6*(B^2*b^3*c*d^2 - B^2*a*b^2*
d^3)*n*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n*x +
 (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2 - 11*B^2*a^3*d^3)
*n + 6*(B^2*b^3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x +
(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n)*log((b*x + a)/(d*
x + c)))*log(e) + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b
^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*
n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*(A*B*b^3*c^3 - 3*A*B*
a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*d
 - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c)...
```

3.17.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$=\frac{\int \frac{A^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx}{g^4}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x)
```

output
```
(Integral(A**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**
4*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a*
*4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Inte
gral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*
a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**4
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1432 vs. 2(440) = 880.

Time = 0.29 (sec) , antiderivative size = 1432, normalized size of antiderivative = 3.20

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="m
axima")
```

output
```
-1/9*A*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c
*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b
^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3
*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4)
 + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
 - a^3*b*d^3)*g^4)) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d +
11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*
d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^
2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*
c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a
^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^
2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b
^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^
3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*
b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a
^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11
*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*...
```

3.17.8 Giac [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.88

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{54}\left(\frac{18\left(B^2b^2n^2-\frac{3(bx+a)B^2bdn^2}{dx+c}+\frac{3(bx+a)^2B^2d^2n^2}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}+\frac{6\left(2B^2b^2n^2-\frac{9(bx+a)B^2bdn^2}{dx+c}+\frac{18(bx+a)\cdots}{(dx+\cdots}\right.}{\cdots}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="g
iac")
```

output
```
-1/54*(18*(B^2*b^2*n^2 - 3*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 3*(b*x + a)^2
*B^2*d^2*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((b*x + a)^3*b^2*c^2*
g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*
d^2*g^4/(d*x + c)^3) + 6*(2*B^2*b^2*n^2 - 9*(b*x + a)*B^2*b*d*n^2/(d*x + c
) + 18*(b*x + a)^2*B^2*d^2*n^2/(d*x + c)^2 + 6*B^2*b^2*n*log(e) - 18*(b*x
+ a)*B^2*b*d*n*log(e)/(d*x + c) + 18*(b*x + a)^2*B^2*d^2*n*log(e)/(d*x + c
)^2 + 6*A*B*b^2*n - 18*(b*x + a)*A*B*b*d*n/(d*x + c) + 18*(b*x + a)^2*A*B*
d^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x
+ c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(
d*x + c)^3) + (4*B^2*b^2*n^2 - 27*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 108*(b
*x + a)^2*B^2*d^2*n^2/(d*x + c)^2 + 12*B^2*b^2*n*log(e) - 54*(b*x + a)*B^2
*b*d*n*log(e)/(d*x + c) + 108*(b*x + a)^2*B^2*d^2*n*log(e)/(d*x + c)^2 + 1
8*B^2*b^2*log(e)^2 - 54*(b*x + a)*B^2*b*d*log(e)^2/(d*x + c) + 54*(b*x + a
)^2*B^2*d^2*log(e)^2/(d*x + c)^2 + 12*A*B*b^2*n - 54*(b*x + a)*A*B*b*d*n/(
d*x + c) + 108*(b*x + a)^2*A*B*d^2*n/(d*x + c)^2 + 36*A*B*b^2*log(e) - 108
*(b*x + a)*A*B*b*d*log(e)/(d*x + c) + 108*(b*x + a)^2*A*B*d^2*log(e)/(d*x
+ c)^2 + 18*A^2*b^2 - 54*(b*x + a)*A^2*b*d/(d*x + c) + 54*(b*x + a)^2*A^2*
d^2/(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*
c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/(b*c - a*
d)^2 - a*d/(b*c - a*d)^2)
```

3.17. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

3.17.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 1038, normalized size of antiderivative = 2.32

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{\frac{18A^2a^2d^2-36A^2abcd+18A^2b^2c^2+66ABa^2d^2n-42ABabcdn+12ABb^2c^2n+85B^2a^2d^2n^2-23B^2abcdn^2+4B^2b^2c^2n^2}{6(ad-bc)}+\frac{x(-5c}{}}{x\left(27a^2b^3cg^4-27a^3b^2dg^4\right)-x^2\left(27a^2b^3dg^4-27ab^4cg^4\right)+x^3\left(9b^5c\right.}$$
$$-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{2AB}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}\right.$$
$$+\frac{2B^2d^3\left(x\left(b\left(\frac{bg^4n(ad-bc)(3ad-bc)}{2d^2}+\frac{abg^4n(ad-bc)}{d}\right)+\frac{2ab^2g^4n(ad-bc)}{d}+\frac{b^2g^4n(ad-bc)(3ad-bc)}{d^2}\right)+a\left(\frac{bg^4n}{}\right.\right.}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\left(3a^3bg^4+\right.}$$
$$-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2\left(\frac{B^2}{3b\left(a^3g^4+3a^2bg^4x+3ab^2g^4x^2+b^3g^4x^3\right)}-\frac{B^2d^3}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}\right)$$
$$-\frac{Bd^3n\,\mathrm{atan}\left(\frac{Bd^3n(6A+11Bn)\left(\frac{a^3bd^3g^4-a^2b^2cd^2g^4-ab^3c^2dg^4+b^4c^3g^4}{a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4}+2bdx\right)\left(a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4\right)1i}{bg^4\left(11B^2d^3n^2+6ABd^3n\right)(ad-bc)^3}\right)(6A+}{9bg^4(ad-bc)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^4,x)
```

output
```
((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
 - 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n
^2 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^
2*c*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*
(11*B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27
*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g
^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a^4*b*d*g^4) - log(e*((a + b*x)/
(c + d*x))^n)*((2*A*B)/(3*a^3*b*g^4 + 3*b^4*g^4*x^3 + 9*a^2*b^2*g^4*x + 9*
a*b^3*g^4*x^2) + (2*B^2*d^3*(x*(b*((b*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*
d^2) + (a*b*g^4*n*(a*d - b*c))/d) + (2*a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g
^4*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + a*((b*g^4*n*(a*d - b*c)*(3*a*d - b*
c))/(2*d^2) + (a*b*g^4*n*(a*d - b*c))/d) + (b*g^4*n*(a*d - b*c)*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/d^3 + (3*b^3*g^4*n*x^2*(a*d - b*c))/d))/(3*b*g^4*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(3*a^3*b*g^4 + 3*b^4*g
^4*x^3 + 9*a^2*b^2*g^4*x + 9*a*b^3*g^4*x^2))) - log(e*((a + b*x)/(c + d*x)
)^n)^2*(B^2/(3*b*(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x)
) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
)) - (B*d^3*n*atan((B*d^3*n*(6*A + 11*B*n)*((b^4*c^3*g^4 + a^3*b*d^3*g^4 -
 a*b^3*c^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b
^2*c*d*g^4) + 2*b*d*x)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*...
```

3.17. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

3.18 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

3.18.1 Optimal result

Integrand size = 35, antiderivative size = 615

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx &= \frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} \\ &+ \frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} \\ &+ \frac{2Bd^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(a+bx)} \\ &- \frac{3bBd^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} \\ &+ \frac{2b^2Bdn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} \\ &- \frac{b^3Bn(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} \\ &+ \frac{d^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)} \\ &- \frac{3bd^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} \\ &+ \frac{b^2d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} \\ &- \frac{b^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}\end{aligned}$$

output
```
2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2
/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/g^5/(
b*x+a)^3-1/32*b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*n*(
d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^
2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2
/3*b^2*B*d*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b
*x+a)^3-1/8*b^3*B*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4
/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/
g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d
+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
(-a*d+b*c)^4/g^5/(b*x+a)^4
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{72\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(36A(bc-ad)^4+9B(bc-ad)^4n+48Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3n(a+bx)+72Ad^2(bc-\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

output
```
-1/288*(72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*A*(b*c - a*
d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-
(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2
*(b*c - a*d)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*
B*d^3*(-(b*c) + a*d)*n*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] -
300*B*d^4*n*(a + b*x)^4*Log[a + b*x] + 72*B*d^4*n*(a + b*x)^4*Log[a + b*x]
^2 + 36*B*(b*c - a*d)^4*Log[e*((a + b*x)/(c + d*x))^n] + 48*B*d*(-(b*c) +
a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 72*B*d^2*(b*c - a*d)^2*(
a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 144*B*d^3*(-(b*c) + a*d)*(a +
b*x)^3*Log[e*((a + b*x)/(c + d*x))^n] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]
*Log[e*((a + b*x)/(c + d*x))^n] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300
*B*d^4*n*(a + b*x)^4*Log[c + d*x] - 144*B*d^4*n*(a + b*x)^4*Log[(d*(a + b*
x))/(-(b*c) + a*d)]*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[e*((a + b*x)/
(c + d*x))^n]*Log[c + d*x] + 72*B*d^4*n*(a + b*x)^4*Log[c + d*x]^2 - 144*B
*d^4*n*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4
*n*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*n*(a +
 b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^4)/(b*g^5*(a +
 b*x)^4)
```

3.18.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^5} \, dx$$

$$\Big\downarrow \; 2949$$

$$\frac{\int \frac{(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5} d\frac{a+bx}{c+dx}}{g^5 (bc-ad)^4}$$

$$\Big\downarrow \; 2795$$

$$\frac{\int \left(\frac{b^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (c+dx)^5}{(a+bx)^5}-\frac{3b^2 d \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (c+dx)^4}{(a+bx)^4}+\frac{3bd^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (c+dx)^3}{(a+bx)^3}-\frac{d^3 \left(A+B \log\right.}{}\right.}{g^5 (bc-ad)^4}$$

$\downarrow$ 2009

$$-\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^4}-\frac{b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8(a+bx)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(a+bx)^3}+\frac{2b^2Bdn(c+dx)^3}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

output
```
((2*B^2*d^3*n^2*(c + d*x))/(a + b*x) - (3*b*B^2*d^2*n^2*(c + d*x)^2)/(4*(a
 + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d*x)^3)/(9*(a + b*x)^3) - (b^3*B^2*n^2*
(c + d*x)^4)/(32*(a + b*x)^4) + (2*B*d^3*n*(c + d*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(a + b*x) - (3*b*B*d^2*n*(c + d*x)^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (2*b^2*B*d*n*(c + d*x)^3*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - (b^3*B*n*(c + d*x)^4*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*(a + b*x)^4) + (d^3*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^3 - (b^3*(c + d*x)^4*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*(a + b*x)^4))/((b*c - a*d)^4*
g^5)
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.18.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2325 vs. $2(599) = 1198$.

Time = 42.23 (sec) , antiderivative size = 2326, normalized size of antiderivative = 3.78

method	result	size
parallelrisch	Expression too large to display	2326

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOS
E)
```

output
```
1/288*(144*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c*d^4*n+576*A*B*x^3*l
n(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c*d^4*n+864*A*B*x^2*ln(e*((b*x+a)/(d*x+c)
)^n)*a^8*b*c*d^4*n+1512*B^2*x^2*a^8*b*c*d^4*n^3-2400*B^2*x^2*a^7*b^2*c^2*d
^3*n^3+1218*B^2*x^2*a^6*b^3*c^3*d^2*n^3-384*B^2*x^2*a^5*b^4*c^4*d*n^3+288*
A^2*x^3*a^7*b^2*c*d^4*n-1152*A^2*x^3*a^6*b^3*c^2*d^3*n+1728*A^2*x^3*a^5*b^
4*c^3*d^2*n-1152*A^2*x^3*a^4*b^5*c^4*d*n+216*A*B*x^2*a^4*b^5*c^5*n^2+288*B
^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^9*c*d^4*n+576*B^2*x*ln(e*((b*x+a)/(d*x+
c))^n)*a^9*c*d^4*n^2-1008*B^2*x*a^8*b*c^2*d^3*n^3+624*B^2*x*a^7*b^2*c^3*d^
2*n^3-228*B^2*x*a^6*b^3*c^4*d*n^3+432*A^2*x^2*a^8*b*c*d^4*n-1728*A^2*x^2*a
^7*b^2*c^2*d^3*n+2592*A^2*x^2*a^6*b^3*c^3*d^2*n-1728*A^2*x^2*a^5*b^4*c^4*d
*n+576*A*B*x*a^9*c*d^4*n^2+144*A*B*x*a^5*b^4*c^5*n^2-432*B^2*ln(e*((b*x+a)
/(d*x+c))^n)^2*a^8*b*c^3*d^2*n+288*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b^2
*c^4*d*n-432*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3*d^2*n^2+192*B^2*ln(e*
((b*x+a)/(d*x+c))^n)*a^7*b^2*c^4*d*n^2-1152*A^2*x*a^8*b*c^2*d^3*n+1728*A^2
*x*a^7*b^2*c^3*d^2*n-1152*A^2*x*a^6*b^3*c^4*d*n+36*B^2*x*a^5*b^4*c^5*n^3+4
32*A^2*x^2*a^4*b^5*c^5*n+288*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^9*c^2*d^3*n
-72*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^3*c^5*n+576*B^2*ln(e*((b*x+a)/(d
*x+c))^n)*a^9*c^2*d^3*n^2-36*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^5*n^2
+288*A^2*x*a^9*c*d^4*n+288*A^2*x*a^5*b^4*c^5*n+72*B^2*x^4*ln(e*((b*x+a)/(d
*x+c))^n)^2*a^6*b^3*c*d^4*n+300*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b...
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1762 vs. $2(599) = 1198$.

Time = 0.33 (sec) , antiderivative size = 1762, normalized size of antiderivative = 2.87

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="f
ricas")
```

output
```
-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 2
88*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^
4)*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B
^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a
^4*d^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b
^2*d^4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)
*n)*x^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*
B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*
a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B
^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3
)*n^2)*log((b*x + a)/(d*x + c))^2 + 12*(3*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d
 + 36*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 25*A*B*a^4*d^4)*n - 4*((7
*B^2*b^4*c^3*d - 60*B^2*a*b^3*c^2*d^2 + 324*B^2*a^2*b^2*c*d^3 - 271*B^2*a^
3*b*d^4)*n^2 + 12*(A*B*b^4*c^3*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*
d^3 - 13*A*B*a^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 7
2*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*
c*d^3 - B^2*a*b^3*d^4)*n*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*
B^2*a^2*b^2*d^4)*n*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a
^2*b^2*c*d^3 - 13*B^2*a^3*b*d^4)*n*x + (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d
 + 36*B^2*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3 + 25*B^2*a^4*d^4)*n - 12...
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2136 vs. $2(599) = 1198$.

Time = 0.34 (sec) , antiderivative size = 2136, normalized size of antiderivative = 3.47

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="m
axima")
```

output
```
1/24*A*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2
+ 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*
d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b
^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b
^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5
*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a
^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7
*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))
+ 1/288*(12*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d
^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2
*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^
3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^
4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 -
a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2
- a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 -
a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*
c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*...
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(599) = 1198$.

Time = 1.69 (sec) , antiderivative size = 1206, normalized size of antiderivative = 1.96

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="g
iac")
```

output
```
-1/288*(72*(B^2*b^3*n^2 - 4*(b*x + a)*B^2*b^2*d*n^2/(d*x + c) + 6*(b*x + a
)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 4*(b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3)*log
((b*x + a)/(d*x + c))^2/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)
^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4
 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4) + 12*(3*B^2*b^3*n^2 - 16*(b*x + a)
*B^2*b^2*d*n^2/(d*x + c) + 36*(b*x + a)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 48*(
b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3 + 12*B^2*b^3*n*log(e) - 48*(b*x + a)*B^
2*b^2*d*n*log(e)/(d*x + c) + 72*(b*x + a)^2*B^2*b*d^2*n*log(e)/(d*x + c)^2
 - 48*(b*x + a)^3*B^2*d^3*n*log(e)/(d*x + c)^3 + 12*A*B*b^3*n - 48*(b*x +
a)*A*B*b^2*d*n/(d*x + c) + 72*(b*x + a)^2*A*B*b*d^2*n/(d*x + c)^2 - 48*(b*
x + a)^3*A*B*d^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/((b*x + a)^4*b^3*
c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x +
 a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4) +
 (9*B^2*b^3*n^2 - 64*(b*x + a)*B^2*b^2*d*n^2/(d*x + c) + 216*(b*x + a)^2*B
^2*b*d^2*n^2/(d*x + c)^2 - 576*(b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3 + 36*B^
2*b^3*n*log(e) - 192*(b*x + a)*B^2*b^2*d*n*log(e)/(d*x + c) + 432*(b*x + a
)^2*B^2*b*d^2*n*log(e)/(d*x + c)^2 - 576*(b*x + a)^3*B^2*d^3*n*log(e)/(d*x
 + c)^3 + 72*B^2*b^3*log(e)^2 - 288*(b*x + a)*B^2*b^2*d*log(e)^2/(d*x + c)
 + 432*(b*x + a)^2*B^2*b*d^2*log(e)^2/(d*x + c)^2 - 288*(b*x + a)^3*B^2*d^
3*log(e)^2/(d*x + c)^3 + 36*A*B*b^3*n - 192*(b*x + a)*A*B*b^2*d*n/(d*x ...
```

3.18.9 Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 1769, normalized size of antiderivative = 2.88

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5
- 48*a*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(25*B^2*d^4*n^2
 + 12*A*B*d^4*n)*(a*d - b*c)^4) + (B*d^5*n*x*(12*A + 25*B*n)*(b^4*c^3*g^5
- a^3*b*d^3*g^5 - 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(25*B^
2*d^4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(12*b*g^5*(a
*d - b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9
*B^2*b^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3
*d^3*n - 36*A*B*b^3*c^3*n + 55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n
^2 + 156*A*B*a*b^2*c^2*d*n - 276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^
2*(163*B^2*a*b^2*d^3*n^2 - 13*B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*
A*B*b^3*c*d^2*n))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*
c^2*d*n^2 - 53*B^2*a*b^2*c*d^2*n^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*
d*n - 60*A*B*a*b^2*c*d^2*n))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2
+ 12*A*B*b^3*d^2*n))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*b^2*d^2*
g^5 - 192*a^4*b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2*g^5 -
192*a^2*b^5*c*d*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48*a*b^6
*c*d*g^5) + x^2*(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*b^4*c
*d*g^5) + 24*a^6*b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5) - lo
g(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*
g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5*x)) - (B^2*d^4)/(4*b*g^5*(a^4...
```

3.18. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

3.19 $\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.19.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.19.2 Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

3.19.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{(ag+bgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

3.19.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.19.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.19.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log(e*((b*x + a)/(d*x +
c))^n) + A), x)
```

3.19.6 Sympy [N/A]

Not integrable

Time = 34.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=g^2\left(\int \frac{a^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{b^2x^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{2abx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) +
Integral(b**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(2*a*b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

3.19.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.19.8 Giac [N/A]

Not integrable

Time = 27.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.19.9 Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.20 $\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.20.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.20.2 Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

3.20.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{ag+bgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

3.20.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.20.4 Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.20.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
integral((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.20.6 Sympy [N/A]

Not integrable

Time = 16.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=g\left(\int \frac{a}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{bx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
g*(Integral(a/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integr
al(b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

3.20.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.20.8 Giac [N/A]

Not integrable

Time = 16.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.20.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.21 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.21.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.21.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

3.21.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

3.21.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.21.4 Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.21.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c)
)^n)), x)
```

3.21.6 Sympy [N/A]

Not integrable

Time = 16.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{Aa+Abx+Ba\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

3.21.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.21.8 Giac [N/A]

Not integrable

Time = 9.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.21.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.22 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.22.1 Optimal result

Integrand size = 35, antiderivative size = 94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

output
```
exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d
*x+c))^n))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

3.22.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a +
b*x))
```

3.22.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2n(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a +
b*x))
```

3.22.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.22.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.66

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="f
ricas")
```

output
```
e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/
(b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

3.22.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2Babx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bb^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n) + 2*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
B*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

3.22.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.22.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.23 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.23.1 Optimal result

Integrand size = 35, antiderivative size = 197

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\mathrm{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)^2} - \frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\mathrm{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)}$$

output
```
b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/g^3/n/(b*x+a)^2-d*exp(A/B/n)*(e*((b
*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B
/(-a*d+b*c)^2/g^3/n/(b*x+a)
```

3.23.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$= \frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\left(be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)-d(a+bx\right.}{B(bc-ad)^2g^3n(a+bx)^2}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*(b*E^(A/(B*n))*(
e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(B*n)] - d*(a + b*x)*ExpIntegralEi[-((A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])/(B*n))]))/(B*(b*c - a*d)^2*g^3*n*(a + b*x)^2
)
```

3.23.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\,dx$$

$$\Big\downarrow \; 2949$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2795$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bn(a+bx)^2}-\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bn(a+bx)}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpInteg
ralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B*n*(a + b*x)^2
) - (d*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegra
lEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*n*(a + b*x)))/((b
*c - a*d)^2*g^3)
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.23.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx =$$

$$-\frac{de^{\left(\frac{B\log(e)+A}{Bn}\right)}\text{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right) - be^{\left(\frac{2(B\log(e)+A)}{Bn}\right)}\text{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2(B\log(e}{Bn}\right.}}{b^2x^2+2\,abx+a^2}\right.}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3n}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="f
ricas")
```

output
```
-(d*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*
n))/(b*x + a)) - b*e^(2*(B*log(e) + A)/(B*n))*log_integral((d^2*x^2 + 2*c*
d*x + c^2)*e^(-2*(B*log(e) + A)/(B*n))/(b^2*x^2 + 2*a*b*x + a^2)))/((B*b^2
*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3*n)
```

3.23.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.23.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.23.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.24 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.24.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.24.2 Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

3.24.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

3.24.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.24.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.24.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.31

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

3.24.6 Sympy [N/A]

Not integrable

Time = 44.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.34

$$\begin{aligned}&\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx\\&=g^2\left(\int \frac{a^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right.\\&\quad+\int \frac{b^2x^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\\&\quad\left.+\int \frac{2abx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)\end{aligned}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g**2*(Integral(a**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)
+ B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b**2*x**
2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*a*b*x/(A**2 + 2*A*B*log(
e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)**2), x))
```

3.24.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 329, normalized size of antiderivative = 9.40

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
-(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c
*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c*n - a*d*n)*
B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d
*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3
+ 3*a^2*b*c*g^2 + a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2
*c*g^2 + a^2*b*d*g^2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n -
a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*
log(e))*B^2), x)
```

3.24.8 Giac [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.24.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.25 $\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.25.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.25.2 Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

3.25.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{ag+bgx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

3.25.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.25.4 Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.25.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral((b*g*x + a*g)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e
*((b*x + a)/(d*x + c))^n) + A^2), x)
```

3.25.6 Sympy [N/A]

Not integrable

Time = 58.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.55

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$=g\left(\int \frac{a}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right.$$
$$\left.+\int \frac{bx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g*(Integral(a/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b*x/(A**2 + 2*
A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)**2), x))
```

3.25.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 251, normalized size of antiderivative = 7.61

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
-(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g
)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x
+ c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integ
rate((3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b
*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n)
+ (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

3.25.8 Giac [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.25.9 Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.26 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.26.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.26.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

3.26.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2955}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

3.26.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.26.4 Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.26.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.43

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(
d*x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n)),
x)
```

3.26.6 Sympy [N/A]

Not integrable

Time = 130.89 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.66

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$=\frac{\int \frac{1}{A^2a+A^2bx+2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2ABbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2+B^2bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral(1/(A**2*a + A**2*b*x + 2*A*B*a*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n) + 2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*a*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)**2 + B**2*b*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)**2), x)/g
```

3.26.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
d*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g
*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d
*g*n*log(e))*B^2), x) - (d*x + c)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n
) - (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (
b*c*g*n*log(e) - a*d*g*n*log(e))*B^2)
```

3.26.8 Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

3.26.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.27 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.27.1 Optimal result

Integrand size = 35, antiderivative size = 153

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(a+bx)} - \frac{c+dx}{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
-exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(
d*x+c))^n))/B/n)/B^2/(-a*d+b*c)/g^2/n^2/(b*x+a)+(-d*x-c)/B/(-a*d+b*c)/g^2/
n/(b*x+a)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

3.27.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{(c+dx)\left(Bn+e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{B^2(bc-ad)g^2n^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
-(((c + d*x)*(B*n + E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpInte
gralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])))/(B^2*(b*c - a*d)*g^2*n^2*(a + b*x)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])))
```

3.27.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2949, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{-\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{c+dx}{Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2747}$$

$$\frac{-\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2(a+bx)}-\frac{c+dx}{Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{-\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(a+bx)}-\frac{c+dx}{Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
(-((E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi
[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B^2*n^2*(a + b*x))) -
(c + d*x)/(B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/((b*c -
a*d)*g^2)
```

3.27.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.27.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.79

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{Bdnx + Bcn + \left(Abx + Aa + (Bbx + Ba)\log(e) + (Bbnx + Ban)\log\left(\frac{bx+a}{dx+c}\right.\right.}{(AB^2b^2c - AB^2abd)g^2n^2x + (AB^2abc - AB^2a^2d)g^2n^2 + ((B^3b^2c - B^3abd)g^2n^2x + (B^3abc - B^3a^2d)g}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"fricas")
```

output
```
-(B*d*n*x + B*c*n + (A*b*x + A*a + (B*b*x + B*a)*log(e) + (B*b*n*x + B*a*n
)*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c
)*e^(-(B*log(e) + A)/(B*n))/(b*x + a)))/((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*n
^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2*n^2 + ((B^3*b^2*c - B^3*a*b*d)*g^2*
n^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^2)*log(e) + ((B^3*b^2*c - B^3*a*b*d)
*g^2*n^3*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^3)*log((b*x + a)/(d*x + c)))
```

3.27.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.27.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"maxima")
```

output
```
-(d*x + c)/((a*b*c*g^2*n - a^2*d*g^2*n)*A*B + (a*b*c*g^2*n*log(e) - a^2*d*
g^2*n*log(e))*B^2 + ((b^2*c*g^2*n - a*b*d*g^2*n)*A*B + (b^2*c*g^2*n*log(e)
 - a*b*d*g^2*n*log(e))*B^2)*x + ((b^2*c*g^2*n - a*b*d*g^2*n)*B^2*x + (a*b*
c*g^2*n - a^2*d*g^2*n)*B^2)*log((b*x + a)^n) - ((b^2*c*g^2*n - a*b*d*g^2*n
)*B^2*x + (a*b*c*g^2*n - a^2*d*g^2*n)*B^2)*log((d*x + c)^n)) + integrate(-
1/(B^2*a^2*g^2*n*log(e) + A*B*a^2*g^2*n + (B^2*b^2*g^2*n*log(e) + A*B*b^2*
g^2*n)*x^2 + 2*(B^2*a*b*g^2*n*log(e) + A*B*a*b*g^2*n)*x + (B^2*b^2*g^2*n*x
^2 + 2*B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((b*x + a)^n) - (B^2*b^2*g^2*n*
x^2 + 2*B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((d*x + c)^n)), x)
```

3.27.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.28 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.28.1 Optimal result

Integrand size = 35, antiderivative size = 314

$$\begin{aligned}
&\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx \\
&= -\frac{2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)^2} \\
&\quad+\frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)} \\
&\quad+\frac{d(c+dx)}{B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} \\
&\quad-\frac{b(c+dx)^2}{B(bc-ad)^2g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}
\end{aligned}$$

output
```
-2*b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)^2+d*exp(A/B/n)
*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))
/B/n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)+d*(d*x+c)/B/(-a*d+b*c)^2/g^3/n/(b*x
+a)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))-b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/n/(b*x+
a)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

3.28.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.81

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{(c+dx)\left(B(-bc+ad)n-2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)(A+B}{B^2(bc-ad)^2g^3n^2(a+b}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
((c + d*x)*(B*(-(b*c) + a*d)*n - 2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*
x))^n)^(2/n)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + d*E^(A/(B*n))*(a +
b*x)*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpIntegralEi[-((A + B*Log[e*((a +
 b*x)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2
*(b*c - a*d)^2*g^3*n^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

3.28.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2949}$$
$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$
$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2n^2(a+bx)^2}+\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{Bn}\right.}{B^2n^2(a+bx)}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
((-2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIn
tegralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B^2*n^2*(a +
 b*x)^2) + (d*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*Exp
IntegralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B^2*n^2*(a +
 b*x)) + (d*(c + d*x))/(B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])) - (b*(c + d*x)^2)/(B*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])))/((b*c - a*d)^2*g^3)
```

3.28.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.28.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(312) = 624$.

Time = 0.29 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.40

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{(Bbcd-Bad^2)nx-\left(Ab^2dx^2+2\,Aabdx+Aa^2d+(Bb^2dx^2+2\,Babdx+Ba\right.}{(AB^2b^4c^2-2\,AB^2ab^3cd+AB^2a^2b^2d^2)g^3n^2x^2+2\,(AB^2ab^3c^2-2\,AB^2a^2b^2cd+AB^2a^3bd^2)g^3n^2x+(A\!B}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"fricas")
```

output
```
-((B*b*c*d - B*a*d^2)*n*x - (A*b^2*d*x^2 + 2*A*a*b*d*x + A*a^2*d + (B*b^2*
d*x^2 + 2*B*a*b*d*x + B*a^2*d)*log(e) + (B*b^2*d*n*x^2 + 2*B*a*b*d*n*x + B
*a^2*d*n)*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral(
(d*x + c)*e^(-(B*log(e) + A)/(B*n))/(b*x + a)) + 2*(A*b^3*x^2 + 2*A*a*b^2*
x + A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log(e) + (B*b^3*n*x^2 +
2*B*a*b^2*n*x + B*a^2*b*n)*log((b*x + a)/(d*x + c)))*e^(2*(B*log(e) + A)/(
B*n))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*(B*log(e) + A)/(B*n))/(
b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c^2 - B*a*c*d)*n)/((A*B^2*b^4*c^2 - 2*A*B
^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*g^3*n^2*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B
^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*g^3*n^2*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2
*a^3*b*c*d + A*B^2*a^4*d^2)*g^3*n^2 + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^
3*a^2*b^2*d^2)*g^3*n^2*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^
3*b*d^2)*g^3*n^2*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3
*n^2)*log(e) + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*n^3*
x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*n^3*x + (B
^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3*n^3)*log((b*x + a)/(d*
x + c)))
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.28.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"maxima")
```

output
```
-(d*x + c)/((a^2*b*c*g^3*n - a^3*d*g^3*n)*A*B + (a^2*b*c*g^3*n*log(e) - a^
3*d*g^3*n*log(e))*B^2 + ((b^3*c*g^3*n - a*b^2*d*g^3*n)*A*B + (b^3*c*g^3*n*
log(e) - a*b^2*d*g^3*n*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3*n - a^2*b*d*g^3*
n)*A*B + (a*b^2*c*g^3*n*log(e) - a^2*b*d*g^3*n*log(e))*B^2)*x + ((b^3*c*g^
3*n - a*b^2*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (
a^2*b*c*g^3*n - a^3*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^3*c*g^3*n - a*b^2
*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (a^2*b*c*g^3
*n - a^3*d*g^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*x + 2*b*c - a*d)
/(((b^4*c*g^3*n - a*b^3*d*g^3*n)*A*B + (b^4*c*g^3*n*log(e) - a*b^3*d*g^3*n
*log(e))*B^2)*x^3 + (a^3*b*c*g^3*n - a^4*d*g^3*n)*A*B + (a^3*b*c*g^3*n*log
(e) - a^4*d*g^3*n*log(e))*B^2 + 3*((a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*A*B +
 (a*b^3*c*g^3*n*log(e) - a^2*b^2*d*g^3*n*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*
g^3*n - a^3*b*d*g^3*n)*A*B + (a^2*b^2*c*g^3*n*log(e) - a^3*b*d*g^3*n*log(e
))*B^2)*x + ((b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^
2*b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*
b*c*g^3*n - a^4*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^4*c*g^3*n - a*b^3*d*g
^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c
*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4*d*g^3*n)*B^2)*log((d*
x + c)^n)), x)
```

3.28.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.29 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.29.1 Optimal result

Integrand size = 33, antiderivative size = 188

$$\begin{aligned}\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= -\frac{B(bc-ad)^4g^4nx}{5b^4}-\frac{B(bc-ad)^3g^4n(c+dx)^2}{10b^3d}\\ &\quad-\frac{B(bc-ad)^2g^4n(c+dx)^3}{15b^2d}-\frac{B(bc-ad)g^4n(c+dx)^4}{20bd}\\ &\quad-\frac{B(bc-ad)^5g^4n\log(a+bx)}{5b^5d}+\frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d}\end{aligned}$$

output
```
-1/5*B*(-a*d+b*c)^4*g^4*n*x/b^4-1/10*B*(-a*d+b*c)^3*g^4*n*(d*x+c)^2/b^3/d-
1/15*B*(-a*d+b*c)^2*g^4*n*(d*x+c)^3/b^2/d-1/20*B*(-a*d+b*c)*g^4*n*(d*x+c)^
4/b/d-1/5*B*(-a*d+b*c)^5*g^4*n*ln(b*x+a)/b^5/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/d
```

3.29.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g^4\left(-\frac{B(bc-ad)n\left(12bd(bc-ad)^3x+6b^2(bc-ad)^2(c+dx)^2+4b^3(bc-ad)(c+dx)^3+3b^4(c+dx)^4+12(bc-ad)^4\log(a+bx)\right)}{12b^5}+(c+dx)^5\left(A+\right.\right.}{5d}$$

input
```
Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^4*(-1/12*(B*(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2
*(c + d*x)^2 + 4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c
 - a*d)^4*Log[a + b*x]))/b^5 + (c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])))/(5*d)
```

3.29.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^4 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \, dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^4 (c+dx)^5 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bn(bc-ad) \int \frac{g^5 (c+dx)^4}{a+bx}dx}{5dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^4 (c+dx)^5 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad) \int \frac{(c+dx)^4}{a+bx}dx}{5d}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^4 (c+dx)^5 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad) \int \left(\frac{(bc-ad)^4}{b^4(a+bx)}+\frac{d(bc-ad)^3}{b^4}+\frac{d(c+dx)(bc-ad)^2}{b^3}+\frac{d(c+dx)^2(bc-ad)}{b^2}+\frac{d(c+dx)^3}{b}\right)dx}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^4 (c+dx)^5 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad) \left(\frac{(bc-ad)^4 \log(a+bx)}{b^5}+\frac{dx(bc-ad)^3}{b^4}+\frac{(c+dx)^2(bc-ad)^2}{2b^3}+\frac{(c+dx)^3(bc-ad)}{3b^2}+\frac{(c+dx)^4}{4b}\right)}{5d}$$

input
```
Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/5*(B*(b*c - a*d)*g^4*n*((d*(b*c - a*d)^3*x)/b^4 + ((b*c - a*d)^2*(c + d
*x)^2)/(2*b^3) + ((b*c - a*d)*(c + d*x)^3)/(3*b^2) + (c + d*x)^4/(4*b) + (
(b*c - a*d)^4*Log[a + b*x])/b^5))/d + (g^4*(c + d*x)^5*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(5*d)
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(176) = 352$.

Time = 17.54 (sec) , antiderivative size = 864, normalized size of antiderivative = 4.60

method	result
parallelrisch	$\frac{60B\,x^4\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^5c\,d^4g^4n-180Aa\,b^4c^4d\,g^4n-54B\,a^4bc\,d^4g^4n^2+90B\,a^3b^2c^2d^3g^4n^2-60B\,a^2b^3c^3d^2g^4n^2-36Ba\,b^4c^4d\,g}{}$

input
```
int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/60*(60*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^4*g^4*n-180*A*a*b^4*c^4*d
*g^4*n-54*B*a^4*b*c*d^4*g^4*n^2+90*B*a^3*b^2*c^2*d^3*g^4*n^2-60*B*a^2*b^3*
c^3*d^2*g^4*n^2-36*B*a*b^4*c^4*d*g^4*n^2-48*B*x*b^5*c^4*d*g^4*n^2+12*B*x^5
*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^4*n+3*B*x^4*a*b^4*d^5*g^4*n^2-3*B*x^4
*b^5*c*d^4*g^4*n^2-4*B*x^3*a^2*b^3*d^5*g^4*n^2-16*B*x^3*b^5*c^2*d^3*g^4*n^
2+6*B*x^2*a^3*b^2*d^5*g^4*n^2-36*B*x^2*b^5*c^3*d^2*g^4*n^2-12*B*x*a^4*b*d^
5*g^4*n^2+60*A*x^4*b^5*c*d^4*g^4*n+120*A*x^3*b^5*c^2*d^3*g^4*n+120*A*x^2*b
^5*c^3*d^2*g^4*n+60*A*x*b^5*c^4*d*g^4*n+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n
)*b^5*c^3*d^2*g^4*n+60*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^4*d*g^4*n+60*B*
x*a^3*b^2*c*d^4*g^4*n^2-120*B*x*a^2*b^3*c^2*d^3*g^4*n^2+120*B*x*a*b^4*c^3*
d^2*g^4*n^2-60*B*ln(b*x+a)*a^4*b*c*d^4*g^4*n^2+120*B*ln(b*x+a)*a^3*b^2*c^2
*d^3*g^4*n^2-120*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^4*n^2+60*B*ln(b*x+a)*a*b^4*
c^4*d*g^4*n^2+20*B*x^3*a*b^4*c*d^4*g^4*n^2-30*B*x^2*a^2*b^3*c*d^4*g^4*n^2+
60*B*x^2*a*b^4*c^2*d^3*g^4*n^2+12*B*a^5*d^5*g^4*n^2+48*B*b^5*c^5*g^4*n^2+1
2*A*x^5*b^5*d^5*g^4*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*g^4*n+12*B*ln
(b*x+a)*a^5*d^5*g^4*n^2-12*B*ln(b*x+a)*b^5*c^5*g^4*n^2+120*B*x^3*ln(e*((b*
x+a)/(d*x+c))^n)*b^5*c^2*d^3*g^4*n-60*A*b^5*c^5*g^4*n)/n/b^5/d
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(176) = 352$.

Time = 0.33 (sec) , antiderivative size = 572, normalized size of antiderivative = 3.04

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 - 12\,Bb^5c^5g^4n\log\left(dx+c\right) + 12\left(5\,Bab^4c^4d - 10\,Ba^2b^3c^3d^2 + 10\,Ba^3b^2c^2d^3 - 5\,Ba^4bcd^4 + \right.}{}$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*b^5*c^5*g^4*n*log(d*x + c) + 12*(5*B*a*b
^4*c^4*d - 10*B*a^2*b^3*c^3*d^2 + 10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4 +
 B*a^5*d^5)*g^4*n*log(b*x + a) + 3*(20*A*b^5*c*d^4*g^4 - (B*b^5*c*d^4 - B*
a*b^4*d^5)*g^4*n)*x^4 + 4*(30*A*b^5*c^2*d^3*g^4 - (4*B*b^5*c^2*d^3 - 5*B*a
*b^4*c*d^4 + B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*A*b^5*c^3*d^2*g^4 - (6*B*b^
5*c^3*d^2 - 10*B*a*b^4*c^2*d^3 + 5*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^4*n)
*x^2 + 12*(5*A*b^5*c^4*d*g^4 - (4*B*b^5*c^4*d - 10*B*a*b^4*c^3*d^2 + 10*B*
a^2*b^3*c^2*d^3 - 5*B*a^3*b^2*c*d^4 + B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^
5*g^4*x^5 + 5*B*b^5*c*d^4*g^4*x^4 + 10*B*b^5*c^2*d^3*g^4*x^3 + 10*B*b^5*c^
3*d^2*g^4*x^2 + 5*B*b^5*c^4*d*g^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*
B*b^5*c*d^4*g^4*n*x^4 + 10*B*b^5*c^2*d^3*g^4*n*x^3 + 10*B*b^5*c^3*d^2*g^4*
n*x^2 + 5*B*b^5*c^4*d*g^4*n*x)*log((b*x + a)/(d*x + c)))/(b^5*d)
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.29.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(176) = 352$.

Time = 0.21 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.60

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\frac{1}{5}\,Bd^4g^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{5}\,Ad^4g^4x^5+Bcd^3g^4x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Acd^3g^4x^4+2\,Bc^2d^2g^4x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+2\,Ac^2d^2g^4x^3+2\,Bc^3dg^4x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+2\,Ac^3dg^4x^2$$
$$+\frac{1}{60}\,Bd^4g^4n\left(\frac{12\,a^5\log(bx+a)}{b^5}-\frac{12\,c^5\log(dx+c)}{d^5}-\frac{3\,(b^4cd^3-ab^3d^4)x^4-4\,(b^4c^2d^2-a^2b^2d^4)x^3+6\,(b\ldots}{b^4d^4}\right.$$
$$-\frac{1}{6}\,Bcd^3g^4n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3\ldots}{b^3d^3}\right.$$
$$+Bc^2d^2g^4n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-2\,Bc^3dg^4n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^4g^4n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^4g^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^4g^4x$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/5*B*d^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*d^4*g^4*x
^5 + B*c*d^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^3*g^4*
x^4 + 2*B*c^2*d^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*c^2
*d^2*g^4*x^3 + 2*B*c^3*d*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
2*A*c^3*d*g^4*x^2 + 1/60*B*d^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log
(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*
d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*
d^4)) - 1/6*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4
 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3
*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*c^2*d^2*g^4*n*(2*a^3*log(b*x + a)/b^3 -
2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*
x)/(b^2*d^2)) - 2*B*c^3*d*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d
^2 + (b*c - a*d)*x/(b*d)) + B*c^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)
/d) + B*c^4*g^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^4*g^4*x
```

3.29. $\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1876 vs. $2(176) = 352$.

Time = 1.09 (sec) , antiderivative size = 1876, normalized size of antiderivative = 9.98

$$\int (cg + dgx)^4 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/60*(12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g
^4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b
*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(b^5*d - 5*(b*x +
 a)*b^4*d^2/(d*x + c) + 10*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - 10*(b*x + a)^
3*b^2*d^4/(d*x + c)^3 + 5*(b*x + a)^4*b*d^5/(d*x + c)^4 - (b*x + a)^5*d^6/
(d*x + c)^5) - (25*B*b^10*c^6*g^4*n - 150*B*a*b^9*c^5*d*g^4*n - 77*(b*x +
a)*B*b^9*c^6*d*g^4*n/(d*x + c) + 375*B*a^2*b^8*c^4*d^2*g^4*n + 462*(b*x +
a)*B*a*b^8*c^5*d^2*g^4*n/(d*x + c) + 94*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d
*x + c)^2 - 500*B*a^3*b^7*c^3*d^3*g^4*n - 1155*(b*x + a)*B*a^2*b^7*c^4*d^3
*g^4*n/(d*x + c) - 564*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 54*
(b*x + a)^3*B*b^7*c^6*d^3*g^4*n/(d*x + c)^3 + 375*B*a^4*b^6*c^2*d^4*g^4*n
+ 1540*(b*x + a)*B*a^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 1410*(b*x + a)^2*B*a^
2*b^6*c^4*d^4*g^4*n/(d*x + c)^2 + 324*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d
*x + c)^3 + 12*(b*x + a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 150*B*a^5*b^5
*c*d^5*g^4*n - 1155*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 1880*(b*
x + a)^2*B*a^3*b^5*c^3*d^5*g^4*n/(d*x + c)^2 - 810*(b*x + a)^3*B*a^2*b^5*c
^4*d^5*g^4*n/(d*x + c)^3 - 72*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^
4 + 25*B*a^6*b^4*d^6*g^4*n + 462*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c)
 + 1410*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 1080*(b*x + a)^3
*B*a^3*b^4*c^3*d^6*g^4*n/(d*x + c)^3 + 180*(b*x + a)^4*B*a^2*b^4*c^4*d^...
```

3.29.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 1045, normalized size of antiderivative = 5.56

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$=x^2\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{c\,d^2\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+B\,a\,d\,n-B\,b\,c}{b}\right.}{10\,b\,d}\right.$$

$$\left.-\frac{a\,c\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)}{2\,b\,d}+\frac{c^2\,d\,g^4\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}\right)$$

$$-x^3\left(\frac{\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)(5\,a\,d+5\,b\,c)}{15\,b\,d}-\frac{c\,d^2\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,b}+\frac{A\,a\,c\,d^3\,g^4}{3\,b}\right)$$

$$+x^4\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{20\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{20\,b}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,c^4\,g^4\,x+2\,B\,c^3\,d\,g^4\,x^2+2\,B\,c^2\,d^2\,g^4\,x^3+B\,c\,d^3\,g^4\,x^4+\frac{B\,d^4\,g^4\,x^5}{5}\right)+x\left(\frac{c^3\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d\,n-2\,B\,b\,c\,n)}{b}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{c\,d^2\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+B\,a\,d\,n-}{b}\right.}{5\,b\,d}\right.}{5\,b}$$

input
```
int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^2*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n)
)/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c
*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^3*g^4)/b))
/(10*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b)
 - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(2*b*d) + (c^2*d*g^4*(5*A*a*d + 5*A
*b*c + B*a*d*n - B*b*c*n))/b) - x^3*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*
d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c)
)/(15*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/(3*b) +
(A*a*c*d^3*g^4)/(3*b)) + x^4*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b
*c*n))/(20*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(20*b)) + log(e*((a + b*x)/(c
+ d*x))^n)*((B*d^4*g^4*x^5)/5 + B*c^4*g^4*x + 2*B*c^3*d*g^4*x^2 + B*c*d^3*
g^4*x^4 + 2*B*c^2*d^2*g^4*x^3) + x*((c^3*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d
*n - 2*B*b*c*n))/b - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a
*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5
*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n -
 B*b*c*n))/b + (A*a*c*d^3*g^4)/b))/(5*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*
A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(b
*d) + (2*c^2*d*g^4*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/b))/(5*b*d) +
(a*c*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*
g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*...
```

3.29. $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.30 $\int (cg+dgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

3.30.1 Optimal result

Integrand size = 33, antiderivative size = 156

$$\int (cg+dgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = -\frac{B(bc-ad)^3g^3nx}{4b^3}-\frac{B(bc-ad)^2g^3n(c+dx)^2}{8b^2d}-\frac{B(bc-ad)g^3n(c+dx)^3}{12bd}-\frac{B(bc-ad)^4g^3n\log(a+bx)}{4b^4d}+\frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}$$

output
```
-1/4*B*(-a*d+b*c)^3*g^3*n*x/b^3-1/8*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2/b^2/d-1
/12*B*(-a*d+b*c)*g^3*n*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*n*ln(b*x+a)/b^
4/d+1/4*g^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (cg+dgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = \frac{g^3\left(-\frac{B(bc-ad)n\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)}{6b^4}+(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{4d}$$

input
```
Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^3*(-1/6*(B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c
+ d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d
*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(4*d)
```

3.30.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bn(bc-ad)\int \frac{g^4(c+dx)^3}{a+bx}dx}{4dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)\int \frac{(c+dx)^3}{a+bx}dx}{4d}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)\int \left(\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)^2}{b^3}+\frac{d(c+dx)(bc-ad)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)\left(\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}\right)}{4d}$$

input
```
Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/4*(B*(b*c - a*d)*g^3*n*((d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x
)^2)/(2*b^2) + (c + d*x)^3/(3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4))/d +
(g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d)
```

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(146) = 292$.

Time = 7.14 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.18

method	result
parallelrisch	$\frac{24B\,x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c\,d^3g^3n+6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^4g^3n-6B\ln(bx+a)a^4d^4g^3n^2-6B\ln(bx+a)b^4c^4g^3n^2+24Ax\,b^4c^3d\,g^3}{}$

input
```
int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/24*(24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*g^3*n+6*B*ln(e*((b*x+a)
/(d*x+c))^n)*b^4*c^4*g^3*n-6*B*ln(b*x+a)*a^4*d^4*g^3*n^2-6*B*ln(b*x+a)*b^4
*c^4*g^3*n^2+24*A*x*b^4*c^3*d*g^3*n-24*A*b^4*c^4*g^3*n+21*B*a^3*b*c*d^3*g^
3*n^2-24*B*a^2*b^2*c^2*d^2*g^3*n^2-9*B*a*b^3*c^3*d*g^3*n^2+6*B*x^4*ln(e*((
b*x+a)/(d*x+c))^n)*b^4*d^4*g^3*n+2*B*x^3*a*b^3*d^4*g^3*n^2-2*B*x^3*b^4*c*d
^3*g^3*n^2-3*B*x^2*a^2*b^2*d^4*g^3*n^2-9*B*x^2*b^4*c^2*d^2*g^3*n^2+6*B*x*a
^3*b*d^4*g^3*n^2-18*B*x*b^4*c^3*d*g^3*n^2+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^
n)*b^4*c^2*d^2*g^3*n+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^3*d*g^3*n-60*A
*a*b^3*c^3*d*g^3*n+24*A*x^3*b^4*c*d^3*g^3*n+36*A*x^2*b^4*c^2*d^2*g^3*n+24*
B*ln(b*x+a)*a^3*b*c*d^3*g^3*n^2-36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g^3*n^2+24*
B*ln(b*x+a)*a*b^3*c^3*d*g^3*n^2+12*B*x^2*a*b^3*c*d^3*g^3*n^2-24*B*x*a^2*b^
2*c*d^3*g^3*n^2+36*B*x*a*b^3*c^2*d^2*g^3*n^2-6*B*a^4*d^4*g^3*n^2+18*B*b^4*
c^4*g^3*n^2+6*A*x^4*b^4*d^4*g^3*n)/b^4/d/n
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(146) = 292$.

Time = 0.30 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.75

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4g^3x^4 - 6\,Bb^4c^4g^3n\log(dx+c) + 6\,(4\,Bab^3c^3d - 6\,Ba^2b^2c^2d^2 + 4\,Ba^3bcd^3 - Ba^4d^4)g^3n\log(bx+}{}$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*b^4*c^4*g^3*n*log(d*x + c) + 6*(4*B*a*b^3*
c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^3*n*log(b*x +
 a) + 2*(12*A*b^4*c*d^3*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(
12*A*b^4*c^2*d^2*g^3 - (3*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)
*g^3*n)*x^2 + 6*(4*A*b^4*c^3*d*g^3 - (3*B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 +
4*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4
*c*d^3*g^3*x^3 + 6*B*b^4*c^2*d^2*g^3*x^2 + 4*B*b^4*c^3*d*g^3*x)*log(e) + 6
*(B*b^4*d^4*g^3*n*x^4 + 4*B*b^4*c*d^3*g^3*n*x^3 + 6*B*b^4*c^2*d^2*g^3*n*x^
2 + 4*B*b^4*c^3*d*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b^4*d)
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.20 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\frac{1}{4}Bd^3g^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{4}Ad^3g^3x^4+Bcd^2g^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Acd^2g^3x^3+\frac{3}{2}Bc^2dg^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}Ac^2dg^3x^2$$
$$-\frac{1}{24}Bd^3g^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3}{b^3d^3}\right.$$
$$+\frac{1}{2}Bcd^2g^3n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{3}{2}Bc^2dg^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^3g^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^3g^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^3g^3x$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/4*B*d^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*d^3*g^3*x
^4 + B*c*d^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^2*g^3*
x^3 + 3/2*B*c^2*d*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*c
^2*d*g^3*x^2 - 1/24*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x +
c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 +
6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*g^3*n*(2*a^3*log(b*x + a
)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2)) - 3/2*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(
d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*g^3*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*c^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^3
*g^3*x
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(146) = 292$.

Time = 0.84 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.99

$$\int (cg + dgx)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/24*(6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^
3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n
)*log((b*x + a)/(d*x + c))/(b^4*d - 4*(b*x + a)*b^3*d^2/(d*x + c) + 6*(b*x
 + a)^2*b^2*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b*d^4/(d*x + c)^3 + (b*x + a)^
4*d^5/(d*x + c)^4) - (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 26*(b*
x + a)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 130*(b*
x + a)*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^5*d^2*g^3*
n/(d*x + c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 260*(b*x + a)*B*a^2*b^5*c^3*
d^3*g^3*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 -
6*(b*x + a)^3*B*b^5*c^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n +
 260*(b*x + a)*B*a^3*b^4*c^2*d^4*g^3*n/(d*x + c) + 210*(b*x + a)^2*B*a^2*b
^4*c^3*d^4*g^3*n/(d*x + c)^2 + 30*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x +
 c)^3 - 11*B*a^5*b^3*d^5*g^3*n - 130*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x
+ c) - 210*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 60*(b*x + a)^
3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 26*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(
d*x + c) + 105*(b*x + a)^2*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 60*(b*x + a
)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x + c)^3 - 21*(b*x + a)^2*B*a^5*b*d^7*g^3*n
/(d*x + c)^2 - 30*(b*x + a)^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 6*(b*x + a
)^3*B*a^5*d^8*g^3*n/(d*x + c)^3 - 6*B*b^8*c^5*g^3*log(e) + 30*B*a*b^7*c^4*
d*g^3*log(e) - 60*B*a^2*b^6*c^3*d^2*g^3*log(e) + 60*B*a^3*b^5*c^2*d^3*g...
```

3.30. $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.30.9 Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= x^3\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b}\right)$$

$$-x^2\left(\frac{\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$\left.-\frac{c\,d\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}+\frac{A\,a\,c\,d^2\,g^3}{2\,b}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,c^3\,g^3\,x+\frac{3\,B\,c^2\,d\,g^3\,x^2}{2}+B\,c\,d^2\,g^3\,x^3+\frac{B\,d^3\,g^3\,x^4}{4}\right)$$

$$+x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{c\,d\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c}{b}\right.}{4\,b\,d}\right.$$

$$+\frac{c^2\,g^3\,(12\,A\,a\,d+8\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n)}{2\,b}$$

$$\left.-\frac{a\,c\left(\frac{d^2\,g^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)}{b\,d}\right)$$

$$-\frac{\ln(a+b\,x)\,(B\,n\,a^4\,d^3\,g^3-4\,B\,n\,a^3\,b\,c\,d^2\,g^3+6\,B\,n\,a^2\,b^2\,c^2\,d\,g^3-4\,B\,n\,a\,b^3\,c^3\,g^3)}{4\,b^4}$$

$$+\frac{A\,d^3\,g^3\,x^4}{4}-\frac{B\,c^4\,g^3\,n\,\ln(c+d\,x)}{4\,d}$$

input
```
int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^3*((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(12*b) - (A*d^2*g^
3*(4*a*d + 4*b*c))/(12*b)) - x^2*((((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n
 - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(
8*b*d) - (c*d*g^3*(4*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/(2*b) + (A*a*c*
d^2*g^3)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d^3*g^3*x^4)/4 + B*c^
3*g^3*x + (3*B*c^2*d*g^3*x^2)/2 + B*c*d^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*(
(((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4
*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*g^3*(4*A*a*d + 6*A*b
*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^2*g^3)/b))/(4*b*d) + (c^2*g^3*(12*A*
a*d + 8*A*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*b) - (a*c*((d^2*g^3*(4*A*a*d +
16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b)))
/(b*d)) - (log(a + b*x)*(B*a^4*d^3*g^3*n - 4*B*a*b^3*c^3*g^3*n - 4*B*a^3*b
*c*d^2*g^3*n + 6*B*a^2*b^2*c^2*d*g^3*n))/(4*b^4) + (A*d^3*g^3*x^4)/4 - (B*
c^4*g^3*n*log(c + d*x))/(4*d)
```

3.30. $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.31 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

3.31.1 Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= -\frac{B(bc-ad)^2g^2nx}{3b^2} - \frac{B(bc-ad)g^2n(c+dx)^2}{6bd} \\ &\quad - \frac{B(bc-ad)^3g^2n\log(a+bx)}{3b^3d} + \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)^2*g^2*n*x/b^2-1/6*B*(-a*d+b*c)*g^2*n*(d*x+c)^2/b/d-1/3*B
*(-a*d+b*c)^3*g^2*n*ln(b*x+a)/b^3/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/d
```

3.31.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{g^2\left(-\frac{B(bc-ad)n(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{2b^3} + (c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{3d}$$

input
```
Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g^2*(-1/2*(B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*
c - a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])))/(3*d)
```

3.31.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^2 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \, dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^2 (c+dx)^3 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bn(bc-ad)\int \frac{g^3(c+dx)^2}{a+bx}dx}{3dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2 (c+dx)^3 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2n(bc-ad)\int \frac{(c+dx)^2}{a+bx}dx}{3d}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2 (c+dx)^3 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2n(bc-ad)\int \left(\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(bc-ad)}{b^2}+\frac{d(c+dx)}{b}\right)dx}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2 (c+dx)^3 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2n(bc-ad)\left(\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}\right)}{3d}$$

input
```
Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/3*(B*(b*c - a*d)*g^2*n*((d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b
*c - a*d)^2*Log[a + b*x])/b^3))/d + (g^2*(c + d*x)^3*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(3*d)
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(116) = 232$.

Time = 2.92 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.73

method	result
parallelrisch	$\frac{6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}c^{2}d\,g^{2}n-6B\ln(bx+a)a^{2}bc\,d^{2}g^{2}n^{2}-6A\,b^{3}c^{3}g^{2}n+6Bxa\,b^{2}c\,d^{2}g^{2}n^{2}+6B\ln(bx+a)a\,b^{2}c^{2}d\,g^{2}n^{2}+2B\,a^{3}d^{3}}{}$

input
```
int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^2*d*g^2*n-6*B*ln(b*x+a)*a^2*b*c
*d^2*g^2*n^2-6*A*b^3*c^3*g^2*n+6*B*x*a*b^2*c*d^2*g^2*n^2+6*B*ln(b*x+a)*a*b
^2*c^2*d*g^2*n^2+2*B*a^3*d^3*g^2*n^2+4*B*b^3*c^3*g^2*n^2+6*B*x^2*ln(e*((b*
x+a)/(d*x+c))^n)*b^3*c*d^2*g^2*n-12*A*a*b^2*c^2*d*g^2*n+6*A*x^2*b^3*c*d^2*
g^2*n+6*A*x*b^3*c^2*d*g^2*n+2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*g^2*
n+B*x^2*a*b^2*d^3*g^2*n^2-B*x^2*b^3*c*d^2*g^2*n^2-2*B*x*a^2*b*d^3*g^2*n^2-
4*B*x*b^3*c^2*d*g^2*n^2-5*B*a^2*b*c*d^2*g^2*n^2-B*a*b^2*c^2*d*g^2*n^2+2*A*
x^3*b^3*d^3*g^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g^2*n+2*B*ln(b*x+a
)*a^3*d^3*g^2*n^2-2*B*ln(b*x+a)*b^3*c^3*g^2*n^2)/b^3/d/n
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(116) = 232$.

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{2\,Ab^3d^3g^2x^3-2\,Bb^3c^3g^2n\log\left(dx+c\right)+2\left(3\,Bab^2c^2d-3\,Ba^2bcd^2+Ba^3d^3\right)g^2n\log\left(bx+a\right)+\left(6\,Ab^3cd^2g\right.}{}$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*b^3*c^3*g^2*n*log(d*x + c) + 2*(3*B*a*b^2*c
^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*g^2*n*log(b*x + a) + (6*A*b^3*c*d^2*g^
2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*A*b^3*c^2*d*g^2 - (2*B*b
^3*c^2*d - 3*B*a*b^2*c*d^2 + B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3
+ 3*B*b^3*c*d^2*g^2*x^2 + 3*B*b^3*c^2*d*g^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n
*x^3 + 3*B*b^3*c*d^2*g^2*n*x^2 + 3*B*b^3*c^2*d*g^2*n*x)*log((b*x + a)/(d*x
 + c)))/(b^3*d)
```

3.31.6 Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time $= 0.19$ (sec) , antiderivative size $= 309$, normalized size of antiderivative $= 2.49$

$$\begin{aligned}
&\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{1}{3}Bd^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Ad^2g^2x^3 \\
&\quad + Bcdg^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acdg^2x^2 \\
&\quad + \frac{1}{6}Bd^2g^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad - Bcdg^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad + Bc^2g^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&\quad + Bc^2g^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^2g^2x
\end{aligned}$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/3*B*d^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*d^2*g^2*x
^3 + B*c*d*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d*g^2*x^2
+ 1/6*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*g^2*n*(a
^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*
g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^2*g^2*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*c^2*g^2*x
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(116) = 232$.

Time = 0.63 (sec) , antiderivative size = 990, normalized size of antiderivative = 7.98

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{6}\left(\frac{2\left(Bb^4c^4g^2n-4\,Bab^3c^3dg^2n+6\,Ba^2b^2c^2d^2g^2n-4\,Ba^3bcd^3g^2n+Ba^4d^4g^2n\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^3d-\frac{3\,(bx+a)b^2d^2}{dx+c}+\frac{3\,(bx+a)^2bd^3}{(dx+c)^2}-\frac{(bx+a)^3d^4}{(dx+c)^3}}-\frac{3\,Bb^6c^4g^2n}{\cdots}\right.$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/6*(2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*
n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3
*d - 3*(b*x + a)*b^2*d^2/(d*x + c) + 3*(b*x + a)^2*b*d^3/(d*x + c)^2 - (b*
x + a)^3*d^4/(d*x + c)^3) - (3*B*b^6*c^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n -
5*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 20*
(b*x + a)*B*a*b^4*c^3*d^2*g^2*n/(d*x + c) + 2*(b*x + a)^2*B*b^4*c^4*d^2*g^
2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*g^2*n - 30*(b*x + a)*B*a^2*b^3*c^2*d^
3*g^2*n/(d*x + c) - 8*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*
a^4*b^2*d^4*g^2*n + 20*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 12*(b*x
 + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*g^2*
n/(d*x + c) - 8*(b*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 2*(b*x + a)^
2*B*a^4*d^6*g^2*n/(d*x + c)^2 - 2*B*b^6*c^4*g^2*log(e) + 8*B*a*b^5*c^3*d*g
^2*log(e) - 12*B*a^2*b^4*c^2*d^2*g^2*log(e) + 8*B*a^3*b^3*c*d^3*g^2*log(e)
 - 2*B*a^4*b^2*d^4*g^2*log(e) - 2*A*b^6*c^4*g^2 + 8*A*a*b^5*c^3*d*g^2 - 12
*A*a^2*b^4*c^2*d^2*g^2 + 8*A*a^3*b^3*c*d^3*g^2 - 2*A*a^4*b^2*d^4*g^2)/(b^5
*d - 3*(b*x + a)*b^4*d^2/(d*x + c) + 3*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - (
b*x + a)^3*b^2*d^4/(d*x + c)^3) + 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2
*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*
log(b - (b*x + a)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^
3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d...
```

3.31.9 Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,c^2\,g^2\,x+B\,c\,d\,g^2\,x^2+\frac{B\,d^2\,g^2\,x^3}{3}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{d\,g^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,b}-\frac{A\,d\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{c\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}+\frac{A\,a\,c\,d\,g^2}{b}\right)$$
$$+x^2\left(\frac{d\,g^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,b}-\frac{A\,d\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b}\right)$$
$$+\frac{\ln(a+b\,x)\,\left(B\,n\,a^3\,d^2\,g^2-3\,B\,n\,a^2\,b\,c\,d\,g^2+3\,B\,n\,a\,b^2\,c^2\,g^2\right)}{3\,b^3}$$
$$+\frac{A\,d^2\,g^2\,x^3}{3}-\frac{B\,c^3\,g^2\,n\,\ln(c+d\,x)}{3\,d}$$

input
```
int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((B*d^2*g^2*x^3)/3 + B*c^2*g^2*x + B*c*d*g^
2*x^2) - x*(((3*a*d + 3*b*c)*((d*g^2*(3*A*a*d + 9*A*b*c + B*a*d*n - B*b*c*
n))/(3*b) - (A*d*g^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*g^2*(3*A*a*d +
3*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d*g^2)/b) + x^2*((d*g^2*(3*A*a*d
+ 9*A*b*c + B*a*d*n - B*b*c*n))/(6*b) - (A*d*g^2*(3*a*d + 3*b*c))/(6*b)) +
 (log(a + b*x)*(B*a^3*d^2*g^2*n + 3*B*a*b^2*c^2*g^2*n - 3*B*a^2*b*c*d*g^2*
n))/(3*b^3) + (A*d^2*g^2*x^3)/3 - (B*c^3*g^2*n*log(c + d*x))/(3*d)
```

3.31. $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.32 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.32.1 Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)gnx}{2b}-\frac{B(bc-ad)^2gn\log(a+bx)}{2b^2d}+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}$$

output
```
-1/2*B*(-a*d+b*c)*g*n*x/b-1/2*B*(-a*d+b*c)^2*g*n*ln(b*x+a)/b^2/d+1/2*g*(d*
x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

3.32.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g\left(-\frac{B(bc-ad)n(bdx+(bc-ad)\log(a+bx))}{b^2}+(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{2d}$$

input
```
Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g*(-((B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bn(bc-ad)\int \frac{g^2(c+dx)}{a+bx}dx}{2dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)\int \frac{c+dx}{a+bx}dx}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)\int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)\left(\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}\right)}{2d}$$

input
```
Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*g*n*((d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2))/d + (g
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)
```

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(80) = 160$.

Time = 1.07 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.91

method	result
parallelrisch	$\frac{B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^2gn+A\,x^2b^2d^2gn-B\ln(bx+a)a^2d^2g\,n^2+2B\ln(bx+a)abcdg\,n^2-B\ln(bx+a)b^2c^2g\,n^2+2Bx\ln\left(e\left(\frac{bx+\cdots}{dx+\cdots}\right.\right.}{2\cdots}$

input
```
int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*g*n+A*x^2*b^2*d^2*g*n-B*ln(b*
x+a)*a^2*d^2*g*n^2+2*B*ln(b*x+a)*a*b*c*d*g*n^2-B*ln(b*x+a)*b^2*c^2*g*n^2+2
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*g*n+B*x*a*b*d^2*g*n^2-B*x*b^2*c*d*g
*n^2+2*A*x*b^2*c*d*g*n+B*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c^2*g*n-B*a^2*d^2*g
*n^2+B*b^2*c^2*g*n^2-3*A*a*b*c*d*g*n-2*A*b^2*c^2*g*n)/b^2/n/d
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(80) = 160$.

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.88

$$\int (cg + dgx) \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \frac{Ab^2d^2gx^2 - Bb^2c^2gn \log (dx + c) + (2\,Babcd - Ba^2d^2)gn \log (bx + a) + (2\,Ab^2cdg - (Bb^2cd - Babd^2)gn}{2\,b^2d}$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*g*x^2 - B*b^2*c^2*g*n*log(d*x + c) + (2*B*a*b*c*d - B*a^2*d
^2)*g*n*log(b*x + a) + (2*A*b^2*c*d*g - (B*b^2*c*d - B*a*b*d^2)*g*n)*x + (
B*b^2*d^2*g*x^2 + 2*B*b^2*c*d*g*x)*log(e) + (B*b^2*d^2*g*n*x^2 + 2*B*b^2*c
*d*g*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d)
```

3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(73) = 146$.

Time = 55.24 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.44

$$\int (cg + dgx) \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \begin{cases} cgx\left(A + B \log \left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Acgx + \frac{Adgx^2}{2} + \frac{Bc^2g \log \left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d} + \frac{Bcgnx}{2} + Bcgx \log \left(e\left(\frac{a}{c+dx}\right)^n\right) + \frac{Bdgnx^2}{4} + \frac{Bdgx^2 \log \left(e\left(\frac{a}{c+dx}\right)^n\right)}{2} \\ cg\left(Ax + \frac{Ba \log \left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right)}{b} - Bnx + Bx \log \left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right)\right) \\ Acgx + \frac{Adgx^2}{2} - \frac{Ba^2dgn \log \left(\frac{c}{d} + x\right)}{2b^2} - \frac{Ba^2dg \log \left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{2b^2} + \frac{Bacgn \log \left(\frac{c}{d} + x\right)}{b} + \frac{Bacg \log \left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{b} + Bc \end{cases}$$

input
```
integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((c*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*g*x +
 A*d*g*x**2/2 + B*c**2*g*log(e*(a/(c + d*x))**n)/(2*d) + B*c*g*n*x/2 + B*c
*g*x*log(e*(a/(c + d*x))**n) + B*d*g*n*x**2/4 + B*d*g*x**2*log(e*(a/(c + d
*x))**n)/2, Eq(b, 0)), (c*g*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x +
 B*x*log(e*(a/c + b*x/c)**n)), Eq(d, 0)), (A*c*g*x + A*d*g*x**2/2 - B*a**2
*d*g*n*log(c/d + x)/(2*b**2) - B*a**2*d*g*log(e*(a/(c + d*x) + b*x/(c + d*
x))**n)/(2*b**2) + B*a*c*g*n*log(c/d + x)/b + B*a*c*g*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/b + B*a*d*g*n*x/(2*b) - B*c**2*g*n*log(c/d + x)/(2*d)
- B*c*g*n*x/2 + B*c*g*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d*g*x*
*2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}\,Bdgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}\,Adgx^2$$
$$-\frac{1}{2}\,Bdgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bcgn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bcgx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acgx$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
1/2*B*d*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*d*g*x^2 - 1/2
*B*d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c*g*x*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + A*c*g*x
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(80) = 160.

Time = 0.44 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.74

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{1}{2}\left(\frac{(Bb^3c^3gn-3\,Bab^2c^2dgn+3\,Ba^2bcd^2gn-Ba^3d^3gn)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d-\frac{2\,(bx+a)bd^2}{dx+c}+\frac{(bx+a)^2d^3}{(dx+c)^2}}-\frac{Bb^4c^3gn-3\,Bab^3c^2dgn-\frac{(bx+a)B}{dx}}{}\right.$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
1/2*((B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^
3*g*n)*log((b*x + a)/(d*x + c))/(b^2*d - 2*(b*x + a)*b*d^2/(d*x + c) + (b*
x + a)^2*d^3/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (b*x +
a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2
*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(
d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - B*b^4*c^3*g*log(e) + 3*B*a*
b^3*c^2*d*g*log(e) - 3*B*a^2*b^2*c*d^2*g*log(e) + B*a^3*b*d^3*g*log(e) - A
*b^4*c^3*g + 3*A*a*b^3*c^2*d*g - 3*A*a^2*b^2*c*d^2*g + A*a^3*b*d^3*g)/(b^3
*d - 2*(b*x + a)*b^2*d^2/(d*x + c) + (b*x + a)^2*b*d^3/(d*x + c)^2) + (B*b
^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*lo
g(-b + (b*x + a)*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g
*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b^2*d)
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=x\left(\frac{g\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,d\,g\,x^2}{2}+B\,c\,g\,x\right)$$
$$-\frac{\ln(a+b\,x)\,\left(B\,a^2\,d\,g\,n-2\,B\,a\,b\,c\,g\,n\right)}{2\,b^2}+\frac{A\,d\,g\,x^2}{2}-\frac{B\,c^2\,g\,n\,\ln(c+d\,x)}{2\,d}$$

input
```
int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((g*(2*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*b) - (A*g*(2*a*d + 2*b*c
))/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d*g*x^2)/2 + B*c*g*x) - (lo
g(a + b*x)*(B*a^2*d*g*n - 2*B*a*b*c*g*n))/(2*b^2) + (A*d*g*x^2)/2 - (B*c^2
*g*n*log(c + d*x))/(2*d)
```

3.32. $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.33 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx$

3.33.1 Optimal result

Integrand size = 33, antiderivative size = 80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d/g-B*n*polylog(
2,d*(b*x+a)/b/(d*x+c))/d/g
```

3.33.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \frac{\log(g(c+dx))\left(2A-2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log(g(c+dx))\right)-2Bn\,\mathrm{PolyLog}\left(2,\frac{b}{}\right.}{2dg}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

output
```
(Log[g*(c + d*x)]*(2*A - 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log
[e*((a + b*x)/(c + d*x))^n] + B*n*Log[g*(c + d*x)]) - 2*B*n*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])/(2*d*g)
```

3.33.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2943, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{cg+dgx}dx$$

$$\Big\downarrow \text{2943}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{Bn(bc-ad)\int \frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2g}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{25}$$

$$-\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{2778}$$

$$\frac{Bn(bc-ad)\int \frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{2005}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}-\frac{Bn\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{dg}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/
(d*g)) - (B*n*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d*g)
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2943
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d)
/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c
*g, 0]
```

3.33.4 Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dgx+cg}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

3.33.5 Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{cg+dgx}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="frica
s")
```

output
```
integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*g*x + c*g), x)
```

3.33.6 Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \frac{\int \frac{A}{c+dx}\,dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g),x)
```

output
```
(Integral(A/(c + d*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n)/(c + d*x), x))/g
```

3.33.7 Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="maxim
a")
```

output
```
-1/2*B*((2*n*log(b*x + a)*log(d*x + c) - n*log(d*x + c)^2 - 2*log(d*x + c)
*log((b*x + a)^n) + 2*log(d*x + c)*log((d*x + c)^n))/(d*g) - 2*integrate((
n*log(b*x + a) + log(e))/(d*g*x + c*g), x)) + A*log(d*g*x + c*g)/(d*g)
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(79) = 158$.

Time = 57.49 (sec) , antiderivative size = 566, normalized size of antiderivative = 7.08

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx$$
$$= \frac{1}{2}\left(\frac{(Bb^3c^3n-3\,Bab^2c^2dn+3\,Ba^2bcd^2n-Ba^3d^3n)\log\left(\frac{bx+a}{dx+c}\right)}{b^2dg-\frac{2\,(bx+a)bd^2g}{dx+c}+\frac{(bx+a)^2d^3g}{(dx+c)^2}} - \frac{Bb^4c^3n-3\,Bab^3c^2dn-\frac{(bx+a)Bb^3c^3dn}{dx+c}+\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="giac"
)
```

output
```
1/2*((B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*l
og((b*x + a)/(d*x + c))/(b^2*d*g - 2*(b*x + a)*b*d^2*g/(d*x + c) + (b*x +
a)^2*d^3*g/(d*x + c)^2) - (B*b^4*c^3*n - 3*B*a*b^3*c^2*d*n - (b*x + a)*B*b
^3*c^3*d*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*n
/(d*x + c) - B*a^3*b*d^3*n - 3*(b*x + a)*B*a^2*b*c*d^3*n/(d*x + c) + (b*x
+ a)*B*a^3*d^4*n/(d*x + c) - B*b^4*c^3*log(e) + 3*B*a*b^3*c^2*d*log(e) - 3
*B*a^2*b^2*c*d^2*log(e) + B*a^3*b*d^3*log(e) - A*b^4*c^3 + 3*A*a*b^3*c^2*d
 - 3*A*a^2*b^2*c*d^2 + A*a^3*b*d^3)/(b^3*d*g - 2*(b*x + a)*b^2*d^2*g/(d*x
+ c) + (b*x + a)^2*b*d^3*g/(d*x + c)^2) + (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n
 + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d
*g) - (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*
log((b*x + a)/(d*x + c))/(b^2*d*g))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2
)^2
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg + dgx}\,dx = \int \frac{A + B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{c\,g + d\,g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x), x)
```

3.34 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx$

3.34.1 Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx=\frac{A(a+bx)}{(bc-ad)g^2(c+dx)}-\frac{Bn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)}$$

output
```
A*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)-B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+B*(b*x
+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)
```

3.34.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx=-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg(cg+dgx)}+\frac{B(bc-ad)n\left(\frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\right)}{dg^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

output
```
-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g*(c*g + d*g*x))) + (B*(b*c -
a*d)*n*(1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Lo
g[c + d*x])/(b*c - a*d)^2))/(d*g^2)
```

3.34.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2951, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(cg+dgx)^2}dx$$

$$\Big\downarrow \ 2951$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

output
```
((A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*
((a + b*x)/(c + d*x))^n])/(c + d*x))/((b*c - a*d)*g^2)
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.34.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{A}{g^{2}(dx+c)d}-\frac{B\left(\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)(bx+a)}{dx+c}-\frac{n(bx+a)}{dx+c}\right)}{g^{2}(ad-cb)}$	81
parts	$-\frac{A}{g^{2}(dx+c)d}-\frac{B\left(\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)(bx+a)}{dx+c}-\frac{n(bx+a)}{dx+c}\right)}{g^{2}(ad-cb)}$	81
parallelrisch	$-\frac{-Bab\,d^{3}n^{2}+B\,b^{2}c\,d^{2}n^{2}+Aab\,d^{3}n-A\,b^{2}c\,d^{2}n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}d^{3}n+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)ab\,d^{3}n}{g^{2}(dx+c)b\,d^{3}n(ad-cb)}$	129

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/g^2*A/(d*x+c)/d-1/g^2*B/(a*d-b*c)*(ln(e*((b*x+a)/(d*x+c))^n)*(b*x+a)/(d
*x+c)-n*(b*x+a)/(d*x+c))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(cg+dgx)^{2}}\,dx=-\frac{Abc-Aad-(Bbc-Bad)n+(Bbc-Bad)\log\left(e\right)-(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)}{(bcd^{2}-ad^{3})g^{2}x+(bc^{2}d-acd^{2})g^{2}}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="fri
cas")
```

output
```
-(A*b*c - A*a*d - (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) - (B*b*d*n*x
+ B*a*d*n)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*g^2*x + (b*c^2*d -
 a*c*d^2)*g^2)
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**2,x)
```

output
```
Timed out
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^2}\,dx = Bn\left(\frac{1}{d^2g^2x + cdg^2} + \frac{b\log(bx + a)}{(bcd - ad^2)g^2} - \frac{b\log(dx + c)}{(bcd - ad^2)g^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{d^2g^2x + cdg^2} - \frac{A}{d^2g^2x + cdg^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="max
ima")
```

output
```
B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) - b*lo
g(d*x + c)/((b*c*d - a*d^2)*g^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(d^2*g^2*x + c*d*g^2) - A/(d^2*g^2*x + c*d*g^2)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = \left(\frac{(bx+a)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2} - \frac{(Bn-B\log(e)-A)(bx+a)}{(dx+c)g^2}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="gia
c")
```

output
```
((b*x + a)*B*n*log((b*x + a)/(d*x + c))/((d*x + c)*g^2) - (B*n - B*log(e)
- A)*(b*x + a)/((d*x + c)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = -\frac{A-B\,n}{x\,d^2\,g^2+c\,d\,g^2} - \frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d\,\left(c\,g^2+d\,g^2\,x\right)} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^2,x)
```

output
```
(B*b*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*g^2*(a*d - b*c))
- (B*log(e*((a + b*x)/(c + d*x))^n))/(d*(c*g^2 + d*g^2*x)) - (A - B*n)/(d^
2*g^2*x + c*d*g^2)
```

3.35 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$

3.35.1 Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx=\frac{Bn}{4dg^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)g^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2g^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}-\frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2g^3}$$

output
```
1/4*B*n/d/g^3/(d*x+c)^2+1/2*b*B*n/d/(-a*d+b*c)/g^3/(d*x+c)+1/2*b^2*B*n*ln(
b*x+a)/d/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^3/(d*x+
c)^2-1/2*b^2*B*n*ln(d*x+c)/d/(-a*d+b*c)^2/g^3
```

3.35.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx=\frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4dg^3(c+dx)^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

output
```
(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(3*b*c - a*
d + 2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c +
d*x]))/(b*c - a*d)^2)/(4*d*g^3*(c + d*x)^2)
```

3.35.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(cg+dgx)^3} \, dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{Bn(bc-ad) \int \frac{1}{g^2(a+bx)(c+dx)^3}dx}{2dg}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bn(bc-ad) \int \frac{1}{(a+bx)(c+dx)^3}dx}{2dg^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{Bn(bc-ad) \int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{db^2}{(bc-ad)^3(c+dx)}-\frac{db}{(bc-ad)^2(c+dx)^2}-\frac{d}{(bc-ad)(c+dx)^3}\right)dx}{2dg^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad) \left(\frac{b^2 \log(a+bx)}{(bc-ad)^3}-\frac{b^2 \log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}\right)}{2dg^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g^3*(c + d*x)^2) + (B*(b*c
- a*d)*n*(1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b
^2*Log[a + b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3))/(2*d*g
^3)
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.35.4 Maple [A] (verified)

Time = 7.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{2A\,a^{2}b\,d^{5}n+2A\,b^{3}c^{2}d^{3}n-B\,a^{2}b\,d^{5}n^{2}-3B\,b^{3}c^{2}d^{3}n^{2}+4Ba\,b^{2}c\,d^{4}n^{2}-4Aa\,b^{2}c\,d^{4}n-2B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{5}n+2Bxa\,b^{2}d^{5}\cdots}{4g^{3}(dx+c)^{2}(a^{2}d^{2}-2abcd+\cdots}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*A*a^2*b*d^5*n+2*A*b^3*c^2*d^3*n-B*a^2*b*d^5*n^2-3*B*b^3*c^2*d^3*n^
2+4*B*a*b^2*c*d^4*n^2-4*A*a*b^2*c*d^4*n-2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
b^3*d^5*n+2*B*x*a*b^2*d^5*n^2-2*B*x*b^3*c*d^4*n^2+2*B*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b*d^5*n-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n-4*B*ln(e*((b
*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n)/g^3/(d*x+c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)
/b/d^4/n
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = -\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\,(Bb^2cd-Babd^2)nx-(3\,Bb^2c^2-4\,Babcd+Ba^2d^2)n+2\,(Bb^2c^2-2}{4\,((b^2c^2d^3-2\,abcd^4+a^2d^5)g^3x^2+2\,(b^2c^3d^2-2\,abc^2d^3+a}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="fri
cas")
```

output
```
-1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*
n*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c
*d + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c
*d - B*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 +
 a^2*d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b
^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. 2(133) = 266.

Time = 127.35 (sec) , antiderivative size = 2103, normalized size of antiderivative = 13.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**3,x)
```

output
```
Piecewise((-A/(2*c**2*d*g**3 + 4*c*d**2*g**3*x + 2*d**3*g**3*x**2) - B*log
(e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n)/(2*c**2*d*g**3 + 4*c*d**2*g**3
*x + 2*d**3*g**3*x**2), Eq(a, b*c/d)), ((A*x + B*a*log(e*(a/c + b*x/c)**n)
/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n))/(c**3*g**3), Eq(d, 0)), (-2*A*a*
*2*d**2/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a**2*d**5*g**3*x
**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b*c*d**4*g**3*x
**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c**2*d**3*g**3
*x**2) + 4*A*a*b*c*d/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**3*x + 4*a**
2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3*x - 8*a*b*
c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**3*x + 4*b**2*c
**2*d**3*g**3*x**2) - 2*A*b**2*c**2/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4
*g**3*x + 4*a**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*
g**3*x - 8*a*b*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g*
*3*x + 4*b**2*c**2*d**3*g**3*x**2) + B*a**2*d**2*n/(4*a**2*c**2*d**3*g**3
+ 8*a**2*c*d**4*g**3*x + 4*a**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16
*a*b*c**2*d**3*g**3*x - 8*a*b*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b*
*2*c**3*d**2*g**3*x + 4*b**2*c**2*d**3*g**3*x**2) - 2*B*a**2*d**2*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n)/(4*a**2*c**2*d**3*g**3 + 8*a**2*c*d**4*g**
3*x + 4*a**2*d**5*g**3*x**2 - 8*a*b*c**3*d**2*g**3 - 16*a*b*c**2*d**3*g**3
*x - 8*a*b*c*d**4*g**3*x**2 + 4*b**2*c**4*d*g**3 + 8*b**2*c**3*d**2*g**...
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$$
$$= \frac{1}{4}Bn\left(\frac{2\,bdx + 3\,bc - ad}{(bcd^3 - ad^4)g^3x^2 + 2\,(bc^2d^2 - acd^3)g^3x + (bc^3d - ac^2d^2)g^3} + \frac{2\,b^2\log(bx+a)}{(b^2c^2d - 2\,abcd^2 + a^2d^3)g^3} - \frac{2}{(b^2c^2d}\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{2\,(d^3g^3x^2 + 2\,cd^2g^3x + c^2dg^3)} - \frac{A}{2\,(d^3g^3x^2 + 2\,cd^2g^3x + c^2dg^3)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="max
ima")
```

output
```
1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^2
 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(
d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A/(d^3*g^3*x^2 + 2*c*d^2*g^
3*x + c^2*d*g^3)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.37

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)Bbn}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2Bdn}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{(Bdn-2\,Bd\log(e)-2\,Ad)(}{(bcg^3-adg^3)(dx+c)^2}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="gia
c")
```

output
```
1/4*(2*(2*(b*x + a)*B*b*n/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^2*B*
d*n/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + (B*d*n -
 2*B*d*log(e) - 2*A*d)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2) - 4*(
B*b*n - B*b*log(e) - A*b)*(b*x + a)/((b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/
(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.46

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx=\frac{B\,b^2\,n\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d\,n+3\,B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^3,x)
```

output
```
(B*b^2*n*atanh((2*a^2*d^3*g^3 - 2*b^2*c^2*d*g^3)/(2*d*g^3*(a*d - b*c)^2) +
 (2*b*d*x)/(a*d - b*c)))/(d*g^3*(a*d - b*c)^2) - (B*log(e*((a + b*x)/(c +
d*x))^n))/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*d*g^3*x)) - ((2*A*a*d - 2*A*b*
c - B*a*d*n + 3*B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/(a*d - b*c))/(2*c^2
*d*g^3 + 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x)
```

3.35. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$

3.36 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$

3.36.1 Optimal result

Integrand size = 33, antiderivative size = 183

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \frac{Bn}{9dg^4(c+dx)^3}+\frac{bBn}{6d(bc-ad)g^4(c+dx)^2} + \frac{b^2Bn}{3d(bc-ad)^2g^4(c+dx)}+\frac{b^3Bn\log(a+bx)}{3d(bc-ad)^3g^4} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}-\frac{b^3Bn\log(c+dx)}{3d(bc-ad)^3g^4}$$

output
```
1/9*B*n/d/g^4/(d*x+c)^3+1/6*b*B*n/d/(-a*d+b*c)/g^4/(d*x+c)^2+1/3*b^2*B*n/d
/(-a*d+b*c)^2/g^4/(d*x+c)+1/3*b^3*B*n*ln(b*x+a)/d/(-a*d+b*c)^3/g^4+1/3*(-A
-B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^4/(d*x+c)^3-1/3*b^3*B*n*ln(d*x+c)/d/(-a*
d+b*c)^3/g^4
```

3.36.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \frac{-6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn((bc-ad)(2a^2d^2-abd(7c+3dx)+b^2(11c^2+15cdx+6d^2x^2))+6b^3(c+dx)^3\log(a+bx)-6b^3(c+dx)^3\log(}{(bc-ad)^3}}{18dg^4(c+dx)^3}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

output
```
(-6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(2*a^2*d^2
- a*b*d*(7*c + 3*d*x) + b^2*(11*c^2 + 15*c*d*x + 6*d^2*x^2)) + 6*b^3*(c +
d*x)^3*Log[a + b*x] - 6*b^3*(c + d*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(18*
d*g^4*(c + d*x)^3)
```

3.36.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(cg+dgx)^4} \, dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{Bn(bc-ad) \int \frac{1}{g^3(a+bx)(c+dx)^4}dx}{3dg}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bn(bc-ad) \int \frac{1}{(a+bx)(c+dx)^4}dx}{3dg^4}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{Bn(bc-ad) \int \left(\frac{b^4}{(bc-ad)^4(a+bx)}-\frac{db^3}{(bc-ad)^4(c+dx)}-\frac{db^2}{(bc-ad)^3(c+dx)^2}-\frac{db}{(bc-ad)^2(c+dx)^3}-\frac{d}{(bc-ad)(c+dx)^4}\right)dx}{3dg^4}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{b^3\log(a+bx)}{(bc-ad)^4}-\frac{b^3\log(c+dx)}{(bc-ad)^4}+\frac{b^2}{(c+dx)(bc-ad)^3}+\frac{b}{2(c+dx)^2(bc-ad)^2}+\frac{1}{3(c+dx)^3(bc-ad)}\right)}{3dg^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

output
```
-1/3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g^4*(c + d*x)^3) + (B*(b*c
- a*d)*n*(1/(3*(b*c - a*d)*(c + d*x)^3) + b/(2*(b*c - a*d)^2*(c + d*x)^2)
+ b^2/((b*c - a*d)^3*(c + d*x)) + (b^3*Log[a + b*x])/(b*c - a*d)^4 - (b^3*
Log[c + d*x])/(b*c - a*d)^4))/(3*d*g^4)
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(174) = 348$.

Time = 16.01 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.40

method	result
parallelrisch	$-\frac{9B\,a^2b^2c\,d^6n^2-18Ba\,b^3c^2d^5n^2-18A\,a^2b^2c\,d^6n+18Aa\,b^3c^2d^5n+18B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c\,d^6n+18Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^2}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(9*B*a^2*b^2*c*d^6*n^2-18*B*a*b^3*c^2*d^5*n^2-18*A*a^2*b^2*c*d^6*n+1
8*A*a*b^3*c^2*d^5*n+18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^6*n+18*B*x*
ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^5*n-18*B*x*a*b^3*c*d^6*n^2-18*B*ln(e*(
(b*x+a)/(d*x+c))^n)*a^2*b^2*c*d^6*n+18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c
^2*d^5*n-2*B*a^3*b*d^7*n^2+11*B*b^4*c^3*d^4*n^2+6*A*a^3*b*d^7*n-6*A*b^4*c^
3*d^4*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*d^7*n+6*B*x^3*ln(e*((b*x+a)/(d
*x+c))^n)*b^4*d^7*n-6*B*x^2*a*b^3*d^7*n^2+6*B*x^2*b^4*c*d^6*n^2+3*B*x*a^2*
b^2*d^7*n^2+15*B*x*b^4*c^2*d^5*n^2)/g^4/(d*x+c)^3/(a^3*d^3-3*a^2*b*c*d^2+3
*a*b^2*c^2*d-b^3*c^3)/n/b/d^5
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(171) = 342$.

Time = 0.28 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.64

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^4}\,dx =$$
$$-\frac{6\,Ab^3c^3 - 18\,Aab^2c^2d + 18\,Aa^2bcd^2 - 6\,Aa^3d^3 - 6\,(Bb^3cd^2 - Bab^2d^3)nx^2 - 3\,(5\,Bb^3c^2d - 6\,Bab^2cd^2 +}{18\,((b^3c^3d^4 - 3\,ab^2c^2d^5 + 3\,a^2bcd^6 - a^3d^7}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="fri
cas")
```

output
```
-1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 - 6
*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(5*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 +
B*a^2*b*d^3)*n*x - (11*B*b^3*c^3 - 18*B*a*b^2*c^2*d + 9*B*a^2*b*c*d^2 - 2*
B*a^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^
3)*log(e) - 6*(B*b^3*d^3*n*x^3 + 3*B*b^3*c*d^2*n*x^2 + 3*B*b^3*c^2*d*n*x +
 (3*B*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*n)*log((b*x + a)/(d*x + c
)))/((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3
*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3
*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*g^4*x + (
b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*g^4)
```

3.36.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**4,x)
```

output
```
Timed out
```

3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(171) = 342$.

Time = 0.21 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.37

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$$
$$= \frac{1}{18}Bn\left(\frac{6\,b^2d^2x^2 + 11\,b^2c^2 - 7\,abcd + 2\,a^2d^2 + 3\,(5\,b^2cd - abd^2)x}{(b^2c^2d^4 - 2\,abcd^5 + a^2d^6)g^4x^3 + 3\,(b^2c^3d^3 - 2\,abc^2d^4 + a^2cd^5)g^4x^2 + 3\,(b^2c^4d^2 - 2\,abc^3d^3 + a^2c^2d\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{3\,(d^4g^4x^3 + 3\,cd^3g^4x^2 + 3\,c^2d^2g^4x + c^3dg^4)}$$
$$- \frac{A}{3\,(d^4g^4x^3 + 3\,cd^3g^4x^2 + 3\,c^2d^2g^4x + c^3dg^4)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="max
ima")
```

output
```
1/18*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c
*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c
^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d
^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) +
 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^
4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
 - a^3*d^4)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*g^4*
x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A/(d^4*g^4*x^3
+ 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(171) = 342$.

Time = 0.68 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.21

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$$
$$=\frac{1}{18}\left(6\left(\frac{3\,(bx+a)Bb^2n}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)}-\frac{3\,(bx+a)^2\,Bbdn}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)^2}+\frac{(bx}{(b^2c^2g^4-2\,abc}\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="gia
c")
```

output
```
1/18*(6*(3*(b*x + a)*B*b^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*
(d*x + c)) - 3*(b*x + a)^2*B*b*d*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2
*g^4)*(d*x + c)^2) + (b*x + a)^3*B*d^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a
^2*d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c)) - 2*(B*d^2*n - 3*B*d^2*
log(e) - 3*A*d^2)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)
*(d*x + c)^3) + 9*(B*b*d*n - 2*B*b*d*log(e) - 2*A*b*d)*(b*x + a)^2/((b^2*c
^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) - 18*(B*b^2*n - B*b^2*l
og(e) - A*b^2)*(b*x + a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x
 + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = \frac{B\,a^2\,d\,n}{9\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{A\,a^2\,d}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{A\,b^2\,c^2}{3\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{3\,d\,g^4\,(c+d\,x)^3} + \frac{2\,A\,a\,b\,c}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} + \frac{B\,b^2\,d\,n\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{7\,B\,a\,b\,c\,n}{18\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} + \frac{11\,B\,b^2\,c^2\,n}{18\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} + \frac{5\,B\,b^2\,c\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} - \frac{B\,a\,b\,d\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3} + \frac{B\,b^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{3\,d\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^4,x)
```

output
```
(B*a^2*d*n)/(9*g^4*(a*d - b*c)^2*(c + d*x)^3) - (A*a^2*d)/(3*g^4*(a*d - b*
c)^2*(c + d*x)^3) - (A*b^2*c^2)/(3*d*g^4*(a*d - b*c)^2*(c + d*x)^3) - (B*l
og(e*((a + b*x)/(c + d*x))^n))/(3*d*g^4*(c + d*x)^3) + (B*b^3*n*atan((a*d*
1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*d*g^4*(a*d - b*c)^3) + (2*A*a*
b*c)/(3*g^4*(a*d - b*c)^2*(c + d*x)^3) + (B*b^2*d*n*x^2)/(3*g^4*(a*d - b*c
)^2*(c + d*x)^3) - (7*B*a*b*c*n)/(18*g^4*(a*d - b*c)^2*(c + d*x)^3) + (11*
B*b^2*c^2*n)/(18*d*g^4*(a*d - b*c)^2*(c + d*x)^3) + (5*B*b^2*c*n*x)/(6*g^4
*(a*d - b*c)^2*(c + d*x)^3) - (B*a*b*d*n*x)/(6*g^4*(a*d - b*c)^2*(c + d*x)
^3)
```

3.37 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$

3.37.1 Optimal result

Integrand size = 33, antiderivative size = 215

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx &= \frac{Bn}{16dg^5(c+dx)^4}+\frac{bBn}{12d(bc-ad)g^5(c+dx)^3}\\ &\quad+\frac{b^2Bn}{8d(bc-ad)^2g^5(c+dx)^2}\\ &\quad+\frac{b^3Bn}{4d(bc-ad)^3g^5(c+dx)}+\frac{b^4Bn\log(a+bx)}{4d(bc-ad)^4g^5}\\ &\quad-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4}-\frac{b^4Bn\log(c+dx)}{4d(bc-ad)^4g^5}\end{aligned}$$

output
```
1/16*B*n/d/g^5/(d*x+c)^4+1/12*b*B*n/d/(-a*d+b*c)/g^5/(d*x+c)^3+1/8*b^2*B*n
/d/(-a*d+b*c)^2/g^5/(d*x+c)^2+1/4*b^3*B*n/d/(-a*d+b*c)^3/g^5/(d*x+c)+1/4*b
^4*B*n*ln(b*x+a)/d/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d
/g^5/(d*x+c)^4-1/4*b^4*B*n*ln(d*x+c)/d/(-a*d+b*c)^4/g^5
```

3.37.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.75

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^4}+\frac{Bn\left(\frac{3(bc-ad)^4}{(c+dx)^4}+\frac{4b(bc-ad)^3}{(c+dx)^3}+\frac{6b^2(bc-ad)^2}{(c+dx)^2}+\frac{12b^3(bc-ad)}{c+dx}+12b^4\log(a+bx)-12b^4\log(c+dx)\right)}{12(bc-ad)^4}}{4dg^5}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x)^4) + (B*n*((3*(b*c - a
*d)^4)/(c + d*x)^4 + (4*b*(b*c - a*d)^3)/(c + d*x)^3 + (6*b^2*(b*c - a*d)^
2)/(c + d*x)^2 + (12*b^3*(b*c - a*d))/(c + d*x) + 12*b^4*Log[a + b*x] - 12
*b^4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*d*g^5)
```

3.37.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(cg+dgx)^5}\,dx$$

$$\Big\downarrow \ 2947$$

$$\frac{Bn(bc-ad)\int \frac{1}{g^4(a+bx)(c+dx)^5}dx}{4dg}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)^5}dx}{4dg^5}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}$$

$$\Big\downarrow \ 54$$

$$\frac{Bn(bc-ad)\int \left(\frac{b^5}{(bc-ad)^5(a+bx)} - \frac{db^4}{(bc-ad)^5(c+dx)} - \frac{db^3}{(bc-ad)^4(c+dx)^2} - \frac{db^2}{(bc-ad)^3(c+dx)^3} - \frac{db}{(bc-ad)^2(c+dx)^4} - \frac{d}{(bc-ad)(c+dx)^5}\right.}{4dg^5}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(\frac{b^4\log(a+bx)}{(bc-ad)^5} - \frac{b^4\log(c+dx)}{(bc-ad)^5} + \frac{b^3}{(c+dx)(bc-ad)^4} + \frac{b^2}{2(c+dx)^2(bc-ad)^3} + \frac{b}{3(c+dx)^3(bc-ad)^2} + \frac{1}{4(c+dx)^4(bc-ad)}\right)}{4dg^5} -$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

output
```
-1/4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g^5*(c + d*x)^4) + (B*(b*c
- a*d)*n*(1/(4*(b*c - a*d)*(c + d*x)^4) + b/(3*(b*c - a*d)^2*(c + d*x)^3)
+ b^2/(2*(b*c - a*d)^3*(c + d*x)^2) + b^3/((b*c - a*d)^4*(c + d*x)) + (b^4
*Log[a + b*x])/(b*c - a*d)^5 - (b^4*Log[c + d*x])/(b*c - a*d)^5))/(4*d*g^5
)
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.37.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(204) = 408$.

Time = 42.25 (sec) , antiderivative size = 1043, normalized size of antiderivative = 4.85

method	result	size
parallelrisch	Expression too large to display	1043

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(96*B*x^2*a^4*b*c^5*d^4*n^2-210*B*x^2*a^3*b^2*c^6*d^3*n^2+240*B*x^2*a
^2*b^3*c^7*d^2*n^2-108*B*x^2*a*b^4*c^8*d*n^2-288*A*x^2*a^4*b*c^5*d^4*n+432
*A*x^2*a^3*b^2*c^6*d^3*n-288*A*x^2*a^2*b^3*c^7*d^2*n+72*A*x^2*a*b^4*c^8*d*
n+48*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^9*n+60*B*x*a^4*b*c^6*d^3*n^2-12
0*B*x*a^3*b^2*c^7*d^2*n^2+120*B*x*a^2*b^3*c^8*d*n^2-192*A*x*a^4*b*c^6*d^3*
n+288*A*x*a^3*b^2*c^7*d^2*n-192*A*x*a^2*b^3*c^8*d*n+48*B*ln(e*((b*x+a)/(d*
x+c))^n)*a^4*b*c^7*d^2*n-72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^8*d*n+16
*B*x^4*a^4*b*c^3*d^6*n^2-36*B*x^4*a^3*b^2*c^4*d^5*n^2+48*B*x^4*a^2*b^3*c^5
*d^4*n^2-25*B*x^4*a*b^4*c^6*d^3*n^2-48*A*x^4*a^4*b*c^3*d^6*n+72*A*x^4*a^3*
b^2*c^4*d^5*n-48*A*x^4*a^2*b^3*c^5*d^4*n+12*A*x^4*a*b^4*c^6*d^3*n+64*B*x^3
*a^4*b*c^4*d^5*n^2-144*B*x^3*a^3*b^2*c^5*d^4*n^2+180*B*x^3*a^2*b^3*c^6*d^3
*n^2-88*B*x^3*a*b^4*c^7*d^2*n^2-192*A*x^3*a^4*b*c^4*d^5*n+288*A*x^3*a^3*b^
2*c^5*d^4*n-192*A*x^3*a^2*b^3*c^6*d^3*n+48*A*x^3*a*b^4*c^7*d^2*n-3*B*x^4*a
^5*c^2*d^7*n^2+12*A*x^4*a^5*c^2*d^7*n-12*B*x^3*a^5*c^3*d^6*n^2+48*A*x^3*a^
5*c^3*d^6*n-18*B*x^2*a^5*c^4*d^5*n^2+72*A*x^2*a^5*c^4*d^5*n-12*B*x*a^5*c^5
*d^4*n^2-48*B*x*a*b^4*c^9*n^2+48*A*x*a^5*c^5*d^4*n+48*A*x*a*b^4*c^9*n-12*B
*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^6*d^3*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^
2*b^3*c^9*n+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^6*d^3*n+48*B*x^3*ln
(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^7*d^2*n+72*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*a*b^4*c^8*d*n)/g^5/(d*x+c)^4/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-...
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. 2(201) = 402.

Time = 0.31 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.42

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = -\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\,(Bb^4cd^3 - Bab^3d^4)nx^3 - 6\,(7\,Bb}{48\,((b^4c^4d^5 - 4\,ab^3c^3d^6 +}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="fri
cas")
```

output
```
-1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b
*c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 - 6*(7*B*b^4*
c^2*d^2 - 8*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 - 4*(13*B*b^4*c^3*d - 18*
B*a*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*n*x - (25*B*b^4*c^4 - 4
8*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 16*B*a^3*b*c*d^3 + 3*B*a^4*d^4)*n
 + 12*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3
 + B*a^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*b^4*c*d^3*n*x^3 + 6*B*b^4
*c^2*d^2*n*x^2 + 4*B*b^4*c^3*d*n*x + (4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^
2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*n)*log((b*x + a)/(d*x + c)))/((b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^4
 + 4*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7
+ a^4*c*d^8)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^
5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*
d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*
d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*g
^5)
```

3.37.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**5,x)
```

output
```
Timed out
```

3.37.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(201) = 402$.

Time = 0.21 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.03

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$$
$$= \frac{1}{48}Bn\left(\frac{12\,b^3d^3x^3+25\,b^3c^3-23\,ab^2c^2d+}{(b^3c^3d^5-3\,ab^2c^2d^6+3\,a^2bcd^7-a^3d^8)g^5x^4+4\,(b^3c^4d^4-3\,ab^2c^3d^5+3\,a^2bc^2d^6-a^3cd^7)g^5x^3+6\,(}\right.$$
$$-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{4\,(d^5g^5x^4+4\,cd^4g^5x^3+6\,c^2d^3g^5x^2+4\,c^3d^2g^5x+c^4dg^5)}$$
$$-\frac{A}{4\,(d^5g^5x^4+4\,cd^4g^5x^3+6\,c^2d^3g^5x^2+4\,c^3d^2g^5x+c^4dg^5)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="max
ima")
```

output
```
1/48*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 -
 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c
*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3
*d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c
*d^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c
^2*d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3
*c^3*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4
*d^4)*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2
*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) -
1/4*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^
3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/4*A/(d^5*g^5*x^4
+ 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5)
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(201) = 402$.

Time = 0.85 (sec) , antiderivative size = 684, normalized size of antiderivative = 3.18

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$$
$$= \frac{1}{48}\left(12\left(\frac{4\,(bx+a)Bb^3n}{(b^3c^3g^5-3\,ab^2c^2dg^5+3\,a^2bcd^2g^5-a^3d^3g^5)(dx+c)}-\frac{6\,(bx+a)^2Bb^2dn}{(b^3c^3g^5-3\,ab^2c^2dg^5+3\,a^2bcd^2g^5-a^3d^3}\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="gia
c")
```

output
```
1/48*(12*(4*(b*x + a)*B*b^3*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*
c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B*b^2*d*n/((b^3*c^3*g^
5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) + 4*
(b*x + a)^3*B*b*d^2*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^
5 - a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B*d^3*n/((b^3*c^3*g^5 - 3*a*b^
2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4))*log((b*x + a)
/(d*x + c)) + 3*(B*d^3*n - 4*B*d^3*log(e) - 4*A*d^3)*(b*x + a)^4/((b^3*c^3
*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) -
 16*(B*b*d^2*n - 3*B*b*d^2*log(e) - 3*A*b*d^2)*(b*x + a)^3/((b^3*c^3*g^5 -
 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) + 36*(B
*b^2*d*n - 2*B*b^2*d*log(e) - 2*A*b^2*d)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b
^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) - 48*(B*b^3*n
 - B*b^3*log(e) - A*b^3)*(b*x + a)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a
^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c -
a*d)^2)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$$
$$= \frac{B\,b^4\,n\,\mathrm{atanh}\left(\frac{4\,a^4\,d^5\,g^5-8\,a^3\,b\,c\,d^4\,g^5+8\,a\,b^3\,c^3\,d^2\,g^5-4\,b^4\,c^4\,d\,g^5}{4\,d\,g^5\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,d\,g^5\,{\left(a\,d-b\,c\right)}^4}$$
$$-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{4\,d\,\left(c^4\,g^5+4\,c^3\,d\,g^5\,x+6\,c^2\,d^2\,g^5\,x^2+4\,c\,d^3\,g^5\,x^3+d^4\,g^5\,x^4\right)}$$
$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3-3\,B\,a^3\,d^3\,n+25\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2-23\,B\,a\,b^2\,c^2\,d\,n+13\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{b\,x\,\left(B\,n\,a^2\,d^3-5\,B\,n\,a\,b\,c\,\ldots\right.}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,\ldots\right.}}{4\,c^4\,d\,g^5+16\,c^3\,d^2\,g^5\,x+24\,c^2\,d^3\,g^5\,x^2+16\,c\,d^4\,\ldots}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^5,x)
```

output
```
(B*b^4*n*atanh((4*a^4*d^5*g^5 - 4*b^4*c^4*d*g^5 - 8*a^3*b*c*d^4*g^5 + 8*a*
b^3*c^3*d^2*g^5)/(4*d*g^5*(a*d - b*c)^4) + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*d*g^5*(a*d - b*c)^4) - (
B*log(e*((a + b*x)/(c + d*x))^n))/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*g^
5*x^3 + 6*c^2*d^2*g^5*x^2 + 4*c^3*d*g^5*x)) - ((12*A*a^3*d^3 - 12*A*b^3*c^
3 - 3*B*a^3*d^3*n + 25*B*b^3*c^3*n + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 -
 23*B*a*b^2*c^2*d*n + 13*B*a^2*b*c*d^2*n)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (b*x*(B*a^2*d^3*n + 13*B*b^2*c^2*d*n - 5*B*a*b*
c*d^2*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (b^2*x
^2*(B*a*d^3*n - 7*B*b*c*d^2*n))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(4*c^4*d*g^5 + 4*d^5*g^5*x^4 + 16*c^3*d^2*g^5*x + 16*c*d^4*g^
5*x^3 + 24*c^2*d^3*g^5*x^2)
```

3.37. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$

3.38 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

3.38.1 Optimal result

Integrand size = 35, antiderivative size = 544

$$\begin{aligned}
&\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4}+\frac{7B^2(bc-ad)^3g^4n^2(c+dx)^2}{60b^3d} \\
&+\frac{B^2(bc-ad)^2g^4n^2(c+dx)^3}{30b^2d}-\frac{2B(bc-ad)^4g^4n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^5} \\
&-\frac{B(bc-ad)^3g^4n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b^3d} \\
&-\frac{2B(bc-ad)^2g^4n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2d} \\
&-\frac{B(bc-ad)g^4n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd} \\
&+\frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} \\
&+\frac{13B^2(bc-ad)^5g^4n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^5d}+\frac{5B^2(bc-ad)^5g^4n^2\log(c+dx)}{6b^5d} \\
&+\frac{2B(bc-ad)^5g^4n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d} \\
&-\frac{2B^2(bc-ad)^5g^4n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d}
\end{aligned}$$

output
```
13/30*B^2*(-a*d+b*c)^4*g^4*n^2*x/b^4+7/60*B^2*(-a*d+b*c)^3*g^4*n^2*(d*x+c)
^2/b^3/d+1/30*B^2*(-a*d+b*c)^2*g^4*n^2*(d*x+c)^3/b^2/d-2/5*B*(-a*d+b*c)^4*
g^4*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^5-1/5*B*(-a*d+b*c)^3*g^4*n
*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-2/15*B*(-a*d+b*c)^2*g^4*n
*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/10*B*(-a*d+b*c)*g^4*n*(
d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2/d+13/30*B^2*(-a*d+b*c)^5*g^4*n^2*ln((b*x+a)/(d*x+c)
)/b^5/d+5/6*B^2*(-a*d+b*c)^5*g^4*n^2*ln(d*x+c)/b^5/d+2/5*B*(-a*d+b*c)^5*g^
4*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^5/d-2/5*B^
2*(-a*d+b*c)^5*g^4*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^5/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 533, normalized size of antiderivative = 0.98

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g^4\left((c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(24Abd(bc-ad)^3x-12B(bc-ad)^3n(bdx+(bc-ad)\log(a+bx))-4B(bc-ad)^2n\right.}{}\right.}{}$$

input
```
Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^4*((c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*
Log[a + b*x]) - 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2
 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x
+ 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[
a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n]
+ 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
+ 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6
*b^4*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*
n*Log[c + d*x] - 12*B*(b*c - a*d)^4*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[
(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))
)/(12*b^5)))/(5*d)
```

3.38.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.32, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.543, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2951}$$

$$g^4(bc-ad)^5\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5d}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}\right)}{5d}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+\cdots}{c+\cdots}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx\cdots}{c+dx}\right.}}{b}\cdots}{5d}\cdots\right.$$

$\downarrow$ 54

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{\cdots}{b(\cdots}\right.}{4d}\cdots\right.}{b}\cdots}{5d}\cdots\right.$$

$\downarrow$ 2009

3.38. $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4}-\frac{\log(}{}\right.}{}\right.}{}\right.}{5d}\right.$$

$\downarrow$ 2789

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+}{c+}\right.\right.}{4d\left(b-\frac{d(a}{c}\right.}\right.}{}\right.}{}\right.$$

$\downarrow$ 2756

3.38. $\int(cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^4(bc-$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{b}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 54

$$g^4(bc-$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c\cdots}{b^3(\cdots}\cdots\right.}{3d}\cdots\right.}{b}}{b}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2009

3.38. $\int(cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^4(bc-$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\right.}{}\right.}{}\right.}{b}\right.}{}\right.$$

$\downarrow$ 2789

$$g^4(bc-$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{3d\left(b-\frac{d(a+b}{c+d}\right.}\right.}{b}\right.}{}\right.$$

$\downarrow$ 2756

3.38. $\int(cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^4(bc-$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{b}}{b}\right.}{}\right.$$

$$\Big\downarrow \; 54$$

$$g^4(bc-$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+}{b}\right.}{}\right.$$

3.38. $\int(cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$\Big\downarrow$ 2009

$$g^4(bc-ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\right.}{}\right.}{}\right.}{b}}{b}\right.}{}\right.$$

$\Big\downarrow$ 2789

3.38. $\int(cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$ad)^5 \left(\frac{\left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{g^4(bc- \quad 2Bn\left(\frac{\frac{\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{2d\left(b-\frac{d(a+b}{c+d}\right.}\right.}{b}}{}\right.}{}\right.$$

$\downarrow$ 2751

$$ad)^5 \left(\frac{\left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{g^4(bc- \quad 2Bn\left(\frac{\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{b}}{}\right.}{}\right.$$

3.38. $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$\Big\downarrow$ 16

$$g^4(bc-$$

$$ad)^5\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(\frac{\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+}{c+d}\right.}{bd}\right.}{b}}{b}}{b}\right.}{}\right.$$

$\Big\downarrow$ 2779

3.38. $\int(cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^4(bc - ad)^5 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\frac{\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}}{b} + \frac{d\left(\frac{(a+bx)(\cdots}{b(c\cdots}\right.}{\cdots} \right.}{\cdots} \right.$$

$$\Big\downarrow \text{2838}$$

$$g^4(bc - ad)^5 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}}{} + \frac{Bn\,\mathrm{Poly}\cdots}{\cdots} \right.}{\cdots} \right.$$

3.38. $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

input
```
Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^5*g^4*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(5*d*(b - (d*(
a + b*x))/(c + d*x))^5) - (2*B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(4*d*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(1/(3*b*(b - (d*(a + b*x))
/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^3*(b - (
d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4 - Log[b - (d*(a +
b*x))/(c + d*x)]/b^4))/(4*d)))/b + ((d*((A + B*Log[e*((a + b*x)/(c + d*x))
^n])/(3*d*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(1/(2*b*(b - (d*(a + b*x
))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(
c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A
 + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2)
 - (B*n*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^
2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c +
d*x))) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B
*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/b)/b)/b))/(5*d))
```

3.38.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.38.4 Maple [F]

$$\int (dgx + cg)^4 \left(A + B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right)\right)^2 dx$$

input
```
int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.38.5 Fricas [F]

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (dgx+cg)^4 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*d^4*g^4*x^4 + 4*A^2*c*d^3*g^4*x^3 + 6*A^2*c^2*d^2*g^4*x^2 + 4
*A^2*c^3*d*g^4*x + A^2*c^4*g^4 + (B^2*d^4*g^4*x^4 + 4*B^2*c*d^3*g^4*x^3 +
6*B^2*c^2*d^2*g^4*x^2 + 4*B^2*c^3*d*g^4*x + B^2*c^4*g^4)*log(e*((b*x + a)/
(d*x + c))^n)^2 + 2*(A*B*d^4*g^4*x^4 + 4*A*B*c*d^3*g^4*x^3 + 6*A*B*c^2*d^2
*g^4*x^2 + 4*A*B*c^3*d*g^4*x + A*B*c^4*g^4)*log(e*((b*x + a)/(d*x + c))^n)
, x)
```

3.38.6 Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2880 vs. $2(519) = 1038$.

Time = 0.72 (sec) , antiderivative size = 2880, normalized size of antiderivative = 5.29

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
2/5*A*B*d^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*d^4*g
^4*x^5 + 2*A*B*c*d^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
c*d^3*g^4*x^4 + 4*A*B*c^2*d^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + 2*A^2*c^2*d^2*g^4*x^3 + 4*A*B*c^3*d*g^4*x^2*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + 2*A^2*c^3*d*g^4*x^2 + 1/30*A*B*d^4*g^4*n*(12*a^5*log(b*x +
 a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^
4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4
 - a^4*d^4)*x)/(b^4*d^4)) - 1/3*A*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 -
6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a
^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*c^2*d^2*g^4*n*
(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^
2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*c^3*d*g^4*n*(a^2*log(b*x +
 a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^4*g^4*n*(a
*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^4*g^4*x*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n) + A^2*c^4*g^4*x - 1/30*(77*a*b^3*c^4*d*g^4*n^2 - 94*a^2
*b^2*c^3*d^2*g^4*n^2 + 54*a^3*b*c^2*d^3*g^4*n^2 - 12*a^4*c*d^4*g^4*n^2 - (
25*g^4*n^2 - 12*g^4*n*log(e))*b^4*c^5)*B^2*log(d*x + c)/(b^4*d) - 2/5*(b^5
*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3
*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x +
 a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*...
```

3.38.8 Giac [F]

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (dgx+cg)^4\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*g*x + c*g)^4*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,g+d\,g\,x)^4\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.39 $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

3.39.1 Optimal result

Integrand size = 35, antiderivative size = 454

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3}+\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d} - \frac{B(bc-ad)^3g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} - \frac{B(bc-ad)^2g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} - \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} + \frac{5B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d}+\frac{11B^2(bc-ad)^4g^3n^2\log(c+dx)}{12b^4d} + \frac{B(bc-ad)^4g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} - \frac{B^2(bc-ad)^4g^3n^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}$$

output
```
5/12*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^
2/b^2/d-1/2*B*(-a*d+b*c)^3*g^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
^4-1/4*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/
d-1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4
*g^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+5/12*B^2*(-a*d+b*c)^4*g
^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d+11/12*B^2*(-a*d+b*c)^4*g^3*n^2*ln(d*x+c)/
b^4/d+1/2*B*(-a*d+b*c)^4*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x
+c)/d/(b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,b*(d*x+c)/d/(b
*x+a))/b^4/d
```

3.39.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.90

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g^3\left((c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2bc\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^3*((c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(
b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*
(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])))/(3*b^4)))/(4*d)
```

3.39.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2951}$$

$$g^3(bc-ad)^4\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2d}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{2d}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc - ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{b}}{2d}\right.$$

$\downarrow$ 54

$$g^3(bc - ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+}{b^3(a}\right.}{3d}\right.}{b}}{2d}\right.$$

$\downarrow$ 2009

3.39. $\int (cg + dgx)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^3(bc - ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b} + \frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{2d \cdots}\right.$$

$\downarrow$ 2789

$$g^3(bc - ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b} + \frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.}{3d\left(b-\frac{d(a+\cdots}{c+\cdots}\right.}\right.}{\cdots}\right.}{2d \cdots}\right.$$

$\downarrow$ 2756

3.39. $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\cdots}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{b}\cdots}{b}\right)}{\cdots}\right.$$

$\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\cdots}{\cdots}}{b}\right)}{\cdots}\right.$$

$\downarrow$ 2009

3.39. $\int(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d}{\cdots}\right.}{b\cdots}\right.}{2\cdots}\right.}{b}}{b}\right.}{\cdots}\right.$$

$\downarrow$ 2789

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.}{2d\left(b-\frac{d(a+bx}{c+dx}\cdots\right.}\right.}{b}\right.}{\cdots}\right.$$

$\downarrow$ 2751

3.39. $\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$g^3(bc -$

$$ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{b}}{b}}{}\right.$$

$\downarrow$ 16

$g^3(bc -$

$$ad)^4 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx}{c+dx}\right.}{bd}\right.}{b}}{b}}{b}}{}\right.$$

$\downarrow$ 2779

3.39. $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\right.}{b(c+\ldots}\right.}{\ldots}}{b}\right.}{\ldots}\right.$$

$\downarrow$ 2838

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{Bn\,\mathrm{PolyL}\ldots}{\ldots}}{\ldots}\right.}{\ldots}\right.$$

input

```
Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

3.39. $\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)^4*g^3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(4*d*(b - (d*(
a + b*x))/(c + d*x))^4) - (B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])
/(3*d*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(1/(2*b*(b - (d*(a + b*x))/(
c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c +
d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B
*Log[e*((a + b*x)/(c + d*x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (
B*n*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 -
Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*P
olyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/b)/b))/(2*d))
```

3.39.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.39.4 Maple [F]

$$\int (dgx+cg)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.39.5 Fricas [F]

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dgx+cg)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*d^3*g^3*x^3 + 3*A^2*c*d^2*g^3*x^2 + 3*A^2*c^2*d*g^3*x + A^2*c
^3*g^3 + (B^2*d^3*g^3*x^3 + 3*B^2*c*d^2*g^3*x^2 + 3*B^2*c^2*d*g^3*x + B^2*
c^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*g^3*x^3 + 3*A*B*c*d
^2*g^3*x^2 + 3*A*B*c^2*d*g^3*x + A*B*c^3*g^3)*log(e*((b*x + a)/(d*x + c))^
n), x)
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(433) = 866$.

Time = 0.71 (sec) , antiderivative size = 2129, normalized size of antiderivative = 4.69

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
1/2*A*B*d^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*d^3*g
^3*x^4 + 2*A*B*c*d^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
c*d^2*g^3*x^3 + 3*A*B*c^2*d*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + 3/2*A^2*c^2*d*g^3*x^2 - 1/12*A*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*
c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*c*d^2*g^3*n*(2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^
2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^3*g^3*n*(a*log(b
*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^3*g^3*x*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n) + A^2*c^3*g^3*x - 1/12*(26*a*b^2*c^3*d*g^3*n^2 - 21*a^2*b*c^2
*d^2*g^3*n^2 + 6*a^3*c*d^3*g^3*n^2 - (11*g^3*n^2 - 6*g^3*n*log(e))*b^3*c^4
)*B^2*log(d*x + c)/(b^3*d) - 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2
+ 6*a^2*b^2*c^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))*B^2/(b^4*d) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 6*B^2*b^4*c^
4*g^3*n^2*log(b*x + a)*log(d*x + c) - 3*B^2*b^4*c^4*g^3*n^2*log(d*x + c)^2
 + 2*(a*b^3*d^4*g^3*n*log(e) - (g^3*n*log(e) - 6*g^3*log(e)^2)*b^4*c*d^3)*
B^2*x^3 + ((g^3*n^2 - 9*g^3*n*log(e) + 18*g^3*log(e)^2)*b^4*c^2*d^2 - 2*(g
^3*n^2 - 6*g^3*n*log(e))*a*b^3*c*d^3 + (g^3*n^2 - 3*g^3*n*log(e))*a^2*b...
```

3.39.8 Giac [F]

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (c\,g+d\,g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.40 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

3.40.1 Optimal result

Integrand size = 35, antiderivative size = 361

$$\begin{aligned}
&\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} - \frac{2B(bc-ad)^2g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} \\
&\quad - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&\quad + \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} \\
&\quad + \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} + \frac{B^2(bc-ad)^3g^2n^2\log(c+dx)}{b^3d} \\
&\quad + \frac{2B(bc-ad)^3g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} \\
&\quad - \frac{2B^2(bc-ad)^3g^2n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2-2/3*B*(-a*d+b*c)^2*g^2*n*(b*x+a)*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/b^3-1/3*B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/b/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/d+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d+B^2*(-a*d+b*c)
^3*g^2*n^2*ln(d*x+c)/b^3/d+2/3*B*(-a*d+b*c)^3*g^2*n*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/d-2/3*B^2*(-a*d+b*c)^3*g^2*n^2*pol
ylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d
```

3.40.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.84

$$\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{g^2\left((c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(2Abd(bc-ad)x-B(bc-ad)n(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx}{}\right.\right.}{}$$

input
```
Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g^2*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a +
b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(
c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c
+ d*x] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*
d)
```

3.40.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2951}$$

$$g^2(bc-ad)^3\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2756}$$

$$g^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)$$

$$\Big\downarrow \text{ 2789}$$

$$g^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)$$

$$\Big\downarrow \text{ 2756}$$

$$g^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx}{c+dx}\right.}}{b}\right.}{3d}\right.$$

$$\Big\downarrow \text{ 54}$$

3.40. $\int(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^2(bc - ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b} + \right.}{3d} \right.$$

$\downarrow$ 2009

$$g^2(bc - ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b} + \frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log(}{}\right.}{}\right.}{b}\right.}{3d} \right.$$

$\downarrow$ 2789

3.40. $\int (cg + dgx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b} + \frac{d\left(\frac{B\log\left(e\left(\frac{a+}{c+}\right.\right.}{2d\left(b-\frac{d(a}{c}\right.}\right.}{} \right.}{3d}\right.$$

$$g^2(bc-$$

$\downarrow$ 2751

$$ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{b} \right.}{3}\right.$$

$$g^2(bc-$$

$\downarrow$ 16

3.40. $\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{g^2(bc- \quad 2Bn\left(\frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+b}{c+d}\right.}{bd}\right.}{b}}{b}\right.}{3d}\right.$$

$\downarrow$ 2779

$$ad)^3 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{g^2(bc- \quad 2Bn\left(\frac{\frac{\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(\right.}{b(c-}\right.}{}}{b}\right.}{}\right.$$

$\downarrow$ 2838

3.40. $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{Bn\ \cdots}{\cdots}\cdots}{\cdots}\right.$$

input
```
Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^3*g^2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*(b - (d*(
a + b*x))/(c + d*x))^3) - (2*B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b - (d*(a + b*x))/(
c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x
)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(a + b*x)
)/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1
 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(d*(a
+ b*x))])/b)/b)/b))/(3*d))
```

3.40.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.40. $\int (cg+dgx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

3.40.4 Maple [F]

$$\int (dgx+cg)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.40.5 Fricas [F]

$$\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dgx+cg)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*d^2*g^2*x^2 + 2*A^2*c*d*g^2*x + A^2*c^2*g^2 + (B^2*d^2*g^2*x^
2 + 2*B^2*c*d*g^2*x + B^2*c^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A
*B*d^2*g^2*x^2 + 2*A*B*c*d*g^2*x + A*B*c^2*g^2)*log(e*((b*x + a)/(d*x + c)
)^n), x)
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. 2(346) = 692.

Time = 0.71 (sec) , antiderivative size = 1473, normalized size of antiderivative = 4.08

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
2/3*A*B*d^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*d^2*g
^2*x^3 + 2*A*B*c*d*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d*g^2*x^2 + 1/3*A*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*
A*B*c*d*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x
/(b*d)) + 2*A*B*c^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^
2*g^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c^2*g^2*x - 1/3*(5*a*
b*c^2*d*g^2*n^2 - 2*a^2*c*d^2*g^2*n^2 - (3*g^2*n^2 - 2*g^2*n*log(e))*b^2*c
^3)*B^2*log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^
2 + 3*a^2*b*c*d^2*g^2*n^2 - a^3*d^3*g^2*n^2)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3
*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + 2*B^2*b^3*c^3*g^2*n^2*log(b*x + a)*log(d*
x + c) - B^2*b^3*c^3*g^2*n^2*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log(e) - (g
^2*n*log(e) - 3*g^2*log(e)^2)*b^3*c*d^2)*B^2*x^2 - (3*a*b^2*c^2*d*g^2*n^2
- 3*a^2*b*c*d^2*g^2*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((g^2*n^2
- 4*g^2*n*log(e) + 3*g^2*log(e)^2)*b^3*c^2*d - 2*(g^2*n^2 - 3*g^2*n*log(e)
)*a*b^2*c*d^2 + (g^2*n^2 - 2*g^2*n*log(e))*a^2*b*d^3)*B^2*x - (2*(2*g^2*n^
2 - 3*g^2*n*log(e))*a*b^2*c^2*d - (7*g^2*n^2 - 6*g^2*n*log(e))*a^2*b*c*d^2
 + (3*g^2*n^2 - 2*g^2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*g
^2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*log((b*x + a)...
```

3.40.8 Giac [F]

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int (dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int (cg + dgx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (c\,g + d\,g\,x)^2 \left(A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.41 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

3.41.1 Optimal result

Integrand size = 33, antiderivative size = 220

$$\begin{aligned}\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
&=-\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2}\\
&+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d}+\frac{B^2(bc-ad)^2gn^2\log(c+dx)}{b^2d}\\
&+\frac{B(bc-ad)^2gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}\\
&-\frac{B^2(bc-ad)^2gn^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}\end{aligned}$$

output
```
-B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/2*g*(d*x+c
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+B^2*(-a*d+b*c)^2*g*n^2*ln(d*x+c)/b
^2/d+B*(-a*d+b*c)^2*g*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^2/d-B^2*(-a*d+b*c)^2*g*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

3.41.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g\left((c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(B(bc-ad)n\log^2(a+bx)-2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad)\cdots\right.\right.}{}\right.}{2d}$$

input
```
Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(g*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*
n*(B*(b*c - a*d)*n*Log[a + b*x]^2 - 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) - 2*(b*c - a*d)*Log
[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(b
*c - a*d)]) + 2*B*(-(b*c) + a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
]))/b^2))/(2*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2951, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$
$$\Big\downarrow \text{2951}$$
$$g(bc-ad)^2\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{2756}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)$$

$\downarrow$ 2789

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)$$

$\downarrow$ 2751

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}\right.}{d}\right.$$

$\downarrow$ 16

$$g(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+}{c+}\right.}{bd}\right.}{b}\right.}{d}\right.$$

$\downarrow$ 2779

3.41. $\int(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g(bc - ad)^2 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b} + \frac{d\left(\frac{(a+bx)}{b(c}\right.}{}\right.}{d}\right.$$

$\downarrow$ 2838

$$g(bc - ad)^2 \left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b} + \frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{b(c+dx)\left(b-\frac{d(a}{c-}\right.}\right.}{}\right.}{d}\right.$$

input
```
Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^2*g*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*d*(b - (d*(a
+ b*x))/(c + d*x))^2) - (B*n*((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(
a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x)
)/(d*(a + b*x))])/b)/b))/d)
```

3.41. $\int (cg + dgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

3.41.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.41. $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

3.41.4 Maple [F]

$$\int (dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.41.5 Fricas [F]

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}dx$$
$$=\int (dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}dx$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n), x)
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}dx=\text{Timed out}$$

input
```
integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(217) = 434$.

Time = 0.68 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.75

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = ABdgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + \frac{1}{2}A^2dgx^2 - ABdgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) + 2ABcgn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) + 2ABcgx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + A^2cgx - \frac{(acdgn^2-(gn^2-gn\log(e))bc^2)B^2\log(dx+c)}{bd} - \frac{(b^2c^2gn^2-2abcdgn^2+a^2d^2gn^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d} + \frac{2B^2b^2c^2gn^2\log(bx+a)\log(dx+c) - B^2b^2c^2gn^2\log(dx+c)^2 + B^2b^2d^2gx^2\log(e)^2 - (2abcdgn^2 - a^2\cdots}{\cdots}$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
A*B*d*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*d*g*x^2 - A*B
*d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + 2*A*B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c*g*x*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*g*x - (a*c*d*g*n^2 - (g*n^2 - g*n*
log(e))*b*c^2)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2 +
 a^2*d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-
(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*g*n^2*log(b*x
 + a)*log(d*x + c) - B^2*b^2*c^2*g*n^2*log(d*x + c)^2 + B^2*b^2*d^2*g*x^2*
log(e)^2 - (2*a*b*c*d*g*n^2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d
^2*g*n*log(e) - (g*n*log(e) - g*log(e)^2)*b^2*c*d)*B^2*x - 2*((g*n^2 - 2*g
*n*log(e))*a*b*c*d - (g*n^2 - g*n*log(e))*a^2*d^2)*B^2*log(b*x + a) + (B^2
*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^
2 + 2*B^2*b^2*c*d*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) -
B^2*b^2*c^2*g*n*log(d*x + c) + (a*b*d^2*g*n - (g*n - 2*g*log(e))*b^2*c*d)*
B^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((b*x + a)^n) -
 2*(B^2*b^2*d^2*g*x^2*log(e) - B^2*b^2*c^2*g*n*log(d*x + c) + (a*b*d^2*g*n
 - (g*n - 2*g*log(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*l
og(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n))*lo
g((d*x + c)^n))/(b^2*d)
```

3.41.8 Giac [F]

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (c\,g+d\,g\,x)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.42 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx$

3.42.1 Optimal result

Integrand size = 35, antiderivative size = 137

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{dg} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d/g-2*B*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/g+2*B^2*n^2*p
olylog(3,d*(b*x+a)/b/(d*x+c))/d/g
```

3.42.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx$$
$$= \frac{A^2\log(c+dx)+2ABn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-2AB\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-B^2\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

output
```
(A^2*Log[c + d*x] + 2*A*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a
*d)/(b*c + b*d*x)] - 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*c - a*d)/
(b*c + b*d*x)] - B^2*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*c
 + b*d*x)] + A*B*n*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*n*Log[e*((a +
b*x)/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*n*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(
c + d*x))])/(d*g)
```

3.42.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2951, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{cg+dgx} \, dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \frac{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{2754}$$

$$\frac{\frac{2Bn\int \frac{(c+dx)\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log \left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log \left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\frac{2Bn\left(Bn\int \frac{(c+dx)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log \left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{g}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{2Bn\left(Bn\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{g}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c +
 d*x))])/d) + (2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))]) + B*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))
]))/d)/g
```

3.42.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.42.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dgx+cg}\,dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

3.42.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="fri
cas")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(d*g*x + c*g), x)
```

3.42.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx$$
$$=\frac{\int \frac{A^2}{c+dx}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g),x)
```

output
```
(Integral(A**2/(c + d*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)/(c + d*x), x))/g
```

3.42.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="max
ima")
```

output
```
B^2*log(d*x + c)*log((d*x + c)^n)^2/(d*g) + A^2*log(d*g*x + c*g)/(d*g) - i
ntegrate(-(B^2*log((b*x + a)^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*l
og(e) + A*B)*log((b*x + a)^n) - 2*(B^2*n*log(d*x + c) + B^2*log((b*x + a)^
n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(d*g*x + c*g), x)
```

3.42.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dgx+cg}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="gia
c")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*g*x + c*g), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,g+d\,g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x), x)
```

3.43 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$

3.43.1 Optimal result

Integrand size = 35, antiderivative size = 163

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = -\frac{2ABn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{2B^2n^2(a+bx)}{(bc-ad)g^2(c+dx)} - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(c+dx)}$$

output
```
-2*A*B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+2*B^2*n^2*(b*x+a)/(-a*d+b*c)/g^2/(
d*x+c)-2*B^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)+(b
*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^2/(d*x+c)
```

3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.03

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$$

$$= \frac{-\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+B \cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

output
```
(-(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d
*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[
c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*g^2*
(c + d*x))
```

3.43.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(cg+dgx)^2} \, dx$$

$$\Big\downarrow \; 2951$$

$$\frac{\int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{g^2 (bc-ad)}$$

$$\Big\downarrow \; 2733$$

$$\frac{\frac{(a+bx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2 B n \int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) d\frac{a+bx}{c+dx}}{g^2 (bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(a+bx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2 B n \left(\frac{A (a+bx)}{c+dx}+\frac{B (a+bx) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{B n (a+bx)}{c+dx}\right)}{g^2 (bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

output
```
(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - 2*B*n*((
A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(c + d*x)))/((b*c - a*d)*g^2)
```

3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.43.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{2B^2ab\,d^3n^3-2B^2b^2c\,d^2n^3+A^2ab\,d^3n-A^2b^2c\,d^2n+2ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^3n+2AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab\,d^3n-2ABab\,d^3n^2}{g^2(dx-}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x,method=_RETURNVERBOS
E)
```

output
```
-(2*B^2*a*b*d^3*n^3-2*B^2*b^2*c*d^2*n^3+A^2*a*b*d^3*n-A^2*b^2*c*d^2*n+2*A*
B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n+2*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*
b*d^3*n-2*A*B*a*b*d^3*n^2+2*A*B*b^2*c*d^2*n^2+B^2*x*ln(e*((b*x+a)/(d*x+c))
^n)^2*b^2*d^3*n-2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n^2+B^2*ln(e*((b
*x+a)/(d*x+c))^n)^2*a*b*d^3*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^3*n^2)
/g^2/(d*x+c)/b/d^3/n/(a*d-b*c)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx =$$
$$-\frac{A^2bc-A^2ad+2\left(B^2bc-B^2ad\right)n^2+\left(B^2bc-B^2ad\right)\log\left(e\right)^2-\left(B^2bdn^2x+B^2adn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^2-2\left(A\right.}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="f
ricas")
```

output
```
-(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(
e)^2 - (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B*b
*c - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d - (B^2*b*c - B^2*a*d)*n - (B^2*b*d*
n*x + B^2*a*d*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a*d*n^2 - A*B*a
*d*n + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^2 -
a*d^3)*g^2*x + (b*c^2*d - a*c*d^2)*g^2)
```

3.43.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$$
$$=\frac{\int \frac{A^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^2+2cdx+d^2x^2}\,dx}{g^2}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**2,x)
```

output
```
(Integral(A**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(B**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integr
al(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**2 + 2*c*d*x + d**2*x*
*2), x))/g**2
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 0.20 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = 2\,ABn\left(\frac{1}{d^2g^2x + cdg^2} + \frac{b\log(bx+a)}{(bcd-ad^2)g^2} - \frac{b\log(dx+c)}{(bcd-ad^2)g^2}\right) + \left(2\,n\left(\frac{1}{d^2g^2x + cdg^2} + \frac{b\log(bx+a)}{(bcd-ad^2)g^2} - \frac{b\log(dx+c)}{(bcd-ad^2)g^2}\right)\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) - \frac{((bdx+bc)\log \cdots}{} - \frac{B^2\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)^2}{d^2g^2x + cdg^2} - \frac{2\,AB\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{d^2g^2x + cdg^2} - \frac{A^2}{d^2g^2x + cdg^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="m
axima")
```

output
```
2*A*B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) -
b*log(d*x + c)/((b*c*d - a*d^2)*g^2)) + (2*n*(1/(d^2*g^2*x + c*d*g^2) + b*
log(b*x + a)/((b*c*d - a*d^2)*g^2) - b*log(d*x + c)/((b*c*d - a*d^2)*g^2))
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + b*c)*log(b*x + a)^2 +
(b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a
) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b*c^2*
d*g^2 - a*c*d^2*g^2 + (b*c*d^2*g^2 - a*d^3*g^2)*x))*B^2 - B^2*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n)^2/(d^2*g^2*x + c*d*g^2) - 2*A*B*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(d^2*g^2*x + c*d*g^2) - A^2/(d^2*g^2*x + c*d*g^2)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = \left(\frac{(bx+a)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)g^2} - \frac{2\,(B^2n^2 - B^2n\log(e) - ABn)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2} + \frac{(2\,B^2n^2 - 2\,B^2n\log(\cdots}{}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="g
iac")
```

output
```
((b*x + a)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)*g^2) - 2*(B^2*n^2
 - B^2*n*log(e) - A*B*n)*(b*x + a)*log((b*x + a)/(d*x + c))/((d*x + c)*g^2
) + (2*B^2*n^2 - 2*B^2*n*log(e) + B^2*log(e)^2 - 2*A*B*n + 2*A*B*log(e) +
A^2)*(b*x + a)/((d*x + c)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,B^2\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{2\,A\,B}{x\,d^2\,g^2+c\,d\,g^2}\right) - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{d\,\left(c\,g^2+d\,g^2\,x\right)} + \frac{B^2\,b}{d\,g^2\,\left(a\,d-b\,c\right)}\right) - \frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{x\,d^2\,g^2+c\,d\,g^2} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,g^2+b\,c\,d\,g^2}{d\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(A-B\,n\right)\,4\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^2,x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(d^2*g^2*x + c*d*g^2) - (2*A*B)/
(d^2*g^2*x + c*d*g^2)) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(d*(c*g^2 +
 d*g^2*x)) + (B^2*b)/(d*g^2*(a*d - b*c))) - (A^2 + 2*B^2*n^2 - 2*A*B*n)/(d
^2*g^2*x + c*d*g^2) + (B*b*n*atan(((2*b*d*x + (a*d^2*g^2 + b*c*d*g^2)/(d*g
^2))*1i)/(a*d - b*c))*(A - B*n)*4i)/(d*g^2*(a*d - b*c))
```

3.44 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$

3.44.1 Optimal result

Integrand size = 35, antiderivative size = 317

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx =& -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2g^3(c+dx)^2} - \frac{2AbBn(a+bx)}{(bc-ad)^2g^3(c+dx)} \\ &+ \frac{2bB^2n^2(a+bx)}{(bc-ad)^2g^3(c+dx)} - \frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2g^3(c+dx)} \\ &+ \frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^3(c+dx)^2} \\ &- \frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^3(c+dx)^2} \\ &+ \frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^3(c+dx)}\end{aligned}$$

output
```
-1/4*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^2/g^3/(d*x+c)^2-2*A*b*B*n*(b*x+a)/(-a*
d+b*c)^2/g^3/(d*x+c)+2*b*B^2*n^2*(b*x+a)/(-a*d+b*c)^2/g^3/(d*x+c)-2*b*B^2*
n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g^3/(d*x+c)+1/2*B*d*n*(b*
x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^3/(d*x+c)^2-1/2*d*(b
*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)^2+b*(b*
x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)
```

3.44.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$= \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\log}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

output
```
(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a
+ b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 + 2*b*(b*c - a*d)*
(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x
]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*n
*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*g^3
*(c + d*x)^2)
```

3.44.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(cg+dgx)^3}\,dx$$

$$\Big\downarrow \text{ 2951}$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{c+dx}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-\frac{d(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(c+dx)^2}-\frac{2AbBn(a+bx)}{c+dx}-\frac{2bB^2}{}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

output
```
(-1/4*(B^2*d*n^2*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*n*(a + b*x))/(c + d*x
) + (2*b*B^2*n^2*(a + b*x))/(c + d*x) - (2*b*B^2*n*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n])/(c + d*x) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(2*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2)/(2*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(c + d*x))/((b*c - a*d)^2*g^3)
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(311) = 622$.

Time = 7.41 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.12

method	result
parallelrisch	$-\frac{-8B^2ab^2cd^4n^3-2ABa^2bd^5n^2-6ABb^3c^2d^3n^2-4A^2ab^2cd^4n+2A^2b^3c^2d^3n+2A^2a^2bd^5n+7B^2b^3c^2d^3n^3+B^2a^2bd^5n^3-4}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/4*(-8*B^2*a*b^2*c*d^4*n^3-2*A*B*a^2*b*d^5*n^2-6*A*B*b^3*c^2*d^3*n^2-4*A
^2*a*b^2*c*d^4*n+2*A^2*b^3*c^2*d^3*n+2*A^2*a^2*b*d^5*n+7*B^2*b^3*c^2*d^3*n
^3+B^2*a^2*b*d^5*n^3-4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n-4*B^2*x
*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^4*n+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
*a*b^2*d^5*n^2+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n^2+4*A*B*x*a*b
^2*d^5*n^2-4*A*B*x*b^3*c*d^4*n^2-4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*c
*d^4*n+8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n^2+4*A*B*ln(e*((b*x+a)
/(d*x+c))^n)*a^2*b*d^5*n+8*A*B*a*b^2*c*d^4*n^2-8*A*B*x*ln(e*((b*x+a)/(d*x+
c))^n)*b^3*c*d^4*n-8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n-2*B^2*x^2
*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^5*n+6*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^3*d^5*n^2-6*B^2*x*a*b^2*d^5*n^3+6*B^2*x*b^3*c*d^4*n^3+2*B^2*ln(e*((b*x+
a)/(d*x+c))^n)^2*a^2*b*d^5*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^5*n^2
)/g^3/(d*x+c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^4/n
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(311) = 622$.

Time = 0.29 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.06

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = -\frac{2\,A^2b^2c^2 - 4\,A^2abcd + 2\,A^2a^2d^2 + (7\,B^2b^2c^2 - 8\,B^2abcd + B^2a^2d^2)n^2 + 2\,(B^2b^2c^2 - 2\,B^2abcd + B^2a^2d}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="f
ricas")
```

output
```
-1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B
^2*a*b*c*d + B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d
^2)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c
*d - B^2*a^2*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*A*B*b^2*c^2 - 4*A
*B*a*b*c*d + A*B*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 - 2*(A*
B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a
^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x - (3*B^2*b^2*c^2 - 4*B^2*a*b*c*
d + B^2*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + (2*B^2*a*b
*c*d - B^2*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((4*B^2*a*b*c*
d - B^2*a^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 - 2*(2*A*
B*a*b*c*d - A*B*a^2*d^2)*n - 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b
*d^2)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*
d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

3.44.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = \frac{\int \frac{A^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{g^3}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**3,x)
```

output
```
(Integral(A**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/g**3
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(311) = 622$.

Time = 0.24 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{2}ABn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)g^3x^2+2\,(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)g^3}-\frac{}{(b^2c^2}\right.$$
$$+\frac{1}{4}\left(2\,n\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)g^3x^2+2\,(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)g^3}-\frac{}{(b}\right.\right.$$
$$-\frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{2\,(d^3g^3x^2+2\,cd^2g^3x+c^2dg^3)}-\frac{AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^3g^3x^2+2\,cd^2g^3x+c^2dg^3}$$
$$-\frac{A^2}{2\,(d^3g^3x^2+2\,cd^2g^3x+c^2dg^3)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="m
axima")
```

output
```
1/2*A*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d
^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*g^3)) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3
- a*d^4)*g^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g
^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2
*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3))*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2
 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2
/(b^2*c^4*d*g^3 - 2*a*b*c^3*d^2*g^3 + a^2*c^2*d^3*g^3 + (b^2*c^2*d^3*g^3 -
 2*a*b*c*d^4*g^3 + a^2*d^5*g^3)*x^2 + 2*(b^2*c^3*d^2*g^3 - 2*a*b*c^2*d^3*g
^3 + a^2*c*d^4*g^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)^2/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A^2/(d^3
*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.28

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)B^2bn^2}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2B^2dn^2}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\left(\frac{(B^2dn^2-2\,B^2dn\log(e)\cdots}{(bcg^3-adg^3)(\cdots}\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="g
iac")
```

output
```
1/4*(2*(2*(b*x + a)*B^2*b*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^
2*B^2*d*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c))^2
+ 2*((B^2*d*n^2 - 2*B^2*d*n*log(e) - 2*A*B*d*n)*(b*x + a)^2/((b*c*g^3 - a*
d*g^3)*(d*x + c)^2) - 4*(B^2*b*n^2 - B^2*b*n*log(e) - A*B*b*n)*(b*x + a)/(
(b*c*g^3 - a*d*g^3)*(d*x + c)))*log((b*x + a)/(d*x + c)) - (B^2*d*n^2 - 2*
B^2*d*n*log(e) + 2*B^2*d*log(e)^2 - 2*A*B*d*n + 4*A*B*d*log(e) + 2*A^2*d)*
(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2) + 4*(2*B^2*b*n^2 - 2*B^2*b*n
*log(e) + B^2*b*log(e)^2 - 2*A*B*b*n + 2*A*B*b*log(e) + A^2*b)*(b*x + a)/(
(b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.59

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)} - \frac{B^2\,b^2}{2\,d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$
$$- \frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+B^2\,a\,d\,n^2-7\,B^2\,b\,c\,n^2-2\,A\,B\,a\,d\,n+6\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)} - \frac{b\,x\,\left(3\,B^2\,d\,n^2-2\,A\,B\,d\,n\right)}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2}$$
$$- \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{c^2\,d\,g^3+2\,c\,d^2\,g^3\,x+d^3\,g^3\,x^2} + \frac{B^2\,b^2\,\left(\frac{d^2\,g^3\,n\,x\,(a\,d-b\,c)}{b} - \frac{d\,g^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2} + \frac{c\,d\,g^3\,n\,(a\,d-b\,c)}{2\,b}\right)}{d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(c^2\,d\,g^3+2\,c\,d^2\,g^3\,x+d^3\,g^3\,x^2\right)}\right)$$
$$- \frac{B\,b^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,(a\,d-b\,c)}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,\left(2\,A-3\,B\,n\right)\,1{}\mathrm{i}}{d\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*
d*g^3*x)) - (B^2*b^2)/(2*d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2
*a*d - 2*A^2*b*c + B^2*a*d*n^2 - 7*B^2*b*c*n^2 - 2*A*B*a*d*n + 6*A*B*b*c*n
)/(2*(a*d - b*c)) - (b*x*(3*B^2*d*n^2 - 2*A*B*d*n))/(a*d - b*c))/(2*c^2*d*
g^3 + 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*
B)/(c^2*d*g^3 + d^3*g^3*x^2 + 2*c*d^2*g^3*x) + (B^2*b^2*((d^2*g^3*n*x*(a*d
 - b*c))/b - (d*g^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (c*d*g^3*n*(a*d
 - b*c))/(2*b)))/(d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*d*g^3 + d^3*g
^3*x^2 + 2*c*d^2*g^3*x))) - (B*b^2*n*atan(((2*b*d*x + (2*a^2*d^3*g^3 - 2*b
^2*c^2*d*g^3)/(2*d*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A - 3*B*n)*1i)/(d
*g^3*(a*d - b*c)^2)
```

3.44. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$

3.45 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$

3.45.1 Optimal result

Integrand size = 35, antiderivative size = 429

$$\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx &= \frac{2B^2d^2n^2(a+bx)^3}{27(bc-ad)^3g^4(c+dx)^3} \\
&\quad - \frac{bB^2dn^2(a+bx)^2}{2(bc-ad)^3g^4(c+dx)^2} + \frac{2b^2B^2n^2(a+bx)}{(bc-ad)^3g^4(c+dx)} \\
&\quad - \frac{2Bd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^4(c+dx)^3} \\
&\quad + \frac{bBdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)^2} \\
&\quad - \frac{2b^2Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(c+dx)} \\
&\quad - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} \\
&\quad + \frac{2b^3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4} \\
&\quad - \frac{b^3B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{3d(bc-ad)^3g^4}
\end{aligned}$$

output
```
2/27*B^2*d^2*n^2*(b*x+a)^3/(-a*d+b*c)^3/g^4/(d*x+c)^3-1/2*b*B^2*d*n^2*(b*x
+a)^2/(-a*d+b*c)^3/g^4/(d*x+c)^2+2*b^2*B^2*n^2*(b*x+a)/(-a*d+b*c)^3/g^4/(d
*x+c)-2/9*B*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g
^4/(d*x+c)^3+b*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^
3/g^4/(d*x+c)^2-2*b^2*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*
c)^3/g^4/(d*x+c)-1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^4/(d*x+c)^3+2/3
*b^3*B*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^
3/g^4-1/3*b^3*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^3/g^4
```

3.45.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.43

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$$
$$= \frac{-18\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(12A(bc-ad)^3-4B(bc-ad)^3n+18Ab(bc-ad)^2(c+dx)-15bB(bc-ad)^2n(c+dx)+36Ab^2(bc-ad)(c}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

output
```
(-18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*A*(b*c - a*d)^3 -
 4*B*(b*c - a*d)^3*n + 18*A*b*(b*c - a*d)^2*(c + d*x) - 15*b*B*(b*c - a*d)
^2*n*(c + d*x) + 36*A*b^2*(b*c - a*d)*(c + d*x)^2 - 66*b^2*B*(b*c - a*d)*n
*(c + d*x)^2 + 36*A*b^3*(c + d*x)^3*Log[a + b*x] - 66*b^3*B*n*(c + d*x)^3*
Log[a + b*x] - 18*b^3*B*n*(c + d*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*
Log[e*((a + b*x)/(c + d*x))^n] + 18*b*B*(b*c - a*d)^2*(c + d*x)*Log[e*((a
+ b*x)/(c + d*x))^n] + 36*b^2*B*(b*c - a*d)*(c + d*x)^2*Log[e*((a + b*x)/(
c + d*x))^n] + 36*b^3*B*(c + d*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x
))^n] - 36*A*b^3*(c + d*x)^3*Log[c + d*x] + 66*b^3*B*n*(c + d*x)^3*Log[c +
 d*x] + 36*b^3*B*n*(c + d*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d
*x] - 36*b^3*B*(c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 1
8*b^3*B*n*(c + d*x)^3*Log[c + d*x]^2 + 36*b^3*B*n*(c + d*x)^3*Log[a + b*x]
*Log[(b*(c + d*x))/(b*c - a*d)] + 36*b^3*B*n*(c + d*x)^3*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)] + 36*b^3*B*n*(c + d*x)^3*PolyLog[2, (b*(c + d*x))/
(b*c - a*d)]))/(b*c - a*d)^3)/(54*d*g^4*(c + d*x)^3)
```

3.45.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2951, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(cg+dgx)^4}dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{2Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{3d}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$\frac{\frac{2Bn\left(-Bn\int \left(\frac{b^3(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}-\frac{1}{6}d\left(18b^2-\frac{9d(a+bx)b}{c+dx}+\frac{2d^2(a+bx)^2}{(c+dx)^2}\right)\right)d\frac{a+bx}{c+dx}+b^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{3b^2d(a+bx)\left(B\log\left(e\left(\frac{a\ldots}{c\ldots}\right.\right.\right.}{c+dx}}{3d}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2Bn\left(b^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{3b^2d(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}-\frac{d^3(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(c+dx)^3}+\frac{3bd^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\ldots\right.\right.}{2(c+dx)^2}}{3d}}{g^4(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

output
```
(-1/3*((b - (d*(a + b*x))/(c + d*x))^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/d + (2*B*n*(-1/3*(d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(c + d*x)^3 + (3*b*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(2*(c + d*x)^2) - (3*b^2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(c + d*x) + b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a +
 b*x)/(c + d*x)] - B*n*(-1/9*(d^3*(a + b*x)^3)/(c + d*x)^3 + (3*b*d^2*(a +
 b*x)^2)/(4*(c + d*x)^2) - (3*b^2*d*(a + b*x))/(c + d*x) + (b^3*Log[(a + b
*x)/(c + d*x)]^2)/2)))/(3*d))/((b*c - a*d)^3*g^4)
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.45. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. $2(417) = 834$.

Time = 16.05 (sec) , antiderivative size = 1146, normalized size of antiderivative = 2.67

method	result	size
parallelrisch	Expression too large to display	1146

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/54*(54*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c*d^6*n-36*B^2*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^3*d^7*n^2-162*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b
^4*c*d^6*n^2-36*A*B*x^2*a*b^3*d^7*n^2+36*A*B*x^2*b^4*c*d^6*n^2+54*B^2*x*ln
(e*((b*x+a)/(d*x+c))^n)^2*b^4*c^2*d^5*n+36*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^
n)*b^4*d^7*n+54*A*B*a^2*b^2*c*d^6*n^2-108*A*B*a*b^3*c^2*d^5*n^2+108*A*B*x^
2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^6*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n
)*a*b^3*c*d^6*n^2+108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^5*n-108*A*
B*x*a*b^3*c*d^6*n^2-108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c*d^6*n+108*
A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^5*n+18*B^2*x*ln(e*((b*x+a)/(d*x+
c))^n)*a^2*b^2*d^7*n^2-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^5*n^2
+162*B^2*x*a*b^3*c*d^6*n^3+18*A*B*x*a^2*b^2*d^7*n^2+90*A*B*x*b^4*c^2*d^5*n
^2-54*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c*d^6*n+54*B^2*ln(e*((b*x+a)
/(d*x+c))^n)^2*a*b^3*c^2*d^5*n+54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c*
d^6*n^2-108*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^5*n^2+36*A*B*ln(e*((
b*x+a)/(d*x+c))^n)*a^3*b*d^7*n-27*B^2*a^2*b^2*c*d^6*n^3+108*B^2*a*b^3*c^2*
d^5*n^3-12*A*B*a^3*b*d^7*n^2+66*A*B*b^4*c^3*d^4*n^2-54*A^2*a^2*b^2*c*d^6*n
+54*A^2*a*b^3*c^2*d^5*n+4*B^2*a^3*b*d^7*n^3-85*B^2*b^4*c^3*d^4*n^3+18*A^2*
a^3*b*d^7*n-18*A^2*b^4*c^3*d^4*n+18*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^
4*d^7*n-66*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^7*n^2+66*B^2*x^2*a*b^3*
d^7*n^3-66*B^2*x^2*b^4*c*d^6*n^3-15*B^2*x*a^2*b^2*d^7*n^3-147*B^2*x*b^4...
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(417) = 834$.

Time = 0.29 (sec) , antiderivative size = 1167, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx =$$

$$-\frac{18\,A^2b^3c^3-54\,A^2ab^2c^2d+54\,A^2a^2bcd^2-18\,A^2a^3d^3+(85\,B^2b^3c^3-108\,B^2ab^2c^2d+27\,B^2a^2bcd^2-4\,B}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="f
ricas")
```

output
```
-1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a
^3*d^3 + (85*B^2*b^3*c^3 - 108*B^2*a*b^2*c^2*d + 27*B^2*a^2*b*c*d^2 - 4*B^
2*a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 - 6*(A*B*b^3*c*
d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*
a^2*b*c*d^2 - B^2*a^3*d^3)*log(e)^2 - 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*b^3*
c*d^2*n^2*x^2 + 3*B^2*b^3*c^2*d*n^2*x + (3*B^2*a*b^2*c^2*d - 3*B^2*a^2*b*c
*d^2 + B^2*a^3*d^3)*n^2)*log((b*x + a)/(d*x + c))^2 - 6*(11*A*B*b^3*c^3 -
18*A*B*a*b^2*c^2*d + 9*A*B*a^2*b*c*d^2 - 2*A*B*a^3*d^3)*n + 3*((49*B^2*b^3
*c^2*d - 54*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n^2 - 6*(5*A*B*b^3*c^2*d -
6*A*B*a*b^2*c*d^2 + A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*
c^2*d + 18*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 - 6*(B^2*b^3*c*d^2 - B^2*a*b^2*
d^3)*n*x^2 - 3*(5*B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n*x -
 (11*B^2*b^3*c^3 - 18*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 - 2*B^2*a^3*d^3)
*n - 6*(B^2*b^3*d^3*n*x^3 + 3*B^2*b^3*c*d^2*n*x^2 + 3*B^2*b^3*c^2*d*n*x +
(3*B^2*a*b^2*c^2*d - 3*B^2*a^2*b*c*d^2 + B^2*a^3*d^3)*n)*log((b*x + a)/(d*
x + c)))*log(e) + 6*((11*B^2*b^3*d^3*n^2 - 6*A*B*b^3*d^3*n)*x^3 + (18*B^2*
a*b^2*c^2*d - 9*B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3)*n^2 - 3*(6*A*B*b^3*c*d^2*
n - (9*B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^3)*n^2)*x^2 - 6*(3*A*B*a*b^2*c^2*d -
3*A*B*a^2*b*c*d^2 + A*B*a^3*d^3)*n - 3*(6*A*B*b^3*c^2*d*n - (6*B^2*b^3*c^2
*d + 6*B^2*a*b^2*c*d^2 - B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c)...
```

3.45.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$$
$$=\frac{\int \frac{A^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{g^4}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**4,x)
```

output
```
(Integral(A**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Inte
gral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**4 + 4*c**3*d*x + 6*
c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/g**4
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(417) = 834$.

Time = 0.28 (sec) , antiderivative size = 1435, normalized size of antiderivative = 3.34

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="m
axima")
```

output
```
1/9*A*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*
c*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*
c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*
d^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4)
+ 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
3 - a^3*d^4)*g^4)) + 1/54*(6*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d +
2*a^2*d^2 + 3*(5*b^2*c*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d
^6)*g^4*x^3 + 3*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2
*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2
 + a^2*c^3*d^3)*g^4) + 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) - (85*b^3*c^3 - 108*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 4*a^3*d^3 + 66*(b^
3*c*d^2 - a*b^2*d^3)*x^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*log(b*x + a)^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3)*log(d*x + c)^2 + 3*(49*b^3*c^2*d - 54*a*b^2*c*d^2 + 5*a^
2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*b^3*c*d^2*x^2 + 33*b^3*c^2*d*x + 11*
b^3*c^3 + 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*l...
```

3.45.8 Giac [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.81

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$$
$$=\frac{1}{54}\left(18\left(\frac{3\,(bx+a)B^2b^2n^2}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)}-\frac{3\,(bx+a)^2B^2bdn^2}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)^2}+\frac{(bx}{(b^2c^2g^4-2\,ab}\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="g
iac")
```

output
```
1/54*(18*(3*(b*x + a)*B^2*b^2*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*
g^4)*(d*x + c)) - 3*(b*x + a)^2*B^2*b*d*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4
+ a^2*d^2*g^4)*(d*x + c)^2) + (b*x + a)^3*B^2*d^2*n^2/((b^2*c^2*g^4 - 2*a*
b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c))^2 - 6*(2*(
B^2*d^2*n^2 - 3*B^2*d^2*n*log(e) - 3*A*B*d^2*n)*(b*x + a)^3/((b^2*c^2*g^4
- 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 9*(B^2*b*d*n^2 - 2*B^2*b*d*n
*log(e) - 2*A*B*b*d*n)*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2
*g^4)*(d*x + c)^2) + 18*(B^2*b^2*n^2 - B^2*b^2*n*log(e) - A*B*b^2*n)*(b*x
+ a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)))*log((b*x + a
)/(d*x + c)) + 2*(2*B^2*d^2*n^2 - 6*B^2*d^2*n*log(e) + 9*B^2*d^2*log(e)^2
- 6*A*B*d^2*n + 18*A*B*d^2*log(e) + 9*A^2*d^2)*(b*x + a)^3/((b^2*c^2*g^4 -
 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 27*(B^2*b*d*n^2 - 2*B^2*b*d*n
*log(e) + 2*B^2*b*d*log(e)^2 - 2*A*B*b*d*n + 4*A*B*b*d*log(e) + 2*A^2*b*d)
*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) + 5
4*(2*B^2*b^2*n^2 - 2*B^2*b^2*n*log(e) + B^2*b^2*log(e)^2 - 2*A*B*b^2*n + 2
*A*B*b^2*log(e) + A^2*b^2)*(b*x + a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d
^2*g^4)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.42

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$$

$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{3\,d\,\left(c^3\,g^4+3\,c^2\,d\,g^4\,x+3\,c\,d^2\,g^4\,x^2+d^3\,g^4\,x^3\right)}+\frac{B^2\,b^3}{3\,d\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)$$

$$-\frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2-12\,A\,B\,a^2\,d^2\,n+42\,A\,B\,a\,b\,c\,d\,n-66\,A\,B\,b^2\,c^2\,n+4\,B^2\,a^2\,d^2\,n^2-23\,B^2\,a\,b\,c\,d\,n^2+85\,B^2\,b^2\,c^2\,n^2}{6\,(a\,d-b\,c)}-\frac{x\,(-}{}}{x\,\left(27\,a\,c^2\,d^3\,g^4-27\,b\,c^3\,d^2\,g^4\right)-x^2\,\left(27\,b\,c^2\,d^3\,g^4-27\,a\,c\,d^4\,g^4\right)+x^3\,\left(9\,a\right.}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,A\,B}{3\,c^3\,d\,g^4+9\,c^2\,d^2\,g^4\,x+9\,c\,d^3\,g^4\,x^2+3\,d^4\,g^4\,x^3}\right.$$

$$+\frac{2\,B^2\,b^3\,\left(x\,\left(d\,\left(\frac{d\,g^4\,n\,(a\,d-b\,c)\,(a\,d-3\,b\,c)}{2\,b^2}-\frac{c\,d\,g^4\,n\,(a\,d-b\,c)}{b}\right)-\frac{2\,c\,d^2\,g^4\,n\,(a\,d-b\,c)}{b}+\frac{d^2\,g^4\,n\,(a\,d-b\,c)\,(a\,d-3\,b\,c)}{b^2}\right)+c\,\left(\frac{d\,g^4\,}{}\right.\right.}{3\,d\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)\,\left(3\,c^3\,d\,g^4+\right.}$$

$$-\frac{B\,b^3\,n\,\mathrm{atan}\left(\frac{B\,b^3\,n\,(6\,A-11\,B\,n)\,\left(\frac{a^3\,d^4\,g^4-a^2\,b\,c\,d^3\,g^4-a\,b^2\,c^2\,d^2\,g^4+b^3\,c^3\,d\,g^4}{a^2\,d^3\,g^4-2\,a\,b\,c\,d^2\,g^4+b^2\,c^2\,d\,g^4}+2\,b\,d\,x\right)\,\left(a^2\,d^3\,g^4-2\,a\,b\,c\,d^2\,g^4+b^2\,c^2\,d\,g^4\right)\,1\mathrm{i}}{d\,g^4\,\left(11\,B^2\,b^3\,n^2-6\,A\,B\,b^3\,n\right)\,(a\,d-b\,c)^3}\right)\,(6\,A-}{9\,d\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^4,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(3*d*(c^3*g^4 + d^3*g^4*x^3 + 3*c*
d^2*g^4*x^2 + 3*c^2*d*g^4*x)) + (B^2*b^3)/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))) - ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 4*B^2
*a^2*d^2*n^2 + 85*B^2*b^2*c^2*n^2 - 36*A^2*a*b*c*d - 12*A*B*a^2*d^2*n - 66
*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n^2 + 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) -
(x*(5*B^2*a*b*d^2*n^2 - 49*B^2*b^2*c*d*n^2 - 6*A*B*a*b*d^2*n + 30*A*B*b^2*
c*d*n))/(2*(a*d - b*c)) + (b*x^2*(11*B^2*b*d^2*n^2 - 6*A*B*b*d^2*n))/(a*d
- b*c))/(x*(27*a*c^2*d^3*g^4 - 27*b*c^3*d^2*g^4) - x^2*(27*b*c^2*d^3*g^4 -
 27*a*c*d^4*g^4) + x^3*(9*a*d^5*g^4 - 9*b*c*d^4*g^4) + 9*a*c^3*d^2*g^4 - 9
*b*c^4*d*g^4) - log(e*((a + b*x)/(c + d*x))^n)*((2*A*B)/(3*c^3*d*g^4 + 3*d
^4*g^4*x^3 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^2) + (2*B^2*b^3*(x*(d*((d*g^4
*n*(a*d - b*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b) - (2*c*
d^2*g^4*n*(a*d - b*c))/b + (d^2*g^4*n*(a*d - b*c)*(a*d - 3*b*c))/b^2) + c*
((d*g^4*n*(a*d - b*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b)
- (d*g^4*n*(a*d - b*c)*(a^2*d^2 + 3*b^2*c^2 - 3*a*b*c*d))/b^3 - (3*d^3*g^4
*n*x^2*(a*d - b*c))/b))/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^
2*b*c*d^2)*(3*c^3*d*g^4 + 3*d^4*g^4*x^3 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^
2))) - (B*b^3*n*atan((B*b^3*n*(6*A - 11*B*n)*((a^3*d^4*g^4 + b^3*c^3*d*g^4
 - a^2*b*c*d^3*g^4 - a*b^2*c^2*d^2*g^4)/(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a
*b*c*d^2*g^4) + 2*b*d*x)*(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a*b*c*d^2*g^4...
```

3.45. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$

3.46 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$

3.46.1 Optimal result

Integrand size = 35, antiderivative size = 536

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx &= -\frac{B^2d^3n^2(a+bx)^4}{32(bc-ad)^4g^5(c+dx)^4}+\frac{2bB^2d^2n^2(a+bx)^3}{9(bc-ad)^4g^5(c+dx)^3}\\
&\quad-\frac{3b^2B^2dn^2(a+bx)^2}{4(bc-ad)^4g^5(c+dx)^2}+\frac{2b^3B^2n^2(a+bx)}{(bc-ad)^4g^5(c+dx)}\\
&\quad+\frac{Bd^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^4g^5(c+dx)^4}\\
&\quad-\frac{2bBd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^5(c+dx)^3}\\
&\quad+\frac{3b^2Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^5(c+dx)^2}\\
&\quad-\frac{2b^3Bn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^5(c+dx)}\\
&\quad-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4}\\
&\quad+\frac{b^4Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{2d(bc-ad)^4g^5}\\
&\quad-\frac{b^4B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{4d(bc-ad)^4g^5}\end{aligned}$$

output
```
-1/32*B^2*d^3*n^2*(b*x+a)^4/(-a*d+b*c)^4/g^5/(d*x+c)^4+2/9*b*B^2*d^2*n^2*(
b*x+a)^3/(-a*d+b*c)^4/g^5/(d*x+c)^3-3/4*b^2*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)
^4/g^5/(d*x+c)^2+2*b^3*B^2*n^2*(b*x+a)/(-a*d+b*c)^4/g^5/(d*x+c)+1/8*B*d^3*
n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x+c)^4-2/3
*b*B*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x
+c)^3+3/2*b^2*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4
/g^5/(d*x+c)^2-2*b^3*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c
)^4/g^5/(d*x+c)-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^5/(d*x+c)^4+1/2*
b^4*B*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^4
/g^5-1/4*b^4*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^4/g^5
```

3.46.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.31

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$$

$$= \frac{-72\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(36A(bc-ad)^4-9B(bc-ad)^4n+48Ab(bc-ad)^3(c+dx)-28bB(bc-ad)^3n(c+dx)+72Ab^2(bc-ad)^2(\right.}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

output
```
(-72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*A*(b*c - a*d)^4 -
 9*B*(b*c - a*d)^4*n + 48*A*b*(b*c - a*d)^3*(c + d*x) - 28*b*B*(b*c - a*d)
^3*n*(c + d*x) + 72*A*b^2*(b*c - a*d)^2*(c + d*x)^2 - 78*b^2*B*(b*c - a*d)
^2*n*(c + d*x)^2 + 144*A*b^3*(b*c - a*d)*(c + d*x)^3 - 300*b^3*B*(b*c - a*
d)*n*(c + d*x)^3 + 144*A*b^4*(c + d*x)^4*Log[a + b*x] - 300*b^4*B*n*(c + d
*x)^4*Log[a + b*x] - 72*b^4*B*n*(c + d*x)^4*Log[a + b*x]^2 + 36*B*(b*c - a
*d)^4*Log[e*((a + b*x)/(c + d*x))^n] + 48*b*B*(b*c - a*d)^3*(c + d*x)*Log[
e*((a + b*x)/(c + d*x))^n] + 72*b^2*B*(b*c - a*d)^2*(c + d*x)^2*Log[e*((a
+ b*x)/(c + d*x))^n] + 144*b^3*B*(b*c - a*d)*(c + d*x)^3*Log[e*((a + b*x)/
(c + d*x))^n] + 144*b^4*B*(c + d*x)^4*Log[a + b*x]*Log[e*((a + b*x)/(c + d
*x))^n] - 144*A*b^4*(c + d*x)^4*Log[c + d*x] + 300*b^4*B*n*(c + d*x)^4*Log
[c + d*x] + 144*b^4*B*n*(c + d*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[
c + d*x] - 144*b^4*B*(c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*
x] - 72*b^4*B*n*(c + d*x)^4*Log[c + d*x]^2 + 144*b^4*B*n*(c + d*x)^4*Log[a
 + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 144*b^4*B*n*(c + d*x)^4*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + 144*b^4*B*n*(c + d*x)^4*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)]))/(b*c - a*d)^4)/(288*d*g^5*(c + d*x)^4)
```

3.46.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2951, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(cg+dgx)^5} \, dx$$

$$\Big\downarrow \; 2951$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{g^5 (bc-ad)^4}$$

$$\Big\downarrow \; 2756$$

$$\frac{\frac{Bn \int \frac{(c+dx) \left(b-\frac{d(a+bx)}{c+dx}\right)^4 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx} d\frac{a+bx}{c+dx}}{2d}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^4 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d}}{g^5 (bc-ad)^4}$$

$$\Big\downarrow \; 2772$$

$$\frac{Bn\left(-Bn\int\left(\frac{(c+dx)\log\left(\frac{a+bx}{c+dx}\right)b^4}{a+bx}-4db^3+\frac{3d^2(a+bx)b^2}{c+dx}-\frac{4d^3(a+bx)^2b}{3(c+dx)^2}+\frac{d^4(a+bx)^3}{4(c+dx)^3}\right)d\frac{a+bx}{c+dx}+b^4\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{4b^3d(a+bx)\left(B\right.}{}\right.}{2d}$$

$\downarrow$ 2009

$$\frac{Bn\left(b^4\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{4b^3d(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}+\frac{3b^2d^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)^2}+\frac{d^4(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{4(c+dx)^4}\right.}{2d}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

output
```
(-1/4*((b - (d*(a + b*x))/(c + d*x))^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/d + (B*n*((d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(4*(c + d*x)^4) - (4*b*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(3*(c + d*x)^3) + (3*b^2*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/(c + d*x)^2 - (4*b^3*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(c + d*x) + b^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a +
 b*x)/(c + d*x)] - B*n*((d^4*(a + b*x)^4)/(16*(c + d*x)^4) - (4*b*d^3*(a +
 b*x)^3)/(9*(c + d*x)^3) + (3*b^2*d^2*(a + b*x)^2)/(2*(c + d*x)^2) - (4*b^
3*d*(a + b*x))/(c + d*x) + (b^4*Log[(a + b*x)/(c + d*x)]^2)/2)))/(2*d))/((
b*c - a*d)^4*g^5)
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.46. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2325 vs. 2(518) = 1036.

Time = 42.18 (sec) , antiderivative size = 2326, normalized size of antiderivative = 4.34

method	result	size
parallelrisch	Expression too large to display	2326

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x,method=_RETURNVERBOS
E)
```

output
```
1/288*(-1056*A*B*x^3*a*b^4*c^7*d^2*n^2+432*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^
n)^2*a*b^4*c^8*d*n+72*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^6*d^3*n^
2-576*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^7*d^2*n^2+2160*A*B*x^3*a
^2*b^3*c^6*d^3*n^2+9*B^2*x^4*a^5*c^2*d^7*n^3+36*B^2*x^3*a^5*c^3*d^6*n^3+72
*A^2*x^4*a^5*c^2*d^7*n+54*B^2*x^2*a^5*c^4*d^5*n^3+288*A^2*x^3*a^5*c^3*d^6*
n+36*B^2*x*a^5*c^5*d^4*n^3+576*B^2*x*a*b^4*c^9*n^3+432*A^2*x^2*a^5*c^4*d^5
*n-72*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*c^6*d^3*n+288*B^2*ln(e*((b*x+a)/
(d*x+c))^n)^2*a^2*b^3*c^9*n+36*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^6*d^3*n
^2-576*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^9*n^2+288*A^2*x*a^5*c^5*d^4
*n+288*A^2*x*a*b^4*c^9*n-64*B^2*x^4*a^4*b*c^3*d^6*n^3+216*B^2*x^4*a^3*b^2*
c^4*d^5*n^3-576*B^2*x^4*a^2*b^3*c^5*d^4*n^3+415*B^2*x^4*a*b^4*c^6*d^3*n^3-
36*A*B*x^4*a^5*c^2*d^7*n^2-256*B^2*x^3*a^4*b*c^4*d^5*n^3+864*B^2*x^3*a^3*b
^2*c^5*d^4*n^3-2004*B^2*x^3*a^2*b^3*c^6*d^3*n^3+1360*B^2*x^3*a*b^4*c^7*d^2
*n^3-1152*A^2*x*a^2*b^3*c^8*d*n-144*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^6*
d^3*n+576*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^9*n+144*A*B*x^4*ln(e*((b
*x+a)/(d*x+c))^n)*a*b^4*c^6*d^3*n+576*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^4*c^7*d^2*n+864*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^8*d*n-288*A^2*
x^4*a^4*b*c^3*d^6*n+432*A^2*x^4*a^3*b^2*c^4*d^5*n-288*A^2*x^4*a^2*b^3*c^5*
d^4*n+72*A^2*x^4*a*b^4*c^6*d^3*n-144*A*B*x^3*a^5*c^3*d^6*n^2-384*B^2*x^2*a
^4*b*c^5*d^4*n^3+1218*B^2*x^2*a^3*b^2*c^6*d^3*n^3-2400*B^2*x^2*a^2*b^3*...
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1768 vs. $2(518) = 1036$.

Time = 0.32 (sec) , antiderivative size = 1768, normalized size of antiderivative = 3.30

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="f
ricas")
```

output
```
-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 2
88*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 + 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^
4)*n^2 - 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (415*B^2*b^4*c^4 - 57
6*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 64*B^2*a^3*b*c*d^3 + 9*B^2*a
^4*d^4)*n^2 + 6*((163*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 13*B^2*a^2*b
^2*d^4)*n^2 - 12*(7*A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)
*n)*x^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*
B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*
b^4*c*d^3*n^2*x^3 + 6*B^2*b^4*c^2*d^2*n^2*x^2 + 4*B^2*b^4*c^3*d*n^2*x + (4
*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4*d^4
)*n^2)*log((b*x + a)/(d*x + c))^2 - 12*(25*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*
d + 36*A*B*a^2*b^2*c^2*d^2 - 16*A*B*a^3*b*c*d^3 + 3*A*B*a^4*d^4)*n + 4*((2
71*B^2*b^4*c^3*d - 324*B^2*a*b^3*c^2*d^2 + 60*B^2*a^2*b^2*c*d^3 - 7*B^2*a^
3*b*d^4)*n^2 - 12*(13*A*B*b^4*c^3*d - 18*A*B*a*b^3*c^2*d^2 + 6*A*B*a^2*b^2
*c*d^3 - A*B*a^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 7
2*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*
c*d^3 - B^2*a*b^3*d^4)*n*x^3 - 6*(7*B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 +
B^2*a^2*b^2*d^4)*n*x^2 - 4*(13*B^2*b^4*c^3*d - 18*B^2*a*b^3*c^2*d^2 + 6*B^
2*a^2*b^2*c*d^3 - B^2*a^3*b*d^4)*n*x - (25*B^2*b^4*c^4 - 48*B^2*a*b^3*c^3*
d + 36*B^2*a^2*b^2*c^2*d^2 - 16*B^2*a^3*b*c*d^3 + 3*B^2*a^4*d^4)*n - 12...
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**5,x)
```

output
```
Timed out
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2138 vs. $2(518) = 1036$.

Time = 0.33 (sec) , antiderivative size = 2138, normalized size of antiderivative = 3.99

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="m
axima")
```

output
```
1/24*A*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2
 - 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2
*c*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a
^3*d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3
*c*d^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3
*c^2*d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a
^3*c^3*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c
^4*d^4)*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b
^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4
*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5))
+ 1/288*(12*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*
d^2 - 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*
b^2*c*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7
- a^3*d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 -
a^3*c*d^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 -
a^3*c^2*d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4
- a^3*c^3*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^
3*c^4*d^4)*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*
c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*...
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(518) = 1036$.

Time = 1.69 (sec) , antiderivative size = 1265, normalized size of antiderivative = 2.36

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="g
iac")
```

output
```
1/288*(72*(4*(b*x + a)*B^2*b^3*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a
^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B^2*b^2*d*n^2/((b
^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c
)^2) + 4*(b*x + a)^3*B^2*b*d^2*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a
^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B^2*d^3*n^2/((b^3
*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^
4))*log((b*x + a)/(d*x + c))^2 + 12*(3*(B^2*d^3*n^2 - 4*B^2*d^3*n*log(e) -
 4*A*B*d^3*n)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^
2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) - 16*(B^2*b*d^2*n^2 - 3*B^2*b*d^2*n*log(
e) - 3*A*B*b*d^2*n)*(b*x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*
b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) + 36*(B^2*b^2*d*n^2 - 2*B^2*b^2*d*
n*log(e) - 2*A*B*b^2*d*n)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 +
3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) - 48*(B^2*b^3*n^2 - B^2*b^3*
n*log(e) - A*B*b^3*n)*(b*x + a)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*
b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))*log((b*x + a)/(d*x + c)) - 9*(B^2*d
^3*n^2 - 4*B^2*d^3*n*log(e) + 8*B^2*d^3*log(e)^2 - 4*A*B*d^3*n + 16*A*B*d^
3*log(e) + 8*A^2*d^3)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^
2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) + 32*(2*B^2*b*d^2*n^2 - 6*B^2*b*
d^2*n*log(e) + 9*B^2*b*d^2*log(e)^2 - 6*A*B*b*d^2*n + 18*A*B*b*d^2*log(e)
+ 9*A^2*b*d^2)*(b*x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*...
```

3.46.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 1765, normalized size of antiderivative = 3.29

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^5,x)
```

output
```
(B*b^4*n*atan((B*b^4*n*(12*A - 25*B*n)*(24*a^4*d^5*g^5 - 24*b^4*c^4*d*g^5
- 48*a^3*b*c*d^4*g^5 + 48*a*b^3*c^3*d^2*g^5)*1i)/(24*d*g^5*(25*B^2*b^4*n^2
 - 12*A*B*b^4*n)*(a*d - b*c)^4) + (B*b^5*n*x*(12*A - 25*B*n)*(a^3*d^4*g^5
- b^3*c^3*d*g^5 - 3*a^2*b*c*d^3*g^5 + 3*a*b^2*c^2*d^2*g^5)*2i)/(g^5*(25*B^
2*b^4*n^2 - 12*A*B*b^4*n)*(a*d - b*c)^4))*(12*A - 25*B*n)*1i)/(12*d*g^5*(a
*d - b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 9*B^2*a^3*d^3*n^2 - 415
*B^2*b^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 - 36*A*B*a^3*
d^3*n + 300*A*B*b^3*c^3*n + 161*B^2*a*b^2*c^2*d*n^2 - 55*B^2*a^2*b*c*d^2*n
^2 - 276*A*B*a*b^2*c^2*d*n + 156*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^
2*(13*B^2*a*b^2*d^3*n^2 - 163*B^2*b^3*c*d^2*n^2 - 12*A*B*a*b^2*d^3*n + 84*
A*B*b^3*c*d^2*n))/(2*(a*d - b*c)) - (x*(7*B^2*a^2*b*d^3*n^2 + 271*B^2*b^3*
c^2*d*n^2 - 53*B^2*a*b^2*c*d^2*n^2 - 12*A*B*a^2*b*d^3*n - 156*A*B*b^3*c^2*
d*n + 60*A*B*a*b^2*c*d^2*n))/(3*(a*d - b*c)) - (b*x^3*(25*B^2*b^2*d^3*n^2
- 12*A*B*b^2*d^3*n))/(a*d - b*c))/(x*(96*a^2*c^3*d^4*g^5 + 96*b^2*c^5*d^2*
g^5 - 192*a*b*c^4*d^3*g^5) + x^3*(96*a^2*c*d^6*g^5 + 96*b^2*c^3*d^4*g^5 -
192*a*b*c^2*d^5*g^5) + x^4*(24*a^2*d^7*g^5 + 24*b^2*c^2*d^5*g^5 - 48*a*b*c
*d^6*g^5) + x^2*(144*a^2*c^2*d^5*g^5 + 144*b^2*c^4*d^3*g^5 - 288*a*b*c^3*d
^4*g^5) + 24*b^2*c^6*d*g^5 + 24*a^2*c^4*d^3*g^5 - 48*a*b*c^5*d^2*g^5) - lo
g(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*
g^5*x^3 + 6*c^2*d^2*g^5*x^2 + 4*c^3*d*g^5*x)) - (B^2*b^4)/(4*d*g^5*(a^4...
```

3.46. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$

3.47 $\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.47.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Unintegrable((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.47.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

3.47.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cg+dgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{(cg+dgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

3.47.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.47.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(dgx+cg)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.47.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(dgx+cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B*log(e*((b*x + a)/(d*x +
c))^n) + A), x)
```

3.47.6 Sympy [N/A]

Not integrable

Time = 24.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=g^2\left(\int \frac{c^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{d^2x^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{2cdx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

input
```
integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
g**2*(Integral(c**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) +
Integral(d**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(2*c*d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

3.47.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(dgx+cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.47.8 Giac [N/A]

Not integrable

Time = 27.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(dgx+cg)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.47.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(c\,g+d\,g\,x)^2}{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

input
```
int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.48 $\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.48.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Unintegrable((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.48.2 Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

3.48.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cg+dgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$
$$\Big\downarrow \; 2955$$
$$\int \frac{cg+dgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

3.48.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.48.4 Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{dgx+cg}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.48.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
integral((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.48.6 Sympy [N/A]

Not integrable

Time = 14.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=g\left(\int \frac{c}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{dx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

input
```
integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
g*(Integral(c/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integr
al(d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

3.48.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.48.8 Giac [N/A]

Not integrable

Time = 15.91 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg + dgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{dgx + cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.48.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg + dgx}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{c\,g + d\,g\,x}{A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

input
```
int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.49 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.49.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Unintegrable(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.49.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

3.49.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \ 2955$$

$$\int \frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

3.49.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.49.4 Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.49.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
integral(1/(A*d*g*x + A*c*g + (B*d*g*x + B*c*g)*log(e*((b*x + a)/(d*x + c)
)^n)), x)
```

3.49.6 Sympy [N/A]

Not integrable

Time = 18.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Ac+Adx+Bc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g}$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*c + A*d*x + B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

3.49.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.49.8 Giac [N/A]

Not integrable

Time = 9.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.49.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.50 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.50.1 Optimal result

Integrand size = 35, antiderivative size = 96

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}$$

output
```
(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)/exp(A/B/n)/g^
2/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)
```

3.50.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}$$

input
```
Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B
*(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x
))
```

3.50.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2737}$$

$$\frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2n(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(c+dx)(bc-ad)}$$

input
```
Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B
*(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x
))
```

3.50.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.50.4 Maple [F]

$$\int \frac{1}{\left(dgx+cg\right)^{2} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

input
```
int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{e^{\left(-\frac{B\log(e)+A}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{B\log(e)+A}{Bn}\right)}}{dx+c}\right)}{(Bbc-Bad)g^2n}$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="f
ricas")
```

output
```
e^(-(B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n))/
(d*x + c))/((B*b*c - B*a*d)*g^2*n)
```

3.50.6 Sympy [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{\int \frac{1}{Ac^2+2Acdx+Ad^2x^2+Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2Bcdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g^2}$$

input
```
integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A*c**2 + 2*A*c*d*x + A*d**2*x**2 + B*c**2*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n) + 2*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
B*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

3.50.7 Maxima [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="m
axima")
```

output
```
integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.50.8 Giac [F]

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="g
iac")
```

output
```
integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(c\,g+d\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.51 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.51.1 Optimal result

Integrand size = 35, antiderivative size = 199

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)} - \frac{de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)^2}$$

output
```
b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(A/B/n
)/g^3/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-d*(b*x+a)^2*Ei(2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n/((e*((b*x+a)
/(d*x+c))^n)^(2/n))/(d*x+c)^2
```

3.51.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$
$$= \frac{e^{-\frac{2A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)-d(a+bx)\right.}{B(bc-ad)^2g^3n(c+dx)^2}$$

input
```
Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((a + b*x)*(b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*Exp
IntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)] - d*(a + b*x)*Exp
IntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]))/(B*(b*c - a
*d)^2*E^((2*A)/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

3.51.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \; 2951$$

$$\frac{\int \frac{b-\frac{d(a+bx)}{c+dx}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2767$$

$$\frac{\int \left(\frac{b}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}-\frac{d(a+bx)}{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bn(c+dx)}-\frac{d(a+bx)^2e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bn(c+dx)^2}}{g^3(bc-ad)^2}$$

input
```
Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
((b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])
/(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)) - (d*(a +
b*x)^2*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B
*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2))/((b*c -
 a*d)^2*g^3)
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.51.4 Maple [F]

$$\int \frac{1}{(dgx+cg)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.74

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}dx = \frac{\left(be^{\left(\frac{B\log(e)+A}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{B\log(e)+A}{Bn}\right)}}{dx+c}\right) - d\,\mathrm{log\_integral}\left(\frac{(b^2x^2+2\,abx+a^2)e^{\left(\frac{2\,(B\log(e)+A)}{Bn}\right)}}{d^2x^2+2\,cdx+c^2}\right)\right)e^{\left(-\frac{2\,(B\mathrm{l}}{}\right.}}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3n}$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="f
ricas")
```

output
```
(b*e^((B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n)
)/(d*x + c)) - d*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*(B*log(e) + A
)/(B*n))/(d^2*x^2 + 2*c*d*x + c^2)))*e^(-2*(B*log(e) + A)/(B*n))/((B*b^2*c
^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3*n)
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}dx = \text{Timed out}$$

input
```
integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.51.7 Maxima [F]

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="m
axima")
```

output
```
integrate(1/((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.51.8 Giac [F]

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(dgx+cg)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="g
iac")
```

output
```
integrate(1/((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(c\,g+d\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.52 $\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.52.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.52.2 Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

3.52.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cg+dgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{(cg+dgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

3.52.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.52.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(dgx+cg)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.52.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.31

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B^2*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

3.52.6 Sympy [N/A]

Not integrable

Time = 45.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.34

$$\begin{aligned}&\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx\\&=g^2\left(\int \frac{c^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right.\\&\quad+\int \frac{d^2x^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\\&\quad\left.+\int \frac{2cdx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)\end{aligned}$$

input
```
integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g**2*(Integral(c**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)
+ B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d**2*x**
2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*c*d*x/(A**2 + 2*A*B*log(
e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)**2), x))
```

3.52.7 Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 329, normalized size of antiderivative = 9.40

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
-(b*d^3*g^2*x^4 + a*c^3*g^2 + (3*b*c*d^2*g^2 + a*d^3*g^2)*x^3 + 3*(b*c^2*d
*g^2 + a*c*d^2*g^2)*x^2 + (b*c^3*g^2 + 3*a*c^2*d*g^2)*x)/((b*c*n - a*d*n)*
B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d
*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*b*d^3*g^2*x^3
+ b*c^3*g^2 + 3*a*c^2*d*g^2 + 3*(3*b*c*d^2*g^2 + a*d^3*g^2)*x^2 + 6*(b*c^2
*d*g^2 + a*c*d^2*g^2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n -
a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*
log(e))*B^2), x)
```

3.52.8 Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(dgx+cg)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.52.9 Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{\left(c\,g+d\,g\,x\right)^2}{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.53 $\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.53.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.53.2 Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

3.53.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cg + dgx}{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2} dx$$
$$\Big\downarrow \text{2955}$$
$$\int \frac{cg + dgx}{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2} dx$$

input
```
Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

3.53.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.53.4 Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{dgx + cg}{\left(A + B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2} dx$$

input
```
int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.53.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{dgx+cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral((d*g*x + c*g)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e
*((b*x + a)/(d*x + c))^n) + A^2), x)
```

3.53.6 Sympy [N/A]

Not integrable

Time = 58.59 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.55

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$=g\left(\int \frac{c}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx+\int \frac{dx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

input
```
integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
g*(Integral(c/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d*x/(A**2 + 2*
A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)**2), x))
```

3.53.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 251, normalized size of antiderivative = 7.61

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{dgx+cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
-(b*d^2*g*x^3 + a*c^2*g + (2*b*c*d*g + a*d^2*g)*x^2 + (b*c^2*g + 2*a*c*d*g
)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x
+ c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integ
rate((3*b*d^2*g*x^2 + b*c^2*g + 2*a*c*d*g + 2*(2*b*c*d*g + a*d^2*g)*x)/((b
*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n)
+ (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

3.53.8 Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{dgx+cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.53.9 Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{c\,g+d\,g\,x}{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.54 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.54.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.54.2 Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

3.54.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

3.54.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.54.4 Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.54.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.43

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(
d*x + c))^n)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n)),
x)
```

3.54.6 Sympy [N/A]

Not integrable

Time = 139.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.66

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$=\frac{\int \frac{1}{A^2c+A^2dx+2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2ABdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2+B^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}}{g}$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral(1/(A**2*c + A**2*d*x + 2*A*B*c*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n) + 2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*c*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)**2 + B**2*d*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)**2), x)/g
```

3.54.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.31

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
b*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g
*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d
*g*n*log(e))*B^2), x) - (b*x + a)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n
) - (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (
b*c*g*n*log(e) - a*d*g*n*log(e))*B^2)
```

3.54.8 Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

3.54.9 Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

input
```
int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.55 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.55.1 Optimal result

Integrand size = 35, antiderivative size = 154

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(c+dx)} - \frac{a+bx}{B(bc-ad)g^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)/exp(A/B/n)/
g^2/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)+(-b*x-a)/B/(-a*d+b*c)/g^2/
n/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

3.55.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\left(Be^{\frac{A}{Bn}}n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} - \text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\right.\right.\right.}{B^2(bc-ad)g^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

input
```
Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
-(((a + b*x)*(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1) - ExpInte
gralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])))/(B^2*(b*c - a*d)*E^(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c
+ d*x))^n)^n^(-1)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))
```

3.55.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2951, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2} \, dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \frac{1}{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{g^2 (bc-ad)}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{a+bx}{Bn(c+dx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2 (bc-ad)}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{(a+bx) \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \int \frac{\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}}{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}d\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2 (c+dx)}-\frac{a+bx}{Bn(c+dx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2 (bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{(a+bx) e^{-\frac{A}{Bn}} \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \text{ExpIntegralEi}\left(\frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2 n^2 (c+dx)}-\frac{a+bx}{Bn(c+dx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^2 (bc-ad)}$$

input
```
Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
(((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(
B^2*E^(A/(B*n))*n^2*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)) - (a + b
*x)/(B*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/((b*c - a*d)*g
^2)
```

3.55.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.55.4 Maple [F]

$$\int \frac{1}{(dgx+cg)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.89

$$\int \frac{1}{(cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{\left((Bbnx+Ban)e^{\left(\frac{B\log(e)+A}{Bn}\right)} - \left(Adx+Ac+(Bdx+Bc)\log(e)+(Bdnx+Bcn)\log\right.\right.}{(AB^2bcd-AB^2ad^2)g^2n^2x+(AB^2bc^2-AB^2acd)g^2n^2+((B^3bcd-B^3ad^2)g^2n^2x+(B^3bc^2-B^3acd)g}$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"fricas")
```

output
```
-((B*b*n*x + B*a*n)*e^((B*log(e) + A)/(B*n)) - (A*d*x + A*c + (B*d*x + B*c
)*log(e) + (B*d*n*x + B*c*n)*log((b*x + a)/(d*x + c)))*log_integral((b*x +
 a)*e^((B*log(e) + A)/(B*n))/(d*x + c)))*e^(-(B*log(e) + A)/(B*n))/((A*B^2
*b*c*d - A*B^2*a*d^2)*g^2*n^2*x + (A*B^2*b*c^2 - A*B^2*a*c*d)*g^2*n^2 + ((
B^3*b*c*d - B^3*a*d^2)*g^2*n^2*x + (B^3*b*c^2 - B^3*a*c*d)*g^2*n^2)*log(e)
 + ((B^3*b*c*d - B^3*a*d^2)*g^2*n^3*x + (B^3*b*c^2 - B^3*a*c*d)*g^2*n^3)*l
og((b*x + a)/(d*x + c)))
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.55.7 Maxima [F]

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"maxima")
```

output
```
-(b*x + a)/((b*c^2*g^2*n - a*c*d*g^2*n)*A*B + (b*c^2*g^2*n*log(e) - a*c*d*
g^2*n*log(e))*B^2 + ((b*c*d*g^2*n - a*d^2*g^2*n)*A*B + (b*c*d*g^2*n*log(e)
 - a*d^2*g^2*n*log(e))*B^2)*x + ((b*c*d*g^2*n - a*d^2*g^2*n)*B^2*x + (b*c^
2*g^2*n - a*c*d*g^2*n)*B^2)*log((b*x + a)^n) - ((b*c*d*g^2*n - a*d^2*g^2*n
)*B^2*x + (b*c^2*g^2*n - a*c*d*g^2*n)*B^2)*log((d*x + c)^n)) - integrate(-
1/(B^2*c^2*g^2*n*log(e) + A*B*c^2*g^2*n + (B^2*d^2*g^2*n*log(e) + A*B*d^2*
g^2*n)*x^2 + 2*(B^2*c*d*g^2*n*log(e) + A*B*c*d*g^2*n)*x + (B^2*d^2*g^2*n*x
^2 + 2*B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((b*x + a)^n) - (B^2*d^2*g^2*n*
x^2 + 2*B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((d*x + c)^n)), x)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)\left(\frac{bx+a}{\left(B^2g^2n^2\log\left(\frac{bx+a}{dx+c}\right)+B^2g^2n\log(e)+ABg^2n\right)(dx+c)}-\frac{\mathrm{Ei}\left(\frac{\log(e)}{n}+\frac{A}{Bn}\right.}{B}\right.$$

input
```
integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"giac")
```

output
```
-(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*((b*x + a)/((B^2*g^2*n^2*log((b*x
 + a)/(d*x + c)) + B^2*g^2*n*log(e) + A*B*g^2*n)*(d*x + c)) - Ei(log(e)/n
+ A/(B*n) + log((b*x + a)/(d*x + c)))*e^(-A/(B*n))/(B^2*e^(1/n)*g^2*n^2))
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(c\,g+d\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.56 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.56.1 Optimal result

Integrand size = 35, antiderivative size = 256

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)} - \frac{2de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)^2} - \frac{a+bx}{B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(A/B
/n)/g^3/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-2*d*(b*x+a)^2*Ei(2*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n^2/((
e*((b*x+a)/(d*x+c))^n)^(2/n))/(d*x+c)^2+(-b*x-a)/B/(-a*d+b*c)/g^3/n/(d*x+c
)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

3.56.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.12

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{e^{-\frac{2A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(-B(bc-ad)e^{\frac{2A}{Bn}}n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}+be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegral}}{B^2(bc-ad)^2g^3}$$

input
```
Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
((a + b*x)*(-(B*(b*c - a*d)*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^
(2/n)) + b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpInt
egralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]) - 2*d*(a + b*x)*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*
c - a*d)^2*E^((2*A)/(B*n))*g^3*n^2*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c +
d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

3.56.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2951, 2757, 2737, 2609, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cg+dgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2951}$$
$$\frac{\int \frac{b-\frac{d(a+bx)}{c+dx}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$
$$\Big\downarrow \text{2757}$$

$$\frac{-\frac{b\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}+\frac{2\int \frac{b-\frac{d(a+bx)}{c+dx}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^3(bc-ad)^2}$$

$\big\downarrow$ 2737

$$\frac{-\frac{b(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2(c+dx)}+\frac{2\int \frac{b-\frac{d(a+bx)}{c+dx}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^3(bc-ad)^2}$$

$\big\downarrow$ 2609

$$\frac{\frac{2\int \frac{b-\frac{d(a+bx)}{c+dx}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(c+dx)}-\frac{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}{Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}}{g^3(bc-ad)^2}$$

$\big\downarrow$ 2767

$$\frac{\frac{2\int \left(\frac{b}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}-\frac{d(a+bx)}{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\right)d\frac{a+bx}{c+dx}}{Bn}-\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(c+dx)}-}{g^3(bc-ad)^2}$$

$\big\downarrow$ 2009

$$\frac{-\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2n^2(c+dx)}+\frac{2\left(\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.}{Bn}\right.}{Bn(c+dx)}\right.}{}}{g^3(bc-ad)^2}$$

input
```
Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

3.56. $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

output
```
(-((b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)
])/(B^2*E^(A/(B*n))*n^2*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))) + (
2*((b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)
])/(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)) - (d*(a
+ b*x)^2*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/
(B*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)))/(B*n
) - ((a + b*x)*(b - (d*(a + b*x))/(c + d*x)))/(B*n*(c + d*x)*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])))/((b*c - a*d)^2*g^3)
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2757
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
 1), x], x] + Simp[d*(q/(b*n*(p + 1)))   Int[(d + e*x)^(q - 1)*(a + b*Log[c
*x^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && Gt
Q[q, 0]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

3.56.4 Maple [F]

$$\int \frac{1}{(dgx+cg)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(256) = 512$.

Time = 0.28 (sec) , antiderivative size = 770, normalized size of antiderivative = 3.01

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{\left(\left(Abd^2x^2 + 2\,Abcdx + Abc^2 + (Bbd^2x^2 + 2\,Bbcdx + Bbc^2)\log(e) + (Bbd^2nx^2 + 2\,Bbc\right.\right.}{(AB^2b^2c^2d^2 - 2\,AB^2abcd^3 + AB^2a^2d^4)g^3n^2x^2 + 2\,(AB^2b^2c^3d - 2\,AB^2abc^2d^2 + AB^2a^2cd^3)g^3n^2x + (AB^2}$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"fricas")
```

output
```
((A*b*d^2*x^2 + 2*A*b*c*d*x + A*b*c^2 + (B*b*d^2*x^2 + 2*B*b*c*d*x + B*b*c
^2)*log(e) + (B*b*d^2*n*x^2 + 2*B*b*c*d*n*x + B*b*c^2*n)*log((b*x + a)/(d*
x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)
/(B*n))/(d*x + c)) - ((B*b^2*c - B*a*b*d)*n*x + (B*a*b*c - B*a^2*d)*n)*e^(
2*(B*log(e) + A)/(B*n)) - 2*(A*d^3*x^2 + 2*A*c*d^2*x + A*c^2*d + (B*d^3*x^
2 + 2*B*c*d^2*x + B*c^2*d)*log(e) + (B*d^3*n*x^2 + 2*B*c*d^2*n*x + B*c^2*d
*n)*log((b*x + a)/(d*x + c)))*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*
(B*log(e) + A)/(B*n))/(d^2*x^2 + 2*c*d*x + c^2)))*e^(-2*(B*log(e) + A)/(B*
n))/((A*B^2*b^2*c^2*d^2 - 2*A*B^2*a*b*c*d^3 + A*B^2*a^2*d^4)*g^3*n^2*x^2 +
 2*(A*B^2*b^2*c^3*d - 2*A*B^2*a*b*c^2*d^2 + A*B^2*a^2*c*d^3)*g^3*n^2*x + (
A*B^2*b^2*c^4 - 2*A*B^2*a*b*c^3*d + A*B^2*a^2*c^2*d^2)*g^3*n^2 + ((B^3*b^2
*c^2*d^2 - 2*B^3*a*b*c*d^3 + B^3*a^2*d^4)*g^3*n^2*x^2 + 2*(B^3*b^2*c^3*d -
 2*B^3*a*b*c^2*d^2 + B^3*a^2*c*d^3)*g^3*n^2*x + (B^3*b^2*c^4 - 2*B^3*a*b*c
^3*d + B^3*a^2*c^2*d^2)*g^3*n^2)*log(e) + ((B^3*b^2*c^2*d^2 - 2*B^3*a*b*c*
d^3 + B^3*a^2*d^4)*g^3*n^3*x^2 + 2*(B^3*b^2*c^3*d - 2*B^3*a*b*c^2*d^2 + B^
3*a^2*c*d^3)*g^3*n^3*x + (B^3*b^2*c^4 - 2*B^3*a*b*c^3*d + B^3*a^2*c^2*d^2)
*g^3*n^3)*log((b*x + a)/(d*x + c)))
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.56.7 Maxima [F]

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"maxima")
```

output
```
-(b*x + a)/((b*c^3*g^3*n - a*c^2*d*g^3*n)*A*B + (b*c^3*g^3*n*log(e) - a*c^
2*d*g^3*n*log(e))*B^2 + ((b*c*d^2*g^3*n - a*d^3*g^3*n)*A*B + (b*c*d^2*g^3*
n*log(e) - a*d^3*g^3*n*log(e))*B^2)*x^2 + 2*((b*c^2*d*g^3*n - a*c*d^2*g^3*
n)*A*B + (b*c^2*d*g^3*n*log(e) - a*c*d^2*g^3*n*log(e))*B^2)*x + ((b*c*d^2*
g^3*n - a*d^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (
b*c^3*g^3*n - a*c^2*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^2*g^3*n - a*d
^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (b*c^3*g^3*n
 - a*c^2*d*g^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*x - b*c + 2*a*d)
/(((b*c*d^3*g^3*n - a*d^4*g^3*n)*A*B + (b*c*d^3*g^3*n*log(e) - a*d^4*g^3*n
*log(e))*B^2)*x^3 + (b*c^4*g^3*n - a*c^3*d*g^3*n)*A*B + (b*c^4*g^3*n*log(e
) - a*c^3*d*g^3*n*log(e))*B^2 + 3*((b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*A*B +
 (b*c^2*d^2*g^3*n*log(e) - a*c*d^3*g^3*n*log(e))*B^2)*x^2 + 3*((b*c^3*d*g^
3*n - a*c^2*d^2*g^3*n)*A*B + (b*c^3*d*g^3*n*log(e) - a*c^2*d^2*g^3*n*log(e
))*B^2)*x + ((b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n -
a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^
4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^3*g^3*n - a*d^4*g
^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g
^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((d*
x + c)^n)), x)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.27

$$\int \frac{1}{(cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \left(\frac{b\text{Ei}\left(\frac{\log(e)}{n}+\frac{A}{Bn}+\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{A}{Bn}\right)}}{(B^2bcg^3n^2-B^2adg^3n^2)e^{\left(\frac{1}{n}\right)}} - \frac{2\,d\text{Ei}\left(\frac{2\log(e)}{n}+\frac{2\,A}{Bn}+2\,\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{2\,A}{Bn}\right)}}{(B^2bcg^3n^2-B^2adg^3n^2)e^{\frac{2}{n}}} - \frac{}{B^2bcg^3n^2\log\left(\frac{bx}{dx}\right.}\right.$$

input
```
integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm=
"giac")
```

output
```
(b*Ei(log(e)/n + A/(B*n) + log((b*x + a)/(d*x + c)))*e^(-A/(B*n))/((B^2*b*
c*g^3*n^2 - B^2*a*d*g^3*n^2)*e^(1/n)) - 2*d*Ei(2*log(e)/n + 2*A/(B*n) + 2*
log((b*x + a)/(d*x + c)))*e^(-2*A/(B*n))/((B^2*b*c*g^3*n^2 - B^2*a*d*g^3*n
^2)*e^(2/n)) - ((b*x + a)*b/(d*x + c) - (b*x + a)^2*d/(d*x + c)^2)/(B^2*b*
c*g^3*n^2*log((b*x + a)/(d*x + c)) - B^2*a*d*g^3*n^2*log((b*x + a)/(d*x +
c)) + B^2*b*c*g^3*n*log(e) - B^2*a*d*g^3*n*log(e) + A*B*b*c*g^3*n - A*B*a*
d*g^3*n))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{{\left(c\,g+d\,g\,x\right)}^3\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

input
```
int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.57 $\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.57.1 Optimal result

Integrand size = 30, antiderivative size = 364

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c^2}{5b^4d^4}$$
$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,nx^2}{10b^3d^3}$$
$$-\frac{B(bc-ad)g^3(5bdf-bcg-adg)nx^3}{15b^2d^2}-\frac{B(bc-ad)g^4nx^4}{20bd}-\frac{B(bf-ag)^5n\log(a+bx)}{5b^5g}$$
$$+\frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g}+\frac{B(df-cg)^5n\log(c+dx)}{5d^5g}$$

output
```
1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*
g^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^
3*f^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f
)+b^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*
(-a*d*g-b*c*g+5*b*d*f)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*n*x^4/b/d-1/5*B
*(-a*g+b*f)^5*n*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/g+1/5*B*(-c*g+d*f)^5*n*ln(d*x+c)/d^5/g
```

3.57.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.78

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{\frac{B(-bc+ad)g^2nx\left(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10f\right.\right.}{12b^4d^4}}{}$$

input
```
Integrate[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((B*(-(b*c) + a*d)*g^2*n*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*
c*g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2
 + 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) -
2*c*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20
*f*g^2*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*n*Log[a + b*x])
/b^5 + (f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(d*f - c*g)
^5*n*Log[c + d*x])/d^5)/(5*g)
```

3.57.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\Big\downarrow \text{ 2947}$$
$$\frac{(f+gx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g}-\frac{Bn(bc-ad)\int\frac{(f+gx)^5}{(a+bx)(c+dx)}dx}{5g}$$
$$\Big\downarrow \text{ 93}$$

$$\frac{(f+gx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g}-\frac{Bn(bc-ad)\int\left(\frac{x^3g^5}{bd}+\frac{(5bdf-bcg-adg)x^2g^4}{b^2d^2}+\frac{((10d^2f^2-5cdgf+c^2g^2)b^2-adg(5df-cg)b+a^2d^2g^2)xg^3}{b^3d^3}+\frac{((10d^3f^3-10cd^2gf^2+5c^2dg^2}{}\right.}{5g}$$

$\downarrow$ 2009

$$\frac{(f+gx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g}-\frac{Bn(bc-ad)\left(\frac{g^3x^2(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2))}{2b^3d^3}-\frac{g^2x(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b}{b^4d^4}\right.}{5g}$$

input
```
Int[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*g) - (B*(b*c - a*d
)*n*(-((g^2*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10*d^2
*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f
*g^2 - c^3*g^3))*x)/(b^4*d^4)) + (g^3*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g)
 + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(2*b^3*d^3) + (g^4*(5*b*d*
f - b*c*g - a*d*g)*x^3)/(3*b^2*d^2) + (g^5*x^4)/(4*b*d) + ((b*f - a*g)^5*L
og[a + b*x])/(b^5*(b*c - a*d)) - ((d*f - c*g)^5*Log[c + d*x])/(d^5*(b*c -
a*d))))/(5*g)
```

3.57.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.57.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(350) = 700$.

Time = 5.46 (sec) , antiderivative size = 1160, normalized size of antiderivative = 3.19

method	result	size
parallelrisch	Expression too large to display	1160

input
```
int((g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/60*(12*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^5*g^4*n+60*A*x^4*a*b^5*
c*d^5*f*g^3*n+12*A*x^5*a*b^5*c*d^5*g^4*n-120*B*ln(b*x+a)*a^3*b^3*c*d^5*f^3
*g*n^2+60*B*ln(b*x+a)*a*b^5*c^5*d*f*g^3*n^2-120*B*ln(b*x+a)*a*b^5*c^4*d^2*
f^2*g^2*n^2+120*B*ln(b*x+a)*a*b^5*c^3*d^3*f^3*g*n^2+60*B*x^4*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^5*c*d^5*f*g^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5
*c*d^5*f^2*g^2*n+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^5*f^3*g*n+2
0*B*x^3*a^2*b^4*c*d^5*f*g^3*n^2-20*B*x^3*a*b^5*c^2*d^4*f*g^3*n^2+120*A*x^3
*a*b^5*c*d^5*f^2*g^2*n-30*B*x^2*a^3*b^3*c*d^5*f*g^3*n^2+60*B*x^2*a^2*b^4*c
*d^5*f^2*g^2*n^2+30*B*x^2*a*b^5*c^3*d^3*f*g^3*n^2-60*B*x^2*a*b^5*c^2*d^4*f
^2*g^2*n^2+120*A*x^2*a*b^5*c*d^5*f^3*g*n+60*B*x*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^5*c*d^5*f^4*n+60*B*x*a^4*b^2*c*d^5*f*g^3*n^2-120*B*x*a^3*b^3*c*d^5*f^2
*g^2*n^2+120*B*x*a^2*b^4*c*d^5*f^3*g*n^2-60*B*x*a*b^5*c^4*d^2*f*g^3*n^2+12
0*B*x*a*b^5*c^3*d^3*f^2*g^2*n^2-120*B*x*a*b^5*c^2*d^4*f^3*g*n^2-60*B*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^5*c^5*d*f*g^3*n+120*B*ln(e*((b*x+a)/(d*x+c))^n)*a
*b^5*c^4*d^2*f^2*g^2*n-120*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^3*d^3*f^3*g
*n-60*B*ln(b*x+a)*a^5*b*c*d^5*f*g^3*n^2+120*B*ln(b*x+a)*a^4*b^2*c*d^5*f^2*
g^2*n^2+12*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^6*g^4*n+12*B*ln(b*x+a)*a^6*
c*d^5*g^4*n^2-12*B*ln(b*x+a)*a*b^5*c^6*g^4*n^2+60*B*ln(e*((b*x+a)/(d*x+c))
^n)*a*b^5*c^2*d^4*f^4*n+60*B*ln(b*x+a)*a^2*b^4*c*d^5*f^4*n^2-60*B*ln(b*x+a
)*a*b^5*c^2*d^4*f^4*n^2+3*B*x^4*a^2*b^4*c*d^5*g^4*n^2-3*B*x^4*a*b^5*c^2...
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(350) = 700$.

Time = 0.66 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.02

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5g^4x^5+3\,(20\,Ab^5d^5fg^3-(Bb^5cd^4-Bab^4d^5)g^4n)x^4+4\,(30\,Ab^5d^5f^2g^2-(5\,(Bb^5cd^4-Bab^4d^5)fg\cdots}{}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^
4*d^5)*g^4*n)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^
5)*f*g^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*n)*x^3 + 6*(20*A*b^5*d^5*f
^3*g - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 - 5*(B*b^5*c^2*d^3 - B*a^2*
b^3*d^5)*f*g^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*n)*x^2 + 12*(5*B*a*b
^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*f^2*g^2 - 5*B*a^4*b
*d^5*f*g^3 + B*a^5*d^5*g^4)*n*log(b*x + a) - 12*(5*B*b^5*c*d^4*f^4 - 10*B*
b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*d*f*g^3 + B*b^5
*c^5*g^4)*n*log(d*x + c) + 12*(5*A*b^5*d^5*f^4 - (10*(B*b^5*c*d^4 - B*a*b^
4*d^5)*f^3*g - 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2 + 5*(B*b^5*c^3*d
^2 - B*a^3*b^2*d^5)*f*g^3 - (B*b^5*c^4*d - B*a^4*b*d^5)*g^4)*n)*x + 12*(B*
b^5*d^5*g^4*x^5 + 5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*
b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^5*f^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 +
5*B*b^5*d^5*f*g^3*n*x^4 + 10*B*b^5*d^5*f^2*g^2*n*x^3 + 10*B*b^5*d^5*f^3*g*
n*x^2 + 5*B*b^5*d^5*f^4*n*x)*log((b*x + a)/(d*x + c)))/(b^5*d^5)
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.73

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{5}Bg^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{5}Ag^4x^5+Bfg^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Afg^3x^4+2\,Bf^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+2\,Af^2g^2x^3+2\,Bf^3gx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+2\,Af^3gx^2$$
$$+\frac{1}{60}Bg^4n\left(\frac{12\,a^5\log(bx+a)}{b^5}-\frac{12\,c^5\log(dx+c)}{d^5}-\frac{3\,(b^4cd^3-ab^3d^4)x^4-4\,(b^4c^2d^2-a^2b^2d^4)x^3+6\,(b^4c}{b^4d^4}\right.$$
$$-\frac{1}{6}Bfg^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3-}{b^3d^3}\right.$$
$$+Bf^2g^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-2\,Bf^3gn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bf^4n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bf^4x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Af^4x$$

input
```
integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
1/5*B*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*g^4*x^5 + B*f
*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^3*x^4 + 2*B*f^2*g^
2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^2*g^2*x^3 + 2*B*f^3*g
*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^3*g*x^2 + 1/60*B*g^4*n
*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^
3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)
*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/6*B*f*g^3*n*(6*a^4*log(b*x
 + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b
^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*f^2*g^
2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2
)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*B*f^3*g*n*(a^2*log(b*x + a
)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^4*n*(a*log(b*x +
 a)/b - c*log(d*x + c)/d) + B*f^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + A*f^4*x
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11996 vs. $2(350) = 700$.

Time = 1.67 (sec) , antiderivative size = 11996, normalized size of antiderivative = 32.96

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
1/60*(12*(5*B*b^6*c^2*d^4*f^4*n - 10*B*a*b^5*c*d^5*f^4*n - 20*(b*x + a)*B*
b^5*c^2*d^5*f^4*n/(d*x + c) + 5*B*a^2*b^4*d^6*f^4*n + 40*(b*x + a)*B*a*b^4
*c*d^6*f^4*n/(d*x + c) + 30*(b*x + a)^2*B*b^4*c^2*d^6*f^4*n/(d*x + c)^2 -
20*(b*x + a)*B*a^2*b^3*d^7*f^4*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^3*c*d^7*
f^4*n/(d*x + c)^2 - 20*(b*x + a)^3*B*b^3*c^2*d^7*f^4*n/(d*x + c)^3 + 30*(b
*x + a)^2*B*a^2*b^2*d^8*f^4*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a*b^2*c*d^8*f
^4*n/(d*x + c)^3 + 5*(b*x + a)^4*B*b^2*c^2*d^8*f^4*n/(d*x + c)^4 - 20*(b*x
 + a)^3*B*a^2*b*d^9*f^4*n/(d*x + c)^3 - 10*(b*x + a)^4*B*a*b*c*d^9*f^4*n/(
d*x + c)^4 + 5*(b*x + a)^4*B*a^2*d^10*f^4*n/(d*x + c)^4 - 10*B*b^6*c^3*d^3
*f^3*g*n + 10*B*a*b^5*c^2*d^4*f^3*g*n + 50*(b*x + a)*B*b^5*c^3*d^4*f^3*g*n
/(d*x + c) + 10*B*a^2*b^4*c*d^5*f^3*g*n - 70*(b*x + a)*B*a*b^4*c^2*d^5*f^3
*g*n/(d*x + c) - 90*(b*x + a)^2*B*b^4*c^3*d^5*f^3*g*n/(d*x + c)^2 - 10*B*a
^3*b^3*d^6*f^3*g*n - 10*(b*x + a)*B*a^2*b^3*c*d^6*f^3*g*n/(d*x + c) + 150*
(b*x + a)^2*B*a*b^3*c^2*d^6*f^3*g*n/(d*x + c)^2 + 70*(b*x + a)^3*B*b^3*c^3
*d^6*f^3*g*n/(d*x + c)^3 + 30*(b*x + a)*B*a^3*b^2*d^7*f^3*g*n/(d*x + c) -
30*(b*x + a)^2*B*a^2*b^2*c*d^7*f^3*g*n/(d*x + c)^2 - 130*(b*x + a)^3*B*a*b
^2*c^2*d^7*f^3*g*n/(d*x + c)^3 - 20*(b*x + a)^4*B*b^2*c^3*d^7*f^3*g*n/(d*x
 + c)^4 - 30*(b*x + a)^2*B*a^3*b*d^8*f^3*g*n/(d*x + c)^2 + 50*(b*x + a)^3*
B*a^2*b*c*d^8*f^3*g*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a*b*c^2*d^8*f^3*g*n/(
d*x + c)^4 + 10*(b*x + a)^3*B*a^3*d^9*f^3*g*n/(d*x + c)^3 - 20*(b*x + a...
```

3.57.9 Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 1433, normalized size of antiderivative = 3.94

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
int((f + g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4
*n)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x^2*((20*A*a*c*f*g^3 +
20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 10*B*a*d*f^2*g^2*n
- 10*B*b*c*f^2*g^2*n)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*
c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*
d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*
g^3 + 20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*
n)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g
^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d +
 5*b*c))/(5*b*d)))/(2*b*d)) - x^3*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d
*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b
*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*
f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(15*b*d) + (
A*a*c*g^4)/(3*b*d)) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g + 20*A*b*c*f^3*g +
30*A*a*c*f^2*g^2 + 10*B*a*d*f^3*g*n - 10*B*b*c*f^3*g*n)/(5*b*d) - ((5*a*d
+ 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f
^2*g^2 + 10*B*a*d*f^2*g^2*n - 10*B*b*c*f^2*g^2*n)/(5*b*d) + ((5*a*d + 5*b*
c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^
4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) -
 (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*...
```

3.57. $\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.58 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

3.58.1 Optimal result

Integrand size = 30, antiderivative size = 235

$$\begin{aligned}
&\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,nx}{4b^3d^3} \\
&\quad -\frac{B(bc-ad)g^2(4bdf-bcg-adg)nx^2}{8b^2d^2}-\frac{B(bc-ad)g^3nx^3}{12bd}-\frac{B(bf-ag)^4n\log(a+bx)}{4b^4g} \\
&\quad +\frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g}+\frac{B(df-cg)^4n\log(c+dx)}{4d^4g}
\end{aligned}$$

output
```
-1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f
*g+6*d^2*f^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*n*x
^2/b^2/d^2-1/12*B*(-a*d+b*c)*g^3*n*x^3/b/d-1/4*B*(-a*g+b*f)^4*n*ln(b*x+a)/
b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/4*B*(-c*g+d*f)^4*n
*ln(d*x+c)/d^4/g
```

3.58.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= \frac{(f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{Bn\left(6bd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+3b^2d^2(bc-ad)g^3(4b}{6b^4d^4}}{4g}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(6*b*d*(b*c - a
*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g
 + c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 +
 2*b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*
(d*f - c*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

3.58.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) dx$$
$$\Big\downarrow \text{ 2947}$$
$$\frac{(f+gx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g} - \frac{Bn(bc-ad)\int \frac{(f+gx)^4}{(a+bx)(c+dx)}dx}{4g}$$
$$\Big\downarrow \text{ 93}$$
$$\frac{(f+gx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g} - $$
$$\frac{Bn(bc-ad)\int \left(\frac{x^2g^4}{bd}+\frac{(4bdf-bcg-adg)xg^3}{b^2d^2}+\frac{\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-adg(4df-cg)b+a^2d^2g^2\right)g^2}{b^3d^3}+\frac{(bf-ag)^4}{b^3(bc-ad)(a+bx)}+\frac{(df-cg}{d^3(ad-bc)}\right.}{4g}$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g}-$$
$$\frac{Bn(bc-ad)\left(\frac{g^2x(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2))}{b^3d^3}+\frac{(bf-ag)^4\log(a+bx)}{b^4(bc-ad)}+\frac{g^3x^2(-adg-bcg+4bdf)}{2b^2d^2}-\frac{(df-cg)^4\log(c+}{d^4(bc-ad)}\right.}{4g}$$

input
```
Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*g) - (B*(b*c - a*d
)*n*((g^2*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*
g + c^2*g^2))*x)/(b^3*d^3) + (g^3*(4*b*d*f - b*c*g - a*d*g)*x^2)/(2*b^2*d^
2) + (g^4*x^3)/(3*b*d) + ((b*f - a*g)^4*Log[a + b*x])/(b^4*(b*c - a*d)) -
((d*f - c*g)^4*Log[c + d*x])/(d^4*(b*c - a*d))))/(4*g)
```

3.58.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.58.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(223) = 446$.

Time = 2.96 (sec) , antiderivative size = 976, normalized size of antiderivative = 4.15

method	result
parallelrisch	$\frac{-24B\ln(bx+a)b^4c^3df\,g^2n^2-6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^4g^3n-6B\ln(bx+a)a^4d^4g^3n^2+6B\ln(bx+a)b^4c^4g^3n^2-3B\,a^3bc\,d^3g^3n^2+3}{}$

input
```
int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-24*B*ln(b*x+a)*b^4*c^3*d*f*g^2*n^2-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^
4*c^4*g^3*n-6*B*ln(b*x+a)*a^4*d^4*g^3*n^2+6*B*ln(b*x+a)*b^4*c^4*g^3*n^2-3*
B*a^3*b*c*d^3*g^3*n^2+3*B*a*b^3*c^3*d*g^3*n^2+6*B*x^4*ln(e*((b*x+a)/(d*x+c
))^n)*b^4*d^4*g^3*n+2*B*x^3*a*b^3*d^4*g^3*n^2-2*B*x^3*b^4*c*d^3*g^3*n^2+12
*B*a^2*b^2*c*d^3*f*g^2*n^2-12*B*a*b^3*c^2*d^2*f*g^2*n^2-36*A*a*b^3*c*d^3*f
^2*g*n-3*B*x^2*a^2*b^2*d^4*g^3*n^2+3*B*x^2*b^4*c^2*d^2*g^3*n^2+6*B*x*a^3*b
*d^4*g^3*n^2-6*B*x*b^4*c^3*d*g^3*n^2+36*B*ln(b*x+a)*b^4*c^2*d^2*f^2*g*n^2+
24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f*g^2*n+36*B*x^2*ln(e*((b*x+a)/
(d*x+c))^n)*b^4*d^4*f^2*g*n+12*B*x^2*a*b^3*d^4*f*g^2*n^2-12*B*x^2*b^4*c*d^
3*f*g^2*n^2-24*B*x*a^2*b^2*d^4*f*g^2*n^2+36*B*x*a*b^3*d^4*f^2*g*n^2+24*B*x
*b^4*c^2*d^2*f*g^2*n^2-36*B*x*b^4*c*d^3*f^2*g*n^2+24*B*ln(e*((b*x+a)/(d*x+
c))^n)*b^4*c^3*d*f*g^2*n-36*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^2*f^2*g*
n+24*B*ln(b*x+a)*a^3*b*d^4*f*g^2*n^2-36*B*ln(b*x+a)*a^2*b^2*d^4*f^2*g*n^2+
24*B*ln(b*x+a)*a*b^3*d^4*f^3*n^2+24*A*x^3*b^4*d^4*f*g^2*n+36*A*x^2*b^4*d^4
*f^2*g*n+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*f^3*n+24*B*ln(e*((b*x+a)
/(d*x+c))^n)*b^4*c*d^3*f^3*n-24*B*ln(b*x+a)*b^4*c*d^3*f^3*n^2+24*B*a^3*b*d
^4*f*g^2*n^2-36*B*a^2*b^2*d^4*f^2*g*n^2-24*B*b^4*c^3*d*f*g^2*n^2+36*B*b^4*
c^2*d^2*f^2*g*n^2+24*A*x*b^4*d^4*f^3*n-24*A*a*b^3*d^4*f^3*n-24*A*b^4*c*d^3
*f^3*n-6*B*a^4*d^4*g^3*n^2+6*B*b^4*c^4*g^3*n^2+6*A*x^4*b^4*d^4*g^3*n)/b^4/
d^4/n
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(223) = 446$.

Time = 0.41 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.22

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4g^3x^4+2\,(12\,Ab^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3n)x^3+3\,(12\,Ab^4d^4f^2g-(4\,(Bb^4cd^3-Bab^3d^4)fg^2}{}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3
*d^4)*g^3*n)*x^3 + 3*(12*A*b^4*d^4*f^2*g - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*
f*g^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*n)*x^2 + 6*(4*B*a*b^3*d^4*f^3
 - 6*B*a^2*b^2*d^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*n*log(b*x
+ a) - 6*(4*B*b^4*c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2
- B*b^4*c^4*g^3)*n*log(d*x + c) + 6*(4*A*b^4*d^4*f^3 - (6*(B*b^4*c*d^3 - B
*a*b^3*d^4)*f^2*g - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 + (B*b^4*c^3*d
 - B*a^3*b*d^4)*g^3)*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 +
 6*B*b^4*d^4*f^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*log(e) + 6*(B*b^4*d^4*g^3*n*x^
4 + 4*B*b^4*d^4*f*g^2*n*x^3 + 6*B*b^4*d^4*f^2*g*n*x^2 + 4*B*b^4*d^4*f^3*n*
x)*log((b*x + a)/(d*x + c)))/(b^4*d^4)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.89

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$\begin{aligned}
&= \frac{1}{4}\,Bg^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}\,Ag^3x^4+Bfg^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)\\
&\quad +Afg^2x^3+\frac{3}{2}\,Bf^2gx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}\,Af^2gx^2\\
&\quad -\frac{1}{24}\,Bg^3n\left(\frac{6\,a^4\log\left(bx+a\right)}{b^4}-\frac{6\,c^4\log\left(dx+c\right)}{d^4}+\frac{2\left(b^3cd^2-ab^2d^3\right)x^3-3\left(b^3c^2d-a^2bd^3\right)x^2+6\left(b^3c^3-\right.}{b^3d^3}\right.\\
&\quad +\frac{1}{2}\,Bfg^2n\left(\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{\left(b^2cd-abd^2\right)x^2-2\left(b^2c^2-a^2d^2\right)x}{b^2d^2}\right)\\
&\quad -\frac{3}{2}\,Bf^2gn\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{\left(bc-ad\right)x}{bd}\right)\\
&\quad +Bf^3n\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)+Bf^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Af^3x
\end{aligned}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
1/4*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*g^3*x^4 + B*f
*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^2*x^3 + 3/2*B*f^2*
g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*f^2*g*x^2 - 1/24*B*g^
3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3)) + 1/2*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 -
 ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*f^
2*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
+ B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f^3*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*f^3*x
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6772 vs. 2(223) = 446.

Time = 1.05 (sec) , antiderivative size = 6772, normalized size of antiderivative = 28.82

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n - 12*(b*x + a)*B*b^
4*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2*b^3*d^5*f^3*n + 24*(b*x + a)*B*a*b^3*c
*d^5*f^3*n/(d*x + c) + 12*(b*x + a)^2*B*b^3*c^2*d^5*f^3*n/(d*x + c)^2 - 12
*(b*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x + c) - 24*(b*x + a)^2*B*a*b^2*c*d^6*f^
3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^2*c^2*d^6*f^3*n/(d*x + c)^3 + 12*(b*x
+ a)^2*B*a^2*b*d^7*f^3*n/(d*x + c)^2 + 8*(b*x + a)^3*B*a*b*c*d^7*f^3*n/(d*
x + c)^3 - 4*(b*x + a)^3*B*a^2*d^8*f^3*n/(d*x + c)^3 - 6*B*b^5*c^3*d^2*f^2
*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 24*(b*x + a)*B*b^4*c^3*d^3*f^2*g*n/(d*x
 + c) + 6*B*a^2*b^3*c*d^4*f^2*g*n - 36*(b*x + a)*B*a*b^3*c^2*d^4*f^2*g*n/(
d*x + c) - 30*(b*x + a)^2*B*b^3*c^3*d^4*f^2*g*n/(d*x + c)^2 - 6*B*a^3*b^2*
d^5*f^2*g*n + 54*(b*x + a)^2*B*a*b^2*c^2*d^5*f^2*g*n/(d*x + c)^2 + 12*(b*x
 + a)^3*B*b^2*c^3*d^5*f^2*g*n/(d*x + c)^3 + 12*(b*x + a)*B*a^3*b*d^6*f^2*g
*n/(d*x + c) - 18*(b*x + a)^2*B*a^2*b*c*d^6*f^2*g*n/(d*x + c)^2 - 24*(b*x
+ a)^3*B*a*b*c^2*d^6*f^2*g*n/(d*x + c)^3 - 6*(b*x + a)^2*B*a^3*d^7*f^2*g*n
/(d*x + c)^2 + 12*(b*x + a)^3*B*a^2*c*d^7*f^2*g*n/(d*x + c)^3 + 4*B*b^5*c^
4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g^2*n - 16*(b*x + a)*B*b^4*c^4*d^2*f*g^2
*n/(d*x + c) + 16*(b*x + a)*B*a*b^3*c^3*d^3*f*g^2*n/(d*x + c) + 24*(b*x +
a)^2*B*b^3*c^4*d^3*f*g^2*n/(d*x + c)^2 - 4*B*a^3*b^2*c*d^4*f*g^2*n + 12*(b
*x + a)*B*a^2*b^2*c^2*d^4*f*g^2*n/(d*x + c) - 36*(b*x + a)^2*B*a*b^2*c^3*d
^4*f*g^2*n/(d*x + c)^2 - 12*(b*x + a)^3*B*b^2*c^4*d^4*f*g^2*n/(d*x + c)...
```

3.58.9 Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 766, normalized size of antiderivative = 3.26

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g\,n-6\,B\,b\,c\,f^2\,g\,n}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\cdots}{\cdots}\right)}{4\,b\,d}$$

$$\left.-\frac{a\,c\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\,g^2+12\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2\,n-4\,B\,b\,c\,f\,g^2\,n}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)$$

$$+x^3\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2+B\,a\,d\,g^3\,n-B\,b\,c\,g^3\,n}{12\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)+\frac{A\,g^3\,x^4}{4}$$

$$-\frac{\ln(a+b\,x)\,\left(B\,n\,a^4\,g^3-4\,B\,n\,a^3\,b\,f\,g^2+6\,B\,n\,a^2\,b^2\,f^2\,g-4\,B\,n\,a\,b^3\,f^3\right)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,n\,c^4\,g^3-4\,B\,n\,c^3\,d\,f\,g^2+6\,B\,n\,c^2\,d^2\,f^2\,g-4\,B\,n\,c\,d^3\,f^3\right)}{4\,d^4}$$

input
```
int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a
*d*f^2*g*n - 6*B*b*c*f^2*g*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 +
 4*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^
3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A
*a*d*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f
*g^2*n)/(4*b*d) + (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b
*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a
*d + 4*b*c))/(4*b*d)))/(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b
*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4
*b*d))*(4*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c
*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f*g^2*n)/(8*b*d) + (A*
a*c*g^3)/(2*b*d)) + x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a
*d*g^3*n - B*b*c*g^3*n)/(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + log
(e*((a + b*x)/(c + d*x))^n)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 +
 B*f*g^2*x^3) + (A*g^3*x^4)/4 - (log(a + b*x)*(B*a^4*g^3*n - 4*B*a*b^3*f^3
*n - 4*B*a^3*b*f*g^2*n + 6*B*a^2*b^2*f^2*g*n))/(4*b^4) + (log(c + d*x)*(B*
c^4*g^3*n - 4*B*c*d^3*f^3*n - 4*B*c^3*d*f*g^2*n + 6*B*c^2*d^2*f^2*g*n))/(4
*d^4)
```

3.58. $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.59 $\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.59.1 Optimal result

Integrand size = 30, antiderivative size = 157

$$\begin{aligned}&\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=-\frac{B(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2}-\frac{B(bc-ad)g^2nx^2}{6bd}-\frac{B(bf-ag)^3n\log(a+bx)}{3b^3g}\\&\quad+\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g}+\frac{B(df-cg)^3n\log(c+dx)}{3d^3g}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^
2*n*x^2/b/d-1/3*B*(-a*g+b*f)^3*n*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/g+1/3*B*(-c*g+d*f)^3*n*ln(d*x+c)/d^3/g
```

3.59.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{Bn(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg}{2b^3d^3}}{3g}\end{aligned}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(2*b*d*(b*c - a
*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*
(b*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3
))/(3*g)
```

3.59.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bc-ad)\int \frac{(f+gx)^3}{(a+bx)(c+dx)}dx}{3g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bc-ad)\int \left(\frac{xg^3}{bd}+\frac{(3bdf-bcg-adg)g^2}{b^2d^2}+\frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)}+\frac{(df-cg)^3}{d^2(ad-bc)(c+dx)}\right)dx}{3g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bc-ad)\left(\frac{(bf-ag)^3\log(a+bx)}{b^3(bc-ad)}+\frac{g^2x(-adg-bcg+3bdf)}{b^2d^2}-\frac{(df-cg)^3\log(c+dx)}{d^3(bc-ad)}+\frac{g^3x^2}{2bd}\right)}{3g}$$

input
```
Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g) - (B*(b*c - a*d
)*n*((g^2*(3*b*d*f - b*c*g - a*d*g)*x)/(b^2*d^2) + (g^3*x^2)/(2*b*d) + ((b
*f - a*g)^3*Log[a + b*x])/(b^3*(b*c - a*d)) - ((d*f - c*g)^3*Log[c + d*x])
/(d^3*(b*c - a*d))))/(3*g)
```

3.59.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.59.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(147) = 294$.

Time = 1.63 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.93

method	result
parallelrisch	$\frac{-6Aa\,b^2c\,d^2fgn+6B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^3fgn+6Bxa\,b^2d^3fg\,n^2-6Bx\,b^3c\,d^2fg\,n^2-6B\ln(bx+a)a^2b\,d^3fg\,n^2+6B\ln(bx+a)}{}$

input
```
int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-6*A*a*b^2*c*d^2*f*g*n+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*f*g*
n+6*B*x*a*b^2*d^3*f*g*n^2-6*B*x*b^3*c*d^2*f*g*n^2-6*B*ln(b*x+a)*a^2*b*d^3*
f*g*n^2+6*B*ln(b*x+a)*b^3*c^2*d*f*g*n^2-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*
c^2*d*f*g*n-6*B*a^2*b*d^3*f*g*n^2+6*B*b^3*c^2*d*f*g*n^2+6*B*ln(e*((b*x+a)/
(d*x+c))^n)*b^3*c*d^2*f^2*n-6*A*a*b^2*d^3*f^2*n-6*A*b^3*c*d^2*f^2*n+6*A*x*
b^3*d^3*f^2*n+2*B*a^3*d^3*g^2*n^2-2*B*b^3*c^3*g^2*n^2+6*A*x^2*b^3*d^3*f*g*
n+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*f^2*n+6*B*ln(b*x+a)*a*b^2*d^3*f^
2*n^2-6*B*ln(b*x+a)*b^3*c*d^2*f^2*n^2+2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^
3*d^3*g^2*n+B*x^2*a*b^2*d^3*g^2*n^2-B*x^2*b^3*c*d^2*g^2*n^2-2*B*x*a^2*b*d^
3*g^2*n^2+2*B*x*b^3*c^2*d*g^2*n^2+B*a^2*b*c*d^2*g^2*n^2-B*a*b^2*c^2*d*g^2*
n^2+2*A*x^3*b^3*d^3*g^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g^2*n+2*B*
ln(b*x+a)*a^3*d^3*g^2*n^2-2*B*ln(b*x+a)*b^3*c^3*g^2*n^2)/b^3/d^3/n
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(147) = 294$.

Time = 0.33 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.13

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3g^2x^3 + (6\,Ab^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2n)x^2 + 2\,(3\,Bab^2d^3f^2 - 3\,Ba^2bd^3fg + Ba^3d^3g^2)n\log(b}{}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*
g^2*n)*x^2 + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3*d^3*g^2)*n*l
og(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c^3*g^2)*n*
log(d*x + c) + 2*(3*A*b^3*d^3*f^2 - (3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g - (
B*b^3*c^2*d - B*a^2*b*d^3)*g^2)*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*
f*g*x^2 + 3*B*b^3*d^3*f^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n*x^3 + 3*B*b^3*d^3
*f*g*n*x^2 + 3*B*b^3*d^3*f^2*n*x)*log((b*x + a)/(d*x + c)))/(b^3*d^3)
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.80

$$\begin{aligned}\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= \frac{1}{3}Bg^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Ag^2x^3\\ &+Bfgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afgx^2\\ &+\frac{1}{6}Bg^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)\\ &-Bfgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)\\ &+Bf^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bf^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Af^2x\end{aligned}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
1/3*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*g^2*x^3 + B*f
*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g*x^2 + 1/6*B*g^2*n*(2
*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*f*g*n*(a^2*log(b*x + a)/b^2 - c^
2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^2*n*(a*log(b*x + a)/b - c*
log(d*x + c)/d) + B*f^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f^2*x
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3408 vs. $2(147) = 294$.

Time = 0.72 (sec) , antiderivative size = 3408, normalized size of antiderivative = 21.71

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*
c^2*d^3*f^2*n/(d*x + c) + 3*B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d
^4*f^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*
x + a)*B*a^2*b*d^5*f^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x
+ c)^2 + 3*(b*x + a)^2*B*a^2*d^6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n +
 3*B*a*b^3*c^2*d^2*f*g*n + 9*(b*x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B
*a^2*b^2*c*d^3*f*g*n - 15*(b*x + a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b
*x + a)^2*B*b^2*c^3*d^3*f*g*n/(d*x + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x +
 a)*B*a^2*b*c*d^4*f*g*n/(d*x + c) + 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*
x + c)^2 + 3*(b*x + a)*B*a^3*d^5*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d
^5*f*g*n/(d*x + c)^2 + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)
*B*b^3*c^4*d*g^2*n/(d*x + c) + 3*(b*x + a)*B*a*b^2*c^3*d^2*g^2*n/(d*x + c)
 + 3*(b*x + a)^2*B*b^2*c^4*d^2*g^2*n/(d*x + c)^2 - B*a^3*b*c*d^3*g^2*n + 3
*(b*x + a)*B*a^2*b*c^2*d^3*g^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c^3*d^3*g
^2*n/(d*x + c)^2 + B*a^4*d^4*g^2*n - 3*(b*x + a)*B*a^3*c*d^4*g^2*n/(d*x +
c) + 3*(b*x + a)^2*B*a^2*c^2*d^4*g^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c
))/(b^3*d^3 - 3*(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a)^2*b*d^5/(d*x + c
)^2 - (b*x + a)^3*d^6/(d*x + c)^3) - (6*B*b^6*c^3*d*f*g*n - 18*B*a*b^5*c^2
*d^2*f*g*n - 12*(b*x + a)*B*b^5*c^3*d^2*f*g*n/(d*x + c) + 18*B*a^2*b^4*c*d
^3*f*g*n + 36*(b*x + a)*B*a*b^4*c^2*d^3*f*g*n/(d*x + c) + 6*(b*x + a)^2...
```

3.59.9 Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.36

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)}{3\,b\,d}\right.$$
$$-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+3\,B\,a\,d\,f\,g\,n-3\,B\,b\,c\,f\,g\,n}{3\,b\,d}$$
$$\left.+\frac{A\,a\,c\,g^2}{b\,d}\right)+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)$$
$$+\frac{A\,g^2\,x^3}{3}+\frac{\ln(a+b\,x)\,\left(B\,n\,a^3\,g^2-3\,B\,n\,a^2\,b\,f\,g+3\,B\,n\,a\,b^2\,f^2\right)}{3\,b^3}$$
$$-\frac{\ln(c+d\,x)\,\left(B\,n\,c^3\,g^2-3\,B\,n\,c^2\,d\,f\,g+3\,B\,n\,c\,d^2\,f^2\right)}{3\,d^3}$$

input
```
int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)
/(6*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*(((3*a*d + 3*b*c)*((3*A*a*
d*g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)/(3*b*d) - (
A*g^2*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*
A*a*d*f*g + 6*A*b*c*f*g + 3*B*a*d*f*g*n - 3*B*b*c*f*g*n)/(3*b*d) + (A*a*c*
g^2)/(b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^2*x^3)/3 + B*f^2*x + B*
f*g*x^2) + (A*g^2*x^3)/3 + (log(a + b*x)*(B*a^3*g^2*n + 3*B*a*b^2*f^2*n -
3*B*a^2*b*f*g*n))/(3*b^3) - (log(c + d*x)*(B*c^3*g^2*n + 3*B*c*d^2*f^2*n -
 3*B*c^2*d*f*g*n))/(3*d^3)
```

3.60 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

3.60.1 Optimal result

Integrand size = 28, antiderivative size = 115

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=-\frac{B(bc-ad)gnx}{2bd}-\frac{B(bf-ag)^2n\log(a+bx)}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g}+\frac{B(df-cg)^2n\log(c+dx)}{2d^2g}$$

output
```
-1/2*B*(-a*d+b*c)*g*n*x/b/d-1/2*B*(-a*g+b*f)^2*n*ln(b*x+a)/b^2/g+1/2*(g*x+
f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/2*B*(-c*g+d*f)^2*n*ln(d*x+c)/d^2/
g
```

3.60.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{-Bd^2(bf-ag)^2n\log(a+bx)+b\left(d(B(-bc+ad)g^2nx+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2b^2d^2g}$$

input
```
Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(-(B*d^2*(b*f - a*g)^2*n*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*n*x +
A*b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[e*((a + b*x)/(c + d*x))^n] +
b*B*(d*f - c*g)^2*n*Log[c + d*x]))/(2*b^2*d^2*g)
```

3.60.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bc-ad)\int \frac{(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bc-ad)\int \left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(ad-bc)(c+dx)}\right)dx}{2g}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bc-ad)\left(\frac{(bf-ag)^2\log(a+bx)}{b^2(bc-ad)}-\frac{(df-cg)^2\log(c+dx)}{d^2(bc-ad)}+\frac{g^2x}{bd}\right)}{2g}$$

input
```
Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*g) - (B*(b*c - a*d
)*n*((g^2*x)/(b*d) + ((b*f - a*g)^2*Log[a + b*x])/(b^2*(b*c - a*d)) - ((d*
f - c*g)^2*Log[c + d*x])/(d^2*(b*c - a*d))))/(2*g)
```

3.60.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(107) = 214$.

Time = 0.81 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.18

method	result
parallelrisch	$\frac{-2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)ab\,d^{2}f-2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}cdf-B\ln(bx+a)a^{2}d^{2}gn+B\ln(dx+c)b^{2}c^{2}gn+B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}d^{2}g-B}{}$

input
```
int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*f-2*B*ln(e*((b*x+a)/(d*x+c))^n
)*b^2*c*d*f-B*ln(b*x+a)*a^2*d^2*g*n+B*ln(d*x+c)*b^2*c^2*g*n+B*x^2*ln(e*((b
*x+a)/(d*x+c))^n)*b^2*d^2*g-B*a^2*d^2*g*n+B*b^2*c^2*g*n-4*B*ln(d*x+c)*b^2*
c*d*f*n+B*x*a*b*d^2*g*n-B*x*b^2*c*d*g*n-B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*
d*g+4*B*ln(b*x+a)*a*b*d^2*f*n+2*B*ln(b*x+a)*b^2*c*d*f*n-2*B*ln(d*x+c)*a*b*
d^2*f*n+B*ln(b*x+a)*a*b*c*d*g*n-B*ln(d*x+c)*a*b*c*d*g*n-A*a*b*c*d*g-2*A*a*
b*d^2*f-2*A*b^2*c*d*f+2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*f+A*x^2*b^2*
d^2*g+2*A*x*b^2*d^2*f)/b^2/d^2
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.56

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{Ab^2d^2gx^2+(2\,Babd^2f-Ba^2d^2g)n\log(bx+a)-(2\,Bb^2cdf-Bb^2c^2g)n\log(dx+c)+(2\,Ab^2d^2f-(Bb^2}{2\,b^2d^2}$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + (2*B*a*b*d^2*f - B*a^2*d^2*g)*n*log(b*x + a) - (2*B
*b^2*c*d*f - B*b^2*c^2*g)*n*log(d*x + c) + (2*A*b^2*d^2*f - (B*b^2*c*d - B
*a*b*d^2)*g*n)*x + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log(e) + (B*b^2*d^2
*g*n*x^2 + 2*B*b^2*d^2*f*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d^2)
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(97) = 194.

Time = 59.23 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.29

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\begin{cases}\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right)\left(fx+\frac{gx^2}{2}\right)\\ Afx+\frac{Agx^2}{2}-\frac{Bc^2g\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d^2}+\frac{Bcf\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}-\frac{Bcgnx}{2d}+Bfnx+Bfx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bgnx^2}{4}+\cdots\\ Afx+\frac{Agx^2}{2}-\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b^2}+\frac{Baf\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}+\frac{Bagnx}{2b}-Bfnx+Bfx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bgnx^2}{4}\cdots\\ Afx+\frac{Agx^2}{2}-\frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b^2}-\frac{Ba^2g\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2}+\frac{Bafn\log\left(\frac{c}{d}+x\right)}{b}+\frac{Baf\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b}+\frac{Bagnx}{2b}\cdots\end{cases}$$

input
```
integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise(((A + B*log(e*(a/c)**n))*(f*x + g*x**2/2), Eq(b, 0) & Eq(d, 0)),
 (A*f*x + A*g*x**2/2 - B*c**2*g*log(e*(a/(c + d*x))**n)/(2*d**2) + B*c*f*l
og(e*(a/(c + d*x))**n)/d - B*c*g*n*x/(2*d) + B*f*n*x + B*f*x*log(e*(a/(c +
 d*x))**n) + B*g*n*x**2/4 + B*g*x**2*log(e*(a/(c + d*x))**n)/2, Eq(b, 0)),
 (A*f*x + A*g*x**2/2 - B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b**2) + B*a*f*l
og(e*(a/c + b*x/c)**n)/b + B*a*g*n*x/(2*b) - B*f*n*x + B*f*x*log(e*(a/c +
b*x/c)**n) - B*g*n*x**2/4 + B*g*x**2*log(e*(a/c + b*x/c)**n)/2, Eq(d, 0)),
 (A*f*x + A*g*x**2/2 - B*a**2*g*n*log(c/d + x)/(2*b**2) - B*a**2*g*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)/(2*b**2) + B*a*f*n*log(c/d + x)/b + B*a*f
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*g*n*x/(2*b) + B*c**2*g*n*
log(c/d + x)/(2*d**2) - B*c*f*n*log(c/d + x)/d - B*c*g*n*x/(2*d) + B*f*x*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*g*x**2*log(e*(a/(c + d*x) + b*x
/(c + d*x))**n)/2, True))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.30

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}\,Bgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}\,Agx^2$$
$$-\frac{1}{2}\,Bgn\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bfn\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)+Bfx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afx$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

output
```
1/2*B*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*g*x^2 - 1/2*B*g
*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B
*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + A*f*x
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(107) = 214$.

Time = 0.57 (sec) , antiderivative size = 1215, normalized size of antiderivative = 10.57

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*
f*n/(d*x + c) + 2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c)
- 2*(b*x + a)*B*a^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n
+ 2*(b*x + a)*B*b^2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*
B*a*b*c^2*d^2*g*n/(d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/
(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*d^3/(d*x + c)
 + (b*x + a)^2*d^4/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (
b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B
*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3
*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - 2*B*b^4*c^2*d*f*log(e
) + 4*B*a*b^3*c*d^2*f*log(e) + 2*(b*x + a)*B*b^3*c^2*d^2*f*log(e)/(d*x + c
) - 2*B*a^2*b^2*d^3*f*log(e) - 4*(b*x + a)*B*a*b^2*c*d^3*f*log(e)/(d*x + c
) + 2*(b*x + a)*B*a^2*b*d^4*f*log(e)/(d*x + c) + B*b^4*c^3*g*log(e) - B*a*
b^3*c^2*d*g*log(e) - 2*(b*x + a)*B*b^3*c^3*d*g*log(e)/(d*x + c) - B*a^2*b^
2*c*d^2*g*log(e) + 4*(b*x + a)*B*a*b^2*c^2*d^2*g*log(e)/(d*x + c) + B*a^3*
b*d^3*g*log(e) - 2*(b*x + a)*B*a^2*b*c*d^3*g*log(e)/(d*x + c) - 2*A*b^4*c^
2*d*f + 4*A*a*b^3*c*d^2*f + 2*(b*x + a)*A*b^3*c^2*d^2*f/(d*x + c) - 2*A*a^
2*b^2*d^3*f - 4*(b*x + a)*A*a*b^2*c*d^3*f/(d*x + c) + 2*(b*x + a)*A*a^2*b*
d^4*f/(d*x + c) + A*b^4*c^3*g - A*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*
g/(d*x + c) - A*a^2*b^2*c*d^2*g + 4*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + ...
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.33

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=x\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g\,n-B\,b\,c\,g\,n}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)-\frac{\ln(a+b\,x)\,\left(B\,a^2\,g\,n-2\,B\,a\,b\,f\,n\right)}{2\,b^2}$$
$$+\frac{\ln(c+d\,x)\,\left(B\,c^2\,g\,n-2\,B\,c\,d\,f\,n\right)}{2\,d^2}+\frac{A\,g\,x^2}{2}$$

input
```
int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((2*A*a*d*g + 2*A*b*c*g + 2*A*b*d*f + B*a*d*g*n - B*b*c*g*n)/(2*b*d) - (
A*g*(2*a*d + 2*b*c))/(2*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*(B*f*x + (B
*g*x^2)/2) - (log(a + b*x)*(B*a^2*g*n - 2*B*a*b*f*n))/(2*b^2) + (log(c + d
*x)*(B*c^2*g*n - 2*B*c*d*f*n))/(2*d^2) + (A*g*x^2)/2
```

3.61 $\int \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

3.61.1 Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = Ax+\frac{B(a+bx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{B(bc-ad)n\log(c+dx)}{bd}$$

output
```
A*x+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b-B*(-a*d+b*c)*n*ln(d*x+c)/b/d
```

3.61.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = Ax+\frac{B(a+bx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{B(bc-ad)n\log(c+dx)}{bd}$$

input
```
Integrate[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

output
```
A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Lo
g[c + d*x])/(b*d)
```

3.61.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

input
```
Int[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

output
```
A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Lo
g[c + d*x])/(b*d)
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.61.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

method	result	size
default	$Ax+B\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)x+n(ad-cb)\left(-\frac{c\ln(dx+c)}{(ad-cb)d}+\frac{a\ln(bx+a)}{(ad-cb)b}\right)\right)$	82
parts	$Ax+B\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)x+n(ad-cb)\left(-\frac{c\ln(dx+c)}{(ad-cb)d}+\frac{a\ln(bx+a)}{(ad-cb)b}\right)\right)$	82
parallelrisch	$\frac{B\left(\ln(bx+a)ad\,n^2-\ln(bx+a)bc\,n^2+x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)bdn+\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)bcn\right)}{bdn}+Ax$	87

input
```
int(A+B*ln(e*((b*x+a)/(d*x+c))^n),x,method=_RETURNVERBOSE)
```

output
```
A*x+B*(ln(e*((b*x+a)/(d*x+c))^n)*x+n*(a*d-b*c)*(-c/(a*d-b*c)/d*ln(d*x+c)+a
/(a*d-b*c)/b*ln(b*x+a)))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{Bbdnx\log\left(\frac{bx+a}{dx+c}\right)+Badn\log\left(bx+a\right)-Bbcn\log\left(dx+c\right)+Bbdx\log\left(e\right)+Abdx}{bd}$$

input
```
integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
(B*b*d*n*x*log((b*x + a)/(d*x + c)) + B*a*d*n*log(b*x + a) - B*b*c*n*log(d
*x + c) + B*b*d*x*log(e) + A*b*d*x)/(b*d)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.68

$$\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=Ax$$
$$+B\left(\begin{cases}x\log\left(e\left(\frac{a}{c}\right)^n\right) & \text{for } b=0\wedge d=0\\ \frac{c\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d}+nx+x\log\left(e\left(\frac{a}{c+dx}\right)^n\right) & \text{for } b=0\\ \frac{a\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-nx+x\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right) & \text{for } d=0\\ \frac{an\log\left(c+dx\right)}{b}+\frac{a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b}-\frac{cn\log\left(c+dx\right)}{d}+x\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(A+B*ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
A*x + B*Piecewise((x*log(e*(a/c)**n), Eq(b, 0) & Eq(d, 0)), (c*log(e*(a/(c
 + d*x))**n)/d + n*x + x*log(e*(a/(c + d*x))**n), Eq(b, 0)), (a*log(e*(a/c
 + b*x/c)**n)/b - n*x + x*log(e*(a/c + b*x/c)**n), Eq(d, 0)), (a*n*log(c +
 d*x)/b + a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b - c*n*log(c + d*x)/d
 + x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), True))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = Bn\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right) + Bx\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+Ax$$

input
```
integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*x*log(e*((b*x + a)/(d*x + c)
)^n) + A*x
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(56) = 112.

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.34

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= B\left(\frac{(b^2c^2n-2\,abcdn+a^2d^2n)\log\left(\frac{bx+a}{dx+c}\right)}{bd-\frac{(bx+a)d^2}{dx+c}}+\frac{b^2c^2\log\left(e\right)-2\,abcd\log\left(e\right)+a^2d^2\log\left(e\right)}{bd-\frac{(bx+a)d^2}{dx+c}}+\frac{(b^2c^2n-2\,abcdn}{}\right.$$
$$+Ax$$

input
```
integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

output
```
B*((b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d - (
b*x + a)*d^2/(d*x + c)) + (b^2*c^2*log(e) - 2*a*b*c*d*log(e) + a^2*d^2*log
(e))/(b*d - (b*x + a)*d^2/(d*x + c)) + (b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*
n)*log(b - (b*x + a)*d/(d*x + c))/(b*d) - (b^2*c^2*n - 2*a*b*c*d*n + a^2*d
^2*n)*log((b*x + a)/(d*x + c))/(b*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)
^2) + A*x
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = A\,x+B\,x\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right) + \frac{B\,a\,n\,\ln\left(a+b\,x\right)}{b}-\frac{B\,c\,n\,\ln\left(c+d\,x\right)}{d}$$

input
```
int(A + B*log(e*((a + b*x)/(c + d*x))^n),x)
```

output
```
A*x + B*x*log(e*((a + b*x)/(c + d*x))^n) + (B*a*n*log(a + b*x))/b - (B*c*n
*log(c + d*x))/d
```

3.62 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx$

3.62.1 Optimal result

Integrand size = 30, antiderivative size = 147

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}$$

output
```
-B*n*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*((b*x+a)/(d*x+c))^n))
*ln(g*x+f)/g+B*n*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*n*polylog(2,b*(g*
x+f)/(-a*g+b*f))/g+B*n*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

3.62.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx$$
$$= \frac{\left(A-Bn\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-Bn\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)+}{g}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

output
```
((A - B*n*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[e*((a + b*x)/(c + d*x)
)^n] + B*n*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*n*PolyLog[2
, (b*(f + g*x))/(b*f - a*g)] + B*n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/
g
```

3.62.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2945, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{f+gx}\,dx$$
$$\Big\downarrow \ 2945$$
$$-\frac{bBn\int \frac{\log(f+gx)}{a+bx}dx}{g}+\frac{Bdn\int \frac{\log(f+gx)}{c+dx}dx}{g}+\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}$$
$$\Big\downarrow \ 2841$$

$$-\frac{bBn\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{g\int\frac{\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{f+gx}dx}{b}\right)}{g}+\frac{Bdn\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{g\int\frac{\log\left(-\frac{g(c+dx)}{df-cg}\right)}{f+gx}dx}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}$$

$$\Big\downarrow \text{2840}$$

$$-\frac{bBn\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(f+gx)}{bf-ag}\right)}{f+gx}d(f+gx)}{b}\right)}{g}+\frac{Bdn\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(f+gx)}{df-cg}\right)}{f+gx}d(f+gx)}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}-\frac{bBn\left(\frac{\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{b}+\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}\right)}{g}+\frac{Bdn\left(\frac{\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{d}+\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}\right)}{g}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

output
```
((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x])/g - (b*B*n*((Log[-((
g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/b + PolyLog[2, (b*(f + g*x))/(b*f
 - a*g)]/b))/g + (B*d*n*((Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/
d + PolyLog[2, (d*(f + g*x))/(d*f - c*g)]/d))/g
```

3.62. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2945
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Simp[b*B*(n/g)   Int[Log[f + g*x]/(a + b
*x), x], x] + Simp[B*d*(n/g)   Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ
[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

3.62.4 Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx+f}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
```

3.62.5 Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f),x)
```

output
```
Timed out
```

3.62.7 Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="maxima")
```

output
```
-B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(g*x + f), x)
 + A*log(g*x + f)/g
```

3.62.8 Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx=\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f+g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x), x)
```

3.63 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx$

3.63.1 Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(f+gx)}+\frac{B(bc-ad)n\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*n*
ln((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

3.63.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}+\frac{Bn(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)) + (B*n*(b*(d*f - c*g)
*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*
x]))/((b*f - a*g)*(d*f - c*g)))/g
```

3.63.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.55, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2953, 2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^2}dx$$

$$\Big\downarrow \ 2953$$

$$(bc-ad)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2751$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{Bn\int \frac{1}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \ 16$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{Bn\log\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}{(bf-ag)(df-cg)}\right)$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f - a*
g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (B*n*Log[b
*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)*(d*f - c*g)))
```

3.63.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(91) = 182$.

Time = 3.51 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.00

method	result
parallelrisch	$\frac{Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)abcd\,f^{2}n-Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}cdfgn+B\ln(bx+a)x\,a^{2}cdfg\,n^{2}-B\ln(bx+a)xab\,c^{2}fg\,n^{2}-B\ln(gx+f)x\,a^{2}c}{}$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
(B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*f^2*n-B*x*ln(e*((b*x+a)/(d*x+c))^n)
*a^2*c*d*f*g*n+B*ln(b*x+a)*x*a^2*c*d*f*g*n^2-B*ln(b*x+a)*x*a*b*c^2*f*g*n^2
-B*ln(g*x+f)*x*a^2*c*d*f*g*n^2+B*ln(g*x+f)*x*a*b*c^2*f*g*n^2+A*x*a^2*c^2*g
^2*n-A*x*a^2*c*d*f*g*n-A*x*a*b*c^2*f*g*n+A*x*a*b*c*d*f^2*n-B*ln(e*((b*x+a)
/(d*x+c))^n)*a^2*c^2*f*g*n+B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c^2*f^2*n+B*ln(
b*x+a)*a^2*c*d*f^2*n^2-B*ln(b*x+a)*a*b*c^2*f^2*n^2-B*ln(g*x+f)*a^2*c*d*f^2
*n^2+B*ln(g*x+f)*a*b*c^2*f^2*n^2)/(a*g-b*f)/(g*x+f)/n/(c*g-d*f)/a/c/f
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(91) = 182.

Time = 3.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.23

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = -\frac{Abdf^2 + Aacg^2 - (Abc + Aad)fg + (Bbdf^2 + Bacg^2 - (Bbc + Bad)fg)n\log\left(\frac{bx+a}{dx+c}\right) - ((Bbdfg - Bbc$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="fricas"
)
```

output
```
-(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g + (B*b*d*f^2 + B*a*c*g^2 - (
B*b*c + B*a*d)*f*g)*n*log((b*x + a)/(d*x + c)) - ((B*b*d*f*g - B*b*c*g^2)*
n*x + (B*b*d*f^2 - B*b*c*f*g)*n)*log(b*x + a) + ((B*b*d*f*g - B*a*d*g^2)*n
*x + (B*b*d*f^2 - B*a*d*f*g)*n)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*n*x +
(B*b*c - B*a*d)*f*g*n)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*
a*d)*f*g)*log(e))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*
g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**2,x)
```

output
```
Timed out
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.56

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx$$
$$= Bn\left(\frac{b\log(bx+a)}{bfg - ag^2} - \frac{d\log(dx+c)}{dfg - cg^2} + \frac{(bc-ad)\log(gx+f)}{bdf^2 + acg^2 - (bc+ad)fg}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{g^2x + fg} - \frac{A}{g^2x + fg}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="maxima"
)
```

output
```
B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*
c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)/(g^2*x + f*g) - A/(g^2*x + f*g)
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(91) = 182$.

Time = 0.55 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.07

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx$$
$$= \left(\frac{(Bb^2c^2n - 2\,Babcdn + Ba^2d^2n)\log\left(-bf + \frac{(bx+a)df}{dx+c} + ag - \frac{(bx+a)cg}{dx+c}\right)}{bdf^2 - bcfg - adfg + acg^2} + \frac{(Bb^2c^2n - 2\,Babcdn}{bdf^2 - \frac{(bx+a)d^2f^2}{dx+c} - bcfg - adf}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="giac")
```

output
```
((B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log(-b*f + (b*x + a)*d*f/(d*x
 + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*
g^2) + (B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c)
)/(b*d*f^2 - (b*x + a)*d^2*f^2/(d*x + c) - b*c*f*g - a*d*f*g + 2*(b*x + a)
*c*d*f*g/(d*x + c) + a*c*g^2 - (b*x + a)*c^2*g^2/(d*x + c)) - (B*b^2*c^2*n
 - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d*f^2 - b*c*f*
g - a*d*f*g + a*c*g^2) + (B*b^2*c^2*log(e) - 2*B*a*b*c*d*log(e) + B*a^2*d^
2*log(e) + A*b^2*c^2 - 2*A*a*b*c*d + A*a^2*d^2)/(b*d*f^2 - (b*x + a)*d^2*f
^2/(d*x + c) - b*c*f*g - a*d*f*g + 2*(b*x + a)*c*d*f*g/(d*x + c) + a*c*g^2
 - (b*x + a)*c^2*g^2/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = \frac{B\,d\,n\,\ln(c+d\,x)}{c\,g^2 - d\,f\,g} - \frac{\ln(f+g\,x)\,(B\,a\,d\,n - B\,b\,c\,n)}{a\,c\,g^2 + b\,d\,f^2 - a\,d\,f\,g - b\,c\,f\,g} - \frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g\,(f+g\,x)} - \frac{B\,b\,n\,\ln(a+b\,x)}{a\,g^2 - b\,f\,g} - \frac{A}{x\,g^2 + f\,g}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^2,x)
```

output
```
(B*d*n*log(c + d*x))/(c*g^2 - d*f*g) - (log(f + g*x)*(B*a*d*n - B*b*c*n))/
(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (B*log(e*((a + b*x)/(c + d*x))^n
))/(g*(f + g*x)) - (B*b*n*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x)
```

3.64 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$

3.64.1 Optimal result

Integrand size = 30, antiderivative size = 190

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx=-\frac{B(bc-ad)n}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}-\frac{Bd^2n\log(c+dx)}{2g(df-cg)^2}+\frac{B(bc-ad)(2bdf-bcg-adg)n\log(f+gx)}{2(bf-ag)^2(df-cg)^2}$$

output
```
-1/2*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*n*ln(b*x+a)/g/
(-a*g+b*f)^2+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^2-1/2*B*d^2*n*
ln(d*x+c)/g/(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*ln(g*x+
f)/(-a*g+b*f)^2/(-c*g+d*f)^2
```

3.64.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx=\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}+B(bc-ad)n\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)}+\frac{d^2\log(c+dx)}{-bc+ad}-\frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2) + B*(b*c - a*d)*n*(
(b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f
 - a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*
c*g + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

3.64.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^3} \, dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{Bn(bc-ad) \int \frac{1}{(a+bx)(c+dx)(f+gx)^2}dx}{2g}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \text{93}$$

$$\frac{Bn(bc-ad) \int \left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(cg-df)^2(c+dx)}-\frac{g^2(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^2}\right)dx}{2g}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad) \left(\frac{b^2 \log(a+bx)}{(bc-ad)(bf-ag)^2}-\frac{d^2 \log(c+dx)}{(bc-ad)(df-cg)^2}-\frac{g}{(f+gx)(bf-ag)(df-cg)}+\frac{g \log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}\right)}{2g}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

output
```
-1/2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(f + g*x)^2) + (B*(b*c - a*
d)*n*(-(g/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*Log[a + b*x])/((b*c
- a*d)*(b*f - a*g)^2) - (d^2*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) + (
g*(2*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/
(2*g)
```

3.64.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1375 vs. $2(183) = 366$.

Time = 14.21 (sec) , antiderivative size = 1376, normalized size of antiderivative = 7.24

method	result	size
parallelrisch	Expression too large to display	1376

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*B*ln(g*x+f)*b^3*c*d^2*f^3*g^2*n-B*ln(b*x+a)*x^2*b^3*c^2*d*g^5*n-B
*x*a^2*b*c*d^2*g^5*n+B*x*a^2*b*d^3*f*g^4*n+B*x*a*b^2*c^2*d*g^5*n-B*x*a*b^2
*d^3*f^2*g^3*n-B*x*b^3*c^2*d*f*g^4*n+B*x*b^3*c*d^2*f^2*g^3*n-2*B*ln(e*((b*
x+a)/(d*x+c))^n)*a^2*b*c*d^2*f*g^4-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c^2
*d*f*g^4+4*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^2*f^2*g^3+A*a^2*b*c^2*d*g
^5+A*a^2*b*d^3*f^2*g^3-2*A*a*b^2*d^3*f^3*g^2+A*b^3*c^2*d*f^2*g^3-2*A*b^3*c
*d^2*f^3*g^2-2*B*ln(d*x+c)*x^2*a*b^2*d^3*f*g^4*n+2*B*ln(g*x+f)*x^2*a*b^2*d
^3*f*g^4*n-2*B*ln(g*x+f)*x^2*b^3*c*d^2*f*g^4*n-2*B*ln(b*x+a)*x*b^3*c^2*d*f
*g^4*n+4*B*ln(b*x+a)*x*b^3*c*d^2*f^2*g^3*n+2*B*ln(d*x+c)*x*a^2*b*d^3*f*g^4
*n-4*B*ln(d*x+c)*x*a*b^2*d^3*f^2*g^3*n-2*B*ln(g*x+f)*x*a^2*b*d^3*f*g^4*n+4
*B*ln(g*x+f)*x*a*b^2*d^3*f^2*g^3*n+2*B*ln(g*x+f)*x*b^3*c^2*d*f*g^4*n-4*B*l
n(g*x+f)*x*b^3*c*d^2*f^2*g^3*n+2*B*ln(b*x+a)*x^2*b^3*c*d^2*f*g^4*n-B*a^2*b
*c*d^2*f*g^4*n+B*a*b^2*c^2*d*f*g^4*n-B*ln(b*x+a)*x^2*b^3*d^3*f^2*g^3*n+B*l
n(d*x+c)*x^2*a^2*b*d^3*g^5*n+B*ln(d*x+c)*x^2*b^3*d^3*f^2*g^3*n-B*ln(g*x+f)
*x^2*a^2*b*d^3*g^5*n+B*ln(g*x+f)*x^2*b^3*c^2*d*g^5*n-2*B*ln(b*x+a)*x*b^3*d
^3*f^3*g^2*n+2*B*ln(d*x+c)*x*b^3*d^3*f^3*g^2*n-B*ln(b*x+a)*b^3*c^2*d*f^2*g
^3*n+2*B*ln(b*x+a)*b^3*c*d^2*f^3*g^2*n+B*ln(d*x+c)*a^2*b*d^3*f^2*g^3*n-2*B
*ln(d*x+c)*a*b^2*d^3*f^3*g^2*n-B*ln(g*x+f)*a^2*b*d^3*f^2*g^3*n+2*B*ln(g*x+
f)*a*b^2*d^3*f^3*g^2*n+B*ln(g*x+f)*b^3*c^2*d*f^2*g^3*n+B*ln(e*((b*x+a)/(d*
x+c))^n)*b^3*d^3*f^4*g+B*a^2*b*d^3*f^2*g^3*n-B*a*b^2*d^3*f^3*g^2*n-B*b^...
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(180) = 360$.

Time = 45.64 (sec) , antiderivative size = 1175, normalized size of antiderivative = 6.18

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="fricas"
)
```

output
```
-1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*(A*b^2*c*d + A*a*b*d^2)*f^3*g + (A
*b^2*c^2 + 4*A*a*b*c*d + A*a^2*d^2)*f^2*g^2 - 2*(A*a*b*c^2 + A*a^2*c*d)*f*
g^3 + ((B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (
B*a*b*c^2 - B*a^2*c*d)*g^4)*n*x + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^
2*c*d + B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 -
 2*(B*a*b*c^2 + B*a^2*c*d)*f*g^3)*n*log((b*x + a)/(d*x + c)) + ((B*b^2*c*d
 - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2*g^2 + (B*a*b*c^2 - B*a^2
*c*d)*f*g^3)*n - ((B*b^2*d^2*f^2*g^2 - 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*
n*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*f^2*g^2 + B*b^2*c^2*f*g^3)*n*x +
(B*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2)*n)*log(b*x + a) +
((B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2*d^2*g^4)*n*x^2 + 2*(B*b^2*
d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)*n*x + (B*b^2*d^2*f^4 -
2*B*a*b*d^2*f^3*g + B*a^2*d^2*f^2*g^2)*n)*log(d*x + c) - ((2*(B*b^2*c*d -
B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2)*g^4)*n*x^2 + 2*(2*(B*b^2*c*d -
B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3)*n*x + (2*(B*b^2*c*d -
B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2*g^2)*n)*log(g*x + f) + (B*b
^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*b*d^2)*f^3*g + (B*b^2*c^2
+ 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2 + B*a^2*c*d)*f*g^3)*log(
e))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + ...
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**3,x)
```

output
```
Timed out
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.87

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$$
$$=\frac{1}{2}\left(\frac{b^2\log(bx+a)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{d^2\log(dx+c)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{(2\,(b^2cd-abd^2)f-(b^2c^2-\right.$$
$$-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(g^3x^2+2\,fg^2x+f^2g)}-\frac{A}{2\,(g^3x^2+2\,fg^2x+f^2g)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="maxima"
)
```

output
```
1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x +
c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c
^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*
b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*
c*d)*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*
f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x))*B*n - 1/2*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*A/(g^3*x^2 + 2*f*g^2
*x + f^2*g)
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2994 vs. 2(180) = 360.

Time = 0.86 (sec) , antiderivative size = 2994, normalized size of antiderivative = 15.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="giac")
```

output
```
1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*
c^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b*f
+ (b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^2*d^2*f^4 -
2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 +
a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) + (2*B*
b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*f*n/(d*x +
 c) + 2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c) - 2*(b*x +
 a)*B*a^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + 2*(b*x +
 a)*B*b^2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*B*a*b*c^2*
d^2*g*n/(d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/(d*x + c))
*log((b*x + a)/(d*x + c))/(b^2*d^2*f^4 - 2*(b*x + a)*b*d^3*f^4/(d*x + c) +
 (b*x + a)^2*d^4*f^4/(d*x + c)^2 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + 6*(
b*x + a)*b*c*d^2*f^3*g/(d*x + c) + 2*(b*x + a)*a*d^3*f^3*g/(d*x + c) - 4*(
b*x + a)^2*c*d^3*f^3*g/(d*x + c)^2 + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 -
 6*(b*x + a)*b*c^2*d*f^2*g^2/(d*x + c) + a^2*d^2*f^2*g^2 - 6*(b*x + a)*a*c
*d^2*f^2*g^2/(d*x + c) + 6*(b*x + a)^2*c^2*d^2*f^2*g^2/(d*x + c)^2 - 2*a*b
*c^2*f*g^3 + 2*(b*x + a)*b*c^3*f*g^3/(d*x + c) - 2*a^2*c*d*f*g^3 + 6*(b*x
+ a)*a*c^2*d*f*g^3/(d*x + c) - 4*(b*x + a)^2*c^3*d*f*g^3/(d*x + c)^2 + a^2
*c^2*g^4 - 2*(b*x + a)*a*c^3*g^4/(d*x + c) + (b*x + a)^2*c^4*g^4/(d*x + c)
^2) - (2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*...
```

3.64.9 Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$$
$$= \frac{\ln(f+g\,x)\,\left(g\,(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n)-2\,B\,a\,b\,d^2\,f\,n+2\,B\,b^2\,c\,d\,f\,n\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c\,d}$$
$$-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-B\,a\,d\,f\,g\,n+B\,b\,c\,f\,g\,n}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{x\,(B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}$$
$$-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,g\,(f^2+2\,f\,g\,x+g^2\,x^2)}+\frac{B\,b^2\,n\,\ln(a+b\,x)}{2\,a^2\,g^3-4\,a\,b\,f\,g^2+2\,b^2\,f^2\,g}-\frac{B\,d^2\,n\,\ln(c+d\,x)}{2\,c^2\,g^3-4\,c\,d\,f\,g^2+2\,d^2\,f^2\,g}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^3,x)
```

output
```
(log(f + g*x)*(g*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*f*n + 2*B*b^2*c
*d*f*n))/(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^
2*g^2 - 4*a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^
3*g + 8*a*b*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g
 - B*a*d*f*g*n + B*b*c*f*g*n)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x
*(B*a*d*g^2*n - B*b*c*g^2*n))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*
f^2*g + 2*g^3*x^2 + 4*f*g^2*x) - (B*log(e*((a + b*x)/(c + d*x))^n))/(2*g*(
f^2 + g^2*x^2 + 2*f*g*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*
g - 4*a*b*f*g^2) - (B*d^2*n*log(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d
*f*g^2)
```

3.64. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$

3.65 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$

3.65.1 Optimal result

Integrand size = 30, antiderivative size = 283

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx &= -\frac{B(bc-ad)n}{6(bf-ag)(df-cg)(f+gx)^2}-\frac{B(bc-ad)(2bdf-bcg-adg)n}{3(bf-ag)^2(df-cg)^2(f+gx)} \\ &\quad+\frac{b^3Bn\log(a+bx)}{3g(bf-ag)^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3}-\frac{Bd^3n\log(c+dx)}{3g(df-cg)^3} \\ &\quad+\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\log(f+gx)}{3(bf-ag)^3(df-cg)^3}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d
*g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*n*ln(b*x+a
)/g/(-a*g+b*f)^3+1/3*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^3-1/3*B*d^
3*n*ln(d*x+c)/g/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3
*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*
f)^3
```

3.65.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$$
$$= \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}+B(bc-ad)n\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2}+\frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log}{(bc-ad)}\right.}{3g}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3) + B*(b*c - a*d)*n*(
-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*
g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a
*d)*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (
g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2
*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

3.65.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^4}\,dx$$
$$\Big\downarrow \ 2947$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}dx}{3g}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}$$
$$\Big\downarrow \ 93$$

$$\frac{Bn(bc-ad)\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(cg-df)^3(c+dx)}+\frac{g^2((3d^2f^2-3cdgf+c^2g^2)b^2-adg(3df-cg)b+a^2d^2g^2)}{(bf-ag)^3(df-cg)^3(f+gx)}-\frac{g^2(-}{(bf-ag)}\right.}{3g}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(\frac{g\log(f+gx)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}-\frac{d^3\log(c+dx)}{(bc-ad)(df-cg)^3}-\frac{g(-adg-bcg}{(f+gx)(bf-ag)}\right.}{3g}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

output
```
-1/3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(f + g*x)^3) + (B*(b*c - a*
d)*n*(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (g*(2*b*d*f - b*c*g -
 a*d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*
c - a*d)*(b*f - a*g)^3) - (d^3*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^3) +
 (g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^
2*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

3.65.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.65.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3256 vs. 2(274) = 548.

Time = 41.68 (sec) , antiderivative size = 3257, normalized size of antiderivative = 11.51

method	result	size
parallelrisch	Expression too large to display	3257

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c*d^3*f^4*g^4*n+2*B*ln(g*x+f)*a
^3*b*d^4*f^3*g^5*n^2-6*B*ln(g*x+f)*a^2*b^2*d^4*f^4*g^4*n^2+6*B*ln(g*x+f)*a
*b^3*d^4*f^5*g^3*n^2-2*B*ln(g*x+f)*b^4*c^3*d*f^3*g^5*n^2+6*B*ln(g*x+f)*b^4
*c^2*d^2*f^4*g^4*n^2-6*B*ln(g*x+f)*b^4*c*d^3*f^5*g^3*n^2-2*B*ln(b*x+a)*x^3
*a^3*b*d^4*g^8*n^2+2*B*ln(b*x+a)*x^3*b^4*c^3*d*g^8*n^2+2*B*ln(g*x+f)*x^3*a
^3*b*d^4*g^8*n^2-2*B*ln(g*x+f)*x^3*b^4*c^3*d*g^8*n^2-3*B*a^3*b*d^4*f^3*g^5
*n^2+8*B*a^2*b^2*d^4*f^4*g^4*n^2-5*B*a*b^3*d^4*f^5*g^3*n^2+3*B*b^4*c^3*d*f
^3*g^5*n^2-8*B*b^4*c^2*d^2*f^4*g^4*n^2+5*B*b^4*c*d^3*f^5*g^3*n^2+2*A*a^3*b
*c^3*d*g^8*n-2*A*a^3*b*d^4*f^3*g^5*n+6*A*a^2*b^2*d^4*f^4*g^4*n-6*A*a*b^3*d
^4*f^5*g^3*n-2*A*b^4*c^3*d*f^3*g^5*n+6*A*b^4*c^2*d^2*f^4*g^4*n-6*A*b^4*c*d
^3*f^5*g^3*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^3*d*g^8*n-2*B*ln(e*((b*
x+a)/(d*x+c))^n)*b^4*c^3*d*f^3*g^5*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2
*d^2*f^4*g^4*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*f^5*g^3*n-2*B*ln(b*
x+a)*a^3*b*d^4*f^3*g^5*n^2+6*B*ln(b*x+a)*a^2*b^2*d^4*f^4*g^4*n^2-6*B*ln(b*
x+a)*a*b^3*d^4*f^5*g^3*n^2+2*B*ln(b*x+a)*b^4*c^3*d*f^3*g^5*n^2-6*B*ln(b*x+
a)*b^4*c^2*d^2*f^4*g^4*n^2+6*B*ln(b*x+a)*b^4*c*d^3*f^5*g^3*n^2-6*B*ln(b*x+
a)*x^2*a^3*b*d^4*f*g^7*n^2+18*B*ln(b*x+a)*x^2*a^2*b^2*d^4*f^2*g^6*n^2-18*B
*ln(b*x+a)*x^2*a*b^3*d^4*f^3*g^5*n^2+6*B*ln(b*x+a)*x^2*b^4*c^3*d*f*g^7*n^2
-18*B*ln(b*x+a)*x^2*b^4*c^2*d^2*f^2*g^6*n^2+18*B*ln(b*x+a)*x^2*b^4*c*d^3*f
^3*g^5*n^2+18*B*ln(g*x+f)*x^2*b^4*c^2*d^2*f^2*g^6*n^2-18*B*ln(g*x+f)*x^...
```

3.65.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="fricas"
)
```

output
```
Timed out
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**4,x)
```

output
```
Timed out
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(271) = 542.

Time = 0.24 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.01

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$$
$$= \frac{1}{6}\left(\frac{2\,b^3\log(bx+a)}{b^3f^3g-3\,ab^2f^2g^2+3\,a^2bfg^3-a^3g^4} - \frac{2\,d^3\log(dx+c)}{d^3f^3g-3\,cd^2f^2g^2+3\,c^2dfg^3-c^3g^4} + \frac{}{b^3d^3f^6+a^3c^3g^6-3\,(b\ldots}\right.$$
$$-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)} - \frac{A}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="maxima"
)
```

output
```
1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*B*n - 1/3*B*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f
^3*g) - 1/3*A/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9692 vs. $2(271) = 542$.

Time = 0.85 (sec) , antiderivative size = 9692, normalized size of antiderivative = 34.25

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="giac")
```

output
```
1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4*f^
2*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*f*
g*n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3*
b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(-b*f + (b*x + a)*d*f/(d*x + c) + a*g
- (b*x + a)*c*g/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*
f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2
- b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^
3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*
g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + 2*(3*B*b^4*c^
2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*c^2*d^3*f^2*n/(d*x
 + c) + 3*B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d^4*f^2*n/(d*x + c)
 + 3*(b*x + a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*x + a)*B*a^2*b*d^5
*f^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x + c)^2 + 3*(b*x +
a)^2*B*a^2*d^6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2
*f*g*n + 9*(b*x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^3*f*g
*n - 15*(b*x + a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*b^2*c^
3*d^3*f*g*n/(d*x + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x + a)*B*a^2*b*c*d^4*
f*g*n/(d*x + c) + 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*x + c)^2 + 3*(b*x
+ a)*B*a^3*d^5*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d^5*f*g*n/(d*x + c)
^2 + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)*B*b^3*c^4*d*g^...
```

3.65.9 Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 1182, normalized size of antiderivative = 4.18

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = \frac{B\,d^3\,n\,\ln\left(c+d\,x\right)}{3\,c^3\,g^4-9\,c^2\,d\,f\,g^3+9\,c\,d^2\,f^2\,g^2-3\,d^3\,f^3\,g}$$
$$-\frac{\ln\left(f+g\,x\right)\,\left(g^2\,\left(B\,a^3\,d^3\,n-B\,b^3\,c\right.\right.}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f\,g^5+27\,a^2\,b\,c^2\,d\,f^2\,g^4-27\,a^2\,b\,c\,d^2\,f}$$
$$-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{3\,g\,\left(f^3+3\,f^2\,g\,x+3\,f\,g^2\,x^2+g^3\,x^3\right)}-\frac{B\,b^3\,n\,\ln\left(a+b\,x\right)}{3\,a^3\,g^4-9\,a^2\,b\,f\,g^3+9\,a\,b^2\,f^2\,g^2-3\,b^3\,f^3\,g}$$
$$-\frac{\frac{2\,A\,a^2\,c^2\,g^4+2\,A\,b^2\,d^2\,f^4+2\,A\,a^2\,d^2\,f^2\,g^2+2\,A\,b^2\,c^2\,f^2\,g^2+3\,B\,a^2\,d^2\,f^2\,g^2\,n-3\,B\,b^2\,c^2\,f^2\,g^2\,n-4\,A\,a\,b\,c^2\,f\,g^3-4\,A\,a\,b\,d^2\,f^3\,g-4\,A\,a^2\,c\,d\,f\,g^3-}{2\,\left(a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+\right.}}{}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^4,x)
```

output
```
(B*d^3*n*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 - 9*c^2*
d*f*g^3) - (log(f + g*x)*(g^2*(B*a^3*d^3*n - B*b^3*c^3*n) - g*(3*B*a^2*b*d
^3*f*n - 3*B*b^3*c^2*d*f*n) + 3*B*a*b^2*d^3*f^2*n - 3*B*b^3*c*d^2*f^2*n))/
(3*a^3*c^3*g^6 + 3*b^3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9
*a^2*b*c^3*f*g^5 - 9*a*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5
*g + 9*a*b^2*c^3*f^2*g^4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b
^3*c^2*d*f^4*g^2 + 27*a*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^
2*b*c*d^2*f^3*g^3 + 27*a^2*b*c^2*d*f^2*g^4) - (B*log(e*((a + b*x)/(c + d*x
))^n))/(3*g*(f^3 + g^3*x^3 + 3*f^2*g*x + 3*f*g^2*x^2)) - (B*b^3*n*log(a +
b*x))/(3*a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*f*g^3) - ((2*A*
a^2*c^2*g^4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f^2*g^2 + 2*A*b^2*c^2*f^2*g^2
+ 3*B*a^2*d^2*f^2*g^2*n - 3*B*b^2*c^2*f^2*g^2*n - 4*A*a*b*c^2*f*g^3 - 4*A*
a*b*d^2*f^3*g - 4*A*a^2*c*d*f*g^3 - 4*A*b^2*c*d*f^3*g + 8*A*a*b*c*d*f^2*g^
2 + B*a*b*c^2*f*g^3*n - 5*B*a*b*d^2*f^3*g*n - B*a^2*c*d*f*g^3*n + 5*B*b^2*
c*d*f^3*g*n)/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2
*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3
*g + 4*a*b*c*d*f^2*g^2)) + (x*(B*a*b*c^2*g^4*n - B*a^2*c*d*g^4*n + 5*B*a^2
*d^2*f*g^3*n - 5*B*b^2*c^2*f*g^3*n - 9*B*a*b*d^2*f^2*g^2*n + 9*B*b^2*c*d*f
^2*g^2*n))/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g
^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^...
```

3.65. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$

3.66 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$

3.66.1 Optimal result

Integrand size = 30, antiderivative size = 388

$$\begin{aligned}
&\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx \\
&= -\frac{B(bc-ad)n}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)n}{8(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&\quad -\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n}{4(bf-ag)^3(df-cg)^3(f+gx)} \\
&\quad +\frac{b^4Bn\log(a+bx)}{4g(bf-ag)^4}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4}-\frac{Bd^4n\log(c+dx)}{4g(df-cg)^4} \\
&\quad -\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\log(f+gx)}{4(bf-ag)^4(df-cg)^4}
\end{aligned}$$

output
```
-1/12*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*
d*g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*
(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n/(-a
*g+b*f)^3/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*n*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-
A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^4-1/4*B*d^4*n*ln(d*x+c)/g/(-c*g+d
*f)^4-1/4*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b
^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

3.66.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$$
$$= \frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}+B(bc-ad)n\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g(a^2d^2g^2+abdg(-3df+cg)+}{(bf-ag)^3(df-}\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4) + B*(b*c - a*d)*n*(
-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*
g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*
g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*
(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4)
- (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g +
a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^
2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

3.66.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2947, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^5}\,dx$$
$$\Big\downarrow \text{2947}$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}dx}{4g}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4g(f+gx)^4}$$
$$\Big\downarrow \text{93}$$

$$\frac{Bn(bc-ad)\int\left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(cg-df)^4(c+dx)}+\frac{g^2(2bdf-bcg-adg)\left(2d^2f^2b^2+c^2g^2b^2-2cdfgb^2-2ad^2fgb+a^2d^2g^2\right)}{(bf-ag)^4(df-cg)^4(f+gx)}\right.}{4g}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4g(f+gx)^4}$$

$\downarrow$ 2009

$$\frac{Bn(bc-ad)\left(-\frac{g\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(f+gx)(bf-ag)^3(df-cg)^3}-\frac{g\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-\left(b^2\left(c^2g^2-2cdfg\right.\right.\right.}{(bf-ag)^4(df-cg)^4}\right.}{4g}$$

$$\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4g(f+gx)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

output
```
-1/4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(f + g*x)^4) + (B*(b*c - a*
d)*n*(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (g*(2*b*d*f - b*c*g -
 a*d*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 - a
*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g
)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)
^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(2*b*d*f - b*c*g
 - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*
g^2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

3.66.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

3.66.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5205 vs. $2(377) = 754$.

Time = 256.76 (sec) , antiderivative size = 5206, normalized size of antiderivative = 13.42

method	result	size
parallelrisch	Expression too large to display	5206

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.66.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="fricas"
)
```

output
```
Timed out
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**5,x)
```

output
```
Timed out
```

3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1761 vs. 2(374) = 748.

Time = 0.31 (sec) , antiderivative size = 1761, normalized size of antiderivative = 4.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="maxima"
)
```

output
```
1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21743 vs. 2(374) = 748.

Time = 1.21 (sec) , antiderivative size = 21743, normalized size of antiderivative = 56.04

$$\int \frac{A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{(f + gx)^5} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="giac")
```

output
```
1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n + 4*B*a^2*b^3*d^5*f
^3*n - 6*B*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 6*B*a^2*b^3*c
*d^4*f^2*g*n - 6*B*a^3*b^2*d^5*f^2*g*n + 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4
*c^3*d^2*f*g^2*n - 4*B*a^3*b^2*c*d^4*f*g^2*n + 4*B*a^4*b*d^5*f*g^2*n - B*b
^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n
)*log(-b*f + (b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^4
*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 +
 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24
*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 +
b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16
*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*
c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2
*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^
4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) + 6*(4*B*b^5*c^2*d^3*f^3*n - 8*
B*a*b^4*c*d^4*f^3*n - 12*(b*x + a)*B*b^4*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2
*b^3*d^5*f^3*n + 24*(b*x + a)*B*a*b^3*c*d^5*f^3*n/(d*x + c) + 12*(b*x + a)
^2*B*b^3*c^2*d^5*f^3*n/(d*x + c)^2 - 12*(b*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x
 + c) - 24*(b*x + a)^2*B*a*b^2*c*d^6*f^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b
^2*c^2*d^6*f^3*n/(d*x + c)^3 + 12*(b*x + a)^2*B*a^2*b*d^7*f^3*n/(d*x + c)^
2 + 8*(b*x + a)^3*B*a*b*c*d^7*f^3*n/(d*x + c)^3 - 4*(b*x + a)^3*B*a^2*d...
```

3.66.9 Mupad [B] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 2569, normalized size of antiderivative = 6.62

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^5,x)
```

output
```
((x^3*(B*a^3*d^3*g^6*n - B*b^3*c^3*g^6*n - 3*B*a^2*b*d^3*f*g^5*n + 3*B*b^3
*c^2*d*f*g^5*n + 3*B*a*b^2*d^3*f^2*g^4*n - 3*B*b^3*c*d^2*f^2*g^4*n))/(a^3*
c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*
g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*
c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*
g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^
3 + 9*a^2*b*c^2*d*f^2*g^4) - (6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*
d^3*f^3*g^3 - 6*A*b^3*c^3*f^3*g^3 + 18*A*a*b^2*c^3*f^2*g^4 + 18*A*a^2*b*d^
3*f^4*g^2 + 18*A*a^3*c*d^2*f^2*g^4 + 18*A*b^3*c^2*d*f^4*g^2 - 11*B*a^3*d^3
*f^3*g^3*n + 11*B*b^3*c^3*f^3*g^3*n - 18*A*a^2*b*c^3*f*g^5 - 18*A*a*b^2*d^
3*f^5*g - 18*A*a^3*c^2*d*f*g^5 - 18*A*b^3*c*d^2*f^5*g + 2*B*a^2*b*c^3*f*g^
5*n - 26*B*a*b^2*d^3*f^5*g*n - 2*B*a^3*c^2*d*f*g^5*n + 26*B*b^3*c*d^2*f^5*
g*n + 54*A*a*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^2*b*c*d
^2*f^3*g^3 + 54*A*a^2*b*c^2*d*f^2*g^4 - 7*B*a*b^2*c^3*f^2*g^4*n + 31*B*a^2
*b*d^3*f^4*g^2*n + 7*B*a^3*c*d^2*f^2*g^4*n - 31*B*b^3*c^2*d*f^4*g^2*n + 15
*B*a*b^2*c^2*d*f^3*g^3*n - 15*B*a^2*b*c*d^2*f^3*g^3*n)/(6*(a^3*c^3*g^6 + b
^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b
^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4
 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b
^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^...
```

3.66. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$

3.67 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

3.67.1 Optimal result

Integrand size = 32, antiderivative size = 923

$$
\begin{aligned}
\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx &= \frac{B^2(bc-ad)^3g^3n^2x}{6b^3d^3} \\
&+ \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)n^2x}{4b^3d^3} + \frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d^4} \\
&- \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4d^3} \\
&- \frac{B(bc-ad)g^2(4bdf-3bcg-adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d^4} \\
&- \frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd^4} \\
&- \frac{(bf-ag)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} \\
&- \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4d^4} \\
&+ \frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4} \\
&+ \frac{B^2(bc-ad)^4g^3n^2\log(c+dx)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)n^2\log(c+dx)}{4b^4d^4} \\
&+ \frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,n^2\log(c+dx)}{2b^4d^4} \\
&- \frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4}
\end{aligned}
$$

output
```
1/6*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*
b*c*g+4*b*d*f)*n^2*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/b^2/d
^4-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8
*c*d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^3-1/4
*B*(-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/b/d^4-1/4*(-a*g+b*f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4/g
+1/4*(g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g-1/2*B*(-a*d+b*c)*(-a*d*
g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f
^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4+1/
6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^
3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/6*B^2*(-a
*d+b*c)^4*g^3*n^2*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c
*g+4*b*d*f)*n^2*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*a*
b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*n^2*ln(d*x+c)/b^4/
d^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b
^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d
^4
```

3.67.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 757, normalized size of antiderivative = 0.82

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{(f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{Bn\left(6Abd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+6Bd(bc-ad)g^2\left(\right.}{}}{}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(6*A*b*d*(b*c
 - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d
*f*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f
+ c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[e*((a + b*x)
/(c + d*x))^n] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 +
a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*Log[c +
d*x] - 6*b^4*(d*f - c*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c +
d*x] + B*(b*c - a*d)*g^4*n*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*
a^3*d^3*Log[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b
*d*f + b*c*g + a*d*g)*n*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x +
 b*c^2*Log[c + d*x])) - 3*B*d^4*(b*f - a*g)^4*n*(Log[a + b*x]*(Log[a + b*x
] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) + 3*b^4*B*(d*f - c*g)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
- Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/
(3*b^4*d^4))/(4*g)
```

3.67.3 Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 1100, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}}{2g(bc-ad)}\right.$$

$$\downarrow 2804$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\left(\frac{(bc-ad)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)g^4}{bd^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(bc-ad)^3(4}{}\right.}{}\right.$$

$$\downarrow 2009$$

$$(bc-ad)\left(\frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(bc-ad)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)g^4}{3bd^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B(bc-ad)^4n\log}{3b^4d^4}\right.}{}\right.$$

input
```
Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

3.67. $\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^4*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(4*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d
*x))^4) - (B*n*(-1/6*(B*(b*c - a*d)^4*g^4*n)/(b^2*d^4*(b - (d*(a + b*x))/(
c + d*x))^2) - (B*(b*c - a*d)^4*g^4*n)/(3*b^3*d^4*(b - (d*(a + b*x))/(c +
d*x))) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*n)/(2*b^3*d^4*(b
 - (d*(a + b*x))/(c + d*x))) + ((b*c - a*d)^4*g^4*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(3*b*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + ((b*c - a*d)^3
*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
2*b^2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(a^2*d^2*g
^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a
 + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^4*d^3*(c + d*x)*(b - (d
*(a + b*x))/(c + d*x))) + ((b*f - a*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2)/(2*b^4*B*n) - (B*(b*c - a*d)^4*g^4*n*Log[(a + b*x)/(c + d*x)])/(3
*b^4*d^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*n*Log[(a + b*
x)/(c + d*x)])/(2*b^4*d^4) + (B*(b*c - a*d)^4*g^4*n*Log[b - (d*(a + b*x))/
(c + d*x)])/(3*b^4*d^4) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)
*n*Log[b - (d*(a + b*x))/(c + d*x)])/(2*b^4*d^4) + (B*(b*c - a*d)^2*g^2*(a
^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*
g^2))*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^4*d^4) + ((b*c - a*d)*g*(2*b*
d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*...
```

3.67.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.67.4 Maple [F]

$$\int (gx+f)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.67.5 Fricas [F]

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^
3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*x +
 c))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*lo
g(e*((b*x + a)/(d*x + c))^n), x)
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2651 vs. $2(892) = 1784$.

Time = 0.71 (sec) , antiderivative size = 2651, normalized size of antiderivative = 2.87

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxim
a")
```

output
```
1/2*A*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*g^3*x^4 +
 2*A*B*f*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g^2*x^3 +
3*A*B*f^2*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*f^2*g*x^2
 - 1/12*A*B*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a
^3*d^3)*x)/(b^3*d^3)) + A*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*
x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)
) - 3*A*B*f^2*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*
d)*x/(b*d)) + 2*A*B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^
3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f^3*x - 1/12*(6*a^3*c*d^3
*g^3*n^2 - 3*(8*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2)*a^2*b + 2*(18*c*d^3*f^2
*g*n^2 - 6*c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2)*a*b^2 + (24*c*d^3*f^3*n*log(
e) - (11*g^3*n^2 + 6*g^3*n*log(e))*c^4 + 12*(3*f*g^2*n^2 + 2*f*g^2*n*log(e
))*c^3*d - 36*(f^2*g*n^2 + f^2*g*n*log(e))*c^2*d^2)*b^3)*B^2*log(d*x + c)/
(b^3*d^4) + 1/2*(4*a*b^3*d^4*f^3*n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d
^4*f*g^2*n^2 - a^4*d^4*g^3*n^2 - (4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 +
4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*b^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + 1/12*(3*
B^2*b^4*d^4*g^3*x^4*log(e)^2 + 6*(4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 +
4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*B^2*b^4*log(b*x + a)*log(d*x + c) - 3*...
```

3.67.8 Giac [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac"
)
```

output
```
Timed out
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (f+g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.68 $\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

3.68.1 Optimal result

Integrand size = 32, antiderivative size = 565

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{B^2(bc-ad)^2g^2n^2x}{3b^2d^2} - \frac{2B(bc-ad)g(3bdf-2bcg-adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3d^2}$$
$$- \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd^3}$$
$$- \frac{(bf-ag)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g}$$
$$+ \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-a}{b(c+d}\right.}{3b^3d^3}$$
$$+ \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{B^2(bc-ad)^3g^2n^2\log(c+dx)}{3b^3d^3}$$
$$+ \frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)n^2\log(c+dx)}{3b^3d^3}$$
$$+ \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2/d^2-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+
3*b*d*f)*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^2-1/3*B*(-a*d+b*c
)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/3*(-a*g+b*f)^3*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))^2/g+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2
*g^2-3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)
/b/(d*x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d
^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)^2*g*(
-a*d*g-2*b*c*g+3*b*d*f)*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*
g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

3.68.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.90

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$=\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{Bn\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{}}{}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
 - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f -
 b*c*g - a*d*g)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*d^2*(b*c -
a*d)*g^3*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*g)^3*
Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*B*(b*c - a*d)^2*g^
2*(-3*b*d*f + b*c*g + a*d*g)*n*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*(b*c - a*d)*g^3*n*(a^2*d^2
*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f
- a*g)^3*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
 - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*n*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

3.68.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \ 2953$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2798$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3g(bc-ad)}\right.$$

$$\Big\downarrow \ 2804$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int \left(\frac{(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)g^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad)^2}{}\right.}{}\right.$$

$$\Big\downarrow \ 2009$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(-\frac{g(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+}{b^3}\right.}{}\right.$$

input
```
Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d
*x))^3) - (2*B*n*(-1/2*(B*(b*c - a*d)^3*g^3*n)/(b^2*d^3*(b - (d*(a + b*x))
/(c + d*x))) + ((b*c - a*d)^3*g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(2*b*d^3*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f -
2*b*c*g - a*d*g)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*d^
2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a*g)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])^2)/(2*b^3*B*n) - (B*(b*c - a*d)^3*g^3*n*Log[(a +
b*x)/(c + d*x)])/(2*b^3*d^3) + (B*(b*c - a*d)^3*g^3*n*Log[b - (d*(a + b*x)
)/(c + d*x)])/(2*b^3*d^3) + (B*(b*c - a*d)^2*g^2*(3*b*d*f - 2*b*c*g - a*d*
g)*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^3*d^3) - ((b*c - a*d)*g*(a^2*d^2
*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(
b^3*d^3) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*
d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
/(b^3*d^3)))/(3*(b*c - a*d)*g))
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.68.4 Maple [F]

$$\int (gx+f)^2 \left(A+B\ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.68.5 Fricas [F]

$$\int (f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (gx+f)^2 \left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x
+ B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x
 + A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n), x)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. $2(544) = 1088$.

Time = 0.69 (sec) , antiderivative size = 1659, normalized size of antiderivative = 2.94

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxim
a")
```

output
```
2/3*A*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*g^2*x^3 +
 2*A*B*f*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g*x^2 + 1/3*
A*B*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*f*g*n*(a^2*log(b
*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*f^2*n*(a
*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^2*x*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^2*n^2 - (6*c*d^2*f*g*n^2 -
 c^2*d*g^2*n^2)*a*b - (6*c*d^2*f^2*n*log(e) + (3*g^2*n^2 + 2*g^2*n*log(e))
*c^3 - 6*(f*g*n^2 + f*g*n*log(e))*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*d^3) +
 2/3*(3*a*b^2*d^3*f^2*n^2 - 3*a^2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2 - (3*c*d
^2*f^2*n^2 - 3*c^2*d*f*g*n^2 + c^3*g^2*n^2)*b^3)*(log(b*x + a)*log((b*d*x
+ a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3)
 + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + 2*(3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^
2 + c^3*g^2*n^2)*B^2*b^3*log(b*x + a)*log(d*x + c) - (3*c*d^2*f^2*n^2 - 3*
c^2*d*f*g*n^2 + c^3*g^2*n^2)*B^2*b^3*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log
(e) - (c*d^2*g^2*n*log(e) - 3*d^3*f*g*log(e)^2)*b^3)*B^2*x^2 - (3*a*b^2*d^
3*f^2*n^2 - 3*a^2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((
g^2*n^2 - 2*g^2*n*log(e))*a^2*b*d^3 - 2*(c*d^2*g^2*n^2 - 3*d^3*f*g*n*log(e
))*a*b^2 - (6*c*d^2*f*g*n*log(e) - 3*d^3*f^2*log(e)^2 - (g^2*n^2 + 2*g^2*n
*log(e))*c^2*d)*b^3)*B^2*x - ((3*g^2*n^2 - 2*g^2*n*log(e))*a^3*d^3 - (c...
```

3.68.8 Giac [F]

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (gx+f)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (f+g\,x)^2 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.69 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

3.69.1 Optimal result

Integrand size = 30, antiderivative size = 290

$$\begin{aligned}\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx &= -\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2d} \\ &\quad -\frac{(bf-ag)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} \\ &\quad +\frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} \\ &\quad +\frac{B^2(bc-ad)^2gn^2\log(c+dx)}{b^2d^2}+\frac{B^2(bc-ad)(2bdf-bcg-adg)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}\end{aligned}$$

output
```
-B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/2*(-a*g+
b*f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2/g+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+B^2*(-a*d+b*c)^2*g*n^2
*ln(d*x+c)/b^2/d^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n^2*polylog(2,d*(
b*x+a)/b/(d*x+c))/b^2/d^2
```

3.69.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.25

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$=\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{Bn\left(2Abd(bc-ad)g^2x+2Bd(bc-ad)g^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2d^2(bf-ag)^2\log(a+bx)\left(A+\right.\right.}{}}{}$$

input
```
Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
 - a*d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x)
)^n] + 2*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]) - 2*B*(b*c - a*d)^2*g^2*n*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*n*(Log[a
 + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (
d*(a + b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*n*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)])))/(b^2*d^2))/(2*g)
```

3.69.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{g(bc-ad)}\right.$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)g^2}{bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{(bc-ad)(2b}{}\right.}{}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(-\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\right.\right.}{b^2d^2}\right.}{}\right.$$

input
```
Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d
*x))^2) - (B*n*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a*g)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*B*n) + (B*(b*c - a*d)^2
*g^2*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^2*d^2) - ((b*c - a*d)*g*(2*b*d
*f - b*c*g - a*d*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a +
 b*x))/(b*(c + d*x))])/(b^2*d^2) - (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d
*g)*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/((b*c - a*d)*g)
)
```

3.69. $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.69.4 Maple [F]

$$\int (gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.69.5 Fricas [F]

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx=\int (gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c))^n
)^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n), x)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.69.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(285) = 570$.

Time = 0.68 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.10

$$\begin{aligned}\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx &= ABgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)\\ &+\frac{1}{2}A^2gx^2-ABgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)\\ &+2ABfn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+2ABfx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)\\ &+A^2fx-\frac{(acdgn^2+(2cdfn\log(e)-(gn^2+gn\log(e))c^2)b)B^2\log(dx+c)}{bd^2}\\ &+\frac{(2abd^2fn^2-a^2d^2gn^2-(2cdfn^2-c^2gn^2)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\text{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2}\\ &+\frac{B^2b^2d^2gx^2\log(e)^2+2(2cdfn^2-c^2gn^2)B^2b^2\log(bx+a)\log(dx+c)-(2cdfn^2-c^2gn^2)B^2b^2\log(dx}{}\end{aligned}$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima"
)
```

output
```
A*B*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*g*x^2 - A*B*g*n
*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A
*B*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f*x*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) + A^2*f*x - (a*c*d*g*n^2 + (2*c*d*f*n*log(e) - (g*n^
2 + g*n*log(e))*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*f*n^2 - a^2*
d^2*g*n^2 - (2*c*d*f*n^2 - c^2*g*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*d)
/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2
*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(b*x
 + a)*log(d*x + c) - (2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(d*x + c)^2 - (2
*a*b*d^2*f*n^2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*g*n*log(e)
 - (c*d*g*n*log(e) - d^2*f*log(e)^2)*b^2)*B^2*x + 2*((g*n^2 - g*n*log(e))*
a^2*d^2 - (c*d*g*n^2 - 2*d^2*f*n*log(e))*a*b)*B^2*log(b*x + a) + (B^2*b^2*
d^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2
*B^2*b^2*d^2*f*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) - (2*c*
d*f*n - c^2*g*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f*
log(e))*b^2)*B^2*x + (2*a*b*d^2*f*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((
b*x + a)^n) - 2*(B^2*b^2*d^2*g*x^2*log(e) - (2*c*d*f*n - c^2*g*n)*B^2*b^2*
log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f*log(e))*b^2)*B^2*x + (2*a
*b*d^2*f*n - a^2*d^2*g*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^
2*d^2*f*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

3.69.8 Giac [F]

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int (gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (f+g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.70 $\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

3.70.1 Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b}+\frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd}+\frac{2B^2(bc-ad)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

output
```
(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+2*B*(-a*d+b*c)*n*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d+2*B^2*(-a*d+b*c)*n^2*polyl
og(2,d*(b*x+a)/b/(d*x+c))/b/d
```

3.70.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.67

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2ad\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2bc\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)-aBdn\left(\log(a-\right.\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*a*d*Log[a + b*x]*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c
*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

3.70.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2935, 2943, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2935}$$

$$\frac{(a+bx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad) \int \frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}dx}{b}$$

$$\Big\downarrow \text{ 2943}$$

$$\frac{(a+bx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad) \left(\frac{Bn(bc-ad) \int \frac{\log \left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{d}-\frac{\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(-\frac{Bn(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2778}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2005}$$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2752}$$

3.70. $\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}-\frac{Bn\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{d}\right)}{b}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/b - (2*B*(b*c - a*d)*
n*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))
])/d) - (B*n*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/d))/b
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2935
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2943
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d)
/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c
*g, 0]
```

3.70.4 Maple [F]

$$\int \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.70.5 Fricas [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

output
```
integral(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2, x)
```

3.70.6 Sympy [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**2, x)
```

3.70.7 Maxima [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

output
```
2*A*B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*x*log(e*((b*x + a)/(
d*x + c))^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x + a)*log(d*x + c) - b*c*n^2
*log(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d*x*log((d*x + c)^n)^2 + 2*
(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log(e))*log((b*x + a)^n)
- 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log((b*x + a)^n) + b*
d*x*log(e))*log((d*x + c)^n))/(b*d) - integrate(-(b^2*d*x^2*log(e)^2 + a*b
*c*log(e)^2 - ((2*n*log(e) - log(e)^2)*b^2*c - (2*n*log(e) + log(e)^2)*a*b
*d)*x - 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2)*log(b*x + a))/(b^2*d*x^2
 + a*b*c + (b^2*c + a*b*d)*x), x))
```

3.70.8 Giac [F]

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.71 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$

3.71.1 Optimal result

Integrand size = 32, antiderivative size = 297

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = &-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g}\\ &+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}\\ &-\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g}\\ &+\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}\\ &+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g}\\ &-\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}\end{aligned}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*((
b*x+a)/(d*x+c))^n))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*n*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*n*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/g+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-2*B^2*n^2*polylog(3,(-c*g
+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g
```

3.71.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1441 vs. $2(297) = 594$.

Time = 0.30 (sec) , antiderivative size = 1441, normalized size of antiderivative = 4.85

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

output
```
(-(B^2*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/(
(d*f - c*g)*(a + b*x))]^2) + A^2*Log[f + g*x] - 2*A*B*n*Log[a/b + x]*Log[f
 + g*x] + B^2*n^2*Log[a/b + x]^2*Log[f + g*x] + 2*A*B*n*Log[c/d + x]*Log[f
 + g*x] - 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + B^2*n^2*Log[c
/d + x]^2*Log[f + g*x] + 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x]
 - 2*B^2*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + 2*B^
2*n*Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + B^2*Log[e*(
(a + b*x)/(c + d*x))^n]^2*Log[f + g*x] + 2*A*B*n*Log[a/b + x]*Log[(b*(f +
g*x))/(b*f - a*g)] - B^2*n^2*Log[a/b + x]^2*Log[(b*(f + g*x))/(b*f - a*g)]
 + 2*B^2*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*(f + g*x))/(
b*f - a*g)] + 2*B^2*n^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log
[(b*(f + g*x))/(b*f - a*g)] - B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*
Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g
)]*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f + g*x))/
(b*f - a*g)] - B^2*n^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))
]^2*Log[(b*(f + g*x))/(b*f - a*g)] - 2*A*B*n*Log[c/d + x]*Log[(d*(f + g*x)
)/(d*f - c*g)] + 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d*
f - c*g)] - B^2*n^2*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*
n*Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g
)] - 2*B^2*n^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f...
```

3.71.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{f+gx}\,dx$$

$$\Big\downarrow \ 2953$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2804$$

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2009$$

$$(bc-ad)\left(\frac{2Bn\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g(bc-ad)}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{g(bc-ad)}\right.$$

input

```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

output

```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/((b*c - a*d)*g)) + ((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c -
a*d)*g) - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a +
 b*x))/(b*(c + d*x))])/((b*c - a*d)*g) + (2*B*n*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/
((b*c - a*d)*g) + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*
c - a*d)*g) - (2*B^2*n^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(
c + d*x))])/((b*c - a*d)*g))
```

3.71. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$

3.71.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.71.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{gx+f}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

3.71.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{f+gx}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="fricas"
)
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g*x + f), x)
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f),x)
```

output
```
Timed out
```

3.71.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="maxima"
)
```

output
```
A^2*log(g*x + f)/g + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)
^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n
) - 2*(B^2*log((b*x + a)^n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(g*x + f
), x)
```

3.71.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = \int \frac{\left(A + B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{f+g\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x), x)
```

3.72 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$

3.72.1 Optimal result

Integrand size = 32, antiderivative size = 206

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx &= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bf-ag)(f+gx)} \\ &+ \frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} \\ &+ \frac{2B^2(bc-ad)n^2\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}\end{aligned}$$

output
```
(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c
)*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*
x+c))/(-a*g+b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*g+d*f)*(b*x
+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

3.72.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 418 vs. $2(206) = 412$.

Time = 0.28 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.03

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$$
$$= \frac{-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx} + \frac{Bn\left(2b(df-cg)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(bf-ag)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+2(bc-ad)}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)) + (B*n*(2*b*(d*f -
c*g)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(b*f - a*g)
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*(b*c - a*d)*g*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - b*B*(d*f - c*g)*n*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*n*((2*Log[(d*(a + b*x))/(
-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)]) - 2*B*(b*c - a*d)*g*n*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - L
og[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/
(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f
- c*g)))/g
```

3.72.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2953, 2755, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)^2}\,dx$$

$$\Big\downarrow \text{ 2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2755

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2Bn\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$\Big\downarrow$ 2754

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2Bn\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{df-cg}-\frac{\log\left(1-\frac{(a+bx)(df-c\ldots}{(c+dx)(bf-a\ldots}\right.}{}\right.}{bf-ag}\right.$$

$\Big\downarrow$ 2838

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2Bn\left(-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{df-cg}-\frac{Bn\,\mathrm{PolyLo}\ldots}{}\right.}{bf-ag}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*f -
a*g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - (2*B*n*(
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/
((b*f - a*g)*(c + d*x))])/(d*f - c*g)) - (B*n*PolyLog[2, ((d*f - c*g)*(a +
 b*x))/((b*f - a*g)*(c + d*x))])/(d*f - c*g)))/(b*f - a*g))
```

3.72. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$

3.72.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.72.4 Maple [F]

$$\int \frac{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{2}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

3.72.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{\left(gx+f\right)^2}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="frica
s")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**2,x)
```

output
```
Timed out
```

3.72.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{\left(gx+f\right)^2}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="maxim
a")
```

output
```
2*A*B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) +
 (b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B^2*(lo
g((d*x + c)^n)^2/(g^2*x + f*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2
 + (d*g*x + c*g)*log((b*x + a)^n)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log((b
*x + a)^n) + 2*(d*f*n + (g*n - g*log(e))*d*x - c*g*log(e) - (d*g*x + c*g)*
log((b*x + a)^n))*log((d*x + c)^n))/(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*
g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - 2*A*B*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n)/(g^2*x + f*g) - A^2/(g^2*x + f*g)
```

3.72.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^2, x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(f+g\,x)^2}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2, x)
```

3.73 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx$

3.73.1 Optimal result

Integrand size = 32, antiderivative size = 389

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx &= \frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(bf-ag)^2}\\
&\quad-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2}+\frac{B^2(bc-ad)^2gn^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}\\
&\quad+\frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}\\
&\quad+\frac{B^2(bc-ad)(2bdf-bcg-adg)n^2\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}\end{aligned}$$

output
```
B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^2/(-c*
g+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^2-1/
2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*n^2*ln(
(g*x+f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*
d*f)*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/
(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n
^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f
)^2
```

3.73.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left(2(bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

output
```
-1/2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c -
a*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*
b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]) + 2*d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 2*B*(b*c - a*d)*g*n*(f
+ g*x)*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*
c - a*d)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*n*(f + g*x)*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)]) - B*d^2*(b*f - a*g)^2*n*(f + g*x)*((2*Log[(d*(a + b*x
))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)]) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*n*(f + g*x
)*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])
*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f +
 g*x))/(d*f - c*g)])))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

3.73.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)^3}\,dx$$

$$\Big\downarrow \textbf{2953}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \textbf{2798}$$

$$(bc-ad)\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2}\right)$$

$$\Big\downarrow \textbf{2804}$$

$$(bc-ad)\left(\frac{Bn\int \left(\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{(bf-ag)^2(a+bx)}+\frac{(bc-ad)g(-2bdf+bcg+adg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^2(df-cg)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{(bf-ag)(df-cg)\left(bf-ag-\frac{(df-c}{c}\right.}\right.}{g(bc-ad)}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$(bc-ad)\left(\frac{Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bn(bf-ag)^2}+\frac{g^2(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)(bf-ag)^2(df-cg)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx}{(c+dx)}\right.}{(bf-ag)^2(df}\right.}{g}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

3.73. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx$

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))^2) + (B*n*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/((b*f - a*g)^2*(d*f - c*g)*(c + d*x)*(b*f - a*g - ((d*f - c*
g)*(a + b*x))/(c + d*x))) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)
/(2*B*(b*f - a*g)^2*n) + (B*(b*c - a*d)^2*g^2*n*Log[b*f - a*g - ((d*f - c*
g)*(a + b*x))/(c + d*x)])/((b*f - a*g)^2*(d*f - c*g)^2) + ((b*c - a*d)*g*(
2*b*d*f - b*c*g - a*d*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((
d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)
^2) + (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*PolyLog[2, ((d*f - c*g)
*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2)))/((b*
c - a*d)*g))
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.73. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx$

3.73.4 Maple [F]

$$\int \frac{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{\left(gx+f\right)^3}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

3.73.5 Fricas [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3} dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}{\left(gx+f\right)^3} dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="frica
s")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3} dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**3,x)
```

output
```
Timed out
```

3.73.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="maxim
a")
```

output
```
(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(
d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 -
 a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^
2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*
g + a*c*g^3 - (b*c + a*d)*f*g^2)*x))*A*B*n - 1/2*B^2*(log((d*x + c)^n)^2/(
g^3*x^2 + 2*f*g^2*x + f^2*g) + 2*integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2
 + (d*g*x + c*g)*log((b*x + a)^n)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log((b
*x + a)^n) + (d*f*n + (g*n - 2*g*log(e))*d*x - 2*c*g*log(e) - 2*(d*g*x + c
*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3
+ c*g^4)*x^3 + 3*(d*f^2*g^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x
)) - A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2
*g) - 1/2*A^2/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

3.73.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^3, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(f+g\,x\right)^3}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3, x)
```

3.74 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx$

3.74.1 Optimal result

Integrand size = 32, antiderivative size = 747

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx \\
&= \frac{B^2(bc-ad)^2g^2n^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)} - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2} \\
&\quad + \frac{2B(bc-ad)g(3bdf-bcg-2adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)} \\
&\quad + \frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&\quad - \frac{B^2(bc-ad)^3g^2n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} + \frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\right.}{3(bf-ag)^3(df-cg)^3} \\
&\quad + \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)n^2\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}
\end{aligned}
$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*n^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-1/3
*B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(
-c*g+d*f)^3/(g*x+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*n*(b*x+a
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))^2/g/(g*x+f)^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c)
)/(-a*g+b*f)^3/(-c*g+d*f)^3-1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((g*x+f)/(d*x+c
))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*
f)*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2
*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g
+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+
b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+
b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

3.74.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.23

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx =$$

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2bd\ldots}{\ldots}}{\ldots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

output
```
-1/3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*((b*c - a*
d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f +
 g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b^3*(d*f - c*g)^3*(f + g*
x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*
g)^3*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*(
b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*
d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f
 + g*x] - 2*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*(f + g*x)^2*(b*(d*
f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Lo
g[f + g*x]) + B*(b*c - a*d)*g*n*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f
- c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f +
 g*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Lo
g[f + g*x]) + b^3*B*(d*f - c*g)^3*n*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x
] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) - B*d^3*(b*f - a*g)^3*n*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)]) + 2*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3
*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f)
 + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2,...
```

3.74.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 909, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)^4}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{2Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3}\right)$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{2Bn\int \left(\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{(bf-ag)^3(a+bx)}+\frac{(bc-ad)g\left(-\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2\right)+adg(3df-cg)b-a^2d^2g^2\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{(bf-ag)^3(df-cg)^2\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{3g(bc-ad)}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^3}{2B(bf-ag)^3n}+\frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^3(df-cg)^2(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}-\frac{(bc-ad)^3g^3\left(A+B\log\left(e\left(\right.\right.\right.}{2(bf-ag)(df-cg)^3\left(bf-ag-\frac{(a}{}\right.}\right.}{}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

3.74. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx$

output
```
(b*c - a*d)*(-1/3*((b - (d*(a + b*x))/(c + d*x))^3*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))^3) + (2*B*n*((B*(b*c - a*d)^3*g^3*n)/(2*(b*f - a*g)^2*(d*f - c*g)^
3*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - ((b*c - a*d)^3*g^3*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*f - a*g)*(d*f - c*g)^3*(b*f -
a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f
- b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f
 - a*g)^3*(d*f - c*g)^2*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))) + (b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*B*(b*f - a*g)
^3*n) + (B*(b*c - a*d)^3*g^3*n*Log[(a + b*x)/(c + d*x)])/(2*(b*f - a*g)^3*
(d*f - c*g)^3) - (B*(b*c - a*d)^3*g^3*n*Log[b*f - a*g - ((d*f - c*g)*(a +
b*x))/(c + d*x)])/(2*(b*f - a*g)^3*(d*f - c*g)^3) + (B*(b*c - a*d)^2*g^2*(
3*b*d*f - b*c*g - 2*a*d*g)*n*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c +
d*x)])/((b*f - a*g)^3*(d*f - c*g)^3) + ((b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d
*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d
*x))])/((b*f - a*g)^3*(d*f - c*g)^3) + (B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b
*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*PolyLog[2, (
(d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^3*(d*f - c*g
)^3)))/(3*(b*c - a*d)*g))
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.74.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{4}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

3.74.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(f+gx)^{4}}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{4}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="frica
s")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^
4), x)
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**4,x)
```

output
```
Timed out
```

3.74.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="maxim
a")
```

output
```
1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*A*B*n - 1/3*B^2*(
log((d*x + c)^n)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integ
rate(-1/3*(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 3*(d*g*x + c*g)*log((b*x +
a)^n)^2 + 6*(d*g*x*log(e) + c*g*log(e))*log((b*x + a)^n) + 2*(d*f*n + (g*n
 - 3*g*log(e))*d*x - 3*c*g*log(e) - 3*(d*g*x + c*g)*log((b*x + a)^n))*log(
(d*x + c)^n))/(d*g^5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f...
```

3.74.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^4, x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(f+g\,x)^4}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4, x)
```

3.75 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx$

3.75.1 Optimal result

Integrand size = 32, antiderivative size = 1208

$$
\begin{aligned}
&\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx=-\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2}\\
&-\frac{B^2(bc-ad)^3g^3n^2(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)n^2(c+dx)}{4(bf-ag)^3(df-cg)^4(f+gx)}\\
&+\frac{B(bc-ad)g^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}\\
&-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}\\
&+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bf-ag)^4(df-cg)^3(f+gx)}\\
&+\frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}\\
&-\frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}+\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}\\
&+\frac{B^2(bc-ad)^4g^3n^2\log\left(\frac{f+gx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}-\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)n^2\log\left(\frac{f+gx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}\\
&+\frac{B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,n^2\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}\\
&-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2(bf-ag)^4(df-cg)^4}\\
&-\frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)n^2\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a}{(bf-ag)(c}\right.}{2(bf-ag)^4(df-cg)^4}
\end{aligned}
$$

output
```
-1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)
^2-1/6*B^2*(-a*d+b*c)^3*g^3*n^2*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+
1/4*B^2*(-a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*(d*x+c)/(-a*g+b*f)^3
/(-c*g+d*f)^4/(g*x+f)+1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a*
d*g-b*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^
2/(-c*g+d*f)^4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g
+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/(-a*g+b*f)^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))^2/g/(-a*g+b*f)^4-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f
)^4-1/6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*
f)^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x
+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((g*x+f)/(d*
x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4
*b*d*f)*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/2*B^2*(-a*d+b*
c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*
f^2))*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B*(-a*d+b*c)*(
-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*
d^2*f^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+
b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*...
```

3.75.2 Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 1329, normalized size of antiderivative = 1.10

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = -\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left(2(bc-ad)g(bf-ag)^3(df-cg)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-3(bc-ad)g(bf-ag)^2(df-cg)^2(-2\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

output
```
-1/12*(3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c
 - a*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) - 3*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*g
)*(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b*c - a*d)*g*(b*f
- a*g)*(d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
- 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 - 6*b^4*(d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*
a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*
x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 6*B*(b*c - a*d)
*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^
2*g^2))*n*(f + g*x)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log
[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + 3*B*(b*c - a*d)*g*(2*b*d*f - b*c
*g - a*d*g)*n*(f + g*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*
f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*
x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + B*
(b*c - a*d)*g*n*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(
b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*
x) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b*f - a*g)^3...
```

3.75.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 1429, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)^5} \, dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad) \int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4}\right)$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{Bn\int \left(\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^4}{(bf-ag)^4(a+bx)}+\frac{(bc-ad)g(2bdf-bcg-adg)(-2d^2f^2b^2-c^2g^2b^2+2cdfgb^2+2ad^2fgb-a^2d^2g^2)\left(A+B\log\left(e\left(\right.\right.\right.}{(bf-ag)^4(df-cg)^3\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^4}{2B(bf-ag)^4n}+\frac{(bc-ad)^2g^2\left((6d^2f^2-4cdgf+c^2g^2)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^4(df-cg)^3(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

3.75. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx$

output
```
(b*c - a*d)*(-1/4*((b - (d*(a + b*x))/(c + d*x))^4*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))^4) + (B*n*(-1/6*(B*(b*c - a*d)^4*g^4*n)/((b*f - a*g)^2*(d*f - c*g)
^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) - (B*(b*c - a*d)^4*g
^4*n)/(3*(b*f - a*g)^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/
(c + d*x))) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*n)/(2*(b*f
- a*g)^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) +
((b*c - a*d)^4*g^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*f - a*g)*
(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3) - ((b*c -
 a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/
(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g
) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/((b*f - a*g)^4*(d*f - c*g)^3*(c + d*x)*(b*f - a*g - ((d
*f - c*g)*(a + b*x))/(c + d*x))) + (b^4*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2)/(2*B*(b*f - a*g)^4*n) - (B*(b*c - a*d)^4*g^4*n*Log[(a + b*x)/(c +
d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b
*c*g - 3*a*d*g)*n*Log[(a + b*x)/(c + d*x)])/(2*(b*f - a*g)^4*(d*f - c*g)^4
) + (B*(b*c - a*d)^4*g^4*n*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*
x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - ...
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.75. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx$

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.75.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{5}}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)
```

3.75.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(f+gx)^{5}}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{5}}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="frica
s")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^
2 + 5*f^4*g*x + f^5), x)
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**5,x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="maxim
a")
```

output
```
1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

3.75.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="giac"
)
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(g*x + f)^5, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^5}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5,x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5, x)
```

3.76 $\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.76.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Unintegrable((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.76.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

3.76.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \text{ 2955}$$

$$\int \frac{(f+gx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

3.76.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.76.4 Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.76.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log(e*((b*x + a)/(d*x + c))^n) + A),
 x)
```

3.76.6 Sympy [N/A]

Not integrable

Time = 28.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

3.76.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.76.8 Giac [N/A]

Not integrable

Time = 26.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.76.9 Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(f+g\,x)^2}{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

input
```
int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.77 $\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.77.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\operatorname{Int}\left(\frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Unintegrable((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.77.2 Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

3.77.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$
$$\Big\downarrow \text{2955}$$
$$\int \frac{f+gx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
$Aborted
```

3.77.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.77.4 Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.77.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

output
```
integral((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.77.6 Sympy [N/A]

Not integrable

Time = 16.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{f+gx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

input
```
integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

3.77.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.77.8 Giac [N/A]

Not integrable

Time = 16.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.77.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

input
```
int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.78 $\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.78.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Int}\left(\frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

output
```
Unintegrable(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.78.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]
```

output
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x]
```

3.78.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2937}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A} \, dx$$

$$\Big\downarrow \text{2937}$$

$$\int \frac{1}{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]
```

output
```
$Aborted
```

3.78.3.1 Defintions of rubi rules used

rule 2937
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x]
```

3.78.4 Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.78.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

output
```
integral(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.78.6 Sympy [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Integral(1/(A + B*log(e*((a + b*x)/(c + d*x))**n)), x)
```

3.78.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

output
```
integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.78.8 Giac [N/A]

Not integrable

Time = 11.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

3.78.9 Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

input
```
int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.79 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.79.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.79.2 Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

3.79.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \ 2955$$

$$\int \frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

3.79.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.79.4 Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.79.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas"
)
```

output
```
integral(1/(A*g*x + A*f + (B*g*x + B*f)*log(e*((b*x + a)/(d*x + c))^n)), x
)
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.79.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima"
)
```

output
```
integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.79.8 Giac [N/A]

Not integrable

Time = 17.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.79.9 Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.80 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.80.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.80.2 Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

3.80.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

3.80.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.80.4 Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.80.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2
)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.80.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.80.8 Giac [N/A]

Not integrable

Time = 26.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.80.9 Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.81 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

3.81.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.81.2 Mathematica [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

3.81.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$
$$\Big\downarrow \text{ 2955}$$
$$\int \frac{1}{(f+gx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

output
```
$Aborted
```

3.81.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.81.4 Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

3.81.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.72

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

3.81.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.81.8 Giac [N/A]

Not integrable

Time = 35.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

3.81.9 Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.82 $\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.82.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.82.2 Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

3.82.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{(f+gx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

3.82.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.82.4 Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.82.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="frica
s")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 +
 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

3.82.6 Sympy [N/A]

Not integrable

Time = 51.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2,
x)
```

3.82.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 342, normalized size of antiderivative = 10.69

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxim
a")
```

output
```
-(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g
+ c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)
/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)
^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate
((4*b*d*g^2*x^3 + b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2
 + 2*c*f*g)*a + 2*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c*n -
 a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c
*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

3.82.8 Giac [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.82.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(f+g\,x)^2}{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.83 $\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.83.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.83.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

3.83.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2955}$$
$$\int \frac{f+gx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
$Aborted
```

3.83.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.83.4 Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.83.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.00

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas"
)
```

output
```
integral((g*x + f)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b
*x + a)/(d*x + c))^n) + A^2), x)
```

3.83.6 Sympy [N/A]

Not integrable

Time = 60.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2, x)
```

3.83.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 8.30

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima"
)
```

output
```
-(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a
)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x
+ c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integ
rate((3*b*d*g*x^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/(
(b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n
) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

3.83.8 Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

3.83.9 Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{f+g\,x}{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

input
```
int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.84 $\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.84.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.84.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
```

output
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x]
```

3.84.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2937}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2937}$$

$$\int \frac{1}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
```

output
```
$Aborted
```

3.84.3.1 Defintions of rubi rules used

rule 2937
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x]
```

3.84.4 Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.84.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

output
```
integral(1/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/
(d*x + c))^n) + A^2), x)
```

3.84.6 Sympy [N/A]

Not integrable

Time = 22.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**(-2), x)
```

3.84.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 8.12

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (
b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e)
- a*d*n*log(e))*B^2) + integrate((2*b*d*x + b*c + a*d)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n
)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

3.84.8 Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^(-2), x)
```

3.84.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.85 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.85.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.85.2 Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

3.85.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2955}$$
$$\int \frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

3.85.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.85.4 Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.85.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="frica
s")
```

output
```
integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c)
)^n)^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

3.85.6 Sympy [N/A]

Not integrable

Time = 146.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Integral(1/((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x)),
 x)
```

3.85.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 500, normalized size of antiderivative = 15.62

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxim
a")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f*n - a*d*f*n)*A*B + (b*c*f*n*log(e
) - a*d*f*n*log(e))*B^2 + ((b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d
*g*n*log(e))*B^2)*x + ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2
)*log((b*x + a)^n) - ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)
*log((d*x + c)^n)) + integrate((b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g
)*a)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n*log(e) - a*d*f^2*n*log(e))*
B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n*log(e) - a*d*g^2*n*log(e))
*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c*f*g*n*log(e) - a*d*f*g*n
*log(e))*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*
g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*n -
 a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f
^2*n)*B^2)*log((d*x + c)^n)), x)
```

3.85.8 Giac [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

3.85.9 Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.85. $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.86 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.86.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.86.2 Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

3.86.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2} \, dx$$
$$\Big\downarrow \text{ 2955}$$
$$\int \frac{1}{(f+gx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

3.86.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.86.4 Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.86.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.75

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g
*x + B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*
g*x + A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.86.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 752, normalized size of antiderivative = 23.50

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n
*log(e) - a*d*f^2*n*log(e))*B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*
n*log(e) - a*d*g^2*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (
b*c*f*g*n*log(e) - a*d*f*g*n*log(e))*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2
*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log(
(b*x + a)^n) - ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n
)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((d*x + c)^n)) - integrate(-(b*c
*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3*n - a*d*g^3
*n)*A*B + (b*c*g^3*n*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*
d*f^3*n)*A*B + (b*c*f^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n
 - a*d*f*g^2*n)*A*B + (b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 +
 3*((b*c*f^2*g*n - a*d*f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*lo
g(e))*B^2)*x + ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g
^2*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3
*n)*B^2)*log((b*x + a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^
2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^
3*n - a*d*f^3*n)*B^2)*log((d*x + c)^n)), x)
```

3.86.8 Giac [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

3.86.9 Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)^2 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.87 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

3.87.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.87.2 Mathematica [N/A]

Not integrable

Time = 18.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

3.87.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2955}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2955}$$

$$\int \frac{1}{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

output
```
$Aborted
```

3.87.3.1 Defintions of rubi rules used

rule 2955
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f + g*x)
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p, x] /; FreeQ[{a, b, c, d, e, f,
 g, A, B, m, n, p}, x]
```

3.87.4 Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

3.87.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.03

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2
*g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)
*log(e*((b*x + a)/(d*x + c))^n)), x)
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

3.87.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 1001, normalized size of antiderivative = 31.28

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B + (b*c*g^3*
n*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*A*B + (b*c
*f^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n - a*d*f*g^2*n)*A*B
 + (b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 + 3*((b*c*f^2*g*n -
a*d*f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*log(e))*B^2)*x + ((b*
c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(
b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((b*x +
 a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*
B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^
2)*log((d*x + c)^n)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*
(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4*n - a*d*g^4*n)*A*B + (b*c*g^4*n*log(
e) - a*d*g^4*n*log(e))*B^2)*x^4 + 4*((b*c*f*g^3*n - a*d*f*g^3*n)*A*B + (b*
c*f*g^3*n*log(e) - a*d*f*g^3*n*log(e))*B^2)*x^3 + (b*c*f^4*n - a*d*f^4*n)*
A*B + (b*c*f^4*n*log(e) - a*d*f^4*n*log(e))*B^2 + 6*((b*c*f^2*g^2*n - a*d*
f^2*g^2*n)*A*B + (b*c*f^2*g^2*n*log(e) - a*d*f^2*g^2*n*log(e))*B^2)*x^2 +
4*((b*c*f^3*g*n - a*d*f^3*g*n)*A*B + (b*c*f^3*g*n*log(e) - a*d*f^3*g*n*log
(e))*B^2)*x + ((b*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^
3*n)*B^2*x^3 + 6*(b*c*f^2*g^2*n - a*d*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n
- a*d*f^3*g*n)*B^2*x + (b*c*f^4*n - a*d*f^4*n)*B^2)*log((b*x + a)^n) - ((b
*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3 +...
```

3.87.8 Giac [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

3.87.9 Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.88 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.88.1 Optimal result

Integrand size = 30, antiderivative size = 180

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{B(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2} - \frac{B(bc-ad)g^4(a+bx)^4}{20bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{B(bc-ad)^5g^4\log(c+dx)}{5bd^5}$$

output
```
1/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+1/15*
B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5
*g^4*(b*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/5*B*(-a*d+b*c)^5*g^4*ln(d*x
+c)/b/d^5
```

3.88.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(bc-ad)(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+}{12d^5}\right.}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(-
12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c) +
 a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(1
2*d^5)))/(5*b)
```

3.88.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\Big\downarrow \; 2948$$
$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-\frac{B(bc-ad)\int \frac{g^5(a+bx)^4}{c+dx}dx}{5bg}$$
$$\Big\downarrow \; 27$$
$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-\frac{Bg^4(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}$$
$$\Big\downarrow \; 49$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-$$
$$\frac{Bg^4(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-$$
$$\frac{Bg^4(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b) - (B*(b*c - a
*d)*g^4*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3)
- ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*L
og[c + d*x])/d^5))/(5*b)
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(168) = 336$.

Time = 1.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.47

method	result
risch	$\frac{g^4 b^3 B a c^2 x^2}{2d^2} - \frac{2g^4 b B a^3 c x}{d} + \frac{2g^4 b^2 B a^2 c^2 x}{d^2} - \frac{g^4 b^3 B a c^3 x}{d^3} + \frac{g^4 B \ln(dx+c) a^5}{5b} + \frac{g^4 b^4 A x^5}{5} + g^4 b^3 A a x^4 +$
parallelrisch	$\frac{16B x^3 a^2 b^3 d^5 g^4 + 4B x^3 b^5 c^2 d^3 g^4 + 120 A x^2 a^3 b^2 d^5 g^4 + 36 B x^2 a^3 b^2 d^5 g^4 - 6B x^2 b^5 c^3 d^2 g^4 + 60 A x a^4 b d^5 g^4 + 48 B x a^4 b d^5 g}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/2*g^4/d^2*b^3*B*a*c^2*x^2-2*g^4/d*b*B*a^3*c*x+2*g^4/d^2*b^2*B*a^2*c^2*x-
g^4/d^3*b^3*B*a*c^3*x+1/5*g^4/b*B*ln(d*x+c)*a^5+1/5*g^4*b^4*A*x^5+g^4*b^3*
A*a*x^4+1/20*g^4*b^3*B*a*x^4-1/20*g^4/d*b^4*B*c*x^4+2*g^4*b^2*A*a^2*x^3+4/
15*g^4*b^2*B*a^2*x^3+1/15*g^4/d^2*b^4*B*c^2*x^3+2*g^4*b*A*a^3*x^2+3/5*g^4*
b*B*a^3*x^2-1/10*g^4/d^3*b^4*B*c^3*x^2+g^4*A*a^4*x+4/5*g^4*B*a^4*x+1/5*g^4
/d^4*b^4*B*c^4*x-g^4/d*B*ln(d*x+c)*a^4*c-1/5*g^4/d^5*b^4*B*ln(d*x+c)*c^5+2
*g^4/d^2*b*B*ln(d*x+c)*a^3*c^2-2*g^4/d^3*b^2*B*ln(d*x+c)*a^2*c^3+g^4/d^4*b
^3*B*ln(d*x+c)*a*c^4-1/3*g^4/d*b^3*B*a*c*x^3-g^4/d*b^2*B*a^2*c*x^2+1/5*(b*
x+a)^5*g^4*B/b*ln(e*(b*x+a)/(d*x+c))
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(168) = 336$.

Time = 0.30 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.39

$$\int (ag+bgx)^4 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 + 12\,Ba^5d^5g^4\log(bx+a) - 3\,(Bb^5cd^4 - (20\,A+B)ab^4d^5)g^4x^4 + 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4 +}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^
4 - (20*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 2
*(15*A + 2*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3
+ 10*B*a^2*b^3*c*d^4 - 2*(10*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 12*(B*b^5*c^4
*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + (5*A
+ 4*B)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c
^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*
b^5*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*
a^3*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b*e*x + a*e)/(d*x + c)))/(
b*d^5)
```

3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(155) = 310$.

Time = 3.52 (sec) , antiderivative size = 969, normalized size of antiderivative = 5.38

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= \frac{Ab^4g^4x^5}{5} + \frac{Ba^5g^4\log\left(x+\frac{\frac{Ba^6d^5g^4}{b}+5Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5Ba^2b^3c^4dg^4+Bab^4c^5g^4}{Ba^5d^5g^4+5Ba^4bcd^4g^4-10Ba^3b^2c^2d^3g^4+10Ba^2b^3c^3d^2g^4-5Bab^4c^4dg^4+Bb^5c^5g^4}\right)}{5b}$$

$$-\frac{Bcg^4\cdot(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4)\log\left(x+\frac{6Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5B}{}\right.}{5d^5}$$

$$+x^4\left(Aab^3g^4+\frac{Bab^3g^4}{20}-\frac{Bb^4cg^4}{20d}\right)+x^3\cdot\left(2Aa^2b^2g^4+\frac{4Ba^2b^2g^4}{15}-\frac{Bab^3cg^4}{3d}+\frac{Bb^4c^2g^4}{15d^2}\right)$$

$$+x^2\cdot\left(2Aa^3bg^4+\frac{3Ba^3bg^4}{5}-\frac{Ba^2b^2cg^4}{d}+\frac{Bab^3c^2g^4}{2d^2}-\frac{Bb^4c^3g^4}{10d^3}\right)$$

$$+x\left(Aa^4g^4+\frac{4Ba^4g^4}{5}-\frac{2Ba^3bcg^4}{d}+\frac{2Ba^2b^2c^2g^4}{d^2}-\frac{Bab^3c^3g^4}{d^3}+\frac{Bb^4c^4g^4}{5d^4}\right)$$

$$+\left(Ba^4g^4x+2Ba^3bg^4x^2+2Ba^2b^2g^4x^3+Bab^3g^4x^4+\frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**4*g**4*x**5/5 + B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*
d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5
*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**
4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2
*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) - B*c*g**4*(5*a*
*4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b*
*4*c**4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B
*a**3*b**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4
 - B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5
*a*b**3*c**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**
3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*
g**4 + 5*B*a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*
b**3*c**3*d**2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5)
 + x**4*(A*a*b**3*g**4 + B*a*b**3*g**4/20 - B*b**4*c*g**4/(20*d)) + x**3*(
2*A*a**2*b**2*g**4 + 4*B*a**2*b**2*g**4/15 - B*a*b**3*c*g**4/(3*d) + B*b**
4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 + 3*B*a**3*b*g**4/5 - B*a**
2*b**2*c*g**4/d + B*a*b**3*c**2*g**4/(2*d**2) - B*b**4*c**3*g**4/(10*d**3)
) + x*(A*a**4*g**4 + 4*B*a**4*g**4/5 - 2*B*a**3*b*c*g**4/d + 2*B*a**2*b**2
*c**2*g**4/d**2 - B*a*b**3*c**3*g**4/d**3 + B*b**4*c**4*g**4/(5*d**4)) + (
B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b*...
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(168) = 336$.

Time = 0.21 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.46

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{5}Ab^4g^4x^5 + Aab^3g^4x^4 + 2Aa^2b^2g^4x^3$$
$$+ 2Aa^3bg^4x^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^4g^4$$
$$+2\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^3bg^4$$
$$+\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d}{b^2d^2}\right.$$
$$+\frac{1}{6}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2}{b}\right.$$
$$+\frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4(}{}\right.$$
$$+Aa^4g^4x$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*
x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d
*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*l
og(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^3*b*g^4
+ (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2
*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x
)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3))*B*a*b^3*g^4 + 1/60*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*
d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x
^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4036 vs. $2(168) = 336$.

Time = 0.53 (sec) , antiderivative size = 4036, normalized size of antiderivative = 22.42

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/60*(12*(B*b^10*c^6*e^6*g^4 - 6*B*a*b^9*c^5*d*e^6*g^4 + 15*B*a^2*b^8*c^4*
d^2*e^6*g^4 - 20*B*a^3*b^7*c^3*d^3*e^6*g^4 + 15*B*a^4*b^6*c^2*d^4*e^6*g^4
- 6*B*a^5*b^5*c*d^5*e^6*g^4 + B*a^6*b^4*d^6*e^6*g^4 - 5*(b*e*x + a*e)*B*b^
9*c^6*d*e^5*g^4/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g^4/(d*x
+ c) - 75*(b*e*x + a*e)*B*a^2*b^7*c^4*d^3*e^5*g^4/(d*x + c) + 100*(b*e*x +
 a*e)*B*a^3*b^6*c^3*d^4*e^5*g^4/(d*x + c) - 75*(b*e*x + a*e)*B*a^4*b^5*c^2
*d^5*e^5*g^4/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^4*c*d^6*e^5*g^4/(d*x + c
) - 5*(b*e*x + a*e)*B*a^6*b^3*d^7*e^5*g^4/(d*x + c) + 10*(b*e*x + a*e)^2*B
*b^8*c^6*d^2*e^4*g^4/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a*b^7*c^5*d^3*e^4*
g^4/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^4/(d*x + c)^
2 - 200*(b*e*x + a*e)^2*B*a^3*b^5*c^3*d^5*e^4*g^4/(d*x + c)^2 + 150*(b*e*x
 + a*e)^2*B*a^4*b^4*c^2*d^6*e^4*g^4/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5
*b^3*c*d^7*e^4*g^4/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*b^2*d^8*e^4*g^4/
(d*x + c)^2 - 10*(b*e*x + a*e)^3*B*b^7*c^6*d^3*e^3*g^4/(d*x + c)^3 + 60*(b
*e*x + a*e)^3*B*a*b^6*c^5*d^4*e^3*g^4/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*
a^2*b^5*c^4*d^5*e^3*g^4/(d*x + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^
6*e^3*g^4/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^4*b^3*c^2*d^7*e^3*g^4/(d*x
 + c)^3 + 60*(b*e*x + a*e)^3*B*a^5*b^2*c*d^8*e^3*g^4/(d*x + c)^3 - 10*(b*e
*x + a*e)^3*B*a^6*b*d^9*e^3*g^4/(d*x + c)^3 + 5*(b*e*x + a*e)^4*B*b^6*c^6*
d^4*e^2*g^4/(d*x + c)^4 - 30*(b*e*x + a*e)^4*B*a*b^5*c^5*d^5*e^2*g^4/(d...
```

3.88. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

3.88.9 Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 1009, normalized size of antiderivative = 5.61

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$=\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,a^4\,g^4\,x+2\,B\,a^3\,b\,g^4\,x^2+2\,B\,a^2\,b^2\,g^4\,x^3+B\,a\,b^3\,g^4\,x^4+\frac{B\,b^4\,g^4\,x^5}{5}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{15\,b\,d}\right.$$

$$\left.-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+x^2\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}\right.}{10\,b\,d}\right.$$

$$+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}$$

$$\left.-\frac{a\,c\,\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}\right)$$

$$+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}\right.}{5\,b\,d}\right.}{5\,b\,d}$$

$$a\,c\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
log((e*(a + b*x))/(c + d*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*
g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a
*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b*c
))/(3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*
A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d
))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*
a*d - B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*
d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d +
 10*A*b*c + 2*B*a*d - 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*(((
(b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c
+ B*a*d - B*b*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d
+ 5*A*b*c + B*a*d - B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d
 - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (
a*c*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(
5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5
*A*b*c + B*a*d - B*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(2
5*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c)...
```

3.88. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.89 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.89.1 Optimal result

Integrand size = 30, antiderivative size = 149

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = -\frac{B(bc-ad)^3g^3x}{4d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2} - \frac{B(bc-ad)g^3(a+bx)^3}{12bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} + \frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4}$$

output
```
-1/4*B*(-a*d+b*c)^3*g^3*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/12*
B*(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)
))/b+1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

3.89.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}\right)}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(6
*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^
3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

3.89.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \; 2948$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{B(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{4bg}$$

$$\Big\downarrow \; 27$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{4b}$$

$$\Big\downarrow \; 49$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}$$

$$\Big\downarrow \; 2009$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b) - (B*(b*c - a
*d)*g^3*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a
+ b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(4*b)
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.89.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(139) = 278$.

Time = 0.88 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx+a)^4 g^3 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{4b} + \frac{g^3 b^3 A x^4}{4} + g^3 b^2 A a x^3 + \frac{g^3 b^2 B a x^3}{12} - \frac{g^3 b^3 B c x^3}{12d} + \frac{3 g^3 b A a^2 x^2}{2} + \frac{3 g^3 b B a^2 x}{8}$
parallelrisch	$24 B x \ln\left(\frac{e(bx+a)}{dx+c}\right) a^3 b d^4 g^3 + 24 B \ln\left(\frac{e(bx+a)}{dx+c}\right) a b^3 c^3 d g^3 - 24 B \ln(bx+a) a^3 b c d^3 g^3 + 9 B a^3 b c d^3 g^3 + 24 B a^2 b^2 c^2 d^2 g^3 - 21$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x+a)^4*g^3*B/b*ln(e*(b*x+a)/(d*x+c))+1/4*g^3*b^3*A*x^4+g^3*b^2*A*a*
x^3+1/12*g^3*b^2*B*a*x^3-1/12*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2+3/8*g^
3*b*B*a^2*x^2-1/2*g^3*b^2/d*B*a*c*x^2+1/8*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*
x+1/4*g^3/b*B*ln(d*x+c)*a^4-g^3/d*B*ln(d*x+c)*a^3*c+3/2*g^3*b/d^2*B*ln(d*x
+c)*a^2*c^2-g^3*b^2/d^3*B*ln(d*x+c)*a*c^3+1/4*g^3*b^3/d^4*B*ln(d*x+c)*c^4+
3/4*g^3*B*a^3*x-3/2*g^3*b/d*B*a^2*c*x+g^3*b^2/d^2*B*a*c^2*x-1/4*g^3*b^3/d^
3*B*c^3*x
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(139) = 278$.

Time = 0.28 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.13

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4g^3x^4 + 6\,Ba^4d^4g^3\log(bx+a) - 2\,(Bb^4cd^3 - (12\,A+B)ab^3d^4)g^3x^3 + 3\,(Bb^4c^2d^2 - 4\,Bab^3cd^3 + 3}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3
- (12*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(
4*A + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a
^2*b^2*c*d^3 - (4*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3
*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^
4*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^
4*g^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^4)
```

3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.

Time = 2.04 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= \frac{Ab^3g^3x^4}{4} + \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{4b}$$

$$- \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-bc)}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g}\right.}{4d^4}$$

$$+ x^3\left(Aab^2g^3+\frac{Bab^2g^3}{12}-\frac{Bb^3cg^3}{12d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}+\frac{3Ba^2bg^3}{8}-\frac{Bab^2cg^3}{2d}+\frac{Bb^3c^2g^3}{8d^2}\right)$$

$$+ x\left(Aa^3g^3+\frac{3Ba^3g^3}{4}-\frac{3Ba^2bcg^3}{2d}+\frac{Bab^2c^2g^3}{d^2}-\frac{Bb^3c^3g^3}{4d^3}\right)$$

$$+ \left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*
d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b*
*3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c
**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) - B*c*g*
*3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c
*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b
**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d
)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**
3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*
g**3 + B*a*b**2*g**3/12 - B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2
+ 3*B*a**2*b*g**3/8 - B*a*b**2*c*g**3/(2*d) + B*b**3*c**2*g**3/(8*d**2)) +
 x*(A*a**3*g**3 + 3*B*a**3*g**3/4 - 3*B*a**2*b*c*g**3/(2*d) + B*a*b**2*c**
2*g**3/d**2 - B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**
3*x**2/2 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)/(c + d
*x))
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(139) = 278$.

Time = 0.20 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.95

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{1}{4}Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}Aa^2bg^3x^2$$
$$+ \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^3g^3$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^2bg^3$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2}{}\right.$$
$$+Aa^3g^3x$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^3*g^3
 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 +
c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x +
 c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B
*a*b^2*g^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(
b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g
^3 + A*a^3*g^3*x
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. $2(139) = 278$.

Time = 0.45 (sec) , antiderivative size = 2776, normalized size of antiderivative = 18.63

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
-1/24*(6*(B*b^8*c^5*e^5*g^3 - 5*B*a*b^7*c^4*d*e^5*g^3 + 10*B*a^2*b^6*c^3*d
^2*e^5*g^3 - 10*B*a^3*b^5*c^2*d^3*e^5*g^3 + 5*B*a^4*b^4*c*d^4*e^5*g^3 - B*
a^5*b^3*d^5*e^5*g^3 - 4*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^3/(d*x + c) + 20*(
b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^
5*c^3*d^3*e^4*g^3/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g^3/(
d*x + c) - 20*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^3/(d*x + c) + 4*(b*e*x +
 a*e)*B*a^5*b^2*d^6*e^4*g^3/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^6*c^5*d^2*e^
3*g^3/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*g^3/(d*x + c)^2
 + 60*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 60*(b*e*x +
a*e)^2*B*a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b^
2*c*d^6*e^3*g^3/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*b*d^7*e^3*g^3/(d*x +
 c)^2 - 4*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3 + 20*(b*e*x +
a*e)^3*B*a*b^4*c^4*d^4*e^2*g^3/(d*x + c)^3 - 40*(b*e*x + a*e)^3*B*a^2*b^3*
c^3*d^5*e^2*g^3/(d*x + c)^3 + 40*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*g^3
/(d*x + c)^3 - 20*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 + 4*(b
*e*x + a*e)^3*B*a^5*d^8*e^2*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/
(b^4*d^4*e^4 - 4*(b*e*x + a*e)*b^3*d^5*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b
^2*d^6*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e/(d*x + c)^3 + (b*e*x +
a*e)^4*d^8/(d*x + c)^4) + (6*A*b^8*c^5*e^5*g^3 + 11*B*b^8*c^5*e^5*g^3 - 30
*A*a*b^7*c^4*d*e^5*g^3 - 55*B*a*b^7*c^4*d*e^5*g^3 + 60*A*a^2*b^6*c^3*d^...
```

3.89. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.89.9 Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\cdots}{\cdots}\right)}{4\,b\,d}+\frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d-3\,B\,b\,c)}{2\,d}-\frac{a\,c\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$
$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$
$$+x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$
$$+\frac{\ln(c+d\,x)\,(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3)}{4\,d^4}$$
$$+\frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{4\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d
) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3
*(6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a
^2*g^3*(8*A*a*d + 12*A*b*c + 3*B*a*d - 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(1
6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4
*d)))/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*
(6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log(
(e*(a + b*x))/(c + d*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3
*x^2)/2 + B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B
*b*c))/(12*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) + (log(c + d*x)*(B*b^3
*c^4*g^3 - 4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3
))/(4*d^4) + (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(4*b)
```

3.89. $\int (ag + bgx)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.90 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

3.90.1 Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{B(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{6bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}$$

output
```
1/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b
*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)/b/d
^3
```

3.90.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(-bc+ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}\right)}{3b}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)
*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d*x
]))/(2*d^3)))/(3*b)
```

3.90.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{B(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{Bg^2(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{Bg^2(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{Bg^2(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) - (B*(b*c - a
*d)*g^2*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log
[c + d*x])/d^3))/(3*b)
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.90.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(bx+a)^3 g^2 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{3b} + \frac{g^2 b^2 A x^3}{3} + g^2 bAa\, x^2 + \frac{g^2 bBa\, x^2}{6} - \frac{g^2 b^2 Bc\, x^2}{6d} + g^2 A\, a^2 x + \frac{g^2 B \ln(dx+c) a}{3b}$
parallelrisch	$\frac{2B x^3 \ln\left(\frac{e(bx+a)}{dx+c}\right) a b^3 c d^3 g^2 + 2A x^3 a b^3 c d^3 g^2 + 6B x^2 \ln\left(\frac{e(bx+a)}{dx+c}\right) a^2 b^2 c d^3 g^2 + 6A x^2 a^2 b^2 c d^3 g^2 + B x^2 a^2 b^2 c d^3 g^2 - B x}{}$
parts	$\frac{A g^2 (bx+a)^3}{3b} - \frac{B g^2 (ad-cb) e \left(2be(a^2d^2-2abcd+b^2c^2)\left(-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)} - \frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d} + \right.\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(A d^2 g^2 (a^2d^2-2abcd+b^2c^2)\left(-\frac{be}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2} + \frac{b^2e^2}{3d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3} + \frac{1}{d^3\left(be-\left(\frac{be}{d}+\right.\right.}\right.\right.}{}$
default	$-\frac{e(ad-cb)\left(A d^2 g^2 (a^2d^2-2abcd+b^2c^2)\left(-\frac{be}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2} + \frac{b^2e^2}{3d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3} + \frac{1}{d^3\left(be-\left(\frac{be}{d}+\right.\right.}\right.\right.}{}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3*g^2*B/b*ln(e*(b*x+a)/(d*x+c))+1/3*g^2*b^2*A*x^3+g^2*b*A*a*x^
2+1/6*g^2*b*B*a*x^2-1/6*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x+1/3*g^2/b*B*ln(d*x+c
)*a^3-g^2/d*B*ln(d*x+c)*a^2*c+g^2*b/d^2*B*ln(d*x+c)*a*c^2-1/3*g^2*b^2/d^3*
B*ln(d*x+c)*c^3+2/3*g^2*B*a^2*x-g^2*b/d*B*a*c*x+1/3*g^2*b^2/d^2*B*c^2*x
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(110) = 220.

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.88

$$\int (ag+bgx)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{2\,Ab^3d^3g^2x^3 + 2\,Ba^3d^3g^2 \log(bx+a) - (Bb^3cd^2 - (6\,A+B)ab^2d^3)g^2x^2 + 2\,(Bb^3c^2d - 3\,Bab^2cd^2 + (3\,A}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (
6*A + B)*a*b^2*d^3)*g^2*x^2 + 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + (3*A + 2*
B)*a^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^
2*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*
d^3*g^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^3)
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.31 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$

$$= \frac{Ab^2g^2x^3}{3} + \frac{Ba^3g^2\log\left(x+\frac{\frac{Ba^4d^3g^2}{b}+3Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3b}$$

$$- \frac{Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{4Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2-Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{Bbc^2g^2\cdot\left(3a^2d^2-3\right.}{d}}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right.}{3d^3}$$

$$+ x^2\left(Aabg^2+\frac{Babg^2}{6}-\frac{Bb^2cg^2}{6d}\right)+x\left(Aa^2g^2+\frac{2Ba^2g^2}{3}-\frac{Babcg^2}{d}+\frac{Bb^2c^2g^2}{3d^2}\right)$$

$$+ \left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**2*g**2*x**3/3 + B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*
d**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2
 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3
*b) - B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**
2*d**2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d +
 b**2*c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**
2*d*g**2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 + B*a*b*g**2/6 -
 B*b**2*c*g**2/(6*d)) + x*(A*a**2*g**2 + 2*B*a**2*g**2/3 - B*a*b*c*g**2/d
+ B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g
**2*x**3/3)*log(e*(a + b*x)/(c + d*x))
```

3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(110) = 220.

Time = 0.21 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.37

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{1}{3}\,Ab^2g^2x^3+Aabg^2x^2$$
$$+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)Ba^2g^2$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log\left(bx+a\right)}{b^2}+\frac{c^2\log\left(dx+c\right)}{d^2}-\frac{(bc-ad)x}{bd}\right)Babg^2$$
$$+\frac{1}{6}\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a}{b^2d^2}\right.$$
$$+Aa^2g^2x$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c
- a*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(110) = 220.

Time = 0.42 (sec) , antiderivative size = 1742, normalized size of antiderivative = 14.76

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/6*(2*(B*b^6*c^4*e^4*g^2 - 4*B*a*b^5*c^3*d*e^4*g^2 + 6*B*a^2*b^4*c^2*d^2*
e^4*g^2 - 4*B*a^3*b^3*c*d^3*e^4*g^2 + B*a^4*b^2*d^4*e^4*g^2 - 3*(b*e*x + a
*e)*B*b^5*c^4*d*e^3*g^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g
^2/(d*x + c) - 18*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) + 12*(
b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*b*d
^5*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2
 - 12*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 + 18*(b*e*x + a*
e)^2*B*a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^3*b*c*
d^5*e^2*g^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x + c)^2)
*log((b*e*x + a*e)/(d*x + c))/(b^3*d^3*e^3 - 3*(b*e*x + a*e)*b^2*d^4*e^2/(
d*x + c) + 3*(b*e*x + a*e)^2*b*d^5*e/(d*x + c)^2 - (b*e*x + a*e)^3*d^6/(d*
x + c)^3) + (2*A*b^6*c^4*e^4*g^2 + 3*B*b^6*c^4*e^4*g^2 - 8*A*a*b^5*c^3*d*e
^4*g^2 - 12*B*a*b^5*c^3*d*e^4*g^2 + 12*A*a^2*b^4*c^2*d^2*e^4*g^2 + 18*B*a^
2*b^4*c^2*d^2*e^4*g^2 - 8*A*a^3*b^3*c*d^3*e^4*g^2 - 12*B*a^3*b^3*c*d^3*e^4
*g^2 + 2*A*a^4*b^2*d^4*e^4*g^2 + 3*B*a^4*b^2*d^4*e^4*g^2 - 6*(b*e*x + a*e)
*A*b^5*c^4*d*e^3*g^2/(d*x + c) - 7*(b*e*x + a*e)*B*b^5*c^4*d*e^3*g^2/(d*x
+ c) + 24*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) + 28*(b*e*x + a*
e)*B*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) - 36*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*
e^3*g^2/(d*x + c) - 42*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) +
 24*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) + 28*(b*e*x + a*e)*...
```

3.90.9 Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$-\frac{\ln(c+d\,x)\,(3\,B\,a^2\,c\,d^2\,g^2-3\,B\,a\,b\,c^2\,d\,g^2+B\,b^2\,c^3\,g^2)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}+\frac{B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d - B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d -
 B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a
*d + 3*A*b*c + B*a*d - B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(a + b*x))/(c
 + d*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) - (log(c + d*x)
*(B*b^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2
*g^2*x^3)/3 + (B*a^3*g^2*log(a + b*x))/(3*b)
```

3.90. $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.91 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.91.1 Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=-\frac{B(bc-ad)gx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}$$

output
```
-1/2*B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/2*
B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

3.91.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)*(
b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

3.91.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}-\frac{B(bc-ad)\int \frac{g^2(a+bx)}{c+dx}dx}{2bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\int \frac{a+bx}{c+dx}dx}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b) - (B*(b*c - a*d
)*g*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(2*b)
```

3.91.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.91.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{gbA\,x^2}{2}+gAax+\frac{B\,a^2g\ln(-bx-a)}{2b}-\frac{gB\ln(dx+c)ac}{d}+\frac{gbB\ln(dx+c)c^2}{2d^2}+\frac{gB}{2}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)ab\,d^2g+2Axab\,d^2g+B\ln(bx+a)a^2d^2g-2B\ln(bx+a)abcdg+B\ln}{2}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-\frac{Bg(ad-cb)e\left(bde(ad-cb)\left(-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}+\frac{\ln\left(\frac{be}{d}\right.}{}\right.\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-A\,d^2g(ad-cb)\left(\frac{be}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-\frac{1}{d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)-B\,d^2g(ad-cb)\left(-\frac{be\left(-\frac{\ln\left(b\right.}{}\right.}{}\right.\right.}{}$
default	$-\frac{e(ad-cb)\left(-A\,d^2g(ad-cb)\left(\frac{be}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-\frac{1}{d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)-B\,d^2g(ad-cb)\left(-\frac{be\left(-\frac{\ln\left(b\right.}{}\right.}{}\right.\right.}{}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*B*x*(b*x+2*a)*ln(e*(b*x+a)/(d*x+c))+1/2*g*b*A*x^2+g*A*a*x+1/2*B*a^2*
g/b*ln(-b*x-a)-g/d*B*ln(d*x+c)*a*c+1/2*g*b/d^2*B*ln(d*x+c)*c^2+1/2*g*B*a*x
-1/2*g*b/d*B*c*x
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+Ba^2d^2g\log(bx+a)-(Bb^2cd-(2\,A+B)abd^2)gx+(Bb^2c^2-2\,Babcd)g\log(dx+c)+(Bb^2}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*log(b*x + a) - (B*b^2*c*d - (2*A + B)*a
*b*d^2)*g*x + (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2
+ 2*B*a*b*d^2*g*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^2)
```

3.91.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.

Time = 0.87 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{Abgx^2}{2} + \frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b}$$
$$- \frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2}$$
$$+ x\left(Aag+\frac{Bag}{2}-\frac{Bbcg}{2d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*
c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) - B*c*g*(2*
a*d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c)
+ B*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2
*g))/(2*d**2) + x*(A*a*g + B*a*g/2 - B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**
2/2)*log(e*(a + b*x)/(c + d*x))
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{1}{2}Abgx^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bag$$
$$+ \frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bbg$$
$$+ Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/2*A*b*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
 - c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*
g + A*a*g*x
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 869 vs. $2(75) = 150$.

Time = 0.37 (sec) , antiderivative size = 869, normalized size of antiderivative = 10.73

$$\int (ag + bgx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx =$$
$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3e^3g - 3\,Bab^3c^2de^3g + 3\,Ba^2b^2cd^2e^3g - Ba^3bd^3e^3g - \frac{2\,(bex+ae)Bb^3c^3de^2g}{dx+c} + \frac{6\,(bex+ae)Bab^2c^2d^2e^2g}{dx+c}\right.}{b^2d^2e^2 - \frac{2\,(bex+ae)bd^3e}{dx+c} + \frac{(bex+ae)^2d^4}{(dx+c)^2}}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
-1/2*((B*b^4*c^3*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*B*a^2*b^2*c*d^2*e^3*g -
 B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*(b*e*
x + a*e)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*B*a^2*b*c*d^3*e
^2*g/(d*x + c) + 2*(b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))*log((b*e*x + a
*e)/(d*x + c))/(b^2*d^2*e^2 - 2*(b*e*x + a*e)*b*d^3*e/(d*x + c) + (b*e*x +
 a*e)^2*d^4/(d*x + c)^2) + (A*b^4*c^3*e^3*g + B*b^4*c^3*e^3*g - 3*A*a*b^3*
c^2*d*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*A*a^2*b^2*c*d^2*e^3*g + 3*B*a^2*b^
2*c*d^2*e^3*g - A*a^3*b*d^3*e^3*g - B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*A*
b^3*c^3*d*e^2*g/(d*x + c) - (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*
(b*e*x + a*e)*A*a*b^2*c^2*d^2*e^2*g/(d*x + c) + 3*(b*e*x + a*e)*B*a*b^2*c^
2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d*x + c) - 3*
(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*g/(d*x + c) + 2*(b*e*x + a*e)*A*a^3*d^4*e^
2*g/(d*x + c) + (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/(b^2*d^2*e^2 - 2*
(b*e*x + a*e)*b*d^3*e/(d*x + c) + (b*e*x + a*e)^2*d^4/(d*x + c)^2) + (B*b^
3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log
(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*d^2) - (B*b^3*c^3*e*g - 3*B*a*b^2*c^
2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log((b*e*x + a*e)/(d*x + c)
)/(b*d^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
- a*d)))
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d} - \frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right) + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right) + \frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,g-2\,B\,a\,c\,d\,g\right)}{2\,d^2} + \frac{A\,b\,g\,x^2}{2} + \frac{B\,a^2\,g\,\ln(a+b\,x)}{2\,b}$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*((g*(4*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(
2*d)) + log((e*(a + b*x))/(c + d*x))*((B*b*g*x^2)/2 + B*a*g*x) + (log(c +
d*x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2*g*log(a +
 b*x))/(2*b)
```

3.92 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx$

3.92.1 Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} + \frac{B\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g+B*polylog(2,1+(-a
*d+b*c)/d/(b*x+a))/b/g
```

3.92.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx = \frac{\log(g(a+bx))\left(-B\log(g(a+bx))+2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+2B\,\text{PolyLog}\left(2,\frac{d(a+b}{-bc+c}\right.}{2bg}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

output
```
(Log[g*(a + b*x)]*(-(B*Log[g*(a + b*x)]) + 2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)] + B*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)])/(2*b*g)
```

3.92.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{ag+bgx} \, dx$$

$$\Big\downarrow \text{2942}$$

$$\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$-\frac{B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$-\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2752}$$

$$\frac{B\operatorname{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(b*g)) + (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

3.92.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(79) = 158$.

Time = 1.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.86

method	result
parts	$\frac{A\ln(bx+a)}{gb}-\frac{B(ad-cb)e\left(-\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)be}+\frac{d^3\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}\cdots\right.}{\cdots}\right.}{d}\cdots\right.}{(ad-cb)be}\cdots\right.}{g\,d^2\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{g(ad-cb)}-\frac{d^2B\left(-\frac{d\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{d^2\cdots}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{g(ad-cb)}-\frac{d^2B\left(-\frac{d\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{d^2\cdots}$
3.92. $\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx$ risch	$\frac{A\ln(bx+a)}{gb}+\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2a}{2g(ad-cb)b}-\frac{B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2c}{2g(ad-cb)}-\frac{Bd\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)a}{g(ad-cb)b}+\frac{B\,\text{dilo}\cdots}{\cdots}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
A/g*ln(b*x+a)/b-B/g/d^2*(a*d-b*c)*e*(-1/2/(a*d-b*c)*d^2/b/e*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)*d^3/b/e*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x
+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d-b*e)/b/e)/d))
```

3.92.5 Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="fricas")
```

output
```
integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(b*g*x + a*g), x)
```

3.92.6 Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(a*e/(c + d*x) + b*e*x/(c + d*x)
)/(a + b*x), x))/g
```

3.92.7 Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="maxima")
```

output
```
-B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x*log(e) + b*c*log(e)
 + (2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g +
 a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

3.92.8 Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x), x)
```

3.93 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx$

3.93.1 Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{B}{bg^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^2/(b*x+a
)
```

3.93.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{-Abc-bBc+aAd+aBd-Bd(a+bx)\log(a+bx)+(-bBc+aBd)\log\left(\frac{e(a+bx)}{c+dx}\right)+aBd\log(c+dx)+}{b(bc-ad)g^2(a+bx)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

output
```
(-(A*b*c) - b*B*c + a*A*d + a*B*d - B*d*(a + b*x)*Log[a + b*x] + (-(b*B*c)
 + a*B*d)*Log[(e*(a + b*x))/(c + d*x)] + a*B*d*Log[c + d*x] + b*B*d*x*Log[
c + d*x])/(b*(b*c - a*d)*g^2*(a + b*x))
```

3.93.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2950, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

output
```
(-((B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(a + b*x))/((b*c - a*d)*g^2)
```

3.93.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.93.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

method	result
norman	$\frac{\frac{(A+B)x}{ga}+\frac{cB\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)g}+\frac{Bdx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)g}}{g(bx+a)}$
parallelrisch	$-\frac{-Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^2-B\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd+Aa\,b^2d^2-A\,b^3cd+Ba\,b^2d^2-B\,b^3cd}{g^2(bx+a)b^3d(ad-cb)}$
parts	$-\frac{A}{g^2(bx+a)b}-\frac{Be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{g^2(ad-cb)}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{b\,g^2(bx+a)}-\frac{-B\ln(-bx-a)bdx+B\ln(dx+c)bdx-B\ln(-bx-a)ad+B\ln(dx+c)ad+Aad-Abc+Bad-Bbc}{(bx+a)g^2b(ad-cb)}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}\right)}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}\right)}{d^2}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A+B)/g/a*x+c*B/(a*d-b*c)/g*ln(e*(b*x+a)/(d*x+c))+B*d/(a*d-b*c)/g*x*ln(e*
(b*x+a)/(d*x+c)))/g/(b*x+a)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{(A+B)bc-(A+B)ad+(Bbdx+Bbc)\log\left(\frac{bex+ae}{dx+c}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="fricas"
)
```

output
```
-((A + B)*b*c - (A + B)*a*d + (B*b*d*x + B*b*c)*log((b*e*x + a*e)/(d*x + c
)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

3.93.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(49) = 98$.

Time = 0.63 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.70

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x} - \frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{-A-B}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

output
```
-B*log(e*(a + b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) - B*d*log(x + (-B*a
**2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2
*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + B*d*log(x +
 (B*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2
*c**2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A -
B)/(a*b*g**2 + b**2*g**2*x)
```

3.93.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(63) = 126$.

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{A}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="maxima"
)
```

output
```
-B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^
2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b
^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.98

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\left(\frac{(dx+c)Be^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2}+\frac{(Ae^2+Be^2)(dx+c)}{(bex+ae)g^2}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

output
```
-((d*x + c)*B*e^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)*g^2) + (A*e^
2 + B*e^2)*(d*x + c)/((b*e*x + a*e)*g^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.93.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^2,x)
```

output
```
- (A + B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x))/(c + d*x)))/(b^2*g^
2*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*
(a*d - b*c))
```

3.94 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$

3.94.1 Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{B}{4bg^3(a+bx)^2}+\frac{Bd}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}$$

output
```
-1/4*B/b/g^3/(b*x+a)^2+1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*ln(b*x+a
)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-1/2*
B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

3.94.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(-3*a*d + b
*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c +
 d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

3.94.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{2bg}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{2bg^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{B(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{2bg^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{2bg^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(b*g^3*(a + b*x)^2) + (B*(b*c -
a*d)*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^
2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3))/(2*b*g^
3)
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.94.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.59

method	result
norman	$\frac{\frac{Bad^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(a^2d^2-2abcd+b^2c^2)g}-\frac{2Aabd-2Ab^2c+3Babd-Bb^2c}{4gb^2(ad-cb)}-\frac{Bdx}{2g(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g(a^2d^2-2abcd+b^2c^2)}+\frac{Bd^2bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2(a^2d^2-2abcd+b^2c^2)g}}{(bx+a)^2g^2}$
parallelrisch	$-\frac{-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^4d^3-4B\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^4cd^2-2Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5d^3+2Bxab^4d^3-2Bxb^5cd^2+2B\ln\left(\frac{e(bx+\cdots}{dx+\cdots}\right.\cdots}{4g^3(bx+a)^2(a^2d^2-2abcd+b^2c^2)b^4d}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2bg^3(bx+a)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)abd^2x-4B\ln(-bx-a)abd^2x+2B\ln(dx\cdots}{4(a^2d^2-2a\cdots}$
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{B(ad-cb)e\left(\frac{d^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^3}-\frac{d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)\cdots}{d(dx+c)}\right.\cdots}\right.\cdots}{(ad-cb)^3}\cdots\right.}{g^3d^2\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2Abe}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2Bbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e\cdots}{d(dx+c)}\right.\cdots}\right.\cdots}{(ad-cb)^3g^3}\cdots\right.}{d^2\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{d^2Abe}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2Bbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e\cdots}{d(dx+c)}\right.\cdots}\right.\cdots}{(ad-cb)^3g^3}\cdots\right.}{d^2\cdots}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
(B*a*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/g*x*ln(e*(b*x+a)/(d*x+c))-1/4*(2*A*a*
b*d-2*A*b^2*c+3*B*a*b*d-B*b^2*c)/g/b^2/(a*d-b*c)-1/2/g*B*d/(a*d-b*c)*x+1/2
*B*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c))+1/2*B
*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/g*b*x^2*ln(e*(b*x+a)/(d*x+c)))/(b*x+a)^2/
g^2
```

3.94. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$

3.94.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.51

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{(2A+B)b^2c^2-4(A+B)abcd+(2A+3B)a^2d^2-2(Bb^2cd-Babd^2)x-2(Bb^2d^2x^2+2Babd^2x-}{4((b^5c^2-2ab^4cd+a^2b^3d^2)g^3x^2+2(ab^4c^2-2a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2a^3b^2c}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="fricas"
)
```

output
```
-1/4*((2*A + B)*b^2*c^2 - 4*(A + B)*a*b*c*d + (2*A + 3*B)*a^2*d^2 - 2*(B*b
^2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B
*a*b*c*d)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.94.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(124) = 248.

Time = 1.06 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2} \\
&\quad - \frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2} \\
&\quad + \frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2} \\
&\quad + \frac{-2Aad+2Abc-3Bad+Bbc-2Bbdx}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x\left(8a^2b^2dg^3-8ab^3cg^3\right)}
\end{aligned}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

output
```
-B*log(e*(a + b*x)/(c + d*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g*
*3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4
/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*
c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)
**2) + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
 - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*
d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
+ (-2*A*a*d + 2*A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*
a**2*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2
*d*g**3 - 8*a*b**3*c*g**3))
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{1}{4}B\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} - \frac{2\,\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3} + \frac{2\,d \cdots}{(b^3c^2 - \cdots}\right.$$
$$- \frac{A}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="maxima"
)
```

output
```
1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e
/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)
/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
- 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^
2*b*g^3)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\left(Bbe^3-\frac{2\,(bex+ae)Bde^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}+\frac{2\,Abe^3+Bbe^3-\frac{4\,(bex+ae)Ade^2}{dx+c}-\frac{4\,(bex+ae)Bde^2}{dx+c}}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}\right)\left(\frac{}{(bce-ad}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

output
```
-1/4*(2*(B*b*e^3 - 2*(b*e*x + a*e)*B*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d
*x + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3/(d
*x + c)^2) + (2*A*b*e^3 + B*b*e^3 - 4*(b*e*x + a*e)*A*d*e^2/(d*x + c) - 4*
(b*e*x + a*e)*B*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b
*e*x + a*e)^2*a*d*g^3/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a
*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.94.9 Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^3,x)
```

output
```
- ((2*A*a*d - 2*A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d
- b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log((e*(a + b*x
))/(c + d*x)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B*d^2*atanh((2*b^3*c
^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c))
)/(b*g^3*(a*d - b*c)^2)
```

3.95 $\int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4} \, dx$

3.95.1 Optimal result

Integrand size = 30, antiderivative size = 175

$$\begin{aligned}\int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4} \, dx = &-\frac{B}{9bg^4(a+bx)^3}+\frac{Bd}{6b(bc-ad)g^4(a+bx)^2} \\ &-\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} \\ &-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}\end{aligned}$$

output
```
-1/9*B/b/g^4/(b*x+a)^3+1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2/b/(-a*
d+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e
*(b*x+a)/(d*x+c)))/b/g^4/(b*x+a)^3+1/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4
```

3.95.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4} \, dx = -\frac{6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log}{(bc-ad)^3}}{18bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

output
```
-1/18*(6*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(11*a^2*d^
2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3*(a
+ b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b
*g^4*(a + b*x)^3)
```

3.95.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{B(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(b*g^4*(a + b*x)^3) + (B*(b*c -
a*d)*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x)^2) -
 d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 + (d^3*L
og[c + d*x])/(b*c - a*d)^4))/(3*b*g^4)
```

3.95.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(166) = 332$.

Time = 1.20 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.06

method	result
parts	$-\frac{A}{3g^4(bx+a)^3 b}-\frac{B(ad-cb)e\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^4}-\frac{2d^3be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb}{d(dx+}\right.}\right.}{(ad-cb)^4}\right.}{g^4d^2}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-bx-a)b^3d^3x^3+6B\ln(dx+c)b^3d^3x^3-18B\ln(-bx-a)a\,b^2d^3x^2+18B\ln(dx+c)a\,b^2d^3x^2-}{}$
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-18B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^6d^4-18Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^5d^4-18Bxa\,b^6c\,d^3+15Bx\,a^2b^5d^4+}{}$
norman	$\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}-\frac{6A\,a^2b\,d^2-12Aa\,b^2cd+6A\,b^3c^2+9B\,a^2b\,d^2-7Ba\,b^2cd+}{18g\,b^2\left(a^2d^2-2abcd+b^2c^2\right)}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A\,b^2e^2}{3(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{d^3Abe}{(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^4A}{(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\,b^2e^2\left(-\right.}{}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A\,b^2e^2}{3(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{d^3Abe}{(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^4A}{(ad-cb)^4g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\,b^2e^2\left(-\right.}{}\right.}{}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/g^4/(b*x+a)^3/b-B/g^4/d^2*(a*d-b*c)*e*(d^4/(a*d-b*c)^4*(-1/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e
/d/(d*x+c)))-2*d^3/(a*d-b*c)^4*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+d^2/(a
*d-b*c)^4*e^2*b^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(163) = 326$.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.32

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{2\,(3\,A+B)b^3c^3 - 9\,(2\,A+B)ab^2c^2d + 18\,(A+B)a^2bcd^2 - (6\,A+11\,B)a^3d^3 + 6\,(Bb^3cd^2 - Bab^2d^3)x^2}{18\,((b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3\,(ab^6c^3 - 3\,a^2b^5c^2d + 3\,a^3b^4cd^2 -}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="fricas"
)
```

output
```
-1/18*(2*(3*A + B)*b^3*c^3 - 9*(2*A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*
d^2 - (6*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*
c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*
d^3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)
*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2
 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2
 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d
^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2
 - a^6*b*d^3)*g^4)
```

3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(151) = 302$.

Time = 1.62 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.75

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3} - \frac{Bd^3\log\left(x + \frac{-\frac{Ba^4d^7}{(ad-bc)^3} + \frac{4Ba^3bcd^6}{(ad-bc)^3} - \frac{6Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{4Bab^3c^3d^4}{(ad-bc)^3} + Bad^4 - \frac{Bb^4c^4d^3}{(ad-bc)^3} + Bbcd^3}{2Bbd^4}\right)}{3bg^4\,(ad-bc)^3} + \frac{Bd^3\log\left(x + \frac{\frac{Ba^4d^7}{(ad-bc)^3} - \frac{4Ba^3bcd^6}{(ad-bc)^3} + \frac{6Ba^2b^2c^2d^5}{(ad-bc)^3} - \frac{4Bab^3c^3d^4}{(ad-bc)^3} + Bad^4 + \frac{Bb^4c^4d^3}{(ad-bc)^3} + Bbcd^3}{2Bbd^4}\right)}{3bg^4\,(ad-bc)^3} + \frac{-6Aa^2d^2 + 12Aabcd - 6Ab^2c^2 - 11Ba^2d^2 + 7Babcd - 2Bb^2c^2 - 6}{18a^5bd^2g^4 - 36a^4b^2cdg^4 + 18a^3b^3c^2g^4 + x^3\cdot(18a^2b^4d^2g^4 - 36ab^5cdg^4 + 18b^6c^2g^4) + x^2\cdot(54a^3b^3d^2g^4 -}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

output
```
-B*log(e*(a + b*x)/(c + d*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b*
*3*g**4*x**2 + 3*b**4*g**4*x**3) - B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c
)**3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b
*c)**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3
/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + B*
d**3*log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**
3 + 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b
*c)**3 + B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d
**4))/(3*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**
2*c**2 - 11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**2*c**2 - 6*B*b**2*d**2*x**2
 + x*(-15*B*a*b*d**2 + 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*
c*d*g**4 + 18*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b*
*5*c*d*g**4 + 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2
*b**4*c*d*g**4 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a*
*3*b**3*c*d*g**4 + 54*a**2*b**4*c**2*g**4))
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 0.21 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.45

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2 \cdots} \cdots\right) - \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="maxima"
)
```

output
```
-1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
6*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*
a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d
+ 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b
^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b
^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(163) = 326$.

Time = 0.46 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.38

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^4}\,dx =$$
$$-\frac{1}{18}\left(\frac{6\left(Bb^2e^4 - \frac{3\,(bex+ae)Bbde^3}{dx+c} + \frac{3\,(bex+ae)^2Bd^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3b^2c^2g^4}{(dx+c)^3} - \frac{2\,(bex+ae)^3abcdg^4}{(dx+c)^3} + \frac{(bex+ae)^3a^2d^2g^4}{(dx+c)^3}} + \frac{6\,Ab^2e^4 + 2\,Bb^2e^4 - \frac{18\,(bex+ae)Abde^3}{dx+c} - \frac{9}{}}{\frac{(bex+ae)^3b^2c^2g^4}{(dx+c)^3} - \frac{2\,(b}{}}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

output
```
-1/18*(6*(B*b^2*e^4 - 3*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 3*(b*e*x + a*e
)^2*B*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b
^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*e*
x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3) + (6*A*b^2*e^4 + 2*B*b^2*e^4 - 18*(b*e
*x + a*e)*A*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 18*(
b*e*x + a*e)^2*A*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*d^2*e^2/(d*x +
 c)^2)/((b*e*x + a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*
d*g^4/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.95.9 Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3} - \frac{B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3} - \frac{11\,B\,a^2\,d^2}{18\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3} - \frac{B\,b\,d^2\,x^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3} + \frac{7\,B\,a\,c\,d}{18\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,x}{6\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3} - \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^4,x)
```

output
```
(2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*
1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4
*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3)
- (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(18*b*g
^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^2*(a + b*
x)^3) - (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a + b
*x))/(c + d*x)))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2
*(a + b*x)^3) + (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.96 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx$

3.96.1 Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{B}{16bg^5(a+bx)^4}+\frac{Bd}{12b(bc-ad)g^5(a+bx)^3} -\frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2}+\frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)} +\frac{Bd^4\log(a+bx)}{4b(bc-ad)^4g^5}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}-\frac{Bd^4\log(c+dx)}{4b(bc-ad)^4g^5}$$

output
```
-1/16*B/b/g^5/(b*x+a)^4+1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2/b/(-
a*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*B*d^4*ln
(b*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*(b*x+a)/(d*x+c)))/b/g^5/(b*x+a)^
4-1/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

3.96.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4}+\frac{B\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}}{4bg^5}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^4) + (B*((-3*(b*c - a*d)
^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2)/
(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d^
4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

3.96.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^5} \, dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{4bg}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{4bg^5}-\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{B(bc-ad)\int \left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+bx)^5}\right.}{4bg^5}$$
$$\frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{4bg^5}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(b*g^5*(a + b*x)^4) + (B*(b*c -
a*d)*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^3) -
 d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (d^4*
Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5))/(4*b*g^5)
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(195) = 390$.

Time = 2.16 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.30

method	result
parts	$-\frac{A}{4g^5(bx+a)^4 b} - \frac{B(ad-cb)e\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^5}-\frac{3d^4be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb}{d(dx+}\right.}\cdots\right.}{(ad-cb)^5}\cdots}{\cdots}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4b\,g^5(bx+a)^4} - \frac{-48Ba\,b^3c\,d^3x^2-72B\,a^2b^2c\,d^3x+24Ba\,b^3c^2d^2x-48A\,a^3bc\,d^3+72A\,a^2b^2c^2d^2-48Aa\,b^3c^3d-48B\cdots}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2Ab^3e^3}{4(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{d^3Ab^2e^2}{(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{3d^4Abe}{2(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d\cdots}{(ad-cb)^5g^5\left(\cdots\right.}\cdots\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{d^2Ab^3e^3}{4(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{d^3Ab^2e^2}{(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{3d^4Abe}{2(ad-cb)^5g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d\cdots}{(ad-cb)^5g^5\left(\cdots\right.}\cdots\right.}{\cdots}$
parallelrisch	$\frac{12B\,x^4\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6b^3c\,d^4+48B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^7b^2c\,d^4+72B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^8bc\,d^4+12A\,x^4a^2b^7c^5+3B\,x^4a^2b^7c^5\cdots}{\cdots}$
norman	$\frac{B\,a^3d^4x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,b^2d^4B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{\left(4A\,a^3d^3-12A\,a^2bc\,d^2+12Aa\cdots\right.}{4g\cdots}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/g^5/(b*x+a)^4/b-B/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e
/d/(d*x+c)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*d^3/
(a*d-b*c)^5*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b^3*e
^3*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1
/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(192) = 384$.

Time = 0.26 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.05

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{3\,(4\,A+B)b^4c^4 - 16\,(3\,A+B)ab^3c^3d + 36\,(2\,A+B)a^2b^2c^2d^2 - 48\,(A+B)a^3bcd^3 + (12\,A+25\,B)a^4d}{48\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4\,(ab^8c^4 - 4\,a^2b^7c^3}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="fricas"
)
```

output
```
-1/48*(3*(4*A + B)*b^4*c^4 - 16*(3*A + B)*a*b^3*c^3*d + 36*(2*A + B)*a^2*b
^2*c^2*d^2 - 48*(A + B)*a^3*b*c*d^3 + (12*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*
d^3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*
d^4)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*
a^3*b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2
 + 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4
*B*a^3*b*c*d^3)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d +
6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4
- 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5
*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*
d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*
c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*
c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

3.96.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(178) = 356$.

Time = 2.41 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.58

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4a^4bg^5 + 16a^3b^2g^5x + 24a^2b^3g^5x^2 + 16ab^4g^5x^3 + 4b^5g^5x^4} - \frac{Bd^4\log\left(x + \frac{-\frac{Ba^5d^9}{(ad-bc)^4} + \frac{5Ba^4bcd^8}{(ad-bc)^4} - \frac{10Ba^3b^2c^2d^7}{(ad-bc)^4} + \frac{10Ba^2b^3c^3d^6}{(ad-bc)^4} - \frac{5Bab^4c^4d^5}{(ad-bc)^4} + Bad^5 + \frac{Bb^5c^5d^4}{(ad-bc)^4} + Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4} + \frac{Bd^4\log\left(x + \frac{\frac{Ba^5d^9}{(ad-bc)^4} - \frac{5Ba^4bcd^8}{(ad-bc)^4} + \frac{10Ba^3b^2c^2d^7}{(ad-bc)^4} - \frac{10Ba^2b^3c^3d^6}{(ad-bc)^4} + \frac{5Bab^4c^4d^5}{(ad-bc)^4} + Bad^5 - \frac{Bb^5c^5d^4}{(ad-bc)^4} + Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4} + \frac{-12Aa^3d^3 + 36Aa^2bcd^2 - 36Aab^2c^2}{48a^7bd^3g^5 - 144a^6b^2cd^2g^5 + 144a^5b^3c^2dg^5 - 48a^4b^4c^3g^5 + x^4\cdot(48a^3b^5d^3g^5 - 144a^2b^6cd^2g^5 + 144ab^7c^2d}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

output
```
-B*log(e*(a + b*x)/(c + d*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a*
*2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4*log(x
 + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*
a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)*
*4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*
d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + B*d**4
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 +
10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d -
b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**
4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (
-12*A*a**3*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3
 - 25*B*a**3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3*c**
3 - 12*B*b**3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) + x*(
-52*B*a**2*b*d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(48*a**7*b*d**3
*g**5 - 144*a**6*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b*
*4*c**3*g**5 + x**4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 +
144*a*b**7*c**2*d*g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**
5 - 576*a**3*b**5*c*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**
3*g**5) + x**2*(288*a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*
a**3*b**5*c**2*d*g**5 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**...
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(192) = 384$.

Time = 0.22 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.14

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{1}{48}B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - 23\,\cdots}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\,(a\cdots}\right.$$
$$- \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="maxima"
)
```

output
```
1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d
^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3
*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b
^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d
^3)*g^5) - 12*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*
g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*
x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a
^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b
^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(192) = 384$.

Time = 0.49 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{1}{48}\left(\frac{12\left(Bb^3e^5 - \frac{4\,(bex+ae)Bb^2de^4}{dx+c} + \frac{6\,(bex+ae)^2Bbd^2e^3}{(dx+c)^2} - \frac{4\,(bex+ae)^3Bd^3e^2}{(dx+c)^3}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4b^3c^3g^5}{(dx+c)^4} - \frac{3\,(bex+ae)^4ab^2c^2dg^5}{(dx+c)^4} + \frac{3\,(bex+ae)^4a^2bcd^2g^5}{(dx+c)^4} - \frac{(bex+ae)^4a^3d^3g^5}{(dx+c)^4}} + \frac{12\,Ab^3e^5 + 3\,Bb^3e^5\cdots}{\cdots}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

output
```
-1/48*(12*(B*b^3*e^5 - 4*(b*e*x + a*e)*B*b^2*d*e^4/(d*x + c) + 6*(b*e*x +
a*e)^2*B*b*d^2*e^3/(d*x + c)^2 - 4*(b*e*x + a*e)^3*B*d^3*e^2/(d*x + c)^3)*
log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*
(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*c*d^
2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) + (12*A*b^3*e
^5 + 3*B*b^3*e^5 - 48*(b*e*x + a*e)*A*b^2*d*e^4/(d*x + c) - 16*(b*e*x + a*
e)*B*b^2*d*e^4/(d*x + c) + 72*(b*e*x + a*e)^2*A*b*d^2*e^3/(d*x + c)^2 + 36
*(b*e*x + a*e)^2*B*b*d^2*e^3/(d*x + c)^2 - 48*(b*e*x + a*e)^3*A*d^3*e^2/(d
*x + c)^3 - 48*(b*e*x + a*e)^3*B*d^3*e^2/(d*x + c)^3)/((b*e*x + a*e)^4*b^3
*c^3*g^5/(d*x + c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(
b*e*x + a*e)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(
d*x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*
c - a*d)))
```

3.96.9 Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.80

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b\right.}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x}$$

$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}$$

$$-\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,\left(a\,d-b\,c\right)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{\left(a\,d-b\,c\right)^4}\right)}{2\,b\,g^5\,\left(a\,d-b\,c\right)^4}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^5,x)
```

output
```
- ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 36*A*a*b^2*
c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a
*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B
*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*b^2*g^5*x + 1
6*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x))/(c + d*x)))/(
4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^
4*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c
*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4)
```

3.96. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx$

3.97 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.97.1 Optimal result

Integrand size = 32, antiderivative size = 365

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} \\
&+\frac{B(bc-ad)^2g^4(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} \\
&-\frac{B(bc-ad)^3g^4(a+bx)^2\left(12A+7B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60bd^3} \\
&+\frac{B(bc-ad)^4g^4(a+bx)\left(12A+13B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^4} \\
&+\frac{B(bc-ad)^5g^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12A+25B+12B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^5} \\
&+\frac{2B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}$$

output
```
-1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/5*g^4*(
b*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/30*B*(-a*d+b*c)^2*g^4*(b*x+a)^3
*(4*A+B+4*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/60*B*(-a*d+b*c)^3*g^4*(b*x+a)^2
*(12*A+7*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+1/30*B*(-a*d+b*c)^4*g^4*(b*x+
a)*(12*A+13*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4+1/30*B*(-a*d+b*c)^5*g^4*ln
((-a*d+b*c)/b/(d*x+c))*(12*A+25*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^5+2/5*B^
2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

3.97.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.40

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(bc-ad)\left(24Abd(bc-ad)^3x+24Bd(bc-ad)^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-12d^2(bc-ad)^2(a+bx\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*
(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x
))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*
c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d
*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x
)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x
 + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*
Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) +
 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

3.97.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2950, 2781, 2784, 2784, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2950}$$

$$g^4(bc-ad)^5\int \frac{(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\int \frac{(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}\right)$$

$$\Big\downarrow \text{2784}$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\int \frac{(a+bx)^3\left(4A+B+4B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4d}\right)}{5b}\right)$$

$$\Big\downarrow \text{2784}$$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2(}{(}}{}}{4d}\right)}{5b}\right)$$

$\downarrow$ 2784

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2(1}{2d(c}}{}}{}\right)}{5b}\right)$$

$\downarrow$ 27

3.97. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$g^4(bc -$$

$$2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx)^2\left(1\right.}{2d(c}}{}\right.$$

$$ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\cdots}{5b}\right.$$

$$\downarrow 2784$$

$$g^4(bc -$$

$$2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx)^2\left(1\right.}{2d(c}}{}\right.$$

$$ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \cdots\right.$$

3.97. $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$\downarrow$ 2754

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(1\right.}{2d(c}}{}}{}\right)}{}\right.$$

$\downarrow$ 2838

3.97. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$g^4(bc - ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx)^2\left(1\cdots\right.}{2d(c\cdots}}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^5*g^4*(((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5
*b*(c + d*x)^5*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*(((a + b*x)^4*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d*(c + d*x)^4*(b - (d*(a + b*x))/(c +
d*x))^4) - (((a + b*x)^3*(4*A + B + 4*B*Log[(e*(a + b*x))/(c + d*x)]))/(3*
d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(12*A + 7*B
 + 12*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))
/(c + d*x))^2) - (((a + b*x)*(12*A + 13*B + 12*B*Log[(e*(a + b*x))/(c + d*
x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((12*A + 25*B + 12*
B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) -
 (12*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d)/(3*d))/(4*d)))/(5
*b))
```

3.97. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.97. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.97.4 Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.97.5 Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b*e*x + a*e)
/(d*x + c))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g
^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((b*e*x + a*e)/(d*x + c)), x)
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.97.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2389 vs. 2(350) = 700.

Time = 0.32 (sec) , antiderivative size = 2389, normalized size of antiderivative = 6.55

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^
3*b*g^4*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
 - c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x +
 c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*
A*B*a^3*b*g^4 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(
b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3
*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4
+ A^2*a^4*g^4*x - 1/30*((12*g^4*log(e) + 25*g^4)*b^4*c^5 - (60*g^4*log(e)
+ 113*g^4)*a*b^3*c^4*d + 4*(30*g^4*log(e) + 49*g^4)*a^2*b^2*c^3*d^2 - 12*(
10*g^4*log(e) + 13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) + 4*g^4)*a^4*c*d^
4)*B^2*log(d*x + c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^
3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*
(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b
*c - a*d)))*B^2/(b*d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 - 6*(b^...
```

3.97.8 Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^4*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx = \int (a\,g+b\,g\,x)^4\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.98 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.98.1 Optimal result

Integrand size = 32, antiderivative size = 309

$$\begin{aligned}\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx &= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} \\ &+ \frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^2} \\ &- \frac{B(bc-ad)^3g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^3} \\ &- \frac{B(bc-ad)^4g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12bd^4} \\ &- \frac{B^2(bc-ad)^4g^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/4*g^3*(b
*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/12*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*
(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/12*B*(-a*d+b*c)^3*g^3*(b*x+a)*(6
*A+5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3-1/12*B*(-a*d+b*c)^4*g^3*ln((-a*d+b
*c)/b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B^2*(-a*d+b*
c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

3.98.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.27

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+3d^2(-bc+ad)(a+bx)^2\left(\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*
(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(
b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2
 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*
d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
/(3*d^4)))/(4*b)
```

3.98.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2950, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2950}$$

$$g^3(bc-ad)^4 \int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}\right)$$

$\downarrow$ 2784

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{2b}\right)$$

$\downarrow$ 2784

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(6A\right.}{(c+}}{}}{3d}\right.}{2b}$$

$\downarrow$ 2784

3.98. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{(a+bx)\left(6B\log\ldots\right.}{d(c+\ldots}}{\ldots}\right)}{2b}\right.$$

$\downarrow$ 2754

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{(a+bx)\left(6B\log\ldots\right.}{d(c+\ldots}}{\ldots}\right)}{\ldots}\right.$$

$\downarrow$ 2838

3.98. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{(a+bx)\left(6B\log\right.}{d(c+\ldots}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^4*g^3*(((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4
*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*
x))^3) - (((a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*
(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B + 6*
B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) - (-(((6*A + 11*B + 6*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/d) - (6*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)
/d)/(2*d))/(3*d)))/(2*b))
```

3.98.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.98.4 Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln \left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.98.5 Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log \left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2
*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)),
x)
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1732 vs. $2(296) = 592$.

Time = 0.32 (sec) , antiderivative size = 1732, normalized size of antiderivative = 5.61

$$\int (ag+bgx)^3 \left(A+B\log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A
*B*a^3*g^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a
)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log
(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d
^2))*A*B*a*b^2*g^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/12*((6*g^3*log(e) + 11*g^3)*b^3*c^4 - 2*(
12*g^3*log(e) + 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) + 5*g^3)*a^2*b*c^2*d
^2 - 6*(4*g^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c
^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a
^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d
*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2
 - 2*(b^4*c*d^3*g^3*log(e) - (6*g^3*log(e)^2 + g^3*log(e))*a*b^3*d^4)*B^2*
x^3 + ((3*g^3*log(e) + g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) + g^3)*a*b^3*c*d
^3 + (18*g^3*log(e)^2 + 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - ((6*g^3
*log(e) + 5*g^3)*b^4*c^3*d - (24*g^3*log(e) + 17*g^3)*a*b^3*c^2*d^2 + (36*
g^3*log(e) + 19*g^3)*a^2*b^2*c*d^3 - (12*g^3*log(e)^2 + 18*g^3*log(e) +...
```

3.98.8 Giac [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.99 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.99.1 Optimal result

Integrand size = 32, antiderivative size = 253

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = -\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} + \frac{B(bc-ad)^2g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^2} + \frac{B(bc-ad)^3g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3} + \frac{2B^2(bc-ad)^3g^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}$$

output
```
-1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/3*g^2*(b
*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/3*B*(-a*d+b*c)^2*g^2*(b*x+a)*(2*
A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2+1/3*B*(-a*d+b*c)^3*g^2*ln((-a*d+b*c)/
b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+2/3*B^2*(-a*d+b*c)^3*
g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

3.99.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(bc-ad)\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-d^2(a+bx)^2\left(A+B\log\left(\frac{e(a}{c}\right.\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*
(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c
+ d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c
- a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])*Log[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + B*
(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c
+ d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

3.99.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2950, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$
$$\Big\downarrow \text{2950}$$
$$g^2(bc-ad)^3\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{2781}$$

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)$$

$\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}\right)$$

$\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{2A+3B+2\ldots}{b-\ldots}}{\ldots}}{2d}\right.}{3b}\right.$$

$\downarrow$ 2754

3.99. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$g^2(bc - ad)^3 \left(\frac{(a+bx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{2B\int\frac{(c+dx)}{}}{}}{}\right)}{3b} \right.$$

$\downarrow$ 2838

$$g^2(bc - ad)^3 \left(\frac{(a+bx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{\log\left(1-\frac{d(a}{b(c}\right.}{}}{}\right)}{3b} \right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^3*g^2*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3
*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c +
d*x))^2) - (((a + b*x)*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c
 + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B + 2*B*Log[(e*(a +
b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*PolyLog[
2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))
```

3.99. $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.99.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.99.4 Maple [F]

$$\int (bgx + ag)^2 \left(A + B\ln\left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.99.5 Fricas [F]

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B
*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))
, x)
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(242) = 484$.

Time = 0.29 (sec) , antiderivative size = 1165, normalized size of antiderivative = 4.60

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d
^2 - (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a
^2*g^2*x - 1/3*((2*g^2*log(e) + 3*g^2)*b^2*c^3 - (6*g^2*log(e) + 7*g^2)*a*
b*c^2*d + 2*(3*g^2*log(e) + 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(
b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*
x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (b^3*c*d^2*g^2*log(
e) - (3*g^2*log(e)^2 + g^2*log(e))*a*b^2*d^3)*B^2*x^2 + ((2*g^2*log(e) + g
^2)*b^3*c^2*d - 2*(3*g^2*log(e) + g^2)*a*b^2*c*d^2 + (3*g^2*log(e)^2 + 4*g
^2*log(e) + g^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3
*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*
b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c
^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + (2*B
^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*d^2*g^2 - (6*g^2*log(e) + g^2)*a*b^2*d^
3)*B^2*x^2 + 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + (3*g^2*log(e) + 2*g^2)
*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + (2*g^2*log...
```

3.99.8 Giac [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.100 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.100.1 Optimal result

Integrand size = 30, antiderivative size = 180

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{B(bc-ad)^2g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd^2} - \frac{B^2(bc-ad)^2g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

output

```
-B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/2*g*(b*x+a)^2*(A
+B*ln(e*(b*x+a)/(d*x+c)))^2/b-B*(-a*d+b*c)^2*g*ln((-a*d+b*c)/b/(d*x+c))*(A
+B+B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b
/(d*x+c))/b/d^2
```

3.100.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B(bc-ad)\left(2Abdx+2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2B(bc-ad)\log(c+dx)-2(bc-ad)\left(A+B\log\right.\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(2
*A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*
Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

3.100.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2950, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$
$$\Big\downarrow \text{ 2950}$$
$$g(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{ 2781}$$
$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)$$

$\Big\downarrow$ 2784

$$g(bc-$$
$$ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)$$

$\Big\downarrow$ 2754

$$g(bc-$$
$$ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx}{b(c+dx}\right.}{}}{d}\right.}{b}\right.$$

$\Big\downarrow$ 2838

$$g(bc-$$
$$ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{d}-\frac{B\,\mathrm{Pol}}{}}{d}\right.}{b}\right.$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^2*g*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b
*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(((a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (
-(((A + B + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c +
d*x))])/d) - (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d))/b)
```

3.100. $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

3.100.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.100.4 Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.100.5 Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)), x)
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(177) = 354$.

Time = 0.29 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.39

$$\begin{aligned}
&\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&=\frac{1}{2}A^2bgx^2+2\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)ABag \\
&\quad+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)ABbg \\
&\quad+A^2agx+\frac{((g\log(e)+g)bc^2-(2g\log(e)+g)acd)B^2\log(dx+c)}{d^2} \\
&\quad+\frac{(b^2c^2g-2abcdg+a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\text{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2} \\
&\quad+\frac{B^2b^2d^2gx^2\log(e)^2-2\left(b^2cdg\log(e)-\left(g\log(e)^2+g\log(e)\right)abd^2\right)B^2x+\left(B^2b^2d^2gx^2+2B^2abd^2gx+\ldots\right.}{}
\end{aligned}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
1/2*A^2*b*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x +
a)/b - c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*
B*b*g + A^2*a*g*x + ((g*log(e) + g)*b*c^2 - (2*g*log(e) + g)*a*c*d)*B^2*lo
g(d*x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((
b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*
d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(b^2*c*d*g*log(e) - (g*log(e)^2
 + g*log(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2
*a^2*d^2*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2
*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) +
((2*g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + ((g*log(e) + g)*a^2*d^2 - a
*b*c*d*g)*B^2)*log(b*x + a) - 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) +
 g)*a*b*d^2 - b^2*c*d*g)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x +
B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

3.100.8 Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.101 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

3.101.1 Optimal result

Integrand size = 32, antiderivative size = 128

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{2B^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*(A+B*ln(e
*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d
*x+c)/d/(b*x+a))/b/g
```

3.101.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. 2(128) = 256.

Time = 0.94 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.58

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{3A^2\log(a+bx)+3AB\left(\log^2\left(\frac{a}{b}+x\right)-2\log(a+bx)\left(\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)-\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2\left(\text{lo}\right.\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

output
```
(3*A^2*Log[a + b*x] + 3*A*B*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x]
 - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(
a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*
(Log[a/b + x]^3 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*L
og[a + b*x]*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^
2 + 3*Log[a/b + x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*
Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[
(e*(a + b*x))/(c + d*x)])*(Log[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*
x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[
3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]
))/(3*b*g)
```

3.101.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2950, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ag+bgx}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{2779}$$

$$\frac{\frac{2B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \ 7143$$

$$\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+B\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b
*x))])/b) + (2*B*((A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c +
d*x))/(d*(a + b*x))] + B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/g
```

3.101.3.1 Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(128) = 256$.

Time = 1.18 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.62

method	result
parts	$\frac{A^2 \ln(bx+a)}{gb} - \frac{B^2 e \left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3be} + \frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 \ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right) + 2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right) \mathrm{Li}_2\left(\frac{d(}{}\right.}{be}\right.}{g}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2 A^2 \left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be} + \frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{g(ad-cb)} - \frac{d^2 B^2 \left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 \ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}{be}\right.}{}\right.}{}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2 A^2 \left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be} + \frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{g(ad-cb)} - \frac{d^2 B^2 \left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 \ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}{be}\right.}{}\right.}{}\right.}{}$
risch	$\frac{A^2 \ln(bx+a)}{gb} + \frac{B^2 \ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3gb} - \frac{B^2 \ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 \ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{gb} - \frac{2B^2 \ln\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}\right.}{}$

3.101. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
A^2/g*ln(b*x+a)/b-B^2/g*e*(-1/3/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+1/b/
e*(ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*
x+c)))+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,1/b/e*d*(b*e/d+(a*d-b*c
)*e/d/(d*x+c)))-2*polylog(3,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))))-2*A*B
/g/d^2*(a*d-b*c)*e*(-1/2/(a*d-b*c)*d^2/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2+1/(a*d-b*c)*d^3/b/e*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/
d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e
)/b/e)/d))
```

3.101.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="fricas"
)
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(b*g*x + a*g), x)
```

3.101.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(a*e/(c + d*x) + b*e*x/(c
+ d*x))**2/(a + b*x), x) + Integral(2*A*B*log(a*e/(c + d*x) + b*e*x/(c + d
*x))/(a + b*x), x))/g
```

3.101.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="maxima"
)
```

output
```
B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integ
rate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log(b*x
 + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e) + A*
B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) - 2*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + (2*B^2*b*d*x + (b*c + a*d)*B^2)*lo
g(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x),
 x)
```

3.101.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x), x)
```

3.102 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

3.102.1 Optimal result

Integrand size = 32, antiderivative size = 126

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
-2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c
)))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c
)/g^2/(b*x+a)
```

3.102.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(a+bx)\left(A+B\log\left(\frac{e}{\ }\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)*(A + B*Log[(
e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a + b
*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[
c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c +
 d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(
b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

3.102.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2950, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2} \, dx$$

$$\Big\downarrow \; 2950$$

$$\frac{\int \frac{(c+dx)^2\left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2742$$

$$\frac{2B\int \frac{(c+dx)^2\left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2741$$

$$\frac{2B\left(-\frac{(c+dx)\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)) + 2*B*(-(
(B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(a + b*x)))/((b*c - a*d)*g^2)
```

3.102.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.102.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.40

method	result
norman	$\frac{\frac{(A^2+2BA+2B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{2cB(A+B)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}+\frac{2Bd(A+B)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{-2AB\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd-2AB\,b^3cd+A^2a\,b^2d^2-A^2b^3cd+2B^2a\,b^2d^2-2B^2b^3cd+2ABa\,b^2d^2-2ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^2\cdots}{g^2(bx+a)b^3d(ad-cb\cdots}$
parts	$-\frac{A^2}{g^2(bx+a)b}-\frac{B^2e\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{g^2(ad-cb)}-\frac{2BAe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\cdots\right.}{g^2(ad-cb)}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A^2}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}+\frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\cdots\right.}{\cdots}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A^2}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}+\frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\cdots\right.}{\cdots}\right.}{d^2}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2e\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)}+\frac{2B^2e\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)}+\frac{2B^2e}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\cdots\right.}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A^2+2*A*B+2*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2+B^2*d/g
/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c))^2+2*c*B*(A+B)/g/(a*d-b*c)*ln(e*(b*x+a)/
(d*x+c))+2*B*d*(A+B)/g/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/g/(b*x+a)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{(A^2+2\,AB+2\,B^2)bc-(A^2+2\,AB+2\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\left((AB+B^2)bdx+\cdots\right.}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="frica
s")
```

output
```
-((A^2 + 2*A*B + 2*B^2)*b*c - (A^2 + 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2
*b*c)*log((b*e*x + a*e)/(d*x + c))^2 + 2*((A*B + B^2)*b*d*x + (A*B + B^2)*
b*c)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a
^2*b*d)*g^2)
```

3.102.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(105) = 210$.

Time = 1.09 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.44

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= -\frac{2Bd(A+B)\log\left(x+\frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd-\frac{2Ba^2d^3(A+B)}{ad-bc}+\frac{4Babcd^2(A+B)}{ad-bc}-\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$+\frac{2Bd(A+B)\log\left(x+\frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd+\frac{2Ba^2d^3(A+B)}{ad-bc}-\frac{4Babcd^2(A+B)}{ad-bc}+\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$+\frac{(-2AB-2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x}+\frac{(B^2c+B^2dx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2-2AB-2B^2}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

output
```
-2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d + 2*B**2*a*d**2 + 2*B**
2*b*c*d - 2*B*a**2*d**3*(A + B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A + B)/(a*d
- b*c) - 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4*A*B*b*d**2 + 4*B**2*b*d**
2))/(b*g**2*(a*d - b*c)) + 2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c
*d + 2*B**2*a*d**2 + 2*B**2*b*c*d + 2*B*a**2*d**3*(A + B)/(a*d - b*c) - 4*
B*a*b*c*d**2*(A + B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4
*A*B*b*d**2 + 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B - 2*B**2)*log
(e*(a + b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log
(e*(a + b*x)/(c + d*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2
*c*g**2*x) + (-A**2 - 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(126) = 252$.

Time = 0.21 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.30

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(2\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{(bdx+ad)\log(bx+a}{}\right.$$
$$-2\,AB\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$-\frac{B^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="maxim
a")
```

output
```
-(2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (
(b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*
d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x
+ a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)
*x))*B^2 - 2*A*B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^
2) + 1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c
))^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.52

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(\frac{(dx+c)B^2e^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)g^2}+\frac{2\,(ABe^2+B^2e^2)(dx+c)\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2}+\frac{(A^2e^2+2\,ABe^2+2\,B^2e^2)(dx}{(bex+ae)g^2}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="giac"
)
```

output
```
-((d*x + c)*B^2*e^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)*g^2) + 2
*(A*B*e^2 + B^2*e^2)*(d*x + c)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)
*g^2) + (A^2*e^2 + 2*A*B*e^2 + 2*B^2*e^2)*(d*x + c)/((b*e*x + a*e)*g^2))*(
b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.102.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{A^2+2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A+B)\,4\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^2,x)
```

output
```
- (A^2 + 2*B^2 + 2*A*B)/(b^2*g^2*x + a*b*g^2) - log((e*(a + b*x))/(c + d*x
))^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a*d - b*c))) - (log((e*(a
+ b*x))/(c + d*x))*((2*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x/d + a/(
b*d)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d -
b*c))*(A + B)*4i)/(b*g^2*(a*d - b*c))
```

3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

3.103.1 Optimal result

Integrand size = 32, antiderivative size = 268

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} + \frac{2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output
```
2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/
g^3/(b*x+a)^2+2*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(
b*x+a)-1/2*b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(b*x
+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2
*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

3.103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.65

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-4d^2(a+bx)^2\log(a+b}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(
a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x]
- d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b
*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2
*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*(a + b*x)
^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2
)
```

3.103.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3}dx$$
$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{2Bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{a+bx}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

output
```
((2*B^2*d*(c + d*x))/(a + b*x) - (b*B^2*(c + d*x)^2)/(4*(a + b*x)^2) + (2*
B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (b*B*(c +
d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + (d*(c + d*x
)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) - (b*(c + d*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2))/((b*c - a*d)^2*g^3)
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.103.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.81

method	result
norman	$\frac{\frac{Bd(2Aad+2Bad+Bbc)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2a\,d^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(2A^2ad-2A^2bc+4ABad-2ABbc+4B^2ad-B^2bc\right)x}{2ag(ad-cb)}+\frac{Bc(4Aad-\cdots}{\cdots}}{\cdots}$
parallelrisch	$-\frac{-4A^2a\,b^4c\,d^2+6AB\,a^2b^3d^3+2AB\,b^5c^2d-8B^2a\,b^4c\,d^2-8ABa\,b^4c\,d^2-4B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5c\,d^2-4B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a\cdots}{\cdots}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}-\frac{B^2(ad-cb)e\left(\frac{d^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^3}-\frac{d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad\cdots}{d(d\cdots}\right)}{2\left(\frac{be}{d}+\frac{(ad\cdots}{d(d\cdots}\right)}\cdots\right)}{\cdots}\cdots\right)}{g^3d^2}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2A^2be}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A^2}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^2ABbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-c\cdots}{d(dx+\cdots}\right)\cdots}\right)}{(ad-cb)^3g^3}\cdots\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{d^2A^2be}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A^2}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^2ABbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-c\cdots}{d(dx+\cdots}\right)\cdots}\right)}{(ad-cb)^3g^3}\cdots\right)}{\cdots}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
(B/g*d*(2*A*a*d+2*B*a*d+B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/
(d*x+c))+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2
+1/2*(2*A^2*a*d-2*A^2*b*c+4*A*B*a*d-2*A*B*b*c+4*B^2*a*d-B^2*b*c)/a/g/(a*d-
b*c)*x+1/2*B*c*(4*A*a*d-2*A*b*c+4*B*a*d-B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(b*x+a)/(d*x+c))+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(b*x+a)/(d*x+c))^2+1/4*(2*A^2*a*d-2*A^2*b*c+6*A*B*a*d-2*A*B*b*c+7*
B^2*a*d-B^2*b*c)/g/a^2*b/(a*d-b*c)*x^2+1/2*b*d^2*B^2/g/(a^2*d^2-2*a*b*c*d+
b^2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c))^2+1/2*B*b/g*d^2*(2*A+3*B)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c)))/(b*x+a)^2/g^2
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.37

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{(2A^2+2AB+B^2)b^2c^2-4(A^2+2AB+2B^2)abcd+(2A^2+6AB+7B^2)a^2d^2-2(B^2b^2d^2x^2+2B}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="frica
s")
```

output
```
-1/4*((2*A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + 2*B^2)*a*b*c*d + (2
*A^2 + 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2
*b^2*c^2 + 2*B^2*a*b*c*d)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((2*A*B + 3*B
^2)*b^2*c*d - (2*A*B + 3*B^2)*a*b*d^2)*x - 2*((2*A*B + 3*B^2)*b^2*d^2*x^2
- (2*A*B + B^2)*b^2*c^2 + 4*(A*B + B^2)*a*b*c*d + 2*(B^2*b^2*c*d + 2*(A*B
+ B^2)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d +
 a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x
+ (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.103.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(241) = 482$.

Time = 2.11 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.34

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$- \frac{Bd^2\cdot(2A+3B)\log\left(x + \frac{2ABad^3+2ABbcd^2+3B^2ad^3+3B^2bcd^2-\frac{Ba^3d^5\cdot(2A+3B)}{(ad-bc)^2}+\frac{3Ba^2bcd^4\cdot(2A+3B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^3\cdot(2A+3B)}{(ad-bc)^2}+\frac{Bb^3 \cdots}{\cdots}}{4ABbd^3+6B^2bd^3}\right)}{2bg^3\,(ad-bc)^2 \cdots}$$

$$+ \frac{Bd^2\cdot(2A+3B)\log\left(x + \frac{2ABad^3+2ABbcd^2+3B^2ad^3+3B^2bcd^2+\frac{Ba^3d^5\cdot(2A+3B)}{(ad-bc)^2}-\frac{3Ba^2bcd^4\cdot(2A+3B)}{(ad-bc)^2}+\frac{3Bab^2c^2d^3\cdot(2A+3B)}{(ad-bc)^2}-\frac{Bb^3 \cdots}{\cdots}}{4ABbd^3+6B^2bd^3}\right)}{2bg^3\,(ad-bc)^2 \cdots}$$

$$+ \frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+\cdots}$$

$$+ \frac{(-2ABad+2ABbc-3B^2ad+B^2bc-2B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^3bdg^3-2a^2b^2cg^3+4a^2b^2dg^3x-4ab^3cg^3x+2ab^3dg^3x^2-2b^4cg^3x^2}$$

$$+ \frac{-2A^2ad+2A^2bc-6ABad+2ABbc-7B^2ad+B^2bc+x(-4ABbd-6B^2bd)}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x\,(8a^2b^2dg^3-8ab^3cg^3)}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

output
```
-B*d**2*(2*A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3
 + 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c
*d**4*(2*A + 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A + 3*B)/(a*d -
 b*c)**2 + B*b**3*c**3*d**2*(2*A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 6*
B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + B*d**2*(2*A + 3*B)*log(x + (2*A*
B*a*d**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3 + 3*B**2*b*c*d**2 + B*a**3*d**5*
(2*A + 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A + 3*B)/(a*d - b*c)**2
+ 3*B*a*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A
+ 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*
c)**2) + (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)
*log(e*(a + b*x)/(c + d*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a
**3*b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**
2*b**2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*
b**4*c**2*g**3*x**2) + (-2*A*B*a*d + 2*A*B*b*c - 3*B**2*a*d + B**2*b*c - 2
*B**2*b*d*x)*log(e*(a + b*x)/(c + d*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g
**3 + 4*a**2*b**2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*
b**4*c*g**3*x**2) + (-2*A**2*a*d + 2*A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 7*
B**2*a*d + B**2*b*c + x*(-4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a*
*2*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d
*g**3 - 8*a*b**3*c*g**3))
```

3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(262) = 524$.

Time = 0.24 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$

$$= \frac{1}{4}\left(2\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{2}{(b^3c^2 -}\right.\right.$$

$$+ \frac{1}{2}\,AB\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} - \frac{2\,\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3} + \frac{}{(b^3c}\right.$$

$$- \frac{B^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)} - \frac{A^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="maxim
a")
```

output
```
1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)*g^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (b^2*c^2 -
 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x +
 a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*
d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*
(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x +
a^2*d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3
+ a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 +
2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + 1/2*A*B*
((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2
*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^
2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b
^2*g^3*x + a^2*b*g^3)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.64

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\left(\frac{2\left(B^2be^3-\frac{2(bex+ae)B^2de^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}+\frac{2\left(2\,ABbe^3+B^2be^3-\frac{4(bex+ae)ABde^2}{dx+c}-\frac{4(bex+ae)B^2de^2}{dx+c}\right)\mathrm{l}}{\frac{(bex+ae)^2bcg^3}{(dx+c)^2}-\frac{(bex+ae)^2adg^3}{(dx+c)^2}}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="giac"
)
```

output
```
-1/4*(2*(B^2*b*e^3 - 2*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e
)/(d*x + c))^2/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*
g^3/(d*x + c)^2) + 2*(2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A*B*d*e^2/
(d*x + c) - 4*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d*x +
c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3/(d*x +
c)^2) + (2*A^2*b*e^3 + 2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A^2*d*e^2
/(d*x + c) - 8*(b*e*x + a*e)*A*B*d*e^2/(d*x + c) - 8*(b*e*x + a*e)*B^2*d*e
^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g
^3/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)
*(b*c - a*d)))
```

3.103.9 Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{\frac{2A^2ad-2A^2bc+7B^2ad-B^2bc+6ABad-2ABbc}{2(ad-bc)}+\frac{x(3bdB^2+2AbdB)}{ad-bc}}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}$$
$$-\ln\left(\frac{e(a+bx)}{c+dx}\right)^2\left(\frac{B^2}{2b^2g^3\left(2ax+bx^2+\frac{a^2}{b}\right)}-\frac{B^2d^2}{2bg^3\left(a^2d^2-2abcd+b^2c^2\right)}\right)$$
$$-\frac{\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(\frac{AB}{b^2dg^3}+\frac{B^2x(ad-bc)}{bg^3(a^2d^2-2abcd+b^2c^2)}+\frac{B^2d^2\left(\frac{2a^2d^2-3abcd+b^2c^2}{2bd^3}+\frac{a(ad-bc)}{2bd^2}\right)}{bg^3(a^2d^2-2abcd+b^2c^2)}\right)}{\frac{bx^2}{d}+\frac{a^2}{bd}+\frac{2ax}{d}}$$
$$-\frac{Bd^2\,\mathrm{atan}\left(\frac{Bd^2\left(2bdx-\frac{b^3c^2g^3-a^2bd^2g^3}{bg^3(ad-bc)}\right)(2A+3B)\,1i}{(ad-bc)(3B^2d^2+2ABd^2)}\right)(2A+3B)\,1i}{bg^3(ad-bc)^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^3,x)
```

output
```
- ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d - B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(
2*(a*d - b*c)) + (x*(3*B^2*b*d + 2*A*B*b*d))/(a*d - b*c))/(2*a^2*b*g^3 + 2
*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(2*b^2
*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(b^2*d*g^3) + (B^2*x*(a*
d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*d^2 +
 b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2
*atan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))
*(2*A + 3*B)*1i)/((a*d - b*c)*(3*B^2*d^2 + 2*A*B*d^2)))*(2*A + 3*B)*1i)/(b
*g^3*(a*d - b*c)^2)
```

3.103. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.104.1 Optimal result

Integrand size = 32, antiderivative size = 418

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} - \frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^4(a+bx)} + \frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)} + \frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}$$

output
```
-2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*
c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-2*B*d
^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)+b*B*d*(d*x
+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^2-2/9*b^2*B*(d*
x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-d^2*(d*x+c)*
(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d*(d*x+c)^2*(A+B*
ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^3
```

3.104.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.39

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{18\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(12A(bc-ad)^3+4B(bc-ad)^3-18Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2(a+bx)+36Ad^2(bc-ad)(a+bx\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

output
```
-1/54*(18*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(12*A*(b*c - a*d)^3
+ 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^
2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a +
 b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a +
 b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a
 + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c
+ d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] +
36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*
(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a
 + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b
*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*Log
[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a
*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*
B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^3)/(
b*g^4*(a + b*x)^3)
```

3.104.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \ 2950$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \ 2795$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}-\frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}-\frac{2Bd^2(c+dx)\left(B\log\left(\frac{e(a+b}{c+da}\right.\right.}{a+bx}}{g^4(bc-}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

output
```
((-2*B^2*d^2*(c + d*x))/(a + b*x) + (b*B^2*d*(c + d*x)^2)/(2*(a + b*x)^2)
- (2*b^2*B^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B*d^2*(c + d*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (b*B*d*(c + d*x)^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))/(a + b*x)^2 - (2*b^2*B*(c + d*x)^3*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(9*(a + b*x)^3) - (d^2*(c + d*x)*(A + B*Log[(e*(a +
b*x))/(c + d*x)])^2)/(a + b*x) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2)/(a + b*x)^2 - (b^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])^2)/(3*(a + b*x)^3))/((b*c - a*d)^3*g^4)
```

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(410) = 820$.

Time = 1.34 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.13

method	result
parts	$-\frac{A^2}{3g^4(bx+a)^3 b}-\frac{B^2(ad-cb)e\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^4}-\frac{2d^3be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(a}{d(}\right.}{2\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{}\right.}{}$
norman	$\frac{B^2a^2d^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{B^2ab\,d^3x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}-\frac{18A^2a^2b^2d^2-36A^2a\,b^3cd+18A^2b^4c^2+66AB\,a^2b^2d^2-}{54g}$
parallelrisch	$-\frac{66AB\,a^3b^4d^4-12AB\,b^7c^3d-108B^2a^2b^5c\,d^3+27B^2a\,b^6c^2d^2-108AB\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^6d^4-108ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A^2/g^4/(b*x+a)^3/b-B^2/g^4/d^2*(a*d-b*c)*e*(d^4/(a*d-b*c)^4*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c)))-2*d^3/(a*d-b*c)^4*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+d^2/(a*d-b*c)^
4*e^2*b^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*B*A/g^4/d^2*(a*d-b*c)*e*(d^4/(a
*d-b*c)^4*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*d^3/(a*d-b*c)^4*b*e*(-1/2/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2)+d^2/(a*d-b*c)^4*e^2*b^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{2\,(9\,A^2+6\,AB+2\,B^2)b^3c^3-27\,(2\,A^2+2\,AB+B^2)ab^2c^2d+54\,(A^2+2\,AB+2\,B^2)a^2bcd^2-(18\,A^2+}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="frica
s")
```

output
```
-1/54*(2*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*
c^2*d + 54*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*
a^3*d^3 + 6*((6*A*B + 11*B^2)*b^3*c*d^2 - (6*A*B + 11*B^2)*a*b^2*d^3)*x^2
+ 18*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*
c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b*e*x + a*e)/(d*x + c))^
2 - 3*((6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2 + (30*A*
B + 49*B^2)*a^2*b*d^3)*x + 6*((6*A*B + 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B + B^
2)*b^3*c^3 - 9*(2*A*B + B^2)*a*b^2*c^2*d + 18*(A*B + B^2)*a^2*b*c*d^2 + 3*
(2*B^2*b^3*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6
*B^2*a*b^2*c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)
))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*
(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a
^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(384) = 768$.

Time = 11.68 (sec) , antiderivative size = 1544, normalized size of antiderivative = 3.69

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

output
```
-B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 + 11*B**2*a*d*
*4 + 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A + 11*B)/(a*d - b*c)**3 + 4*B*a**3
*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A + 11*
B)/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A + 11*B)/(a*d - b*c)**3 - B*b
**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 + 22*B**2*b*d**4
))/(9*b*g**4*(a*d - b*c)**3) + B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 +
 6*A*B*b*c*d**3 + 11*B**2*a*d**4 + 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A + 1
1*B)/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 + 6*B*
a**2*b**2*c**2*d**5*(6*A + 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*
A + 11*B)/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(
12*A*B*b*d**4 + 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*
c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B
**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)/(c + d*x))**2/(3*a**6
*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**
2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**
3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x
**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*
c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2
+ 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 +
 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 11*B**2*a**2*d**2 + 7*B**2*a*b*c*d ...
```

3.104.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1419 vs. $2(410) = 820$.

Time = 0.29 (sec) , antiderivative size = 1419, normalized size of antiderivative = 3.39

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="maxim
a")
```

output
```
-1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5
*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c
*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3)*g^4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a
*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2
 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x +
a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*
x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3
*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3
*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3
 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/
(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g
^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*
g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 -
a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^
3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 - 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c
^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*...
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.73

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{54}\left(\frac{18\left(B^2b^2e^4-\frac{3\,(bex+ae)B^2bde^3}{dx+c}+\frac{3\,(bex+ae)^2B^2d^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3b^2c^2g^4}{(dx+c)^3}-\frac{2\,(bex+ae)^3abcdg^4}{(dx+c)^3}+\frac{(bex+ae)^3a^2d^2g^4}{(dx+c)^3}}+\frac{6\left(6\,ABb^2e^4+2\,B^2b^2e^4-\frac{18\,(be\cdots}{d\cdots}\right.}{(be\cdots}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="giac"
)
```

output
```
-1/54*(18*(B^2*b^2*e^4 - 3*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 3*(b*e*x
+ a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x +
 a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^
3 + (b*e*x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3) + 6*(6*A*B*b^2*e^4 + 2*B^2*b^
2*e^4 - 18*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B^2*b*d*e
^3/(d*x + c) + 18*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*
e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^
3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b
*e*x + a*e)^3*a^2*d^2*g^4/(d*x + c)^3) + (18*A^2*b^2*e^4 + 12*A*B*b^2*e^4
+ 4*B^2*b^2*e^4 - 54*(b*e*x + a*e)*A^2*b*d*e^3/(d*x + c) - 54*(b*e*x + a*e
)*A*B*b*d*e^3/(d*x + c) - 27*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 54*(b*e
*x + a*e)^2*A^2*d^2*e^2/(d*x + c)^2 + 108*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x
 + c)^2 + 108*(b*e*x + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)/((b*e*x + a*e)^3*b^
2*c^2*g^4/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*e*x
 + a*e)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a
*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.104.9 Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.55

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2-42\,A\,B\,a\,b\,c\,d+12\,A\,B\,b^2\,c^2+85\,B^2\,a^2\,d^2-23\,B^2\,a\,b\,c\,d+4\,B^2\,b^2\,c^2}{6\,(a\,d-b\,c)}+\frac{x\,(-5\,c\,B^2\,b^2\,d+49\,a\,\cdots}{\cdots}}{x\,(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4)-x^2\,(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4)+x^3\,(9\,b^5\,c\,g^4-9\,a\,b^4\cdots}$$

$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{3\,b^2\,g^4\,\left(3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2\right)}-\frac{B^2\,d^3}{3\,b\,g^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{2\,A\,B}{3\,b^2\,d\,g^4}+\frac{2\,B^2\,d^3\left(a\left(\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2}{6\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{3\,b\,d^2}\right)+\frac{3\,a^3\,d^3-6\,a^2\,b\,c\,d^2+4\,a\,b^2\,c^2\,d-b^3\,c^3}{3\,b\,d^4}\right)}{3\,b\,g^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{2\,B^2\,d^3\,x^2\left(\frac{b^2\,c-\cdots}{3\,\cdots}\cdots\right)}{3\,b\,g^4\,(a^3\,d^3-3\,a^2\,b\cdots}\cdots\right)}{\frac{3\,a^2\,x}{d}+\frac{a^3}{b\,d}+\frac{b^2\,x^3}{d}+\frac{3\,a\cdots}{\cdots}}$$

$$-\frac{B\,d^3\,\mathrm{atan}\left(\frac{B\,d^3\left(\frac{a^3\,b\,d^3\,g^4-a^2\,b^2\,c\,d^2\,g^4-a\,b^3\,c^2\,d\,g^4+b^4\,c^3\,g^4}{a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4}+2\,b\,d\,x\right)(6\,A+11\,B)\,(a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4)\,1\mathrm{i}}{b\,g^4\,(a\,d-b\,c)^3\,(11\,B^2\,d^3+6\,A\,B\,d^3)}\right)(6\,A+11\,\cdots}{9\,b\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^4,x)
```

output
```
((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 + 66*A*
B*a^2*d^2 + 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d - 42*A*B*a*b*
c*d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 30*A*B*a*b*d^2
 - 6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d + 6*A*B*b^2*d))/
(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*
g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^
4 - 9*a^4*b*d*g^4) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(3*b^2*g^4*(3*a^2
*x + a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x))/(c + d*x))*((2*A*
B)/(3*b^2*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d
^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
6*a^2*b*c*d^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) - (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c
))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
 + (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d
- b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d
 - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*
atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^
2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A ...
```

3.104. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.105.1 Optimal result

Integrand size = 32, antiderivative size = 575

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} + \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} + \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

output
```
2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b
*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/3
2*b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*(d*x+c)*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/3*b^2*B*d*(d*x+c)^3*(A+B*ln
(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3*B*(d*x+c)^4*(A+B*l
n(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

3.105.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{72\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(36A(bc-ad)^4+9B(bc-ad)^4+48Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3(a+bx)+72Ad^2(bc-ad)^2(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

output
```
-1/288*(72*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(36*A*(b*c - a*d)^4
 + 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c)
+ a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c -
a*d)^2*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b
*c) + a*d)*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(a
 + b*x)^4*Log[a + b*x] + 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 36*B*(b*c -
 a*d)^4*Log[(e*(a + b*x))/(c + d*x)] + 48*B*d*(-(b*c) + a*d)^3*(a + b*x)*L
og[(e*(a + b*x))/(c + d*x)] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log[(e*(a
 + b*x))/(c + d*x)] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[(e*(a + b*x
))/(c + d*x)] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e*(a + b*x))/(c +
d*x)] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*(a + b*x)^4*Log[c +
 d*x] - 144*B*d^4*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*
x] + 144*B*d^4*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] + 72*
B*d^4*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[
(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

3.105.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \ 2950$$

$$\frac{\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \ 2795$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\log\left(\frac{e(a+b}{c+dx}\right.}{(a+bx)}\right.}{g^5(bc-ad)^4}$$

$\big\downarrow$ 2009

$$-\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}-\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8(a+bx)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(a+bx)^3}+\frac{2b^2Bd(c+dx)^3\left(B\log\left(\frac{e(a}{c}\right.\right.}{3(a+bx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

output
```
((2*B^2*d^3*(c + d*x))/(a + b*x) - (3*b*B^2*d^2*(c + d*x)^2)/(4*(a + b*x)^
2) + (2*b^2*B^2*d*(c + d*x)^3)/(9*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(32
*(a + b*x)^4) + (2*B*d^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
a + b*x) - (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2
*(a + b*x)^2) + (2*b^2*B*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(3*(a + b*x)^3) - (b^3*B*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(8*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2)/(a + b*x)^3 - (b^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(4*(a + b*x)^4))/((b*c - a*d)^4*g^5)
```

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.105. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(559) = 1118$.

Time = 2.32 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.05

method	result	size
parts	Expression too large to display	1179
derivativedivides	Expression too large to display	1393
default	Expression too large to display	1393
norman	Expression too large to display	1796
parallelrisch	Expression too large to display	2035
risch	Expression too large to display	3080

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A^2/g^5/(b*x+a)^4/b-B^2/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*d^3/(a*d-b*c
)^5*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b^3*e^3*(-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^4))-2*B*A/g^5/d^2*(a*d-b*c)*e*(d^5/(a*d-b*c)^5*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*
c)*e/d/(d*x+c)))-3*d^4/(a*d-b*c)^5*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+3*
d^3/(a*d-b*c)^5*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^2/(a*d-b*c)^5*b
^3*e^3*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1035, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{9\,(8\,A^2+4\,AB+B^2)b^4c^4-32\,(9\,A^2+6\,AB+2\,B^2)ab^3c^3d+216\,(2\,A^2+2\,AB+B^2)a^2b^2c^2d^2-288\,(}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="frica
s")
```

output
```
-1/288*(9*(8*A^2 + 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 + 6*A*B + 2*B^2)*a*b^3
*c^3*d + 216*(2*A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 + 2*A*B + 2*
B^2)*a^3*b*c*d^3 + (72*A^2 + 300*A*B + 415*B^2)*a^4*d^4 - 12*((12*A*B + 25
*B^2)*b^4*c*d^3 - (12*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((12*A*B + 13*B^2)*
b^4*c^2*d^2 - 16*(6*A*B + 11*B^2)*a*b^3*c*d^3 + (84*A*B + 163*B^2)*a^2*b^2
*d^4)*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*
x^2 + 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*
c^2*d^2 + 4*B^2*a^3*b*c*d^3)*log((b*e*x + a*e)/(d*x + c))^2 - 4*((12*A*B +
 7*B^2)*b^4*c^3*d - 12*(6*A*B + 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B + 3*B^2)
*a^2*b^2*c*d^3 - (156*A*B + 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B + 25*B^2)*
b^4*d^4*x^4 - 3*(4*A*B + B^2)*b^4*c^4 + 16*(3*A*B + B^2)*a*b^3*c^3*d - 36*
(2*A*B + B^2)*a^2*b^2*c^2*d^2 + 48*(A*B + B^2)*a^3*b*c*d^3 + 4*(3*B^2*b^4*
c*d^3 + 2*(6*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b
^3*c*d^3 - 6*(2*A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - 6*B^2*a
*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*(A*B + B^2)*a^3*b*d^4)*x)*log((b*
e*x + a*e)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a
^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a
^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 -
 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*
x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*...
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

3.105.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2123 vs. 2(559) = 1118.

Time = 0.36 (sec) , antiderivative size = 2123, normalized size of antiderivative = 3.69

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="maxim
a")
```

output
```
1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d
^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^
5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^
4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*
b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^
6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*
b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^
2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*l
og(b*e*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^
2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4
)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b
^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4
)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 +
 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d
^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4
*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*
b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x +
25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*...
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.76

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="giac"
)
```

output
```
-1/288*(72*(B^2*b^3*e^5 - 4*(b*e*x + a*e)*B^2*b^2*d*e^4/(d*x + c) + 6*(b*e
*x + a*e)^2*B^2*b*d^2*e^3/(d*x + c)^2 - 4*(b*e*x + a*e)^3*B^2*d^3*e^2/(d*x
 + c)^3)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x
+ c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4
*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) +
12*(12*A*B*b^3*e^5 + 3*B^2*b^3*e^5 - 48*(b*e*x + a*e)*A*B*b^2*d*e^4/(d*x +
 c) - 16*(b*e*x + a*e)*B^2*b^2*d*e^4/(d*x + c) + 72*(b*e*x + a*e)^2*A*B*b*
d^2*e^3/(d*x + c)^2 + 36*(b*e*x + a*e)^2*B^2*b*d^2*e^3/(d*x + c)^2 - 48*(b
*e*x + a*e)^3*A*B*d^3*e^2/(d*x + c)^3 - 48*(b*e*x + a*e)^3*B^2*d^3*e^2/(d*
x + c)^3)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x +
 c)^4 - 3*(b*e*x + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*
a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a^3*d^3*g^5/(d*x + c)^4) + (
72*A^2*b^3*e^5 + 36*A*B*b^3*e^5 + 9*B^2*b^3*e^5 - 288*(b*e*x + a*e)*A^2*b^
2*d*e^4/(d*x + c) - 192*(b*e*x + a*e)*A*B*b^2*d*e^4/(d*x + c) - 64*(b*e*x
+ a*e)*B^2*b^2*d*e^4/(d*x + c) + 432*(b*e*x + a*e)^2*A^2*b*d^2*e^3/(d*x +
c)^2 + 432*(b*e*x + a*e)^2*A*B*b*d^2*e^3/(d*x + c)^2 + 216*(b*e*x + a*e)^2
*B^2*b*d^2*e^3/(d*x + c)^2 - 288*(b*e*x + a*e)^3*A^2*d^3*e^2/(d*x + c)^3 -
 576*(b*e*x + a*e)^3*A*B*d^3*e^2/(d*x + c)^3 - 576*(b*e*x + a*e)^3*B^2*d^3
*e^2/(d*x + c)^3)/((b*e*x + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*e*x + a*
e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*e*x + a*e)^4*a^2*b*c*d^2*g^5/(d...
```

3.105.9 Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 1881, normalized size of antiderivative = 3.27

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*atan((B*d^4*(12*A + 25*B)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 - 48*a
*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(a*d - b*c)^4*(25*B^2
*d^4 + 12*A*B*d^4)) + (B*d^5*x*(12*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5
- 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*
d^4 + 12*A*B*d^4)))*(12*A + 25*B)*1i)/(12*b*g^5*(a*d - b*c)^4) - log((e*(a
 + b*x))/(c + d*x))^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b
*x^2 + 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c
^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x))/(c + d*x))*
((A*B)/(2*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(
12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^
2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*
d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(
b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*
d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2
)) - a*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c^2 + 4*
a^2*b*d^2 - 5*a*b^2*c*d)/(4*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b
*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2*d)/(4*d^2)))/(2*b*
g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*...
```

3.105. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.106 $\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx$

3.106.1 Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{bc-ad}{b(c+dx)}\right)}{df}$$

output
```
polylog(2,(-a*d+b*c)/b/(d*x+c))/d/f
```

3.106.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.07

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx =\frac{\log\left(\frac{bc-ad}{bc+bdx}\right)\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-2\log\left(\frac{d(a+bx)}{b(c+dx)}\right)+\log\left(\frac{bc-ad}{bc+bdx}\right)\right)-2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2df}$$

input
```
Integrate[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

output
```
(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 2*L
og[(d*(a + b*x))/(b*(c + d*x))] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])/(2*d*f)
```

3.106.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx$$

$$\Big\downarrow \; 2897$$

$$\frac{\text{PolyLog}\left(2,1-\frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

input
```
Int[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

output
```
PolyLog[2, 1 - (d*(a + b*x))/(b*(c + d*x))]/(d*f)
```

3.106.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.106.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30
default	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30
risch	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30
parts	$\frac{\ln\left(\frac{d(bx+a)}{b(dx+c)}\right)\ln(dx+c)}{df}-\frac{b\left(-\frac{d^2\ln(dx+c)^2}{2b}+d^2\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}\right)\right)}{d^3f}$	132

input
```
int(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x,method=_RETURNVERBOSE)
```

output
```
1/d/f*dilog(1+(a*d-b*c)/b/(d*x+c))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx=\frac{\operatorname{Li}_2\left(-\frac{bdx+ad}{bdx+bc}+1\right)}{df}$$

input
```
integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="fricas")
```

output
```
dilog(-(b*d*x + a*d)/(b*d*x + b*c) + 1)/(d*f)
```

3.106.6 Sympy [F]

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx=\frac{\int\frac{\log\left(\frac{ad}{bc+bdx}+\frac{bdx}{bc+bdx}\right)}{c+dx}\,dx}{f}$$

input
```
integrate(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)
```

output
```
Integral(log(a*d/(b*c + b*d*x) + b*d*x/(b*c + b*d*x))/(c + d*x), x)/f
```

3.106.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.64

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = -\frac{b\left(\frac{\log(dx+c)^2}{bf} - \frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bf}\right)}{2\,d} - \frac{b\left(\frac{d\log(bx+a)}{b} - \frac{d\log(dx+c)}{b}\right)\log\left(dfx+cf\right)}{d^2f} + \frac{\log\left(dfx+cf\right)\log\left(\frac{(bx+a)d}{(dx+c)b}\right)}{df}$$

input
```
integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="maxima")
```

output
```
-1/2*b*(log(d*x + c)^2/(b*f) - 2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*
d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*f))/d - b*(d*log(b*x + a)/
b - d*log(d*x + c)/b)*log(d*f*x + c*f)/(d^2*f) + log(d*f*x + c*f)*log((b*x
 + a)*d/((d*x + c)*b))/(d*f)
```

3.106.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(27) = 54$.

Time = 34.96 (sec) , antiderivative size = 1203, normalized size of antiderivative = 42.96

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \text{Too large to display}$$

input
```
integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="giac")
```

output
```
-1/2*(b^2*c*d/(b*c - a*d)^2 - a*b*d^2/(b*c - a*d)^2)^2*((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(log(abs(b*d*x + a*d)/abs(b*d*x + b*c))/
(b^3*d^4*f) - log(abs((b*d*x + a*d)/(b*d*x + b*c) - 1))/(b^3*d^4*f) - 1/(b
^3*d^4*f*((b*d*x + a*d)/(b*d*x + b*c) - 1))) - (b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*log((a + b*((a*d - b*((b*d*x + a*d)*b*c/((b*d*x +
 b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*
(b*c - a*d)) - b*d/(b*c - a*d)))*b*c/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*
b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((
b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))) - a*d/(b*c - a*d))/(b*d/(b*
c - a*d) - (a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(
b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c -
a*d)))*b*d/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c -
a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d))
- b*d/(b*c - a*d))))))*d/(b*(c + ((a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*
c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*
c - a*d)) - b*d/(b*c - a*d)))*b*c/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c
/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d
*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))) - a*d/(b*c - a*d))*d/(b*d/(b*c
 - a*d) - (a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b
*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c ...
```

3.106.9 Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx=\frac{\mathrm{Li}_2\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right)}{d\,f}$$

input
```
int(log((d*(a + b*x))/(b*(c + d*x)))/(c*f + d*f*x),x)
```

output
```
dilog((d*(a + b*x))/(b*(c + d*x)))/(d*f)
```

3.107 $\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$

3.107.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

output
```
polylog(2,-1/(b*x+a))/b
```

3.107.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

input
```
Integrate[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

output
```
PolyLog[2, -(a + b*x)^(-1)]/b
```

3.107.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{1}{a+bx}+1\right)}{a+bx}\,dx$$

$$\Big\downarrow \; 2897$$

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

input
```
Int[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

output
```
PolyLog[2, -(a + b*x)^(-1)]/b
```

3.107.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.107.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
default	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
risch	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
parts	$\frac{\ln\left(1+\frac{1}{bx+a}\right)\ln(bx+a)}{b}+\frac{\frac{\ln(bx+a)^2}{2}-\operatorname{dilog}(bx+a+1)-\ln(bx+a)\ln(bx+a+1)}{b}$	61

input
```
int(ln(1+1/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*dilog(1+1/(b*x+a))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\log \left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\mathrm{Li}_2\left(-\frac{bx+a+1}{bx+a}+1\right)}{b}$$

input
```
integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

output
```
dilog(-(b*x + a + 1)/(b*x + a) + 1)/b
```

3.107.6 Sympy [F]

$$\int \frac{\log \left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\int \frac{\log \left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$$

input
```
integrate(ln(1+1/(b*x+a))/(b*x+a),x)
```

output
```
Integral(log(1 + 1/(a + b*x))/(a + b*x), x)
```

3.107.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(14) = 28.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.07

$$\int \frac{\log \left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{2\log (bx+a+1)\log (bx+a)-\log (bx+a)^2}{2\,b}-\frac{\log (bx+a+1)\log (bx+a)+\mathrm{Li}_2(-bx-a)}{b}$$

input
```
integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*log(b*x + a + 1)*log(b*x + a) - log(b*x + a)^2)/b - (log(b*x + a +
1)*log(b*x + a) + dilog(-b*x - a))/b
```

3.107.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(14) = 28$.

Time = 3.75 (sec) , antiderivative size = 320, normalized size of antiderivative = 21.33

$$\int \frac{\log \left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$$

$$= \frac{1}{2}\left((a+1)b-ab\right)^2 \left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a|}\right)}{b^4} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a}-1\right|\right)}{b^4} - \frac{1}{b^4\left(\frac{bx+a+1}{bx+a}-1\right)} - \frac{\log\left(\frac{1}{a-\frac{\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}\right)\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)}{\frac{(bx+a+1)b}{bx+a}-b}\right)}{\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)}{\frac{(bx+a+1)b}{bx+a}-b}\right)\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-\right)}{\frac{(bx+a+1)b}{bx+a}-b}\right)}}\right)}{b^4\left(\frac{bx+a+1}{bx+a}-1\right)}\right)$$

input
```
integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

3.107. $\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$

output
```
1/2*((a + 1)*b - a*b)^2*(log(abs(b*x + a + 1)/abs(b*x + a))/b^4 - log(abs(
(b*x + a + 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a + 1)/(b*x + a) - 1))
- log(1/(a - ((a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a + 1)*b/
(b*x + a) - b) + 1)*a/(a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a
 + 1)*b/(b*x + a) - b)) - a - 1)*b/((a - ((b*x + a + 1)*a/(b*x + a) - a -
1)*b/((b*x + a + 1)*b/(b*x + a) - b) + 1)*b/(a - ((b*x + a + 1)*a/(b*x + a
) - a - 1)*b/((b*x + a + 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a +
1)/(b*x + a) - 1)^2))
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\mathrm{polylog}\left(2,-\frac{1}{a+b\,x}\right)}{b}$$

input
```
int(log(1/(a + b*x) + 1)/(a + b*x),x)
```

output
```
polylog(2, -1/(a + b*x))/b
```

3.108 $\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$

3.108.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

output
```
polylog(2,1/(b*x+a))/b
```

3.108.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

input
```
Integrate[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
```

output
```
PolyLog[2, (a + b*x)^(-1)]/b
```

3.108.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$$

$$\Big\downarrow \; 2897$$

$$\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

input
```
Int[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
```

output
```
PolyLog[2, (a + b*x)^(-1)]/b
```

3.108.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.108.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
default	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
risch	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
parts	$\frac{\ln\left(1-\frac{1}{bx+a}\right)\ln(bx+a)}{b}-\frac{-\frac{\ln(bx+a)^2}{2}-\operatorname{dilog}(bx+a)}{b}$	48

input
```
int(ln(1-1/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*dilog(1-1/(b*x+a))
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{Li}_2\left(-\frac{bx+a-1}{bx+a}+1\right)}{b}$$

input
```
integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

output
```
dilog(-(b*x + a - 1)/(b*x + a) + 1)/b
```

3.108.6 Sympy [F]

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = \int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$$

input
```
integrate(ln(1-1/(b*x+a))/(b*x+a),x)
```

output
```
Integral(log(1 - 1/(a + b*x))/(a + b*x), x)
```

3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(12) = 24.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.54

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = -\frac{\log(bx+a)^2-2\log(bx+a)\log(bx+a-1)}{2b} - \frac{\log(bx+a)\log(-bx-a+1)+\text{Li}_2(bx+a)}{b}$$

input
```
integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x + a - 1))/b - (log(b*x + a)*
log(-b*x - a + 1) + dilog(b*x + a))/b
```

3.108.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(12) = 24$.

Time = 3.57 (sec) , antiderivative size = 322, normalized size of antiderivative = 24.77

$$\int \frac{\log \left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx =$$

$$-\frac{1}{2}\left((a-1)b-ab\right)^2\left(\frac{\log\left(\frac{|bx+a-1|}{|bx+a|}\right)}{b^4}-\frac{\log\left(\left|\frac{bx+a-1}{bx+a}-1\right|\right)}{b^4}-\frac{1}{b^4\left(\frac{bx+a-1}{bx+a}-1\right)}-\frac{\log\left(-\frac{1}{a-\frac{\left(a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-\cdots\right)}{\frac{(bx+a-1)b}{bx+a}}\cdots\right)}{\cdots}}\cdots\right)}{b^4\left(\frac{bx+a-1}{bx+a}\cdots\right)}\cdots\right)$$

input

```
integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

3.108. $\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$

output
```
-1/2*((a - 1)*b - a*b)^2*(log(abs(b*x + a - 1)/abs(b*x + a))/b^4 - log(abs
((b*x + a - 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a - 1)/(b*x + a) - 1))
 - log(-1/(a - ((a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x + a - 1)*
b/(b*x + a) - b) - 1)*a/(a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x +
 a - 1)*b/(b*x + a) - b)) - a + 1)*b/((a - ((b*x + a - 1)*a/(b*x + a) - a
+ 1)*b/((b*x + a - 1)*b/(b*x + a) - b) - 1)*b/(a - ((b*x + a - 1)*a/(b*x +
 a) - a + 1)*b/((b*x + a - 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a
- 1)/(b*x + a) - 1)^2))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\mathrm{polylog}\left(2,\frac{1}{a+b\,x}\right)}{b}$$

input
```
int(log(1 - 1/(a + b*x))/(a + b*x),x)
```

output
```
polylog(2, 1/(a + b*x))/b
```

3.109 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

3.109.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.109.2 Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

3.109.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
$Aborted
```

3.109.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.109.4 Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.109.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b*e*x + a*e)/(d*x +
 c)) + A), x)
```

3.109.6 Sympy [N/A]

Not integrable

Time = 8.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.97

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=g^2\left(\int \frac{a^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{b^2x^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2abx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Int
egral(b**2*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integra
l(2*a*b*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

3.109.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.109.8 Giac [N/A]

Not integrable

Time = 13.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.109.9 Mupad [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.110 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

3.110.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.110.2 Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

3.110.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{ag+bgx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
$Aborted
```

3.110.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.110.4 Maple [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.110.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

3.110.6 Sympy [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=g\left(\int \frac{a}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
g*(Integral(a/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(
b*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

3.110.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.110.8 Giac [N/A]

Not integrable

Time = 11.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.110.9 Mupad [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.111 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

3.111.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.111.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

3.111.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
$Aborted
```

3.111.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.111.4 Maple [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.111.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c
))), x)
```

3.111.6 Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bbx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b*x
*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g
```

3.111.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.111.8 Giac [N/A]

Not integrable

Time = 9.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.111.9 Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.112 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

3.112.1 Optimal result

Integrand size = 32, antiderivative size = 50

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)g^2}$$

output
```
e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)/g^2
```

3.112.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{bBcg^2-aBdg^2}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
(e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(b*B*
c*g^2 - a*B*d*g^2)
```

3.112.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2950, 2746, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2746}$$

$$\frac{e\int \frac{c+dx}{e(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\log\left(\frac{e(a+bx)}{c+dx}\right)}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{ee^{A/B}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
(e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(B*(b
*c - a*d)*g^2)
```

3.112.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2746
```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Simp[1/c^
(m + 1)   Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[m]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.112.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{e\,{\mathrm e}^{\frac{A}{B}}\,\operatorname{Ei}_{1}\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^{2}B}$	61
default	$\frac{e\,{\mathrm e}^{\frac{A}{B}}\,\operatorname{Ei}_{1}\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^{2}B}$	61
risch	$\frac{e\,{\mathrm e}^{\frac{A}{B}}\,\operatorname{Ei}_{1}\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^{2}B}$	61

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
e/(a*d-b*c)/g^2/B*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{\frac{A}{B}}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(Bbc-Bad)g^2}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="frica
s")
```

output
```
e*e^(A/B)*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e))/((B*b*c - B*a*d)*
g^2)
```

3.112.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) +
b*e*x/(c + d*x)) + 2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2
*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**2
```

3.112.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.112.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.113 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

3.113.1 Optimal result

Integrand size = 32, antiderivative size = 107

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{be^2e^{\frac{2A}{B}}\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B(bc-ad)^2g^3}-\frac{dee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)^2g^3}$$

output
```
b*e^2*exp(2*A/B)*Ei(-2*(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3-d
*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3
```

3.113.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\frac{ee^{A/B}\left(bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)-d\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)\right)}{B(bc-ad)^2g^3}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
(e*E^(A/B)*(b*e*E^(A/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/B] - d*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)]))/(B
*(b*c - a*d)^2*g^3)
```

3.113.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{be^2e^{\frac{2A}{B}}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B}-\frac{dee^{A/B}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
((b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/B])/B - (d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)]
)/B)])/B)/((b*c - a*d)^2*g^3)
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.113.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{e\left(\frac{d\,{\mathrm e}^{\frac{A}{B}} \operatorname{Ei}_{1}\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B}-\frac{be\,{\mathrm e}^{\frac{2A}{B}} \operatorname{Ei}_{1}\left(2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{B}\right)}{\left(ad-cb\right)^{2} g^{3}}$	117
default	$\frac{e\left(\frac{d\,{\mathrm e}^{\frac{A}{B}} \operatorname{Ei}_{1}\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B}-\frac{be\,{\mathrm e}^{\frac{2A}{B}} \operatorname{Ei}_{1}\left(2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{B}\right)}{\left(ad-cb\right)^{2} g^{3}}$	117
risch	$-\frac{e^{2} b\,{\mathrm e}^{\frac{2A}{B}} \operatorname{Ei}_{1}\left(2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{\left(ad-cb\right)^{2} g^{3} B}+\frac{ed\,{\mathrm e}^{\frac{A}{B}} \operatorname{Ei}_{1}\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{\left(ad-cb\right)^{2} g^{3} B}$	131

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
e/(a*d-b*c)^2/g^3*(d/B*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)-
b*e/B*exp(2*A/B)*Ei(1,2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*A/B))
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{be^2e^{\left(\frac{2A}{B}\right)}\,\text{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2A}{B}\right)}}{b^2e^2x^2+2\,abe^2x+a^2e^2}\right)-dee^{\frac{A}{B}}\,\text{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="frica
s")
```

output
```
(b*e^2*e^(2*A/B)*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^
2*x^2 + 2*a*b*e^2*x + a^2*e^2)) - d*e*e^(A/B)*log_integral((d*x + c)*e^(-A
/B)/(b*e*x + a*e)))/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3)
```

3.113.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Ba^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Bab^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}}{g^3}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a**2*b*x*log(a*e/(c + d*x) + b
*e*x/(c + d*x)) + 3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B
*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**3
```

3.113.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.113.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.114 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.114.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.114.2 Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

3.114.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
$Aborted
```

3.114.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.114.4 Maple [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.114.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b*e*x + a*e)/(d*x
 + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.114.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 305, normalized size of antiderivative = 9.53

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
-(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c
*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*
log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e
) - a*d*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*
g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)
/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d
)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

3.114.8 Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.114.9 Mupad [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.115 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.115.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.115.2 Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

3.115.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$
$$\Big\downarrow \ 2956$$
$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
$Aborted
```

3.115.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.115.4 Maple [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.115.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral((b*g*x + a*g)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*
e*x + a*e)/(d*x + c)) + A^2), x)
```

3.115.6 Sympy [N/A]

Not integrable

Time = 15.89 (sec) , antiderivative size = 274, normalized size of antiderivative = 9.13

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\frac{a^2cg+a^2dgx+2abcgx+2abdgx^2+b^2cgx^2+b^2dgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2b^2cx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{3b^2dx^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\right.}{B\,(ad-bc)}$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b*
*2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(c
 + d*x))) - g*(Integral(a**2*d/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))
), x) + Integral(2*a*b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
+ Integral(2*b**2*c*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + I
ntegral(3*b**2*d*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + I
ntegral(4*a*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*
d - b*c))
```

3.115.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 7.57

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
-(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g
)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c -
 a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*
a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x +
a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*lo
g(e))*B^2), x)
```

3.115.8 Giac [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.115.9 Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.116 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.116.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.116.2 Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

3.116.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
$Aborted
```

3.116.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.116.4 Maple [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.116.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.59

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/
(d*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c))), x)
```

3.116.6 Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.81

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$=\frac{c+dx}{ABadg-ABbcg+(B^2adg-B^2bcg)\log\left(\frac{e(a+bx)}{c+dx}\right)}-\frac{d\int \frac{1}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg\,(ad-bc)}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(c + d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(a + b*
x)/(c + d*x))) - d*Integral(1/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))
, x)/(B*g*(a*d - b*c))
```

3.116.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(
d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2), x) -
(d*x + c)/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x
+ c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

3.116.8 Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

3.116.9 Mupad [N/A]

Not integrable

Time = 5.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.117 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.117.1 Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = -\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)g^2} - \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}$$

output
```
-e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)/g^2+(-d*x-c)
/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

3.117.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right) + \frac{B(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}}{B^2(-bc+ad)g^2}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
(e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)] + (B*(
c + d*x))/((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(B^2*(-(b*c) +
 a*d)*g^2)
```

3.117.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2950, 2743, 2746, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{-\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\frac{a+bx}{c+dx}}{B}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2746}$$

$$\frac{-\frac{e\int \frac{c+dx}{e(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}d\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{-\frac{ee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
(-((e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/B^
2) - (c + d*x)/(B*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/((b*c -
 a*d)*g^2)
```

3.117.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2746
```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Simp[1/c^
(m + 1)   Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[m]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.117.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{dx+c}{(ad-cb)B(bx+a)g^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}-\frac{e\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{g^2B^2(ad-cb)}$	113
derivativedivides	$-\frac{e\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)}{(ad-cb)g^2}$	132
default	$-\frac{e\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)}{(ad-cb)g^2}$	132

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)/B/(b*x+a)*(d*x+c)/g^2/(A+B*ln(e*(b*x+a)/(d*x+c)))-1/g^2/B^2*e/
(a*d-b*c)*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx =$$
$$-\frac{Bdx+Bc+\left((Bbex+Bae)e^{\frac{A}{B}}\log\left(\frac{bex+ae}{dx+c}\right)+(Abex+Aae)e^{\frac{A}{B}}\right)\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(AB^2b^2c-AB^2abd)g^2x+(AB^2abc-AB^2a^2d)g^2+((B^3b^2c-B^3abd)g^2x+(B^3abc-B^3a^2d)g^2)\log\Big(}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fri
cas")
```

output
```
-(B*d*x + B*c + ((B*b*e*x + B*a*e)*e^(A/B)*log((b*e*x + a*e)/(d*x + c)) +
(A*b*e*x + A*a*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e)))
/((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2 + ((
B^3*b^2*c - B^3*a*b*d)*g^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2)*log((b*e*x + a
*e)/(d*x + c)))
```

3.117.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$= \frac{c+dx}{ABa^2dg^2 - ABabcg^2 + ABabdg^2x - ABb^2cg^2x + (B^2a^2dg^2 - B^2abcg^2 + B^2abdg^2x - B^2b^2cg^2x)\log\left(\frac{e(}{c}\right.}$$
$$- \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(c + d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2*
c*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2*
b**2*c*g**2*x)*log(e*(a + b*x)/(c + d*x))) - Integral(1/(A*a**2 + 2*A*a*b*
x + A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 2*B*a*b*x*
log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2*x**2*log(a*e/(c + d*x) + b*e
*x/(c + d*x))), x)/(B*g**2)
```

3.117.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="max
ima")
```

output
```
-(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*lo
g(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*l
og(e))*B^2)*x + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B
^2)*log(b*x + a) - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2
)*B^2)*log(d*x + c)) + integrate(-1/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 + (B
^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g^2
)*x + (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) - (B^
2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

3.117.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.118 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.118.1 Optimal result

Integrand size = 32, antiderivative size = 212

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = -\frac{2be^2e^{\frac{2A}{B}}\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2(bc-ad)^2g^3} + \frac{dee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)^2g^3} + \frac{d(c+dx)}{B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} - \frac{b(c+dx)^2}{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}$$

output
```
-2*b*e^2*exp(2*A/B)*Ei(-2*(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/
g^3+d*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/g^3+d
*(d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))-b*(d*x+c)^
2/B/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

3.118.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$= \frac{-2be^2e^{\frac{2A}{B}}\,\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)+dee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)-\frac{B(bc}{(a+bx)^2\left(A\right.}}{B^2(bc-ad)^2g^3}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
(-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/B] + d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/
B)] - (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])))/(B^2*(b*c - a*d)^2*g^3)
```

3.118.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}\,dx$$
$$\Big\downarrow \text{ 2950}$$
$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$
$$\Big\downarrow \text{ 2795}$$
$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2be^2e^{\frac{2A}{B}}\text{ExpIntegralEi}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2}+\frac{dee^{A/B}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2}-\frac{b(c+dx)^2}{B(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
((-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/B])/B^2 + (d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c +
d*x)])/B)])/B^2 + (d*(c + d*x))/(B*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])) - (b*(c + d*x)^2)/(B*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])))/((b*c - a*d)^2*g^3)
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.118.4 Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.86

method	result
risch	$\frac{dx+c}{(ad-cb)B(bx+a)^2 g^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}+\frac{2e^2 b\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{Ei}_1\left(2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{g^3B^2(ad-cb)^2}-\frac{ed\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.\right.}{g^3B^2(ad-cb)^2}$
derivativedivides	$\frac{e\left(-d\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)+be\left(-\frac{}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 B\left(A\right.}\right.\right.}{(ad-cb)^2g^3}$
default	$\frac{e\left(-d\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{Ei}_1\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)+be\left(-\frac{}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 B\left(A\right.}\right.\right.}{(ad-cb)^2g^3}$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)/B/(b*x+a)^2*(d*x+c)/g^3/(A+B*ln(e*(b*x+a)/(d*x+c)))+2*e^2/g^3/
B^2/(a*d-b*c)^2*b*exp(2*A/B)*Ei(1,2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*A/B)
-e/g^3/B^2/(a*d-b*c)^2*d*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B
)
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(210) = 420.

Time = 0.27 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.69

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx =$$

$$-\frac{Bbc^2-Bacd+(Bbcd-Bad^2)x-\left((Bb^2dex^2+2\,Babdex+Ba^2de)e^{\frac{A}{B}}\log\left(\frac{bex+ae}{dx+c}\right)+(Ab^2dex^2+2\,A\right.}{(AB^2b^4c^2-2\,AB^2ab^3cd+AB^2a^2b^2d^2)g^3x^2+2\,(AB^2ab^3c^2-2\,AB^2a^2b^2cd+AB^2a^3bd^2)g^3x+}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fri
cas")
```

output
```
-(B*b*c^2 - B*a*c*d + (B*b*c*d - B*a*d^2)*x - ((B*b^2*d*e*x^2 + 2*B*a*b*d*
e*x + B*a^2*d*e)*e^(A/B)*log((b*e*x + a*e)/(d*x + c)) + (A*b^2*d*e*x^2 + 2
*A*a*b*d*e*x + A*a^2*d*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x
+ a*e)) + 2*((B*b^3*e^2*x^2 + 2*B*a*b^2*e^2*x + B*a^2*b*e^2)*e^(2*A/B)*log
((b*e*x + a*e)/(d*x + c)) + (A*b^3*e^2*x^2 + 2*A*a*b^2*e^2*x + A*a^2*b*e^2
)*e^(2*A/B))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^2*x^
2 + 2*a*b*e^2*x + a^2*e^2)))/((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a
^2*b^2*d^2)*g^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3
*b*d^2)*g^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*g^
3 + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*x^2 + 2*(B^3*a*
b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*x + (B^3*a^2*b^2*c^2 - 2*
B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3)*log((b*e*x + a*e)/(d*x + c)))
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.118.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="max
ima")
```

output
```
-(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*g^
3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^2
*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^
3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2
 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*lo
g(b*x + a) - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d
*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate((b*
d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e) - a
*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3
*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (
a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 -
a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x + ((
b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2
 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*
log(b*x + a) - ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b
^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 -
 a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

3.118.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.118. $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.119 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

3.119.1 Optimal result

Integrand size = 32, antiderivative size = 182

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{2B(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3} + \frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2} - \frac{B(bc-ad)g^4(a+bx)^4}{10bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b} - \frac{2B(bc-ad)^5g^4\log(c+dx)}{5bd^5}$$

output
```
2/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+2/15*B
*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*
g^4*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/5*B*(-a*d+b*c)^5*g^4*ln(
d*x+c)/b/d^5
```

3.119.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(bc-ad)\left(12bd(bc-ad)^3x-6d^2(bc-ad)^2(a+bx)^2+4d^3(bc-ad)(a+bx)^3-3d^4(a+bx)^4-12}{6d^5}\right.}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(b*c - a*d
)*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c -
 a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/(6
*d^5)))/(5*b)
```

3.119.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$
$$\Big\downarrow \text{2948}$$
$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2B(bc-ad)\int\frac{g^5(a+bx)^4}{c+dx}dx}{5bg}$$
$$\Big\downarrow \text{27}$$
$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2Bg^4(bc-ad)\int\frac{(a+bx)^4}{c+dx}dx}{5b}$$
$$\Big\downarrow \text{49}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2Bg^4(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2Bg^4(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b) - (2*B*(b
*c - a*d)*g^4*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2
*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*
d)^4*Log[c + d*x])/d^5))/(5*b)
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.119. $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.119.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(170) = 340$.

Time = 1.20 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.45

method	result
risch	$-\frac{4g^4bBa^3cx}{d}+\frac{4g^4b^2Ba^2c^2x}{d^2}-\frac{2g^4b^3Bac^3x}{d^3}-\frac{4g^4b^2B\ln(dx+c)a^2c^3}{d^3}+\frac{2g^4b^3B\ln(dx+c)ac^4}{d^4}+\frac{4g^4bB\ln(dx}{d^2}$
parallelrisch	$6Bx^5\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab^5cd^5g^4+6B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab^5c^6g^4+12B\ln(bx+a)a^6cd^5g^4-12B\ln(bx+a)ab^5c^6g^4+4Bx^3ab^5$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
-4*g^4/d*b*B*a^3*c*x+4*g^4/d^2*b^2*B*a^2*c^2*x-2*g^4/d^3*b^3*B*a*c^3*x-4*g
^4/d^3*b^2*B*ln(d*x+c)*a^2*c^3+2*g^4/d^4*b^3*B*ln(d*x+c)*a*c^4+4*g^4/d^2*b
*B*ln(d*x+c)*a^3*c^2+1/5*(b*x+a)^5*g^4*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)-2/3*g
^4/d*b^3*B*a*c*x^3-2*g^4/d*b^2*B*a^2*c*x^2+g^4/d^2*b^3*B*a*c^2*x^2+2/15*g^
4/d^2*b^4*B*c^2*x^3+8/15*g^4*b^2*B*a^2*x^3+2*g^4*b*A*a^3*x^2+6/5*g^4*b*B*a
^3*x^2-1/5*g^4/d^3*b^4*B*c^3*x^2+g^4*A*a^4*x+8/5*g^4*B*a^4*x+2/5*g^4/d^4*b
^4*B*c^4*x-2*g^4/d*B*ln(d*x+c)*a^4*c-2/5*g^4/d^5*b^4*B*ln(d*x+c)*c^5+1/5*g
^4*b^4*A*x^5+g^4*b^3*A*a*x^4+1/10*g^4*b^3*B*a*x^4-1/10*g^4/d*b^4*B*c*x^4+2
*g^4*b^2*A*a^2*x^3+2/5*g^4/b*B*ln(d*x+c)*a^5
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(170) = 340$.

Time = 0.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.49

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$=\frac{6\,Ab^5d^5g^4x^5+12\,Ba^5d^5g^4\log(bx+a)-3\,(Bb^5cd^4-(10\,A+B)ab^4d^5)g^4x^4+4\,(Bb^5c^2d^3-5\,Bab^4cd^4+}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
1/30*(6*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4
 - (10*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + (1
5*A + 4*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 - 2*(5*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 6*(2*B*b^5*c^4*d
- 10*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 + (5*A +
8*B)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3
*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5
*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3
*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e
)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^5)
```

3.119.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(163) = 326$.

Time = 3.61 (sec) , antiderivative size = 998, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{Ab^4g^4x^5}{5}$$
$$+\frac{2Ba^5g^4\log\left(x+\frac{\frac{2Ba^6d^5g^4}{b}+10Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-10Ba^2b^3c^4dg^4+2Bab^4c^5g^4}{2Ba^5d^5g^4+10Ba^4bcd^4g^4-20Ba^3b^2c^2d^3g^4+20Ba^2b^3c^3d^2g^4-10Bab^4c^4dg^4+2Bb^5c^5g^4}\right)}{5b}$$
$$-\frac{2Bcg^4\cdot(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4)\log\left(x+\frac{12Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-\cdots}{\cdots}\right.}{5\cdots}$$
$$+x^4\left(Aab^3g^4+\frac{Bab^3g^4}{10}-\frac{Bb^4cg^4}{10d}\right)+x^3\cdot\left(2Aa^2b^2g^4+\frac{8Ba^2b^2g^4}{15}-\frac{2Bab^3cg^4}{3d}+\frac{2Bb^4c^2g^4}{15d^2}\right)$$
$$+x^2\cdot\left(2Aa^3bg^4+\frac{6Ba^3bg^4}{5}-\frac{2Ba^2b^2cg^4}{d}+\frac{Bab^3c^2g^4}{d^2}-\frac{Bb^4c^3g^4}{5d^3}\right)$$
$$+x\left(Aa^4g^4+\frac{8Ba^4g^4}{5}-\frac{4Ba^3bcg^4}{d}+\frac{4Ba^2b^2c^2g^4}{d^2}-\frac{2Bab^3c^3g^4}{d^3}+\frac{2Bb^4c^4g^4}{5d^4}\right)$$
$$+\left(Ba^4g^4x+2Ba^3bg^4x^2+2Ba^2b^2g^4x^3+Bab^3g^4x^4+\frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*b**4*g**4*x**5/5 + 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a*
*5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**
4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4
 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**
3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) -
2*B*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*
b**3*c**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2
*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 +
2*B*a*b**4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a
**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**
4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**
4*c**4)/d)/(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*
c**2*d**3*g**4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 +
 2*B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 + B*a*b**3*g**4/10 -
B*b**4*c*g**4/(10*d)) + x**3*(2*A*a**2*b**2*g**4 + 8*B*a**2*b**2*g**4/15 -
 2*B*a*b**3*c*g**4/(3*d) + 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*
b*g**4 + 6*B*a**3*b*g**4/5 - 2*B*a**2*b**2*c*g**4/d + B*a*b**3*c**2*g**4/d
**2 - B*b**4*c**3*g**4/(5*d**3)) + x*(A*a**4*g**4 + 8*B*a**4*g**4/5 - 4*B*
a**3*b*c*g**4/d + 4*B*a**2*b**2*c**2*g**4/d**2 - 2*B*a*b**3*c**3*g**4/d**3
 + 2*B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2...
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(170) = 340$.

Time = 0.22 (sec) , antiderivative size = 885, normalized size of antiderivative = 4.86

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$

$$= \frac{1}{5}Ab^4g^4x^5 + Aab^3g^4x^4 + 2Aa^2b^2g^4x^3 + 2Aa^3bg^4x^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{2a\log(bx+a)}{b}-\frac{2c\log(dx-$$
$$+2\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)-\frac{2a^2\log(bx+a)}{b^2}+\frac{2c^2\log($$
$$+2\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log($$
$$+\frac{1}{3}\left(3x^4\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log$$
$$+\frac{1}{30}\left(6x^5\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{12a^5\log(bx+a)}{b^5}-\frac{12c$$
$$+Aa^4g^4x$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*
x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*
c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c
*log(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2
) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
 - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d)
)*B*a^3*b*g^4 + 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x
/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(
b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(b^2*e*x^2/(d^2
*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x
^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(b^2*e*x^2/(d^2*x^2
 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 +
 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*
(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c
^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A
*a^4*g^4*x
```

3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(170) = 340$.

Time = 59.29 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.69

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$$
$$= \frac{1}{5}Ab^4g^4x^5 + \frac{2\,Ba^5g^4\log(bx+a)}{5\,b} - \frac{(Bb^4cg^4 - 10\,Aab^3dg^4 - Bab^3dg^4)x^4}{10\,d} + \frac{2\,(Bb^4c^2g^4 - 5\,Bab^3cdg^4 + 15\,Aa^2b^2d^2g^4 + 4\,Ba^2b^2d^2g^4)x^3}{15\,d^2} + \frac{1}{5}\left(Bb^4g^4x^5 + 5\,Bab^3g^4x^4 + 10\,Ba^2b^2g^4x^3 + 10\,Ba^3bg^4x^2 + 5\,Ba^4g^4x\right)\log\left(\frac{b^2ex^2 + 2\,abex + a^2e}{d^2x^2 + 2\,cdx + c^2}\right) - \frac{(Bb^4c^3g^4 - 5\,Bab^3c^2dg^4 + 10\,Ba^2b^2cd^2g^4 - 10\,Aa^3bd^3g^4 - 6\,Ba^3bd^3g^4)x^2}{5\,d^3} + \frac{(2\,Bb^4c^4g^4 - 10\,Bab^3c^3dg^4 + 20\,Ba^2b^2c^2d^2g^4 - 20\,Ba^3bcd^3g^4 + 5\,Aa^4d^4g^4 + 8\,Ba^4d^4g^4)x}{5\,d^4} - \frac{2\,(Bb^4c^5g^4 - 5\,Bab^3c^4dg^4 + 10\,Ba^2b^2c^3d^2g^4 - 10\,Ba^3bc^2d^3g^4 + 5\,Ba^4cd^4g^4)\log(-dx-c)}{5\,d^5}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
1/5*A*b^4*g^4*x^5 + 2/5*B*a^5*g^4*log(b*x + a)/b - 1/10*(B*b^4*c*g^4 - 10*
A*a*b^3*d*g^4 - B*a*b^3*d*g^4)*x^4/d + 2/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d
*g^4 + 15*A*a^2*b^2*d^2*g^4 + 4*B*a^2*b^2*d^2*g^4)*x^3/d^2 + 1/5*(B*b^4*g^
4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*
B*a^4*g^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) - 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 - 10
*A*a^3*b*d^3*g^4 - 6*B*a^3*b*d^3*g^4)*x^2/d^3 + 1/5*(2*B*b^4*c^4*g^4 - 10*
B*a*b^3*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 + 5*A*
a^4*d^4*g^4 + 8*B*a^4*d^4*g^4)*x/d^4 - 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c^4*
d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^4*
g^4)*log(-d*x - c)/d^5
```

3.119. $\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

3.119.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 1025, normalized size of antiderivative = 5.63

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= x^2\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\cdots}{d}\cdots}{10\,b\,d}\right.$$

$$\left.+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}-\frac{a\,c\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{15\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+4\,B\,a\,d-4\,B\,b\,c)}{d}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\cdots}{d}\cdots}{5\,b\,d}\cdots}{5\,b\,d\cdots}$$

$$\frac{a\,c\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)}{b\,d}$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c)
)/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a
*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))
/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d - (a*c*(
(b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a
*d + 5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*
a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c)
)/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(3*d) +
(A*a*b^3*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c + 4*B*a*d - 4*B*b
*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*
c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d
+ 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d
 + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d
 - 2*B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/
(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3
*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c
+ 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(a + b*x)^2)
/(c + d*x)^2)*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b
^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c +...
```

3.119. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.120 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

3.120.1 Optimal result

Integrand size = 32, antiderivative size = 151

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = -\frac{B(bc-ad)^3g^3x}{2d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2}-\frac{B(bc-ad)g^3(a+bx)^3}{6bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b}+\frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4}$$

output
```
-1/2*B*(-a*d+b*c)^3*g^3*x/d^3+1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/6*B
*(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c
)^2))/b+1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

3.120.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B(bc-ad)(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx))}{3d^4}\right)}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(b*c - a*d
)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b
*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4)))/(4*b)
```

3.120.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{B(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{2bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*b) - (B*(b*c
 - a*d)*g^3*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) +
 (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(2*b)
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.120.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(141) = 282$.

Time = 0.88 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx+a)^4 g^3 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{4b}+\frac{g^3 b^3 A x^4}{4}+g^3 b^2 A a x^3+\frac{g^3 b^2 B a x^3}{6}-\frac{g^3 b^3 B c x^3}{6d}+\frac{3 g^3 b A a^2 x^2}{2}+\frac{3 g^3 b B a^2}{4}$
parallelrisch	$\frac{-24 B \ln(bx+a) a^3 b c d^3 g^3+12 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right) a^3 b c d^3 g^3-18 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right) a^2 b^2 c^2 d^2 g^3+12 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right) a b^3 c^3 d g^3}{}$
parts	$\frac{A g^3 (bx+a)^4}{4b}-\frac{B g^3\left(\left(-\frac{(dx+c)^4 \ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{(ad-cb)^3 \ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^4}-\frac{(-ad+cb)(d}{2b^2}\right.\right.\right.}{}$
derivativedivides	$-\frac{A g^3\left(-\frac{b^3(dx+c)^4}{4}-\left(a^3 d^3-3a^2 b c d^2+3 a b^2 c^2 d-b^3 c^3\right)(dx+c)-\frac{3b\left(a^2 d^2-2abcd+b^2 c^2\right)(dx+c)^2}{2}-b^2(ad-cb)(dx+c)^3\right)}{d^3}+\frac{B g^3 (}{}$
default	$-\frac{A g^3\left(-\frac{b^3(dx+c)^4}{4}-\left(a^3 d^3-3a^2 b c d^2+3 a b^2 c^2 d-b^3 c^3\right)(dx+c)-\frac{3b\left(a^2 d^2-2abcd+b^2 c^2\right)(dx+c)^2}{2}-b^2(ad-cb)(dx+c)^3\right)}{d^3}+\frac{B g^3 (}{}$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x+a)^4*g^3*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)+1/4*g^3*b^3*A*x^4+g^3*b^2*
A*a*x^3+1/6*g^3*b^2*B*a*x^3-1/6*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2+3/4*
g^3*b*B*a^2*x^2-g^3*b^2/d*B*a*c*x^2+1/4*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*x+
1/2*g^3/b*B*ln(d*x+c)*a^4-2*g^3/d*B*ln(d*x+c)*a^3*c+3*g^3*b/d^2*B*ln(d*x+c
)*a^2*c^2-2*g^3*b^2/d^3*B*ln(d*x+c)*a*c^3+1/2*g^3*b^3/d^4*B*ln(d*x+c)*c^4+
3/2*g^3*B*a^3*x-3*g^3*b/d*B*a^2*c*x+2*g^3*b^2/d^2*B*a*c^2*x-1/2*g^3*b^3/d^
3*B*c^3*x
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.26

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$=\frac{3\,Ab^4d^4g^3x^4+6\,Ba^4d^4g^3\log(bx+a)-2\,(Bb^4cd^3-(6\,A+B)ab^3d^4)g^3x^3+3\,(Bb^4c^2d^2-4\,Bab^3cd^3+3\,(}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
1/12*(3*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3
- (6*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(2
*A + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^
2*b^2*c*d^3 - (2*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*
d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4
*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4
*g^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b
*d^4)
```

3.120.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(131) = 262$.

Time = 2.04 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.68

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \frac{Ab^3g^3x^4}{4} + \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{2b}$$

$$- \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-bc)}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g}\right.}{2d^4}$$

$$+ x^3\left(Aab^2g^3+\frac{Bab^2g^3}{6}-\frac{Bb^3cg^3}{6d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}+\frac{3Ba^2bg^3}{4}-\frac{Bab^2cg^3}{d}+\frac{Bb^3c^2g^3}{4d^2}\right)$$

$$+ x\left(Aa^3g^3+\frac{3Ba^3g^3}{2}-\frac{3Ba^2bcg^3}{d}+\frac{2Bab^2c^2g^3}{d^2}-\frac{Bb^3c^3g^3}{2d^3}\right)$$

$$+ \left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*
d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b*
*3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c
**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) - B*c*g*
*3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c
*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b
**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d
)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**
3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*
g**3 + B*a*b**2*g**3/6 - B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 +
3*B*a**2*b*g**3/4 - B*a*b**2*c*g**3/d + B*b**3*c**2*g**3/(4*d**2)) + x*(A*
a**3*g**3 + 3*B*a**3*g**3/2 - 3*B*a**2*b*c*g**3/d + 2*B*a*b**2*c**2*g**3/d
**2 - B*b**3*c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2
 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**
2)
```

3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(141) = 282.

Time = 0.22 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.28

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{4}\,Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}\,Aa^2bg^3x^2$$
$$+ \left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx-$$
$$+ \frac{3}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\log$$
$$+ \left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log(d}{d^3}$$
$$+ \frac{1}{12}\left(3\,x^4\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{6\,a^4\log(bx+a)}{b^4} + \frac{6\,c^4\,l$$
$$+ Aa^3g^3x$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b^2*e*x
^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e
/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a
^3*g^3 + 3/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x +
 a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x
^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x
+ c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3
*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b
^2*d^2))*B*a*b^2*g^3 + 1/12*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2)
 + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
- 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*
d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*B*b^3*g^3 + A*a^3*g^3*x
```

3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(141) = 282$.

Time = 10.23 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.35

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{1}{4}Ab^3g^3x^4 + \frac{Ba^4g^3\log(bx+a)}{2b} - \frac{(Bb^3cg^3 - 6Aab^2dg^3 - Bab^2dg^3)x^3}{6d}$$
$$+ \frac{1}{4}\left(Bb^3g^3x^4 + 4Bab^2g^3x^3 + 6Ba^2bg^3x^2 + 4Ba^3g^3x\right)\log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)$$
$$+ \frac{(Bb^3c^2g^3 - 4Bab^2cdg^3 + 6Aa^2bd^2g^3 + 3Ba^2bd^2g^3)x^2}{4d^2}$$
$$- \frac{(Bb^3c^3g^3 - 4Bab^2c^2dg^3 + 6Ba^2bcd^2g^3 - 2Aa^3d^3g^3 - 3Ba^3d^3g^3)x}{2d^3}$$
$$+ \frac{(Bb^3c^4g^3 - 4Bab^2c^3dg^3 + 6Ba^2bc^2d^2g^3 - 4Ba^3cd^3g^3)\log(dx+c)}{2d^4}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
1/4*A*b^3*g^3*x^4 + 1/2*B*a^4*g^3*log(b*x + a)/b - 1/6*(B*b^3*c*g^3 - 6*A*
a*b^2*d*g^3 - B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*B*a*b^2*g^3*x^
3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3 + 6*A
*a^2*b*d^2*g^3 + 3*B*a^2*b*d^2*g^3)*x^2/d^2 - 1/2*(B*b^3*c^3*g^3 - 4*B*a*b
^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 - 2*A*a^3*d^3*g^3 - 3*B*a^3*d^3*g^3)*x/
d^3 + 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4
*B*a^3*c*d^3*g^3)*log(d*x + c)/d^4
```

3.120. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.120.9 Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.75

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+x\left(\frac{(2\,a\,d+2\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\right.}{2\,b\,d}\right.$$

$$\left.+\frac{a^2\,g^3\,(4\,A\,a\,d+6\,A\,b\,c+3\,B\,a\,d-3\,B\,b\,c)}{d}-\frac{a\,c\,\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)}{b\,d}\right)$$

$$+x^3\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{6\,d}\right)$$

$$+\frac{\ln(c+d\,x)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{2\,d^4}$$

$$+\frac{A\,b^3\,g^3\,x^4}{4}+\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{2\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a
^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c +
B*a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c)
)/(4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g
^3)/(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d -
B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d
) - (2*a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d)
)/(2*b*d) + (a^2*g^3*(4*A*a*d + 6*A*b*c + 3*B*a*d - 3*B*b*c))/d - (a*c*((b
^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*
b*c))/(2*d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/
(6*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) + (log(c + d*x)*(B*b^3*c^4*g^3
- 4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4
) + (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(2*b)
```

3.120. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.121 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

3.121.1 Optimal result

Integrand size = 32, antiderivative size = 120

$$\begin{aligned}
&\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx \\
&= \frac{2B(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{3bd} \\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b} - \frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}
\end{aligned}$$

output
```
2/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b
*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)
/b/d^3
```

3.121.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\begin{aligned}
&\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx \\
&= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) + \frac{B(-bc+ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{d^3}\right)}{3b}
\end{aligned}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(-(b*c) +
a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c +
 d*x]))/d^3))/(3*b)
```

3.121.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2B(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2Bg^2(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2Bg^2(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2Bg^2(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b) - (2*B*(b
*c - a*d)*g^2*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)
^2*Log[c + d*x])/d^3))/(3*b)
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.121.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(bx+a)^3 g^2 B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3b}+\frac{g^2b^2Ax^3}{3}+g^2bAax^2+\frac{g^2bBax^2}{3}-\frac{g^2b^2Bcx^2}{3d}+g^2Aa^2x+\frac{2g^2B\ln(dx+}{3b}$
parts	$\frac{Ag^2(bx+a)^3}{3b}-\frac{Bg^2\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)\right.\right.}{}$
parallelrisch	$\frac{-12B\ln(bx+a)a^2bcd^2g^2+12B\ln(bx+a)ab^2c^2dg^2+6Bx^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab^2d^3g^2+6Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^2bd^3g^2+10Ba}{}$
derivativedivides	$-\frac{Ag^2\left(-\left(a^2d^2-2abcd+b^2c^2\right)(dx+c)-\frac{b^2(dx+c)^3}{3}-b(ad-cb)(dx+c)^2\right)}{d^2}+\frac{Bg^2\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2\right.\right.}{}$
default	$-\frac{Ag^2\left(-\left(a^2d^2-2abcd+b^2c^2\right)(dx+c)-\frac{b^2(dx+c)^3}{3}-b(ad-cb)(dx+c)^2\right)}{d^2}+\frac{Bg^2\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2\right.\right.}{}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3*g^2*B/b*ln(e*(b*x+a)^2/(d*x+c)^2)+1/3*g^2*b^2*A*x^3+g^2*b*A*
a*x^2+1/3*g^2*b*B*a*x^2-1/3*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x+2/3*g^2/b*B*ln(d
*x+c)*a^3-2*g^2/d*B*ln(d*x+c)*a^2*c+2*g^2*b/d^2*B*ln(d*x+c)*a*c^2-2/3*g^2*
b^2/d^3*B*ln(d*x+c)*c^3+4/3*g^2*B*a^2*x-2*g^2*b/d*B*a*c*x+2/3*g^2*b^2/d^2*
B*c^2*x
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.02

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$=\frac{Ab^3d^3g^2x^3+2\,Ba^3d^3g^2\log(bx+a)-(Bb^3cd^2-(3\,A+B)ab^2d^3)g^2x^2+(2\,Bb^3c^2d-6\,Bab^2cd^2+(3\,A+}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
1/3*(A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (3*
A + B)*a*b^2*d^3)*g^2*x^2 + (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + (3*A + 4*B)
*a^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*
log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*
d^3)
```

3.121.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(107) = 214$.

Time = 1.39 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.31

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \frac{Ab^2g^2x^3}{3} + \frac{2Ba^3g^2\log\left(x+\frac{\frac{2Ba^4d^3g^2}{b}+6Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right)}{3b}$$

$$- \frac{2Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{8Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2-2Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{2Bbc^2g^2\cdot(3a^2}{}}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right.}{3d^3}$$

$$+ x^2\left(Aabg^2+\frac{Babg^2}{3}-\frac{Bb^2cg^2}{3d}\right)+x\left(Aa^2g^2+\frac{4Ba^2g^2}{3}-\frac{2Babcg^2}{d}+\frac{2Bb^2c^2g^2}{3d^2}\right)$$

$$+ \left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*b**2*g**2*x**3/3 + 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**
3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d
**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3
*g**2))/(3*b) - 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (
8*B*a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B
*a*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*
d**2 - 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g
**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b
*g**2 + B*a*b*g**2/3 - B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 + 4*B*a**2*g*
*2/3 - 2*B*a*b*c*g**2/d + 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x +
B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3)*log(e*(a + b*x)**2/(c + d*x)**2)
```

3.121.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(112) = 224$.

Time = 0.36 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.64

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{3}\,Ab^2g^2x^3 + Aabg^2x^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx-\right.}{d}\right.$$
$$+\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log\left(bx+a\right)}{b^2}+\frac{2\,c^2\log\left(d\right.}{d^2}\right.$$
$$+\frac{1}{3}\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log}{d}\right.$$
$$+Aa^2g^2x$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^
2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b^2*e*
x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*
e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)
/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2
+ 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 +
2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2
*g^2*x
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(112) = 224$.

Time = 1.81 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.05

$$\begin{aligned}
&\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx\\
&=\frac{1}{3}Ab^2g^2x^3+\frac{2Ba^3g^2\log(bx+a)}{3b}-\frac{(Bb^2cg^2-3Aabdg^2-Babdg^2)x^2}{3d}\\
&\quad+\frac{1}{3}\left(Bb^2g^2x^3+3Babg^2x^2+3Ba^2g^2x\right)\log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)\\
&\quad+\frac{(2Bb^2c^2g^2-6Babcdg^2+3Aa^2d^2g^2+4Ba^2d^2g^2)x}{3d^2}\\
&\quad-\frac{2(Bb^2c^3g^2-3Babc^2dg^2+3Ba^2cd^2g^2)\log(-dx-c)}{3d^3}
\end{aligned}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
1/3*A*b^2*g^2*x^3 + 2/3*B*a^3*g^2*log(b*x + a)/b - 1/3*(B*b^2*c*g^2 - 3*A*
a*b*d*g^2 - B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*b*g^2*x^2 + 3*
B*a^2*g^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 + 3*A*a^2*d^2*g^2 + 4*B*a^2*d^2
*g^2)*x/d^2 - 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g^2)*
log(-d*x - c)/d^3
```

3.121.9 Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.47

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right) - x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d} - \frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right) + \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right) - \frac{\ln(c+d\,x)\,(6\,B\,a^2\,c\,d^2\,g^2-6\,B\,a\,b\,c^2\,d\,g^2+2\,B\,b^2\,c^3\,g^2)}{3\,d^3} + \frac{A\,b^2\,g^2\,x^3}{3}+\frac{2\,B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
x^2*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/(6*d) - (A*b*g^2*(3*a
*d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B
*a*d - 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^
2*(3*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(
a + b*x)^2)/(c + d*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2)
 - (log(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2)
)/(3*d^3) + (A*b^2*g^2*x^3)/3 + (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

3.122 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.122.1 Optimal result

Integrand size = 30, antiderivative size = 78

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=-\frac{B(bc-ad)gx}{d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}$$

output
```
-B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b+B*(-
a*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

3.122.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{2B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(-(b*c) +
a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

3.122.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}-\frac{B(bc-ad)\int\frac{g^2(a+bx)}{c+dx}dx}{bg}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\int\frac{a+bx}{c+dx}dx}{b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\int\left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(g*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*b) - (B*(b*c -
 a*d)*g*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/b
```

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.122.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.38

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{2gB\ln(dx+c)ac}{d}+\frac{gbB\ln(dx+c)c^2}{d^2}+\frac{B\,a^2g\ln(-bx-a)}{b}+$
parallelrisch	$B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab\,d^2g+2Axab\,d^2g+2B\ln(bx+a)a^2d^2g-4B\ln(bx+a)abcdg+$
parts	$Ag(\frac{1}{2}b\,x^2+ax)-$ $Bg\Bigg(\Bigg(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-ad+cb)\Bigg(\frac{(ad-cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}-\frac{dx+c}{b}+$
derivativedivides	$-\frac{Ag\left(-\frac{b(dx+c)^2}{2}-(ad-cb)(dx+c)\right)}{d}+$ $Bg\Bigg(\Bigg(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-ad+cb)\Bigg(\frac{(ad-cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}-$
default	$-\frac{Ag\left(-\frac{b(dx+c)^2}{2}-(ad-cb)(dx+c)\right)}{d}+$ $Bg\Bigg(\Bigg(-\frac{(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2}-(-ad+cb)\Bigg(\frac{(ad-cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b^2}-$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*B*x*(b*x+2*a)*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*g*b*A*x^2+g*A*a*x-2*g/d*
B*ln(d*x+c)*a*c+g*b/d^2*B*ln(d*x+c)*c^2+B*a^2*g/b*ln(-b*x-a)+g*B*a*x-g*b/d
*B*c*x
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$=\frac{Ab^2d^2gx^2+2\,Ba^2d^2g\log(bx+a)-2\,(Bb^2cd-(A+B)abd^2)gx+2\,(Bb^2c^2-2\,Babcd)g\log(dx+c)+(}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + 2*B*a^2*d^2*g*log(b*x + a) - 2*(B*b^2*c*d - (A + B)
*a*b*d^2)*g*x + 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*
x^2 + 2*B*a*b*d^2*g*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2)))/(b*d^2)
```

3.122.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(68) = 136$.

Time = 0.84 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag + bgx)\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$$
$$= \frac{Abgx^2}{2} + \frac{Ba^2g\log\left(x + \frac{\frac{Ba^3d^2g}{b} + 2Ba^2cdg - Babc^2g}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{b}$$
$$- \frac{Bcg(2ad-bc)\log\left(x + \frac{3Ba^2cdg - Babc^2g - Bacg(2ad-bc) + \frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{d^2}$$
$$+ x\left(Aag + Bag - \frac{Bbcg}{d}\right) + \left(Bagx + \frac{Bbgx^2}{2}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*
c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b - B*c*g*(2*a*d
- b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*
b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/d**2 + x*(A*a*g + B*a*g - B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(a
+ b*x)**2/(c + d*x)**2)
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{2}\,Abgx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx\right.}{d}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log\left(bx+a\right)}{b^2}+\frac{2\,c^2\log}{c}\right.$$
$$+\,Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxim
a")
```

output
```
1/2*A*b*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x +
a)/b - 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
 + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)
*x/(b*d))*B*b*g + A*a*g*x
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.65

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{1}{2}\,Abgx^2+\frac{Ba^2g\log\left(bx+a\right)}{b}+\frac{1}{2}\left(Bbgx^2+2\,Bagx\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)$$
$$-\frac{(Bbcg-Aadg-Badg)x}{d}+\frac{(Bbc^2g-2\,Bacdg)\log\left(dx+c\right)}{d^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac"
)
```

output
```
1/2*A*b*g*x^2 + B*a^2*g*log(b*x + a)/b + 1/2*(B*b*g*x^2 + 2*B*a*g*x)*log((
b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) - (B*b*c*g - A*a
*d*g - B*a*d*g)*x/d + (B*b*c^2*g - 2*B*a*c*d*g)*log(d*x + c)/d^2
```

3.122.9 Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.54

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = x\left(\frac{g\,(2\,A\,a\,d+A\,b\,c+B\,a\,d-B\,b\,c)}{d} - \frac{A\,g\,(a\,d+b\,c)}{d}\right) + \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right) + \frac{A\,b\,g\,x^2}{2} + \frac{B\,a^2\,g\,\ln(a+b\,x)}{b} - \frac{B\,c\,g\,\ln(c+d\,x)\,(2\,a\,d-b\,c)}{d^2}$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
x*((g*(2*A*a*d + A*b*c + B*a*d - B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e
*(a + b*x)^2)/(c + d*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 + (B*
a^2*g*log(a + b*x))/b - (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

3.123 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx$

3.123.1 Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg} + \frac{2B\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g+2*B*polylog(2
,1+(-a*d+b*c)/d/(b*x+a))/b/g
```

3.123.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx = \frac{\log(a+bx)\left(A-B\log(a+bx)+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)+2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bg}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

output
```
(Log[a + b*x]*(A - B*Log[a + b*x] + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 2
*B*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)])/(b*g)
```

3.123.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{ag+bgx}dx$$

$$\Big\downarrow \text{2942}$$

$$\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$-\frac{2B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$-\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2752}$$

$$\frac{2B\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]))/(b*g)) + (2*B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

3.123.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(82) = 164$.

Time = 0.70 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

method	result
parts	$\frac{A\ln(bx+a)}{gb}+\frac{B\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\right)}{b}-\cdots\right)}{g}$
derivativedivides	$-\frac{\frac{dA\left(\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)}{g}+dB\left(\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\cdots\right)}{b}\cdots\right)}{d}$
default	$-\frac{\frac{dA\left(\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}+\frac{\ln\left(\frac{1}{dx+c}\right)}{b}\right)}{g}+dB\left(\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\cdots\right)}{b}\cdots\right)}{d}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{gb}+\frac{2B\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)ad}{gb(ad-cb)}-\frac{2B\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)c}{g(ad-cb)}+2B\cdots$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
A/g*ln(b*x+a)/b+B/g*(-(ln(1/(d*x+c))*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^
2)-(2*a*d-2*b*c)*(dilog(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)+ln(1/(d*x+c))*l
n(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)))/b-(ln((a*d-b*c)/(d*x+c)+b)/(a*d-b*c
)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/(a*d-b*c)*ln((a*d-b*c)/(d*x+c)
+b)^2)*(-a*d+b*c)/b)
```

3.123.5 Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="frica
s")
```

output
```
integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
 + A)/(b*g*x + a*g), x)
```

3.123.6 Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x
**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*
x + d**2*x**2))/(a + b*x), x))/g
```

3.123.7 Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="maxim
a")
```

output
```
-B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x*log(e) + b*c*log(
e) + 2*(2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c
*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

3.123.8 Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(b*g*x + a*g), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x), x)
```

3.124 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx$

3.124.1 Optimal result

Integrand size = 32, antiderivative size = 65

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx=-\frac{2B}{bg^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-2*B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)/g^2/
(b*x+a)
```

3.124.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx=-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{2B(bc-ad)\left(-\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

output
```
-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^2*(a + b*x))) + (2*B*(b*c
- a*d)*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d
*Log[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

3.124.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2950, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{a+bx}-\frac{2B(c+dx)}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

output
```
((-2*B*(c + d*x))/(a + b*x) - ((c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(a + b*x))/((b*c - a*d)*g^2)
```

3.124.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.124.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

method	result
norman	$\frac{\frac{(A+2B)x}{ga}+\frac{cB\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(ad-cb)g}+\frac{Bdx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parts	$-\frac{A}{g^2(bx+a)b}+\frac{\frac{2Bx}{ag}+\frac{cB\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(ad-cb)g}+\frac{Bdx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{-2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3d^2-2B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3cd+2Aa\,b^2d^2-2A\,b^3cd+4Ba\,b^2d^2-4B\,b^3cd}{2g^2(bx+a)b^3d(ad-cb)}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{b\,g^2(bx+a)}-\frac{-2B\ln(-bx-a)bdx+2B\ln(dx+c)bdx-2B\ln(-bx-a)ad+2B\ln(dx+c)ad+Aad-Abc+2Bad-2}{g^2(bx+a)b(ad-cb)}$
derivativedivides	$-\frac{-\frac{d^2A}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{2d^2B}{bg(dx+c)}-\frac{d^2B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$
default	$-\frac{-\frac{d^2A}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{2d^2B}{bg(dx+c)}-\frac{d^2B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A+2*B)/g/a*x+c*B/(a*d-b*c)/g*ln(e*(b*x+a)^2/(d*x+c)^2)+1/g*B*d/(a*d-b*c)
*x*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(b*x+a)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{(A+2\,B)bc-(A+2\,B)ad+(Bbdx+Bbc)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="fri
cas")
```

output
```
-((A + 2*B)*b*c - (A + 2*B)*a*d + (B*b*d*x + B*b*c)*log((b^2*e*x^2 + 2*a*b
*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^
2*c - a^2*b*d)*g^2)
```

3.124.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(54) = 108$.

Time = 0.65 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.92

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2+b^2g^2x} - \frac{2Bd\log\left(x+\frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)} + \frac{2Bd\log\left(x+\frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)} + \frac{-A-2B}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**2,x)
```

output
```
-B*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) - 2*B*d*log(x
 + (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 -
 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c
)) + 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c)
 + 2*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*
g**2*(a*d - b*c)) + (-A - 2*B)/(a*b*g**2 + b**2*g**2*x)
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^2g^2x+abg^2} + \frac{2}{b^2g^2x+abg^2} + \frac{2\,d\log(bx+a)}{(b^2c-abd)g^2} - \frac{2\,d\log(dx+}{(b^2c-abd)g}\right. - \frac{A}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="max
ima")
```

output
```
-B*(log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x
 + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) + 2/(b^2*
g^2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*log(d*x +
c)/((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(65) = 130.

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = \left(2\left(b^2cg^2 - abdg^2\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag} - \frac{adg}{bgx+ag} + d\right|\right)}{b^4c^2g^4 - 2\,ab^3cdg^4 + a^2b^2d^2g^4} - \frac{1}{(b^2cg^2 - abdg^2)(bgx+ag)bg}\right) - \frac{\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)}{(bgx+ag)bg}\right) - \frac{A}{(bgx+ag)bg}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="gia
c")
```

output
```
(2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x +
 a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c*
g^2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) - log((b*x + a)^2*e/(d*x + c)^2)/((b*
g*x + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

3.124.9 Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx=-\frac{A+2\,B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^2,x)
```

output
```
- (A + 2*B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(
b^2*g^2*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(
b*g^2*(a*d - b*c))
```

3.125 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$

3.125.1 Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{B}{2bg^3(a+bx)^2}+\frac{Bd}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}$$

output
```
-1/2*B/b/g^3/(b*x+a)^2+B*d/b/(-a*d+b*c)/g^3/(b*x+a)+B*d^2*ln(b*x+a)/b/(-a*
d+b*c)^2/g^3+1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^3/(b*x+a)^2-B*d^2*ln
(d*x+c)/b/(-a*d+b*c)^2/g^3
```

3.125.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+\frac{B\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{2bg^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + (B*((b*c - a*d)*(-3*a*d + b
*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c +
 d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

3.125.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{bg}-\frac{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{bg^3}-\frac{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{B(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{bg^3}-\frac{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{bg^3}-\frac{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^3*(a + b*x)^2) + (B*(b*
c - a*d)*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) +
 (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3))/(b*
g^3)
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.125.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

method	result
parallelrisch	$-\frac{2Bxab^4d^3-2Bxb^5cd^2-Bx^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^5d^3+Aa^2b^3d^3+Ab^5c^2d+3Ba^2b^3d^3+Bb^5c^2d-2Aab^4cd^2-4Bab^4cd^2}{2g^3(bx+a)^2(a^2d^2-2abcd+b^2c^2)b^4d}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2bg^3(bx+a)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)abd^2x-4B\ln(-bx-a)abd^2x+2B\ln(}{2(a^2d^2-2}$
parts	$-\frac{A}{2g^3(bx+a)^2b}+\frac{\frac{(2Bad-Bbc)x}{ag(ad-cb)}+\frac{Bad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(a^2d^2-2abcd+b^2c^2)g}+\frac{Bc(2ad-cb)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2g(a^2d^2-2abcd+b^2c^2)}+\frac{(3Bad-Bbc)bx^2}{2ga^2(ad-cb)}+\frac{Bd^2bx^2\ln\left(\frac{e(b}{(d}\right.}{2(a^2d^2-2abcd+}}{g^2(bx+a)^2}$
norman	$\frac{\frac{(Aad-Abc+2Bad-Bbc)x}{ag(ad-cb)}+\frac{Bad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(a^2d^2-2abcd+b^2c^2)g}+\frac{Bc(2ad-cb)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2g(a^2d^2-2abcd+b^2c^2)}+\frac{(Aad-Abc+3Bad-Bbc)bx^2}{2a^2g(ad-cb)}+\frac{Bd^2bx^2\ln\left(\frac{e(b}{(d}\right.}{2(a^2d^2-2abcd+}}{g^2(bx+a)^2}$
derivativedivides	$-\frac{\frac{d^3A\left(\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^3}+\frac{\frac{Bd^3}{g(ad-cb)(dx+c)}+\frac{3Bd^3}{2bg(dx+c)^2}-\frac{bBd^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{b}{d}\right.}{d^2}\right.}{2g(a^2d^2-2abcd+}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+\right.}}{d}$
default	$-\frac{\frac{d^3A\left(\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^3}+\frac{\frac{Bd^3}{g(ad-cb)(dx+c)}+\frac{3Bd^3}{2bg(dx+c)^2}-\frac{bBd^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{b}{d}\right.}{d^2}\right.}{2g(a^2d^2-2abcd+}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+\right.}}{d}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*B*x*a*b^4*d^3-2*B*x*b^5*c*d^2-B*x^2*ln(e*(b*x+a)^2/(d*x+c)^2)*b^5*
d^3+A*a^2*b^3*d^3+A*b^5*c^2*d+3*B*a^2*b^3*d^3+B*b^5*c^2*d-2*A*a*b^4*c*d^2-
4*B*a*b^4*c*d^2+B*ln(e*(b*x+a)^2/(d*x+c)^2)*b^5*c^2*d-2*B*x*ln(e*(b*x+a)^2
/(d*x+c)^2)*a*b^4*d^3-2*B*ln(e*(b*x+a)^2/(d*x+c)^2)*a*b^4*c*d^2)/g^3/(b*x+
a)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/d
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{(A+B)b^2c^2-2\,(A+2\,B)abcd+(A+3\,B)a^2d^2-2\,(Bb^2cd-Babd^2)x-(Bb^2d^2x^2+2\,Babd^2x-Bb^2}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="fri
cas")
```

output
```
-1/2*((A + B)*b^2*c^2 - 2*(A + 2*B)*a*b*c*d + (A + 3*B)*a^2*d^2 - 2*(B*b^2
*c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b
*c*d)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.125.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(122) = 244$.

Time = 1.03 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.03

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag + bgx)^3}\,dx$$
$$= -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2a^2bg^3 + 4ab^2g^3x + 2b^3g^3x^2} - \frac{Bd^2\log\left(x + \frac{-\frac{Ba^3d^5}{(ad-bc)^2} + \frac{3Ba^2bcd^4}{(ad-bc)^2} - \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 + \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{bg^3(ad-bc)^2} + \frac{Bd^2\log\left(x + \frac{\frac{Ba^3d^5}{(ad-bc)^2} - \frac{3Ba^2bcd^4}{(ad-bc)^2} + \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 - \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{bg^3(ad-bc)^2} + \frac{-Aad + Abc - 3Bad + Bbc - 2Bbdx}{2a^3bdg^3 - 2a^2b^2cg^3 + x^2\cdot(2ab^3dg^3 - 2b^4cg^3) + x\,(4a^2b^2dg^3 - 4ab^3cg^3)}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**3,x)
```

output
```
-B*log(e*(a + b*x)**2/(c + d*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b
**3*g**3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*
c*d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B
*b**3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d -
b*c)**2) + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/
(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c
**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2
) + (-A*a*d + A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a*
*2*b**2*c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d
*g**3 - 4*a*b**3*c*g**3))
```

3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(134) = 268$.

Time = 0.20 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.22

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$$

$$=\frac{1}{2}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{}{d^2\cdots}\right.}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3\cdots}\right.$$

$$-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="max
ima")
```

output
```
1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - log(b^2*e*x^2/(d^2*x^2 + 2*
c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*
d*x + c^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)
/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
- 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^
2*b*g^3)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)} + \frac{2\,Bbdx-Abc-Bbc+Aad+3\,Bad}{2\,(b^4cg^3x^2-ab^3dg^3x^2+2\,ab^3cg^3x-2\,a^2b^2dg^3x+a^2b^2cg^3-a^3bdg^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="gia
c")
```

output
```
B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - B*d^2
*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log(
(b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 +
2*a*b^2*g^3*x + a^2*b*g^3) + 1/2*(2*B*b*d*x - A*b*c - B*b*c + A*a*d + 3*B*
a*d)/(b^4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*
x + a^2*b^2*c*g^3 - a^3*b*d*g^3)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.49

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{\frac{A\,a\,d-A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^3,x)
```

output
```
- ((A*a*d - A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d - b*
c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*log((e*(a + b*x)^2)/(c
+ d*x)^2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (2*B*d^2*atanh((b^3*c^2*g
^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*g^3
*(a*d - b*c)^2)
```

3.125. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$

3.126 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx$

3.126.1 Optimal result

Integrand size = 32, antiderivative size = 177

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{2B}{9bg^4(a+bx)^3}+\frac{Bd}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}$$

output
```
-2/9*B/b/g^4/(b*x+a)^3+1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-2/3*B*d^2/b/(-a*
d+b*c)^2/g^4/(b*x+a)-2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e
*(b*x+a)^2/(d*x+c)^2))/b/g^4/(b*x+a)^3+2/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/
g^4
```

3.126.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+d}{(bc-ad)^3}}{9bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

output
```
-1/9*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(2*(b*c - a*d)^3 - 3
*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*
x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b*g^4
*(a + b*x)^3)
```

3.126.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(ag+bgx)^4}dx$$

$$\Big\downarrow \; 2948$$

$$\frac{2B(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2B(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \; 54$$

$$\frac{2B(bc-ad)\int\left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2B(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^4*(a + b*x)^3) + (2*B*(
b*c - a*d)*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a + b*x
)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d)^4 +
(d^3*Log[c + d*x])/(b*c - a*d)^4))/(3*b*g^4)
```

3.126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.126. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx$

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(168) = 336$.

Time = 1.57 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.14

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-bx-a)b^3d^3x^3+6B\ln(dx+c)b^3d^3x^3-18B\ln(-bx-a)a\,b^2d^3x^2+18B\ln(dx+c)a\,b^2d^3x^2\cdots}{\cdots}$
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-36Bxa\,b^6c\,d^3-18B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a\,b^6d^4-18Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^2b^5d^4-18B\ln\left(\frac{e(b\cdots}{(d\cdots}\right)\cdots}{\cdots}$
derivativedivides	$-\frac{\frac{d^4A\left(-\frac{b^2}{3(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{b}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^4}+\frac{11B\,d^4}{9bg(dx+c)^3}-\frac{b^2B\,d^4\cdots}{3g\left(a^3d\cdots\right.}\cdots}{\cdots}$
default	$-\frac{\frac{d^4A\left(-\frac{b^2}{3(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{b}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^4}+\frac{11B\,d^4}{9bg(dx+c)^3}-\frac{b^2B\,d^4\cdots}{3g\left(a^3d\cdots\right.}\cdots}{\cdots}$
parts	$-\frac{A}{3g^4(bx+a)^3b}+\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{\left(6B\,a^2d^2-6Babcd+2B\,b^2c^2\right)x}{3ga\left(a^2d^2-2abcd+b^2c^2\right)}\cdots}{\cdots}$
norman	$\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{\left(3A\,a^2d^2-6Aabcd+3A\,b^2c^2+6B\,a^2d^2-6Babcd+2B\,b^2\cdots\right.}{3ga\left(a^2d^2-2abcd+b^2c^2\right)}\cdots}{\cdots}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*B/b/g^4/(b*x+a)^3*ln(e*(b*x+a)^2/(d*x+c)^2)-1/9*(-6*B*ln(-b*x-a)*b^3*
d^3*x^3+6*B*ln(d*x+c)*b^3*d^3*x^3-18*B*ln(-b*x-a)*a*b^2*d^3*x^2+18*B*ln(d*
x+c)*a*b^2*d^3*x^2-18*B*ln(-b*x-a)*a^2*b*d^3*x+18*B*ln(d*x+c)*a^2*b*d^3*x+
6*B*a*b^2*d^3*x^2-6*B*b^3*c*d^2*x^2-6*B*ln(-b*x-a)*a^3*d^3+6*B*ln(d*x+c)*a
^3*d^3+15*B*a^2*b*d^3*x-18*B*a*b^2*c*d^2*x+3*B*b^3*c^2*d*x+3*A*a^3*d^3-9*A
*a^2*b*c*d^2+9*A*a*b^2*c^2*d-3*A*b^3*c^3+11*B*a^3*d^3-18*B*a^2*b*c*d^2+9*B
*a*b^2*c^2*d-2*B*c^3*b^3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/g^
4/(b*x+a)^3/b
```

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(165) = 330$.

Time = 0.27 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.43

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{(3A+2B)b^3c^3 - 9(A+B)ab^2c^2d + 9(A+2B)a^2bcd^2 - (3A+11B)a^3d^3 + 6(Bb^3cd^2 - Bab^2d^3)x^2}{9((b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="fri
cas")
```

output
```
-1/9*((3*A + 2*B)*b^3*c^3 - 9*(A + B)*a*b^2*c^2*d + 9*(A + 2*B)*a^2*b*c*d^
2 - (3*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^
3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*l
og((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 -
 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3
*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 -
 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3
*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

3.126.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(162) = 324$.

Time = 1.66 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.82

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3}$$

$$-\frac{2Bd^3\log\left(x + \frac{-\frac{2Ba^4d^7}{(ad-bc)^3} + \frac{8Ba^3bcd^6}{(ad-bc)^3} - \frac{12Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{8Bab^3c^3d^4}{(ad-bc)^3} + 2Bad^4 - \frac{2Bb^4c^4d^3}{(ad-bc)^3} + 2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}$$

$$+\frac{2Bd^3\log\left(x + \frac{\frac{2Ba^4d^7}{(ad-bc)^3} - \frac{8Ba^3bcd^6}{(ad-bc)^3} + \frac{12Ba^2b^2c^2d^5}{(ad-bc)^3} - \frac{8Bab^3c^3d^4}{(ad-bc)^3} + 2Bad^4 + \frac{2Bb^4c^4d^3}{(ad-bc)^3} + 2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}$$

$$+\frac{-3Aa^2d^2 + 6Aabcd - 3Ab^2c^2 - 11Ba^2d^2 + 7Babcd - 2Bb^2c^2 - 6Bb^2}{9a^5bd^2g^4 - 18a^4b^2cdg^4 + 9a^3b^3c^2g^4 + x^3\cdot(9a^2b^4d^2g^4 - 18ab^5cdg^4 + 9b^6c^2g^4) + x^2\cdot(27a^3b^3d^2g^4 - 54a^2}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**4,x)
```

output
```
-B*log(e*(a + b*x)**2/(c + d*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x +
9*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) - 2*B*d**3*log(x + (-2*B*a**4*d**7/
(a*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d*
*5/(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B
*b**4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*
d - b*c)**3) + 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b
*c*d**6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b
**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)
**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d
**2 + 6*A*a*b*c*d - 3*A*b**2*c**2 - 11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**
2*c**2 - 6*B*b**2*d**2*x**2 + x*(-15*B*a*b*d**2 + 3*B*b**2*c*d))/(9*a**5*b
*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*
b**4*d**2*g**4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b*
*3*d**2*g**4 - 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b
**2*d**2*g**4 - 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

3.126.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(165) = 330$.

Time = 0.21 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.71

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{9}\,B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d\right.$$

$$-\frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="max
ima")
```

output
```
-1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
 + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 3
*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x +
c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3
*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*
b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*
b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(165) = 330$.

Time = 0.34 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.69

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{2\,Bd^3\log\left(bx+a\right)}{3\left(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4\right)} + \frac{2\,Bd^3\log\left(dx+c\right)}{3\left(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4\right)} - \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{3\left(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4\right)} - \frac{6\,Bb^2d^2x^2-3\,Bb^2cdx+15\,Babd^2x+3\,Ab^2c^2+2\,Bb^2c^2-6\,Aabcd-7\,\ldots}{9\left(b^6c^2g^4x^3-2\,ab^5cdg^4x^3+a^2b^4d^2g^4x^3+3\,ab^5c^2g^4x^2-6\,a^2b^4cdg^4x^2+3\,a^3b^3d^2g^4x^2+3\,a^2b^4c^2g^4x-6\,\ldots\right)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="gia
c")
```

output
```
-2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2
*g^4 - a^3*b*d^3*g^4) + 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*
d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((b^2*e*x^2 + 2*a*
b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 +
 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*
B*a*b*d^2*x + 3*A*b^2*c^2 + 2*B*b^2*c^2 - 6*A*a*b*c*d - 7*B*a*b*c*d + 3*A*
a^2*d^2 + 11*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d
^2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g
^4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x +
 a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{2\,B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{11\,B\,a^2\,d^2}{9\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{5\,B\,a\,d^2\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{2\,B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{3\,b\,g^4\,(a+b\,x)^3} + \frac{7\,B\,a\,c\,d}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,b\,c\,d\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^4,x)
```

output
```
(2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*
1i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4
*(a*d - b*c)^2*(a + b*x)^3) - (2*B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3
) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(9*b*
g^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(3*g^4*(a*d - b*c)^2*(a + b
*x)^3) - (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a
+ b*x)^2)/(c + d*x)^2))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(9*g^4*(a*d -
b*c)^2*(a + b*x)^3) + (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.127 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$

3.127.1 Optimal result

Integrand size = 32, antiderivative size = 208

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{B}{8bg^5(a+bx)^4}+\frac{Bd}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2}+\frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)} + \frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}-\frac{Bd^4\log(c+dx)}{2b(bc-ad)^4g^5}$$

output
```
-1/8*B/b/g^5/(b*x+a)^4+1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/4*B*d^2/b/(-a*
d+b*c)^2/g^5/(b*x+a)^2+1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/2*B*d^4*ln(b
*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^5/(b*x+a
)^4-1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

3.127.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{6\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B\left(3(bc-ad)^4+4d(-bc+ad)^3(a+bx)+6d^2(bc-ad)^2(a+bx)^2+12d^3(-bc+ad)(a+bx)^3-12d^4(a+bx)^4\log(\right.}{(bc-ad)^4}}{24bg^5(a+bx)^4}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

output
```
-1/24*(6*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(3*(b*c - a*d)^4 +
4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*
(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*d^4*(a +
 b*x)^4*Log[c + d*x]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

3.127.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{2bg}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{2bg^5}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{B(bc-ad)\int\left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+bx)^5}\right.}{2bg^5}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{2bg^5}-$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^5*(a + b*x)^4) + (B*(b*
c - a*d)*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a + b*x)^
3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b*x)) + (
d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5))/(2*b*
g^5)
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.127.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(197) = 394$.

Time = 2.71 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.53

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{4b\,g^5(bx+a)^4}-\frac{-48Bab^3cd^3x^2-72Ba^2b^2cd^3x+24Bab^3c^2d^2x-24Aa^3bcd^3+36Aa^2b^2c^2d^2-24Aab^3c^3d-48\ldots}{\ldots}$
derivativedivides	$-\frac{d^5A\left(-\frac{1}{(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b^3}{4(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^4}-\frac{b^2}{(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{3b}{2(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}\ldots\right.}\ldots\right.}{g^5}$
default	$-\frac{d^5A\left(-\frac{1}{(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b^3}{4(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^4}-\frac{b^2}{(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{3b}{2(ad-cb)^4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}\ldots\right.}\ldots\right.}{g^5}$
parts	$-\frac{A}{4g^5(bx+a)^4b}+\frac{\frac{Ba^3d^4x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)g}+\frac{ad^4Bb^2x^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)g}+\frac{(4Ba^3\ldots}{2ga\ldots}}{\ldots}$
parallelrisch	$\frac{6Bx^4\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^6b^3cd^4+24Bx^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^7b^2cd^4+36Bx^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^8bcd^4+24Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)a^9c\ldots}{\ldots}$
norman	$\frac{\frac{Ba^3d^4x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)g}+\frac{ad^4Bb^2x^3\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)g}+\frac{\left(2Aa^3d^3-6Aa^2bcd^2+6Aab^2\ldots\right.}{2ga\ldots}}{\ldots}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*B/b/g^5/(b*x+a)^4*ln(e*(b*x+a)^2/(d*x+c)^2)-1/24*(-48*B*a*b^3*c*d^3*x
^2-72*B*a^2*b^2*c*d^3*x+24*B*a*b^3*c^2*d^2*x-24*A*a^3*b*c*d^3+36*A*a^2*b^2
*c^2*d^2-24*A*a*b^3*c^3*d-48*B*ln(-b*x-a)*a*b^3*d^4*x^3+48*B*ln(d*x+c)*a*b
^3*d^4*x^3-72*B*ln(-b*x-a)*a^2*b^2*d^4*x^2+72*B*ln(d*x+c)*a^2*b^2*d^4*x^2-
48*B*ln(-b*x-a)*a^3*b*d^4*x+48*B*ln(d*x+c)*a^3*b*d^4*x+12*B*ln(d*x+c)*a^4*
d^4+12*B*a*b^3*d^4*x^3-12*B*b^4*c*d^3*x^3+42*B*a^2*b^2*d^4*x^2+6*B*b^4*c^2
*d^2*x^2+52*B*a^3*b*d^4*x-4*B*b^4*c^3*d*x-12*B*ln(-b*x-a)*a^4*d^4+3*B*b^4*
c^4+36*B*a^2*b^2*c^2*d^2+6*A*a^4*d^4+25*B*a^4*d^4-16*B*a*b^3*c^3*d+6*A*b^4
*c^4-48*B*a^3*b*c*d^3-12*B*ln(-b*x-a)*b^4*d^4*x^4+12*B*ln(d*x+c)*b^4*d^4*x
^4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/g^5/(b
*x+a)^4/b
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(194) = 388$.

Time = 0.29 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{3\,(2\,A+B)b^4c^4 - 8\,(3\,A+2\,B)ab^3c^3d + 36\,(A+B)a^2b^2c^2d^2 - 24\,(A+2\,B)a^3bcd^3 + (6\,A+25\,B)a^4d^4}{24\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4\,(ab^8c^4 - 4\,a^2b}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="fri
cas")
```

output
```
-1/24*(3*(2*A + B)*b^4*c^4 - 8*(3*A + 2*B)*a*b^3*c^3*d + 36*(A + B)*a^2*b^
2*c^2*d^2 - 24*(A + 2*B)*a^3*b*c*d^3 + (6*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*
d^3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*
d^4)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*
a^3*b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2
+ 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*
B*a^3*b*c*d^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^
2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4
*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4
*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c
^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*
g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d
^3 + a^8*b*d^4)*g^5)
```

3.127.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(182) = 364$.

Time = 2.42 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.55

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4} - \frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4} + \frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4} + \frac{-6Aa^3d^3+18Aa^2bcd^2-18Aab^2c^2d+}{24a^7bd^3g^5-72a^6b^2cd^2g^5+72a^5b^3c^2dg^5-24a^4b^4c^3g^5+x^4\cdot(24a^3b^5d^3g^5-72a^2b^6cd^2g^5+72ab^7c^2dg^5-}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**5,x)
```

output
```
-B*log(e*(a + b*x)**2/(c + d*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
 24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4
*log(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 -
 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d -
 b*c)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d*
*4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) +
B*d**4*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)
**4 + 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(
a*d - b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c*
*5*d**4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**
4) + (-6*A*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*
c**3 - 25*B*a**3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3
*c**3 - 12*B*b**3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) +
 x*(-52*B*a**2*b*d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(24*a**7*b*
d**3*g**5 - 72*a**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*
b**4*c**3*g**5 + x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 +
 72*a*b**7*c**2*d*g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5
 - 288*a**3*b**5*c*d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*
g**5) + x**2*(144*a**5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a*
*3*b**5*c**2*d*g**5 - 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g...
```

3.127.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(194) = 388$.

Time = 0.22 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.36

$$\int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{1}{24}\,B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - 23}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\,(a\right.$$
$$- \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="max
ima")
```

output
```
1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d
^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3
*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b
^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d
^3)*g^5) - 6*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2
+ 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4
*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b
*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*
b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(194) = 388$.

Time = 0.66 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.01

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$$
$$= -\frac{Bd^4\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{2\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)}$$
$$+\frac{Bd^3}{2\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}$$
$$-\frac{Bd^2}{4\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2bg^2}$$
$$-\frac{B\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)}{4\,(bgx+ag)^4bg}$$
$$+\frac{Bd}{6\,(bgx+ag)^3(bc-ad)bg^2}-\frac{2\,Ab^3g^3+Bb^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="gia
c")
```

output
```
-1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^
5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*
b*d^4*g^5) + 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g
^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g
 + a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log(b^2*e/(b^2*c^2*g^2/(b*g*x
 + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 +
2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)^4
*b*g) + 1/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 + B
*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

3.127. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$

3.127.9 Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c\cdots}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3\cdots} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)} - \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{b\,g^5\,(a\,d-b\,c)^4}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^5,x)
```

output
```
- ((6*A*a^3*d^3 - 6*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 18*A*a*b^2*c^
2*d - 18*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a*b
^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b
^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*
b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(
4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^
4*atanh((2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*b^2*c
*d^3*g^5)/(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4)
```

3.128 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

3.128.1 Optimal result

Integrand size = 34, antiderivative size = 377

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} \\ &+ \frac{2B(bc-ad)^2g^4(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^2} \\ &- \frac{B(bc-ad)^3g^4(a+bx)^2\left(6A+7B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^3} \\ &+ \frac{2B(bc-ad)^4g^4(a+bx)\left(6A+13B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{15bd^4} \\ &+ \frac{2B(bc-ad)^5g^4\left(6A+25B+6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{15bd^5} \\ &+ \frac{8B^2(bc-ad)^5g^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}\end{aligned}$$

output
```
-1/5*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/5*g^
4*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+2/15*B*(-a*d+b*c)^2*g^4*(b
*x+a)^3*(2*A+B+2*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/15*B*(-a*d+b*c)^3*g^
4*(b*x+a)^2*(6*A+7*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3+2/15*B*(-a*d+b*c
)^4*g^4*(b*x+a)*(6*A+13*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4+2/15*B*(-a*
d+b*c)^5*g^4*(6*A+25*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x
+c))/b/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

3.128.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.39

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{B(bc-ad)\left(12Abd(bc-ad)^3x+12Bd(bc-ad)^3(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-6d^2(bc-ad)^2(a+b\right.}{}}{}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(b*c - a
*d)*(12*A*b*d*(b*c - a*d)^3*x + 12*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a +
 b*x)^2)/(c + d*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a +
b*x)^2)/(c + d*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 12*(b*c - a*d)^4*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c
- a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*
x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c)
 + a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])))/(3*d^5)))/(5*b)
```

3.128.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2950, 2781, 2784, 27, 2784, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2950}$$

$$g^4(bc-ad)^5 \int \frac{(a+bx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$g^4(bc-ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{4B\int \frac{(a+bx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b}\right)$$

$$\Big\downarrow \text{2784}$$

$$g^4(bc-ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{4B\left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\int \frac{2(a+bx)^3 \left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+}{c+}}{4d}\right.}{5b}\right.$$

$$\Big\downarrow \text{27}$$

$$g^4(bc- ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\int \frac{(a+bx)^3\left(2A+B+2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2d}\right.}{5b}\right.$$

$\downarrow$ 2784

$$g^4(bc- ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \int \frac{(a+}{}}{2d}\right.}{5b}\right.$$

$\downarrow$ 2784

3.128. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

$$g^4(bc - ad)^5 \left(\frac{(a+bx)^5 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{5b(c+dx)^5 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{4B \left(\frac{(a+bx)^4 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{4d(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3 \left(2B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + 2A + B\right)}{3d(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+b}{}}{} \right.}{5b} \right.$$

$\downarrow$ 27

$$g^4(bc - ad)^5 \left(\frac{(a+bx)^5 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{5b(c+dx)^5 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{4B \left(\frac{(a+bx)^4 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{4d(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3 \left(2B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + 2A + B\right)}{3d(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+b}{}}{} \right.}{5b} \right.$$

$\downarrow$ 2784

3.128. $\int (ag + bgx)^4 \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

$g^4(bc -$

$ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \right.$

$4B \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \ldots}{\ldots} \right.$

$\downarrow$ 2754

3.128. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

$$g^4(bc - ad)^5 \left(\frac{(a+bx)^5 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{(a+bx}{}}{}\right.}{}\right.$$

$\downarrow$ 2838

3.128. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

$$g^4(bc-ad)^5\left(\frac{(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{4B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4d(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{\frac{(a+bx)^3\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{(a+bx}{}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)^5*g^4*(((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2
)/(5*b*(c + d*x)^5*(b - (d*(a + b*x))/(c + d*x))^5) - (4*B*(((a + b*x)^4*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*d*(c + d*x)^4*(b - (d*(a + b*x
))/(c + d*x))^4) - (((a + b*x)^3*(2*A + B + 2*B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2
*(6*A + 7*B + 6*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*d*(c + d*x)^2*(b -
 (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 13*B + 6*B*Log[(e*(a + b
*x)^2)/(c + d*x)^2]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6
*A + 25*B + 6*B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - (d*(a + b*x))/(b
*(c + d*x))])/d) - (12*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d)
/(3*d))/(2*d)))/(5*b))
```

3.128. $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.128. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

3.128.4 Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.128.5 Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b^2*e*x^2 +
2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A
*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4
)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. 2(362) = 724.

Time = 0.35 (sec) , antiderivative size = 2650, normalized size of antiderivative = 7.03

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^
3*b*g^4*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x +
a)/b - 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2
*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c -
a*d)*x/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^
2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*lo
g(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2
) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(
d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*
x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(
b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(
d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d
^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d
^4))*A*B*b^4*g^4 + A^2*a^4*g^4*x - 2/15*((6*g^4*log(e) + 25*g^4)*b^4*c^5 -
 (30*g^4*log(e) + 113*g^4)*a*b^3*c^4*d + 4*(15*g^4*log(e) + 49*g^4)*a^2...
```

3.128.8 Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^4*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^4\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.129 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

3.129.1 Optimal result

Integrand size = 34, antiderivative size = 319

$$\begin{aligned}\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b}\\ &+\frac{B(bc-ad)^2g^3(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{6bd^2}\\ &-\frac{B(bc-ad)^3g^3(a+bx)\left(3A+5B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3}\\ &-\frac{B(bc-ad)^4g^3\left(3A+11B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^4}\\ &-\frac{2B^2(bc-ad)^4g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/4*g^
3*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+1/6*B*(-a*d+b*c)^2*g^3*(b*
x+a)^2*(3*A+2*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/3*B*(-a*d+b*c)^3*g^
3*(b*x+a)*(3*A+5*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*B*(-a*d+b*c)^4
*g^3*(3*A+11*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d
^4-2*B^2*(-a*d+b*c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

3.129.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.26

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B(bc-ad)\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3d^2(-bc+ad)(a+b\ldots\right.}{\ldots}\right.}{\ldots}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(b*c -
 a*d)*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a +
 b*x)^2)/(c + d*x)^2] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c
+ d*x)^2]) - 12*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(2*b*d*(b*c -
 a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 6*B*(b*c - a*d
)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*(b*c - a*d)^3*((2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

3.129.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2950, 2781, 2784, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2dx$$
$$\Big\downarrow \text{2950}$$
$$g^3(bc-ad)^4\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \; 2781$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}\right)$$

$$\Big\downarrow \; 2784$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+2B+3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right.}{b}$$

$$\Big\downarrow \; 2784$$

$$g^3(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{2(a+\cdots}{\cdots}}{\cdots}}{3d}\right.}{b}$$

$$\Big\downarrow \; 27$$

3.129. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$

$$ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{g^3(bc- B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\int \frac{(a+}{}}{}}{3d}\right.}{b}\right.$$

$\downarrow$ 2784

$$ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{g^3(bc- B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{(a+bx}{}}{}\right.}{b}\right.$$

$\downarrow$ 2754

3.129. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

$$g^3(bc - ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{(a+bx}{}}{}\right.}{}\right.$$

$\downarrow$ 2838

$$g^3(bc - ad)^4 \left(\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{(a+bx}{}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

3.129. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

output
```
(b*c - a*d)^4*g^3*(((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2
)/(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))
/(c + d*x))^3) - (((a + b*x)^2*(3*A + 2*B + 3*B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(
3*A + 5*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(d*(c + d*x)*(b - (d*(a
 + b*x))/(c + d*x))) - (-(((3*A + 11*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6*B*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/d)/d)/d)/(3*d)))/b)
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.129.4 Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.129.5 Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g
^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.129.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(308) = 616$.

Time = 0.33 (sec) , antiderivative size = 1948, normalized size of antiderivative = 6.11

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
 + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c
)/d)*A*B*a^3*g^3 + 3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*
e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*l
og(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a^2*
b*g^3 + 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^
2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)
/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^
2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2
*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d
^3)*x)/(b^3*d^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/3*((3*g^3*log(e) + 11*g^
3)*b^3*c^4 - 2*(6*g^3*log(e) + 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) + 5*g
^3)*a^2*b*c^2*d^2 - 6*(2*g^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d
^4 + 2*(b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*
c*d^3*g^3 + a^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3
*x^4*log(e)^2 - 4*(b^4*c*d^3*g^3*log(e) - (3*g^3*log(e)^2 + g^3*log(e))...
```

3.129.8 Giac [F]

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx = \int (bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int (ag + bgx)^3 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (a\,g + b\,g\,x)^3 \left(A + B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.130 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

3.130.1 Optimal result

Integrand size = 34, antiderivative size = 255

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd} \\ &+ \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} \\ &+ \frac{4B(bc-ad)^2g^2(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^2} \\ &+ \frac{4B(bc-ad)^3g^2\left(A+3B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3bd^3} \\ &+ \frac{8B^2(bc-ad)^3g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}\end{aligned}$$

output
```
-2/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/3*g^
2*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4/3*B*(-a*d+b*c)^2*g^2*(b*
x+a)*(A+B+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2+4/3*B*(-a*d+b*c)^3*g^2*(A+3*B
+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d
+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

3.130.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.17

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + \frac{2B(bc-ad)\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-d^2(a+bx)^2\left(A+B\log\right.\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(b*c -
 a*d)*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x
)^2)/(c + d*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]) - 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b
*x)^2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d
)*Log[c + d*x]) + 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
- Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/
d^3))/(3*b)
```

3.130.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2950, 2781, 2784, 27, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2950}$$

$$g^2(bc-ad)^3 \int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2781}$$

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)$$

$\downarrow$ 2784

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{2(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}\right)$$

$\downarrow$ 27

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(A+B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)}{3b}\right)$$

$\downarrow$ 2784

3.130. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{A+3B}{\cdots}}{\cdots}}{d}\right)}{3b}\right.$$

$$\downarrow \text{2754}$$

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\int \frac{(c+\cdots}{\cdots}}{\cdots}}{\cdots}\right)}{3b}\right.$$

$$\downarrow \text{2838}$$

$$g^2(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(1-\cdots\right)}{\cdots}}{\cdots}\right)}{3b}\right.$$

3.130. $\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)^3*g^2*(((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2
)/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (4*B*(((a + b*x)^2*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x
))/(c + d*x))^2) - (((a + b*x)*(A + B + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A + 3*B + B*Log[(e*(
a + b*x)^2)/(c + d*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*
PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d))/(3*b))
```

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.130.4 Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.130.5 Fricas [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^
2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2
*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.130.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. $2(244) = 488$.

Time = 0.32 (sec) , antiderivative size = 1326, normalized size of antiderivative = 5.20

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*
d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*
x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*
log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c
^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*lo
g(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(b^2*e*x
^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e
/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/
d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b
^2*g^2 + A^2*a^2*g^2*x - 4/3*((g^2*log(e) + 3*g^2)*b^2*c^3 - (3*g^2*log(e)
 + 7*g^2)*a*b*c^2*d + (3*g^2*log(e) + 4*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d
^3 - 8/3*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)
/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (2*b^3*c*
d^2*g^2*log(e) - (3*g^2*log(e)^2 + 2*g^2*log(e))*a*b^2*d^3)*B^2*x^2 + (4*(
g^2*log(e) + g^2)*b^3*c^2*d - 4*(3*g^2*log(e) + 2*g^2)*a*b^2*c*d^2 + (3*g^
2*log(e)^2 + 8*g^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x
^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*lo
g(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^
2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B...
```

3.130.8 Giac [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.131 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$

3.131.1 Optimal result

Integrand size = 32, antiderivative size = 188

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx = -\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - \frac{2B(bc-ad)^2g\left(A+2B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd^2} - \frac{4B^2(bc-ad)^2g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

output
```
-2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/2*g*(b*x+a
)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b-2*B*(-a*d+b*c)^2*g*(A+2*B+B*ln(e*(
b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-4*B^2*(-a*d+b*c)^2*g*p
olylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

3.131.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-\frac{4B(bc-ad)\left(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-2B(bc-ad)\log(c+dx)-(bc-ad)\left(A+B\log\right.\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(b*c - a
*d)*(A*b*d*x + B*d*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] - 2*B*(b*c -
 a*d)*Log[c + d*x] - (b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*
Log[c + d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(
2*b)
```

3.131.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2950, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2dx$$
$$\Big\downarrow \text{2950}$$
$$g(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{2781}$$

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)$$

$\downarrow$ 2784

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+2B+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)$$

$\downarrow$ 2754

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{2B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\cdots\right.}{\cdots}}{d}\cdots\right.}{b}\cdots\right.$$

$\downarrow$ 2838

$$g(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+2B\right)}{d}\cdots}{d}\cdots\right.}{b}\cdots\right.$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

3.131. $\int(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$

output
```
(b*c - a*d)^2*g*(((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/
(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (2*B*(((a + b*x)*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c +
d*x))) - (-(((A + 2*B + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - (d*(a
+ b*x))/(b*(c + d*x))])/d) - (2*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
/d)/d))/b)
```

3.131.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.131. $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

3.131.4 Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.131.5 Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)\left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*e*x^2 + 2*a*
b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log(
(b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.131.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(185) = 370$.

Time = 0.31 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.87

$$\int (ag + bgx)\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{1}{2}A^2bgx^2$$
$$+2\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx}{d}\right.$$
$$+\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\log(d}{d^2}\right.$$
$$+A^2agx + \frac{2\left((g\log(e)+2\,g)bc^2 - 2\,(g\log(e)+g)acd\right)B^2\log(dx+c)}{d^2}$$
$$+\frac{4\,(b^2c^2g - 2\,abcdg + a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}$$
$$+\frac{B^2b^2d^2gx^2\log(e)^2 - 2\left(2\,b^2cdg\log(e) - \left(g\log(e)^2 + 2\,g\log(e)\right)abd^2\right)B^2x + 4\,(B^2b^2d^2gx^2 + 2\,B^2abd^2g}{}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="max
ima")
```

output
```
1/2*A^2*b*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x
/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*
x + a)/b - 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c
*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d
*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*
d)*x/(b*d))*A*B*b*g + A^2*a*g*x + 2*((g*log(e) + 2*g)*b*c^2 - 2*(g*log(e)
+ g)*a*c*d)*B^2*log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)
*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(2*b^2*c*d*
g*log(e) - (g*log(e)^2 + 2*g*log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2
 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^
2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 4*
(B^2*b^2*d^2*g*x^2*log(e) + 2*((g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x +
 ((g*log(e) + 2*g)*a^2*d^2 - 2*a*b*c*d*g)*B^2)*log(b*x + a) - 4*(B^2*b^2*d
^2*g*x^2*log(e) + 2*((g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + 2*(B^2*b^
2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c
))/(b*d^2)
```

3.131.8 Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.132 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx$

3.132.1 Optimal result

Integrand size = 34, antiderivative size = 132

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{8B^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+4*B*(A+B*
ln(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylo
g(3,b*(d*x+c)/d/(b*x+a))/b/g
```

3.132.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{-2AB\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+A^2\log(a+bx)-2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-B^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{e(a+bx)}{(c+dx)}\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

output
```
(-2*A*B*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + A^2*Log[a + b*x] - 2*A*B*Log
[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^2)/(c + d*x)^2] - B^2*Log[
(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^2)/(c + d*x)^2]^2 - 4*A*B*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] + 4*A*B*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 4*B^2*Log[(e*(a + b*x)^2)/(c + d*
x)^2]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d
*x))/(d*(a + b*x))])/(b*g)
```

3.132.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2950, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{ag+bgx}\,dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{2779}$$

$$\frac{\frac{4B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \; 2821$$

$$\frac{\frac{4B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-2B\int \frac{(c+dx)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \; 7143$$

$$\frac{\frac{4B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+2B\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[1 - (b*(c + d*x))/(d*(a
 + b*x))])/b) + (4*B*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))] + 2*B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]
))/b)/g
```

3.132.3.1 Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.132.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{bgx+ag}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

3.132.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="fri
cas")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(b*g*x + a*g), x)
```

3.132.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2}\right.}{a+bx}}{g}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(a**2*e/(c**2 + 2*c*d*x +
d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
2*c*d*x + d**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(a**2*e/(c**2 +
 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x*
*2/(c**2 + 2*c*d*x + d**2*x**2))/(a + b*x), x))/g
```

3.132.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="max
ima")
```

output
```
4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - int
egrate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + 4*(B^2*b*d*x + B^2*b*c)*log
(b*x + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 4*(B^2*b*c*log(e)
+ A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) - 4*(B^2*b*c*log(e)
 + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + 2*(2*B^2*b*d*x + (b*c + a*d)*B
^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*
g)*x), x)
```

3.132.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x), x)
```

3.133 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

3.133.1 Optimal result

Integrand size = 34, antiderivative size = 130

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{4B(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
-8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-4*B*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x
+c)^2))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(
-a*d+b*c)/g^2/(b*x+a)
```

3.133.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.47

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B\left((bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-d(a+bx)\left(A+B\log \cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*((b*c - a*d)*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2]) + d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*
(a + b*x)^2)/(c + d*x)^2]) - d*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a +
b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B
*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a
+ b*x)))
```

3.133.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{4B\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{4B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{a+bx}-\frac{2B(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(a + b*x)) + 4*B
*((-2*B*(c + d*x))/(a + b*x) - ((c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2]))/(a + b*x)))/((b*c - a*d)*g^2)
```

3.133.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.133. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

3.133.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.45

method	result
norman	$\frac{\frac{(A^2+4BA+8B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{2cB(A+2B)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}+\frac{2Bd(A+2B)x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{2A^2ab^2d^2-2A^2b^3cd-2B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2b^3d^2-8B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3d^2+16B^2ab^2d^2-16B^2b^3cd-4ABx\ln\left(\frac{e(bx}{(dx}\right.}{2g^2(bx+a)b^3}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{4B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}+\frac{4B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}+\cdots$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{4B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}+\frac{4B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}+\cdots$
derivativedivides	$-\frac{-\frac{d^2A^2}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{8d^2B^2}{bg(dx+c)}-\frac{4d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}-\frac{d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)^2}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{4d^2AB}{bg(dx+c)}-\cdots}{d}$
default	$-\frac{-\frac{d^2A^2}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{8d^2B^2}{bg(dx+c)}-\frac{4d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}-\frac{d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)^2}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{4d^2AB}{bg(dx+c)}-\cdots}{d}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOS
E)
```

output
```
((A^2+4*A*B+8*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(b*x+a)^2/(d*x+c)^2)^2+B^2
*d/g/(a*d-b*c)*x*ln(e*(b*x+a)^2/(d*x+c)^2)^2+2*c*B*(A+2*B)/g/(a*d-b*c)*ln(
e*(b*x+a)^2/(d*x+c)^2)+2*B*d*(A+2*B)/g/(a*d-b*c)*x*ln(e*(b*x+a)^2/(d*x+c)^
2))/g/(b*x+a)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.54

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{(A^2+4\,AB+8\,B^2)bc-(A^2+4\,AB+8\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,((AB+2}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="f
ricas")
```

output
```
-((A^2 + 4*A*B + 8*B^2)*b*c - (A^2 + 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2
*b*c)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2
*((A*B + 2*B^2)*b*d*x + (A*B + 2*B^2)*b*c)*log((b^2*e*x^2 + 2*a*b*e*x + a^
2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2
*b*d)*g^2)
```

3.133.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(112) = 224$.

Time = 1.18 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{4Bd(A+2B)\log\left(x+\frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd-\frac{4Ba^2d^3(A+2B)}{ad-bc}+\frac{8Babcd^2(A+2B)}{ad-bc}-\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$+\frac{4Bd(A+2B)\log\left(x+\frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd+\frac{4Ba^2d^3(A+2B)}{ad-bc}-\frac{8Babcd^2(A+2B)}{ad-bc}+\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2\,(ad-bc)}$$
$$+\frac{(-2AB-4B^2)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2+b^2g^2x}+\frac{(B^2c+B^2dx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2-4AB-8B^2}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**2,x)
```

output
```
-4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d + 8*B**2*a*d**2 + 8*B
**2*b*c*d - 4*B*a**2*d**3*(A + 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A + 2*B)
/(a*d - b*c) - 4*B*b**2*c**2*d*(A + 2*B)/(a*d - b*c))/(8*A*B*b*d**2 + 16*B
**2*b*d**2))/(b*g**2*(a*d - b*c)) + 4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2
+ 4*A*B*b*c*d + 8*B**2*a*d**2 + 8*B**2*b*c*d + 4*B*a**2*d**3*(A + 2*B)/(a*
d - b*c) - 8*B*a*b*c*d**2*(A + 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A + 2*B
)/(a*d - b*c))/(8*A*B*b*d**2 + 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2
*A*B - 4*B**2)*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) +
 (B**2*c + B**2*d*x)*log(e*(a + b*x)**2/(c + d*x)**2)**2/(a**2*d*g**2 - a*
b*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 - 4*A*B - 8*B**2)/(a*b*g
**2 + b**2*g**2*x)
```

3.133.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(130) = 260$.

Time = 0.23 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.42

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-4\left(\left(\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{}{d}\right.\right.\right.$$

$$-2\,AB\left(\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^2g^2x+abg^2}+\frac{2}{b^2g^2x+abg^2}+\frac{2\,d\log(bx+a)}{(b^2c-abd)g^2}-\frac{2\,d\log(d}{(b^2c-ab}\right.$$

$$-\frac{B^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="m
axima")
```

output
```
-4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2)
+ 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) -
 ((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*
a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*
x + a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^
2)*x))*B^2 - 2*A*B*(log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d
^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*
b*g^2) + 2/(b^2*g^2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2)
- 2*d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b^2*e*x^2/(d^2*x^2 + 2
*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2))^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(130) = 260$.

Time = 0.72 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.92

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{(bgx+ag)bg}\right)\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)^2$$
$$+\frac{4\,(ABd+2\,B^2d)\log\left(\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right)}{b^2cg^2-abdg^2}$$
$$-\frac{2\,(AB+2\,B^2)\log\left(\frac{b^2e}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)}{(bgx+ag)bg}-\frac{A^2+4\,AB+8\,B^2}{(bgx+ag)bg}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="g
iac")
```

output
```
-(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log(b^2*e/(b^2*
c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g
*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))^2
+ 4*(A*B*d + 2*B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a*g) + d)/(
b^2*c*g^2 - a*b*d*g^2) - 2*(A*B + 2*B^2)*log(b^2*e/(b^2*c^2*g^2/(b*g*x + a
*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*
c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)*b*g)
- (A^2 + 4*A*B + 8*B^2)/((b*g*x + a*g)*b*g)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.75

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) - \frac{A^2+4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2} - \frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{4\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)1\mathrm{i}}{a\,d-b\,c}\right)(A+2\,B)\,8\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^2,x)
```

output
```
- log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b
*g^2*(a*d - b*c))) - (A^2 + 8*B^2 + 4*A*B)/(b^2*g^2*x + a*b*g^2) - (log((e
*(a + b*x)^2)/(c + d*x)^2)*((4*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x
/d + a/(b*d)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)
/(a*d - b*c))*(A + 2*B)*8i)/(b*g^2*(a*d - b*c))
```

3.134 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

3.134.1 Optimal result

Integrand size = 34, antiderivative size = 272

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = \frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2} + \frac{4Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)} - \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output
```
8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/
(b*x+a)^2+4*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(
b*x+a)-b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(b*x
+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)
-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a
)^2
```

3.134.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.66

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B\left((bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-2d^2(a+bx)^2\log(}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

output
```
-1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*((b*c - a*d)^2*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^2*(a + b*x)^2*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a +
b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(
b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2
*Log[c + d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) -
 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/
(b*g^3*(a + b*x)^2)
```

3.134.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(ag+bgx)^3}dx$$

$$\Big\downarrow \; 2950$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$\Big\downarrow$ 2795

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(a+bx)^2}+\frac{4Bd(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{a+bx}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\cdots\right.\right.}{a+bx}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

output
```
((8*B^2*d*(c + d*x))/(a + b*x) - (b*B^2*(c + d*x)^2)/(a + b*x)^2 + (4*B*d*
(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(a + b*x) - (b*B*(c +
d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(a + b*x)^2 + (d*(c + d*x
)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(a + b*x) - (b*(c + d*x)^2*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*(a + b*x)^2))/((b*c - a*d)^2
*g^3)
```

3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.134. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.134.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.80

method	result
norman	$\frac{\frac{\left(A^2ad-A^2bc+4ABad-2ABbc+8B^2ad-2B^2bc\right)x}{ag(ad-cb)}+\frac{Bc(2Aad-Abc+4Bad-Bbc)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2ad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{bB\,d}{\cdots}}{\cdots}$
parallelrisch	$-\frac{-2ABx^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^5d^3-2B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2ab^4d^3-8B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)ab^4d^3-4B^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^5cd^2-\cdots}{\cdots}$
derivativedivides	$-\frac{\frac{d^3A^2\left(-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}\right)}{g^3}+\frac{\frac{7B^2d^3}{bg(dx+c)^2}-\frac{3bB^2d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\cdots}{\cdots}}{\cdots}$
default	$-\frac{\frac{d^3A^2\left(-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}\right)}{g^3}+\frac{\frac{7B^2d^3}{bg(dx+c)^2}-\frac{3bB^2d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\cdots}{\cdots}}{\cdots}$
risch	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b\left(7B^2ad-B^2bc\right)x^2}{a^2g(ad-cb)}+\frac{(4ad-cb)B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)^2}+\frac{B^2ad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2\left(4B^2ad-B^2bc\right)x}{ag(ad-cb)}+\frac{B^2c(\cdots}{2g\cdots}}{g^{\cdots}}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b\left(7B^2ad-B^2bc\right)x^2}{a^2g(ad-cb)}+\frac{(4ad-cb)B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)^2}+\frac{B^2ad^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2\left(4B^2ad-B^2bc\right)x}{ag(ad-cb)}+\frac{B^2c(\cdots}{2g\cdots}}{g^{\cdots}}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOS
E)
```

output
```
((A^2*a*d-A^2*b*c+4*A*B*a*d-2*A*B*b*c+8*B^2*a*d-2*B^2*b*c)/a/g/(a*d-b*c)*x
+B*c*(2*A*a*d-A*b*c+4*B*a*d-B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x
+a)^2/(d*x+c)^2)+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)^2/
(d*x+c)^2)^2+b*B/g*d^2*(A+3*B)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^2*ln(e*(b*x+a
)^2/(d*x+c)^2)+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b
*x+a)^2/(d*x+c)^2)^2+1/2*(A^2*a*d-A^2*b*c+6*A*B*a*d-2*A*B*b*c+14*B^2*a*d-2
*B^2*b*c)/a^2/g*b/(a*d-b*c)*x^2+2*B/g*d*(A*a*d+2*B*a*d+B*b*c)/(a^2*d^2-2*a
*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*b*d^2*B^2/g/(a^2*d^2-2*a*b
*c*d+b^2*c^2)*x^2*ln(e*(b*x+a)^2/(d*x+c)^2)^2)/g^2/(b*x+a)^2
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.51

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{(A^2+2\,AB+2\,B^2)b^2c^2-2\,(A^2+4\,AB+8\,B^2)abcd+(A^2+6\,AB+14\,B^2)a^2d^2-(B^2b^2d^2x^2+2\,B^2a}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="f
ricas")
```

output
```
-1/2*((A^2 + 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 + 4*A*B + 8*B^2)*a*b*c*d + (A
^2 + 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^
2*c^2 + 2*B^2*a*b*c*d)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2))^2 - 4*((A*B + 3*B^2)*b^2*c*d - (A*B + 3*B^2)*a*b*d^2)*x - 2*((
A*B + 3*B^2)*b^2*d^2*x^2 - (A*B + B^2)*b^2*c^2 + 2*(A*B + 2*B^2)*a*b*c*d +
 2*(B^2*b^2*c*d + (A*B + 2*B^2)*a*b*d^2)*x)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g
^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2
- 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.134.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(252) = 504$.

Time = 2.08 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.23

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$- \frac{2Bd^2(A+3B)\log\left(x + \frac{2ABad^3+2ABbcd^2+6B^2ad^3+6B^2bcd^2-\frac{2Ba^3d^5(A+3B)}{(ad-bc)^2}+\frac{6Ba^2bcd^4(A+3B)}{(ad-bc)^2}-\frac{6Bab^2c^2d^3(A+3B)}{(ad-bc)^2}+\frac{2Bb^3c^3d^2(}{(ad-b}}{4ABbd^3+12B^2bd^3}\right.}{bg^3(ad-bc)^2}$$

$$+ \frac{2Bd^2(A+3B)\log\left(x + \frac{2ABad^3+2ABbcd^2+6B^2ad^3+6B^2bcd^2+\frac{2Ba^3d^5(A+3B)}{(ad-bc)^2}-\frac{6Ba^2bcd^4(A+3B)}{(ad-bc)^2}+\frac{6Bab^2c^2d^3(A+3B)}{(ad-bc)^2}-\frac{2Bb^3c^3d^2(}{(ad-b}}{4ABbd^3+12B^2bd^3}\right.}{bg^3(ad-bc)^2}$$

$$+ \frac{(2B^2acd + 2B^2ad^2x - B^2bc^2 + B^2bd^2x^2)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2}{2a^4d^2g^3 - 4a^3bcdg^3 + 4a^3bd^2g^3x + 2a^2b^2c^2g^3 - 8a^2b^2cdg^3x + 2a^2b^2d^2g^3x^2 + 4ab^3c^2g^3x - 4ab^3cdg^3x^2 +}$$

$$+ \frac{(-ABad + ABbc - 3B^2ad + B^2bc - 2B^2bdx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a^3bdg^3 - a^2b^2cg^3 + 2a^2b^2dg^3x - 2ab^3cg^3x + ab^3dg^3x^2 - b^4cg^3x^2}$$

$$+ \frac{-A^2ad + A^2bc - 6ABad + 2ABbc - 14B^2ad + 2B^2bc + x(-4ABbd - 12B^2bd)}{2a^3bdg^3 - 2a^2b^2cg^3 + x^2\cdot(2ab^3dg^3 - 2b^4cg^3) + x(4a^2b^2dg^3 - 4ab^3cg^3)}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**3,x)
```

output
```
-2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3
 + 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A + 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c
*d**4*(A + 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c
)**2 + 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**
2*b*d**3))/(b*g**3*(a*d - b*c)**2) + 2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d
**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3 + 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A
+ 3*B)/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A + 3*B)/(a*d - b*c)**2 + 6*B*a
*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A + 3*B)/(a
*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (
2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(a
+ b*x)**2/(c + d*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*
b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b*
*2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4
*c**2*g**3*x**2) + (-A*B*a*d + A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*
d*x)*log(e*(a + b*x)**2/(c + d*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 +
2*a**2*b**2*d*g**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**
3*x**2) + (-A**2*a*d + A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 14*B**2*a*d + 2*
B**2*b*c + x*(-4*A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*
g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*
a*b**3*c*g**3))
```

3.134.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(270) = 540$.

Time = 0.26 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.68

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$=\Bigg(\Bigg(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2\,d^2\,l}{(b^3c^2-2\,a}$$
$$+AB\Bigg(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\right.}{b^3g^3x^2+2\,ab^2g^3x+a^2b}$$
$$-\frac{B^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{A^2}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="m
axima")
```

output
```
(((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^
2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2)*g^3))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (b^2*c^2 - 8*a
*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^
2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d -
a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b
^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*
d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^
4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a
*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + A*B*((2*b*d*
x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*
x + (a^2*b^2*c - a^3*b*d)*g^3) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) +
 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b
^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2)*g^3)) - 1/2*B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a
*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^3
*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*...
```

3.134.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="g
iac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^3, x)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c+6\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{2\,x\,(3\,b\,d\,B^2+A\,b\,d\,B)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}$$
$$-\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{A\,B}{b^2\,d\,g^3}+\frac{2\,B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,(a\,d-b\,c)}{b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)(A+3\,B)\,2\mathrm{i}}{(a\,d-b\,c)\,(6\,B^2\,d^2+2\,A\,B\,d^2)}\right)(A+3\,B)\,4\mathrm{i}}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^3,x)
```

output
```
- ((A^2*a*d - A^2*b*c + 14*B^2*a*d - 2*B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(2
*(a*d - b*c)) + (2*x*(3*B^2*b*d + A*B*b*d))/(a*d - b*c))/(a^2*b*g^3 + b^3*
g^3*x^2 + 2*a*b^2*g^3*x) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(2*b^2*
g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((A*B)/(b^2*d*g^3) + (2*B^2*
x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*
d^2 + b^2*c^2 - 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^
2*atan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c))
)*(A + 3*B)*2i)/((a*d - b*c)*(6*B^2*d^2 + 2*A*B*d^2)))*(A + 3*B)*4i)/(b*g^
3*(a*d - b*c)^2)
```

3.135 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.135.1 Optimal result

Integrand size = 34, antiderivative size = 429

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2}-\frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}-\frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)}+\frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2}-\frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3}-\frac{d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)}+\frac{bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^3g^4(a+bx)^2}-\frac{b^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3(bc-ad)^3g^4(a+bx)^3}$$

output
```
-8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)
^3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-4*B*d^2
*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B*d*
(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2-4/9*b
^2*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-
d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d
*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/
3*b^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)
^3
```

3.135.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.39

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{9\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B\left(6A(bc-ad)^3+4B(bc-ad)^3-9Ad(bc-ad)^2(a+bx)-15Bd(bc-ad)^2(a+bx)+18Ad^2(bc-ad)(a+bx)^2\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

output
```
-1/27*(9*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(6*A*(b*c - a*d
)^3 + 4*B*(b*c - a*d)^3 - 9*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*
d)^2*(a + b*x) + 18*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(
a + b*x)^2 + 18*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[
a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 6*B*(b*c - a*d)^3*Log[(e*
(a + b*x)^2)/(c + d*x)^2] - 9*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x)
^2)/(c + d*x)^2] + 18*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x)^2)/(c
 + d*x)^2] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x)^2)/(c + d*
x)^2] - 18*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d
*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]
- 18*B*d^3*(a + b*x)^3*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[c + d*x] - 18*
B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(
b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)]))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

3.135.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \ 2950$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \ 2795$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \ 2009$$

$$-\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3(a+bx)^3}-\frac{4b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{9(a+bx)^3}-\frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{a+bx}-\frac{4Bd^2(c+dx)\left(B\log\ldots\right.}{a+\ldots}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

output
```
((-8*B^2*d^2*(c + d*x))/(a + b*x) + (2*b*B^2*d*(c + d*x)^2)/(a + b*x)^2 -
(8*b^2*B^2*(c + d*x)^3)/(27*(a + b*x)^3) - (4*B*d^2*(c + d*x)*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2]))/(a + b*x) + (2*b*B*d*(c + d*x)^2*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2]))/(a + b*x)^2 - (4*b^2*B*(c + d*x)^3*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2]))/(9*(a + b*x)^3) - (d^2*(c + d*x)*(A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(a + b*x) + (b*d*(c + d*x)^2*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(a + b*x)^2 - (b^2*(c + d*x)^3*(A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*(a + b*x)^3))/((b*c - a*d)^3*g^4)
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.135.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(423) = 846$.

Time = 2.11 (sec) , antiderivative size = 1019, normalized size of antiderivative = 2.38

method	result	size
derivativedivides	Expression too large to display	1019
default	Expression too large to display	1019
parallelrisch	Expression too large to display	1025
norman	Expression too large to display	1054
risch	Expression too large to display	1309
parts	Expression too large to display	1309

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/d*(d^4/g^4*A^2*(-1/3*b^2/(a*d-b*c)^3/(a*d/(d*x+c)-b*c/(d*x+c)+b)^3+b/(a
*d-b*c)^3/(a*d/(d*x+c)-b*c/(d*x+c)+b)^2-1/(a*d-b*c)^3/(a*d/(d*x+c)-b*c/(d*
x+c)+b))+(170/27*B^2/b*d^4/g/(d*x+c)^3-22/9*b^2*B^2*d^4/g/(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+44/9
*B^2*b*d^4/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+98/9*B^2*d^4/g/(a*d-b*c)/
(d*x+c)^2-4*B^2*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)
^2/d^2)-1/3*B^2*b^2*d^4/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln
(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-B^2*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e
*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-6*B^2*d^4*b/g/(a^2*d^2-2*a*b*c*d+b^2
*c^2)/(d*x+c)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-B^2*b*d^4/g/(a^2*d^2
-2*a*b*c*d+b^2*c^2)/(d*x+c)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2)/(a*
d/(d*x+c)-b*c/(d*x+c)+b)^3/g^3+(22/9*A*B/b*d^4/g/(d*x+c)^3-2/3*b^2*A*B*d^4
/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(d*x+c)-b*c/(d*
x+c)+b)^2/d^2)+4/3*A*B*b*d^4/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+10/3*A*
B*d^4/g/(a*d-b*c)/(d*x+c)^2-2*A*B*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e*(a*d/(d*x
+c)-b*c/(d*x+c)+b)^2/d^2)-2*A*B*d^4*b/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c
)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2))/(a*d/(d*x+c)-b*c/(d*x+c)+b)^3/g
^3)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.68

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{(9\,A^2+12\,AB+8\,B^2)b^3c^3-27\,(A^2+2\,AB+2\,B^2)ab^2c^2d+27\,(A^2+4\,AB+8\,B^2)a^2bcd^2-(9\,A^2+6}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="f
ricas")
```

output
```
-1/27*((9*A^2 + 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 + 2*A*B + 2*B^2)*a*b^2*c
^2*d + 27*(A^2 + 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 + 66*A*B + 170*B^2)*a
^3*d^3 + 12*((3*A*B + 11*B^2)*b^3*c*d^2 - (3*A*B + 11*B^2)*a*b^2*d^3)*x^2
+ 9*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c
^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 - 6*((3*A*B + 5*B^2)*b^3*c^2*d - 18*(A*
B + 3*B^2)*a*b^2*c*d^2 + (15*A*B + 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B + 11*B
^2)*b^3*d^3*x^3 + (3*A*B + 2*B^2)*b^3*c^3 - 9*(A*B + B^2)*a*b^2*c^2*d + 9*
(A*B + 2*B^2)*a^2*b*c*d^2 + 3*(2*B^2*b^3*c*d^2 + 3*(A*B + 3*B^2)*a*b^2*d^3
)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 3*(A*B + 2*B^2)*a^2*b*d^3)*
x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c
^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3
 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c
^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3
 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

3.135.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. $2(406) = 812$.

Time = 12.15 (sec) , antiderivative size = 1561, normalized size of antiderivative = 3.64

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**4,x)
```

output
```
-4*B*d**3*(3*A + 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*
a*d**4 + 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A + 11*B)/(a*d - b*c)**3 + 16
*B*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3
*A + 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A + 11*B)/(a*d - b*c)
**3 - 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 + 88*
B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + 4*B*d**3*(3*A + 11*B)*log(x + (1
2*A*B*a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a*d**4 + 44*B**2*b*c*d**3 + 4*B*a
**4*d**7*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A + 11*B)/(a*
d - b*c)**3 + 24*B*a**2*b**2*c**2*d**5*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*
a*b**3*c**3*d**4*(3*A + 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A + 1
1*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 + 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*
c)**3) + (3*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*
B**2*a*b*d**3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)
**2/(c + d*x)**2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d
**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4
*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x
- 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4
*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**
3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4
*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 22*B**2...
```

3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1575 vs. $2(423) = 846$.

Time = 0.31 (sec) , antiderivative size = 1575, normalized size of antiderivative = 3.67

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="m
axima")
```

output
```
-2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5
*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c
*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3)*g^4))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + (4*b^3*c^3 - 27
*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x
^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x
+ a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(
d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d
^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b
^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x
^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c)
)/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3
*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^
3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4
- a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*
b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 - 2/9*A*B*((6*b^2*d^2*x^2 + 2*...
```

3.135.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="g
iac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^4, x)
```

3.135.9 Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{\frac{9A^2a^2d^2-18A^2abcd+9A^2b^2c^2+66ABa^2d^2-42ABabcd+12ABb^2c^2+170B^2a^2d^2-46B^2abcd+8B^2b^2c^2}{3(ad-bc)}+\frac{2x(-5cB^2b^2d+49a}{}}{x\left(27a^2b^3cg^4-27a^3b^2dg^4\right)-x^2\left(27a^2b^3dg^4-27ab^4cg^4\right)+x^3\left(9b^5cg^4-9ab^4\right.}$$
$$-\ln\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2\left(\frac{B^2}{3b^2g^4\left(3a^2x+\frac{a^3}{b}+b^2x^3+3abx^2\right)}-\frac{B^2d^3}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}\right)$$
$$-\frac{\ln\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\left(\frac{2AB}{3b^2dg^4}+\frac{2B^2d^3\left(a\left(\frac{3a^2d^2-4abcd+b^2c^2}{3bd^3}+\frac{2a(ad-bc)}{3bd^2}\right)+\frac{2\left(3a^3d^3-6a^2bcd^2+4ab^2c^2d-b^3c^3\right)}{3bd^4}\right)}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}-\frac{2B^2d^3x^2}{3bg^4(a^3d^3}\right.}{\frac{3a^2x}{d}+\frac{a^3}{bd}+\frac{b^2x^3}{d}}$$
$$-\frac{Bd^3\,\mathrm{atan}\left(\frac{Bd^3\left(\frac{a^3bd^3g^4-a^2b^2cd^2g^4-ab^3c^2dg^4+b^4c^3g^4}{a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4}+2bdx\right)(3A+11B)\left(a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4\right)4\mathrm{i}}{bg^4(ad-bc)^3\left(44B^2d^3+12ABd^3\right)}\right)(3A+11}{9bg^4(ad-bc)^3}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^4,x)
```

output
```
((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 66*A*B
*a^2*d^2 + 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d - 42*A*B*a*b*c
*d)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 15*A*B*a*b*d^
2 - 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d + 3*A*B*b^2*d))/(
a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g
^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4
 - 9*a^4*b*d*g^4) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(3*b^2*g^4*(3*
a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x)^2)/(c + d*x)^2)
*((2*A*B)/(3*b^2*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)
/(3*b*d^3) + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*
b^2*c^2*d - 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) - (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2)
- (4*b*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) + (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*
b*d^3) + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*
d))/(3*d^3) + (4*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (
3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c
^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*...
```

3.135. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.136 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.136.1 Optimal result

Integrand size = 34, antiderivative size = 587

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2} + \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4} + \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)} - \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)^2} + \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4(bc-ad)^4g^5(a+bx)^4} + \frac{d^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)} - \frac{3bd^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2(bc-ad)^4g^5(a+bx)^2} + \frac{b^2d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)^4g^5(a+bx)^3} - \frac{b^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4(bc-ad)^4g^5(a+bx)^4}$$

output
```
8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c
)^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b
^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+4*B*d^3*(d*x+c)*(A+B*ln(e*(b*x
+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(
b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^2+4/3*b^2*B*d*(d*x+c)^3*(A+B
*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*B*(d*x+c)^4
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A
+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)
^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x
+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3
*(d*x+c)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

3.136.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.16

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{18\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{B\left(18A(bc-ad)^4+9B(bc-ad)^4+24Ad(-bc+ad)^3(a+bx)+28Bd(-bc+ad)^3(a+bx)+36Ad^2(bc-ad)^2\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

output
```
-1/72*(18*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(18*A*(b*c - a*d
)^4 + 9*B*(b*c - a*d)^4 + 24*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*
c) + a*d)^3*(a + b*x) + 36*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c
 - a*d)^2*(a + b*x)^2 + 72*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-
(b*c) + a*d)*(a + b*x)^3 - 72*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(
a + b*x)^4*Log[a + b*x] + 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 18*B*(b*c
- a*d)^4*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 24*B*d*(-(b*c) + a*d)^3*(a + b
*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 36*B*d^2*(b*c - a*d)^2*(a + b*x)^2*
Log[(e*(a + b*x)^2)/(c + d*x)^2] + 72*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log
[(e*(a + b*x)^2)/(c + d*x)^2] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e*(
a + b*x)^2)/(c + d*x)^2] + 72*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*(
a + b*x)^4*Log[c + d*x] - 144*B*d^4*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c)
+ a*d)]*Log[c + d*x] + 72*B*d^4*(a + b*x)^4*Log[(e*(a + b*x)^2)/(c + d*x)^
2]*Log[c + d*x] + 72*B*d^4*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*(a + b*x
)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*(a + b*x)^4*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

3.136.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\log\right.}{}\right.}{g^5(bc-ad)^4}$$

$\downarrow$ 2009

$$-\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4(a+bx)^4}-\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4(a+bx)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(a+bx)^3}+\frac{4b^2Bd(c+dx)^3\left(B\right.}{3(a}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

output
```
((8*B^2*d^3*(c + d*x))/(a + b*x) - (3*b*B^2*d^2*(c + d*x)^2)/(a + b*x)^2 +
 (8*b^2*B^2*d*(c + d*x)^3)/(9*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(8*(a +
 b*x)^4) + (4*B*d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(a
 + b*x) - (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))
/(a + b*x)^2 + (4*b^2*B*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]))/(3*(a + b*x)^3) - (b^3*B*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2]))/(4*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c
+ d*x)^2])^2)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/
(c + d*x)^2])^2)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[(e*(a + b
*x)^2)/(c + d*x)^2])^2)/(a + b*x)^3 - (b^3*(c + d*x)^4*(A + B*Log[(e*(a +
b*x)^2)/(c + d*x)^2])^2)/(4*(a + b*x)^4))/((b*c - a*d)^4*g^5)
```

3.136.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.136. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1485 vs. $2(575) = 1150$.

Time = 3.50 (sec) , antiderivative size = 1486, normalized size of antiderivative = 2.53

method	result	size
derivativedivides	Expression too large to display	1486
default	Expression too large to display	1486
norman	Expression too large to display	1816
parallelrisch	Expression too large to display	2110
risch	Expression too large to display	2235
parts	Expression too large to display	2235

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOS
E)
```

output
```
-1/d*(d^5/g^5*A^2*(-1/(a*d-b*c)^4/(a*d/(d*x+c)-b*c/(d*x+c)+b)+1/4*b^3/(a*d
-b*c)^4/(a*d/(d*x+c)-b*c/(d*x+c)+b)^4-b^2/(a*d-b*c)^4/(a*d/(d*x+c)-b*c/(d*
x+c)+b)^3+3/2*b/(a*d-b*c)^4/(a*d/(d*x+c)-b*c/(d*x+c)+b)^2)+(415/72*B^2/b*d
^5/g/(d*x+c)^4-25/12*b^3*B^2*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^
2-4*a*b^3*c^3*d+b^4*c^4)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+25/6*B^2*
b^2*d^5/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+163/12*B^2
*b*d^5/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2+271/18*B^2*d^5/g/(a*d-b*c)/
(d*x+c)^3-4*B^2*d^5/g/(a*d-b*c)/(d*x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)
^2/d^2)-1/4*B^2*b^3*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3
*c^3*d+b^4*c^4)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-B^2*d^5/g/(a*d-b
*c)/(d*x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-9*B^2*d^5*b/g/(a^2
*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-
22/3*B^2*d^5*b^2/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*l
n(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-3/2*B^2*b*d^5/g/(a^2*d^2-2*a*b*c*d+
b^2*c^2)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)^2-B^2*b^2*d^5/g
/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln(e*(a*d/(d*x+c)-b
*c/(d*x+c)+b)^2/d^2)^2)/(a*d/(d*x+c)-b*c/(d*x+c)+b)^4/g^4+(A*B*b^2*d^5/g/(
a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+25/12*A*B/b*d^5/g/(d*
x+c)^4-1/2*b^3*A*B*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+7/2*A*B*b*d^5/g/...
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{9\,(2\,A^2+2\,AB+B^2)b^4c^4-8\,(9\,A^2+12\,AB+8\,B^2)ab^3c^3d+108\,(A^2+2\,AB+2\,B^2)a^2b^2c^2d^2-72\,(A}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="f
ricas")
```

output
```
-1/72*(9*(2*A^2 + 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 + 12*A*B + 8*B^2)*a*b^3*
c^3*d + 108*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 + 4*A*B + 8*B^
2)*a^3*b*c*d^3 + (18*A^2 + 150*A*B + 415*B^2)*a^4*d^4 - 12*((6*A*B + 25*B^
2)*b^4*c*d^3 - (6*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((6*A*B + 13*B^2)*b^4*c
^2*d^2 - 16*(3*A*B + 11*B^2)*a*b^3*c*d^3 + (42*A*B + 163*B^2)*a^2*b^2*d^4)
*x^2 - 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d
^2 + 4*B^2*a^3*b*c*d^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c
*d*x + c^2))^2 - 4*((6*A*B + 7*B^2)*b^4*c^3*d - 12*(3*A*B + 5*B^2)*a*b^3*c
^2*d^2 + 108*(A*B + 3*B^2)*a^2*b^2*c*d^3 - (78*A*B + 271*B^2)*a^3*b*d^4)*x
 - 6*((6*A*B + 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B + B^2)*b^4*c^4 + 8*(3*A*B +
2*B^2)*a*b^3*c^3*d - 36*(A*B + B^2)*a^2*b^2*c^2*d^2 + 24*(A*B + 2*B^2)*a^3
*b*c*d^3 + 4*(3*B^2*b^4*c*d^3 + 2*(3*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2
*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 6*(A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B
^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 6*(A*B + 2*B^2
)*a^3*b*d^4)*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c
^2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^
4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 -
4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d
+ 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b...
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(575)=1150$.

Time = 0.37 (sec) , antiderivative size = 2279, normalized size of antiderivative = 3.88

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="m
axima")
```

output
```
1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*
d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^
3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*
b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*
d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*
d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
 + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^
2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4
)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b
^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4
)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 +
 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d
^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4
*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*
b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x...
```

3.136. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.136.8 Giac [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.49

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$$

$$=\frac{1}{4}\left(\frac{B^2d^4}{b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5}-\frac{B^2}{(bgx+ag)^4bg}\right)\log\left(\frac{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcd}{(bgx+a}}{}\right.$$

$$+\frac{1}{12}\left(\frac{12\,B^2d^3}{(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}-\frac{6\,B^2d^2}{(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2}\right.$$

$$-\frac{(6\,ABd^4+25\,B^2d^4)\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{6\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)}$$

$$+\frac{6\,ABd^3+25\,B^2d^3}{6\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}$$

$$-\frac{6\,ABbd^2+13\,B^2bd^2}{12\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2b^2g^2}$$

$$+\frac{6\,ABb^2dg+7\,B^2b^2dg}{18\,(bgx+ag)^3(bc-ad)b^3g^3}-\frac{2\,A^2b^3g^3+2\,ABb^3g^3+B^2b^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="g
iac")
```

output
```
1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*
a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log(b^2*e/
(b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2
/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2
))^2 + 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*
g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g - 2*a*b*c*d*
g + a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3*(b*c - a*
d)*b*g^2) - 3*(2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4))*log
(b^2*e/(b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*
d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g
) + d^2)) - 1/6*(6*A*B*d^4 + 25*B^2*d^4)*log(-b*c*g/(b*g*x + a*g) + a*d*g/
(b*g*x + a*g) - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^
5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) + 1/6*(6*A*B*d^3 + 25*B^2*d^3)/((
b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x
+ a*g)*b*g) - 1/12*(6*A*B*b*d^2 + 13*B^2*b*d^2)/((b^2*c^2*g - 2*a*b*c*d*g
+ a^2*d^2*g)*(b*g*x + a*g)^2*b^2*g^2) + 1/18*(6*A*B*b^2*d*g + 7*B^2*b^2*d*
g)/((b*g*x + a*g)^3*(b*c - a*d)*b^3*g^3) - 1/8*(2*A^2*b^3*g^3 + 2*A*B*b^3*
g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

3.136.9 Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 1883, normalized size of antiderivative = 3.21

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*atan((B*d^4*(6*A + 25*B)*(6*b^5*c^4*g^5 - 6*a^4*b*d^4*g^5 - 12*a*b^
4*c^3*d*g^5 + 12*a^3*b^2*c*d^3*g^5)*1i)/(6*b*g^5*(a*d - b*c)^4*(25*B^2*d^4
 + 6*A*B*d^4)) + (B*d^5*x*(6*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3*a*
b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4 +
6*A*B*d^4)))*(6*A + 25*B)*1i)/(3*b*g^5*(a*d - b*c)^4) - log((e*(a + b*x)^2
)/(c + d*x)^2)^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2
+ 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*(
(A*B)/(2*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6
*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*
d - 10*a^2*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2
 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*c^4
 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(b*(
b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)
) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a*(a*d - b*c))/d^2) - a*(
(b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c^2 + 4*a^2*b*d^2 -
5*a*b^2*c*d)/(2*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4
*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(2*d^2)
- (b*(a*d - b*c))/d^2) + (b^3*c - a*b^2*d)/(2*d^2)))/(2*b*g^5*(a^4*d^4 + b
^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4...
```

3.136. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.137 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

3.137.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.137.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

3.137.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}\,dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
$Aborted
```

3.137.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.137.4 Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\left(bgx+ag\right)^{2}}{A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.137.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b^2*e*x^2 + 2*a*b*e
*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

3.137.6 Sympy [N/A]

Not integrable

Time = 14.06 (sec) , antiderivative size = 258, normalized size of antiderivative = 7.59

$$\begin{aligned}&\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx\\&=g^2\left(\int \frac{a^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right.\\&\quad+\int \frac{b^2x^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\\&\quad\left.+\int \frac{2abx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right)\end{aligned}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b
*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**
2))), x) + Integral(b**2*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**
2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x
+ d**2*x**2))), x) + Integral(2*a*b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x +
d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
2*c*d*x + d**2*x**2))), x))
```

3.137.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.137.8 Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.137.9 Mupad [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.138 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

3.138.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.138.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

3.138.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{ag+bgx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
$Aborted
```

3.138.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.138.4 Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.138.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="frica
s")
```

output
```
integral((b*g*x + a*g)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2
*c*d*x + c^2)) + A), x)
```

3.138.6 Sympy [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.16

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$
$$=g\left(\int \frac{a}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
g*(Integral(a/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(
c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))),
x) + Integral(b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e
*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)
)), x))
```

3.138.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxim
a")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.138.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.138.9 Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.139 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

3.139.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.139.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

3.139.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
$Aborted
```

3.139.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.139.4 Maple [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.139.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fri
cas")
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b^2*e*x^2 + 2*a*b*e*x
 + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.139.6 Sympy [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+Bbx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*
a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*
x**2)) + B*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 +
 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)/g
```

3.139.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="max
ima")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.139.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.139.9 Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.140 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

3.140.1 Optimal result

Integrand size = 34, antiderivative size = 94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)}$$

output
```
1/2*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b*
x+a)^2/(d*x+c)^2)^(1/2)/B/(-a*d+b*c)/g^2/(b*x+a)
```

3.140.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

3.140.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\int \frac{1}{\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g^2(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
(E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIntegralEi[-1/
2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(2*B*(b*c - a*d)*g^2*(a + b
*x))
```

3.140.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.140.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.140.5 Fricas [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="f
ricas")
```

output
```
integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2
*B*a*b*g^2*x + B*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2
*c*d*x + c^2))), x)
```

3.140.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right.}}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2
*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2
/(c**2 + 2*c*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d*
*2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*
c*d*x + d**2*x**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2)
+ 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d
**2*x**2))), x)/g**2
```

3.140.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.140.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.141 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

3.141.1 Optimal result

Integrand size = 34, antiderivative size = 152

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = -\frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)} + \frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B(bc-ad)^2g^3}$$

output
```
1/2*b*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B/(-a*d+b*c)^2/g^3
-1/2*d*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*
(b*x+a)^2/(d*x+c)^2)^(1/2)/B/(-a*d+b*c)^2/g^3/(b*x+a)
```

3.141.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

3.141.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{2950}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B}-\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(a+bx)}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
((b*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B)]
)/(2*B) - (d*E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIn
tegralEi[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(2*B*(a + b*x))
)/((b*c - a*d)^2*g^3)
```

3.141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.141.4 Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{3} \left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.141.5 Fricas [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="f
ricas")
```

output
```
integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^
3 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(
(b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.141.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

$$=\frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+3Ba^2bx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\cdots\right.}}{g\cdots}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**
2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a**2*b*x*log(a**
2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2)
+ b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a*b**2*x**2*log(a**2*e/(
c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**
2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + B*b**3*x**3*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/
(c**2 + 2*c*d*x + d**2*x**2))), x)/g**3
```

3.141.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.141.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.142 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.142.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.142.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

3.142.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
$Aborted
```

3.142.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.142.4 Maple [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.142.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.68

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b^2*e*x^2 + 2*a*b
*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*
e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.142.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 308, normalized size of antiderivative = 9.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
-1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b
^2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d
)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b
*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*
g^2 + a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2
*b*d*g^2)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x +
 c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

3.142.8 Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.142.9 Mupad [N/A]

Not integrable

Time = 6.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.143 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.143.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.143.2 Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

3.143.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
$Aborted
```

3.143.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.143.4 Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.143.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.34

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fri
cas")
```

output
```
integral((b*g*x + a*g)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 +
 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 +
2*c*d*x + c^2)) + A^2), x)
```

3.143.6 Sympy [N/A]

Not integrable

Time = 22.19 (sec) , antiderivative size = 558, normalized size of antiderivative = 17.44

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\frac{a^2cg+a^2dgx+2abcgx+2abdgx^2+b^2cgx^2+b^2dgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}$$
$$-\frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b*
*2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a +
 b*x)**2/(c + d*x)**2)) - g*(Integral(a**2*d/(A + B*log(a**2*e/(c**2 + 2*c
*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(a**2*e/(c*
*2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*
e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*log
(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x*
*2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*b**2*d*x
**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c
*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integ
ral(4*a*b*d*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(
c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))),
x))/(2*B*(a*d - b*c))
```

3.143.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 230, normalized size of antiderivative = 7.19

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="max
ima")
```

output
```
-1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2
*d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c)
+ (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b^2*
d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c
*log(e) - a*d*log(e))*B^2), x)
```

3.143.8 Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.143.9 Mupad [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.144 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.144.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.144.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

3.144.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
$Aborted
```

3.144.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.144.4 Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.144.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.79

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*e*x^2 + 2
*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*l
og((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.144.6 Sympy [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.59

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$
$$= \frac{c+dx}{2ABadg-2ABbcg+(2B^2adg-2B^2bcg)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{d\int \frac{1}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{2Bg\,(ad-bc)}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(c + d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(e
*(a + b*x)**2/(c + d*x)**2)) - d*Integral(1/(A + B*log(a**2*e/(c**2 + 2*c*
d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c
**2 + 2*c*d*x + d**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

3.144.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 4.88

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="m
axima")
```

output
```
d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^
2*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2),
 x) - 1/2*(d*x + c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g
)*B^2*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B
^2)
```

3.144.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

3.144.9 Mupad [N/A]

Not integrable

Time = 7.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.145 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.145.1 Optimal result

Integrand size = 34, antiderivative size = 150

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = -\frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)} - \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}$$

output
```
1/2*(-d*x-c)/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))-1/4*
exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b*x+a)
^2/(d*x+c)^2)^(1/2)/B^2/(-a*d+b*c)/g^2/(b*x+a)
```

3.145.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x
]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),
x]
```

3.145.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2950, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2743}$$

$$\frac{-\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\frac{a+bx}{c+dx}}{2B}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{-\frac{(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\int \frac{1}{\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}d\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4B(a+bx)}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{-\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(a+bx)}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
(-1/4*(E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIntegral
Ei[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(B^2*(a + b*x)) - (c
+ d*x)/(2*B*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])))/((b*c - a
*d)*g^2)
```

3.145.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.145. $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.145.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.145.5 Fricas [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2
*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*
a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))),
x)
```

3.145.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

$$= \frac{c+dx}{2ABa^2dg^2 - 2ABabcg^2 + 2ABabdg^2x - 2ABb^2cg^2x + (2B^2a^2dg^2 - 2B^2abcg^2 + 2B^2abdg^2x - 2B^2b^2cg^2\cdots}$$

$$- \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d\cdots}\right.}}{2Bg^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(c + d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*A
*B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d*
g**2*x - 2*B**2*b**2*c*g**2*x)*log(e*(a + b*x)**2/(c + d*x)**2)) - Integra
l(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2*c*d*x
+ d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
+ 2*c*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2
) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x +
 d**2*x**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b
*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**
2))), x)/(2*B*g**2)
```

3.145.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"maxima")
```

output
```
-1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^
2*log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g
^2*log(e))*B^2)*x + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*
g^2)*B^2)*log(b*x + a) - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a
^2*d*g^2)*B^2)*log(d*x + c)) + integrate(-1/2/(B^2*a^2*g^2*log(e) + A*B*a^
2*g^2 + (B^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A
*B*a*b*g^2)*x + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*
x + a) - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c))
, x)
```

3.145.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.146 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.146.1 Optimal result

Integrand size = 34, antiderivative size = 266

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)} - \frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2g^3} + \frac{d(c+dx)}{2B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} - \frac{b(c+dx)^2}{2B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}$$

output
```
-1/2*b*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B^2/(-a*d+b*c)^2/
g^3+1/2*d*(d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2
))-1/2*b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c
)^2))+1/4*d*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B
)*(e*(b*x+a)^2/(d*x+c)^2)^(1/2)/B^2/(-a*d+b*c)^2/g^3/(b*x+a)
```

3.146.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x
]
```

output
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),
x]
```

3.146.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2950, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2950}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3}{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}-\frac{d(c+dx)^2}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right)d\frac{a+bx}{c+dx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(a+bx)}-\frac{bee^{A/B}\,\text{ExpIntegralEi}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2}-\frac{b(c+dx)^2}{2B(a+bx)^2\left(B\log\left(\frac{e(a}{(c}\right.\right.}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
(-1/2*(b*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
)/B)])/B^2 + (d*E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*Ex
pIntegralEi[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(4*B^2*(a +
b*x)) + (d*(c + d*x))/(2*B*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2])) - (b*(c + d*x)^2)/(2*B*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2])))/((b*c - a*d)^2*g^3)
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2950
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x]
, x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &
& EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && E
qQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.146.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.146.5 Fricas [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^
2*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B
^2*a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^
3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.146.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"maxima")
```

output
```
-1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*
d*g^3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a
*b^2*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*
c*g^3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B
^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B
^2)*log(b*x + a) - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 -
 a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integ
rate(1/2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^
3*log(e) - a*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B +
(a^3*b*c*g^3*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*
g^3)*A*B + (a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*
b^2*c*g^3 - a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))
*B^2)*x + 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*
d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^
4*d*g^3)*B^2)*log(b*x + a) - 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b
^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x
+ (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

3.146.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.147 $\int (a+bx)^4 (A + B \log (e(a + bx)^n (c + dx)^{-n})) \, dx$

3.147.1 Optimal result

Integrand size = 31, antiderivative size = 171

$$\begin{aligned}\int (a+bx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx &= \frac{B(bc-ad)^4nx}{5d^4}-\frac{B(bc-ad)^3n(a+bx)^2}{10bd^3}+\frac{B(bc-ad)^2n(a+bx)^3}{15bd^2}\\ &\quad -\frac{B(bc-ad)n(a+bx)^4}{20bd}-\frac{B(bc-ad)^5n\log(c+dx)}{5bd^5}\\ &\quad +\frac{(a+bx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5b}\end{aligned}$$

output
```
1/5*B*(-a*d+b*c)^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*n*(b*x+a)^2/b/d^3+1/15*B*(-
a*d+b*c)^2*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*n*(b*x+a)^4/b/d-1/5*B*(-a*d
+b*c)^5*n*ln(d*x+c)/b/d^5+1/5*(b*x+a)^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/
b
```

3.147.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 364 vs. 2(171) = 342.

Time = 0.54 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.13

$$\int (a+bx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{bdx(12a^4d^4(5A+4Bn)+12a^3bd^3(-10Bcn+10Adx+3Bdnx)+4a^2b^2d^2(30Ad^2x^2+Bn(30c^2-15cdx}{}$$

input
```
Integrate[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(12*a^4*d^4*(5*A + 4*B*n) + 12*a^3*b*d^3*(-10*B*c*n + 10*A*d*x + 3*
B*d*n*x) + 4*a^2*b^2*d^2*(30*A*d^2*x^2 + B*n*(30*c^2 - 15*c*d*x + 4*d^2*x^
2)) + b^4*(12*A*d^4*x^4 + B*c*n*(12*c^3 - 6*c^2*d*x + 4*c*d^2*x^2 - 3*d^3*
x^3)) + a*b^3*d*(60*A*d^3*x^3 + B*n*(-60*c^3 + 30*c^2*d*x - 20*c*d^2*x^2 +
 3*d^3*x^3))) - 48*a^5*B*d^5*n*Log[a + b*x] - 12*B*(b^5*c^5 - 5*a*b^4*c^4*
d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - 5*a^5*d^5)*n
*Log[c + d*x] + 12*B*d^5*(5*a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*
x^3 + 5*a*b^4*x^4 + b^5*x^5)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(60*b*d^5)
```

3.147.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(a+bx)^5 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad) \int \frac{(a+bx)^4}{c+dx}dx}{5b}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)^5 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad) \int \left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^5 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad) \left(\frac{(bc-ad)^4 \log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/5*(B*(b*c - a*d)*n*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*
x)^2)/(2*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((
b*c - a*d)^4*Log[c + d*x])/d^5))/b + ((a + b*x)^5*(A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]))/(5*b)
```

3.147.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.147.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(159) = 318$.

Time = 134.47 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.87

method	result
parallelrisch	$\frac{120A\,x^2a^4b^2c\,d^5n+48Bx\,a^5bc\,d^5n^2-120Bx\,a^4b^2c^2d^4n^2+120Bx\,a^3b^3c^3d^3n^2-60Bx\,a^2b^4c^4d^2n^2+12Bxa\,b^5c^5d\,n^2+60Ax\,a^5bc}{}$
risch	Expression too large to display

input
```
int((b*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/60*(120*A*x^2*a^4*b^2*c*d^5*n+48*B*x*a^5*b*c*d^5*n^2-120*B*x*a^4*b^2*c^2
*d^4*n^2+120*B*x*a^3*b^3*c^3*d^3*n^2-60*B*x*a^2*b^4*c^4*d^2*n^2+12*B*x*a*b
^5*c^5*d*n^2+60*A*x*a^5*b*c*d^5*n+60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b*c
^2*d^4*n-120*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^2*c^3*d^3*n+120*B*ln(e*(b
*x+a)^n/((d*x+c)^n))*a^3*b^3*c^4*d^2*n-60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^
2*b^4*c^5*d*n-60*B*ln(b*x+a)*a^5*b*c^2*d^4*n^2+120*B*ln(b*x+a)*a^4*b^2*c^3
*d^3*n^2-120*B*ln(b*x+a)*a^3*b^3*c^4*d^2*n^2+60*B*ln(b*x+a)*a^2*b^4*c^5*d*
n^2+12*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^6*n+12*B*ln(b*x+a)*a^6*c*d^5*
n^2-12*B*ln(b*x+a)*a*b^5*c^6*n^2+12*B*x^5*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^
5*c*d^5*n+12*A*x^5*a*b^5*c*d^5*n+3*B*x^4*a^2*b^4*c*d^5*n^2-3*B*x^4*a*b^5*c
^2*d^4*n^2+60*A*x^4*a^2*b^4*c*d^5*n+16*B*x^3*a^3*b^3*c*d^5*n^2-20*B*x^3*a^
2*b^4*c^2*d^4*n^2+4*B*x^3*a*b^5*c^3*d^3*n^2+120*A*x^3*a^3*b^3*c*d^5*n+36*B
*x^2*a^4*b^2*c*d^5*n^2-60*B*x^2*a^3*b^3*c^2*d^4*n^2+30*B*x^2*a^2*b^4*c^3*d
^3*n^2-6*B*x^2*a*b^5*c^4*d^2*n^2+60*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*
b^4*c*d^5*n+120*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^3*c*d^5*n+120*B*x^
2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^2*c*d^5*n+60*B*x*ln(e*(b*x+a)^n/((d*x+
c)^n))*a^5*b*c*d^5*n)/b/a/c/d^5/n
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(159) = 318$.

Time = 0.26 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.29

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5x^5 + 3\,(20\,Aab^4d^5 - (Bb^5cd^4 - Bab^4d^5)n)x^4 + 4\,(30\,Aa^2b^3d^5 + (Bb^5c^2d^3 - 5\,Bab^4cd^4 + 4\,Ba^2b^3c}{}$$

input
```
integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/60*(12*A*b^5*d^5*x^5 + 3*(20*A*a*b^4*d^5 - (B*b^5*c*d^4 - B*a*b^4*d^5)*n
)*x^4 + 4*(30*A*a^2*b^3*d^5 + (B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 4*B*a^2*b
^3*d^5)*n)*x^3 + 6*(20*A*a^3*b^2*d^5 - (B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3
+ 10*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*n)*x^2 + 12*(5*A*a^4*b*d^5 + (B*b^
5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 +
4*B*a^4*b*d^5)*n)*x + 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*n*x^4 + 10*B*a^2
*b^3*d^5*n*x^3 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x + B*a^5*d^5*n)
*log(b*x + a) - 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*n*x^4 + 10*B*a^2*b^3*d
^5*n*x^3 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x + (B*b^5*c^5 - 5*B*a
*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4
)*n)*log(d*x + c) + 12*(B*b^5*d^5*x^5 + 5*B*a*b^4*d^5*x^4 + 10*B*a^2*b^3*d
^5*x^3 + 10*B*a^3*b^2*d^5*x^2 + 5*B*a^4*b*d^5*x)*log(e))/(b*d^5)
```

3.147.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(159) = 318$.

Time = 0.22 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.92

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = \frac{1}{5} Bb^4x^5 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{5} Ab^4x^5 + Bab^3x^4 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aab^3x^4 + 2\,Ba^2b^2x^3 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 2\,Aa^2b^2x^3 + 2\,Ba^3bx^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 2\,Aa^3bx^2 + Ba^4x \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aa^4x + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Ba^4}{e} - \frac{2\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Ba^3b}{e} + \frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3} - \frac{2\,c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Ba^2b^2}{e} - \frac{\left(\frac{6\,a^4en\log(bx+a)}{b^4} - \frac{6\,c^4en\log(dx+c)}{d^4} + \frac{2\,(b^3cd^2en-ab^2d^3en)x^3-3\,(b^3c^2den-a^2bd^3en)x^2+6\,(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bab^3}{6\,e} + \frac{\left(\frac{12\,a^5en\log(bx+a)}{b^5} - \frac{12\,c^5en\log(dx+c)}{d^5} - \frac{3\,(b^4cd^3en-ab^3d^4en)x^4-4\,(b^4c^2d^2en-a^2b^2d^4en)x^3+6\,(b^4c^3den-a^3bd^4en)x^2-12\,(b^4c^4en-}{b^4d^4}\right.}{60\,e}$$

input
```
integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/5*B*b^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*b^4*x^5 + B*a*b^3*x^4
*log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^3*x^4 + 2*B*a^2*b^2*x^3*log((b*x +
 a)^n*e/(d*x + c)^n) + 2*A*a^2*b^2*x^3 + 2*B*a^3*b*x^2*log((b*x + a)^n*e/(
d*x + c)^n) + 2*A*a^3*b*x^2 + B*a^4*x*log((b*x + a)^n*e/(d*x + c)^n) + A*a
^4*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^4/e - 2*(a^2*e*n*
log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))
*B*a^3*b/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((
b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2
))*B*a^2*b^2/e - 1/6*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/
d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^
3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*a*b^3/e + 1/60*
(12*a^5*e*n*log(b*x + a)/b^5 - 12*c^5*e*n*log(d*x + c)/d^5 - (3*(b^4*c*d^3
*e*n - a*b^3*d^4*e*n)*x^4 - 4*(b^4*c^2*d^2*e*n - a^2*b^2*d^4*e*n)*x^3 + 6*
(b^4*c^3*d*e*n - a^3*b*d^4*e*n)*x^2 - 12*(b^4*c^4*e*n - a^4*d^4*e*n)*x)/(b
^4*d^4))*B*b^4/e
```

3.147. $\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

3.147.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(159) = 318$.

Time = 7.18 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.96

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{Ba^5n\log(bx+a)}{5\,b} + \frac{1}{5}\left(Bb^4\log(e)+Ab^4\right)x^5 - \frac{(Bb^4cn - Bab^3dn - 20\,Bab^3d\log(e) - 20\,Aab^3d)x^4}{20\,d} + \frac{(Bb^4c^2n - 5\,Bab^3cdn + 4\,Ba^2b^2d^2n + 30\,Ba^2b^2d^2\log(e) + 30\,Aa^2b^2d^2)x^3}{15\,d^2} + \frac{1}{5}\left(Bb^4nx^5 + 5\,Bab^3nx^4 + 10\,Ba^2b^2nx^3 + 10\,Ba^3bnx^2 + 5\,Ba^4nx\right)\log(bx+a) - \frac{1}{5}\left(Bb^4nx^5 + 5\,Bab^3nx^4 + 10\,Ba^2b^2nx^3 + 10\,Ba^3bnx^2 + 5\,Ba^4nx\right)\log(dx+c) - \frac{(Bb^4c^3n - 5\,Bab^3c^2dn + 10\,Ba^2b^2cd^2n - 6\,Ba^3bd^3n - 20\,Ba^3bd^3\log(e) - 20\,Aa^3bd^3)x^2}{10\,d^3} + \frac{(Bb^4c^4n - 5\,Bab^3c^3dn + 10\,Ba^2b^2c^2d^2n - 10\,Ba^3bcd^3n + 4\,Ba^4d^4n + 5\,Ba^4d^4\log(e) + 5\,Aa^4d^4)x}{5\,d^4} - \frac{(Bb^4c^5n - 5\,Bab^3c^4dn + 10\,Ba^2b^2c^3d^2n - 10\,Ba^3bc^2d^3n + 5\,Ba^4cd^4n)\log(-dx-c)}{5\,d^5}$$

input
```
integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
1/5*B*a^5*n*log(b*x + a)/b + 1/5*(B*b^4*log(e) + A*b^4)*x^5 - 1/20*(B*b^4*
c*n - B*a*b^3*d*n - 20*B*a*b^3*d*log(e) - 20*A*a*b^3*d)*x^4/d + 1/15*(B*b^
4*c^2*n - 5*B*a*b^3*c*d*n + 4*B*a^2*b^2*d^2*n + 30*B*a^2*b^2*d^2*log(e) +
30*A*a^2*b^2*d^2)*x^3/d^2 + 1/5*(B*b^4*n*x^5 + 5*B*a*b^3*n*x^4 + 10*B*a^2*
b^2*n*x^3 + 10*B*a^3*b*n*x^2 + 5*B*a^4*n*x)*log(b*x + a) - 1/5*(B*b^4*n*x^
5 + 5*B*a*b^3*n*x^4 + 10*B*a^2*b^2*n*x^3 + 10*B*a^3*b*n*x^2 + 5*B*a^4*n*x)
*log(d*x + c) - 1/10*(B*b^4*c^3*n - 5*B*a*b^3*c^2*d*n + 10*B*a^2*b^2*c*d^2
*n - 6*B*a^3*b*d^3*n - 20*B*a^3*b*d^3*log(e) - 20*A*a^3*b*d^3)*x^2/d^3 + 1
/5*(B*b^4*c^4*n - 5*B*a*b^3*c^3*d*n + 10*B*a^2*b^2*c^2*d^2*n - 10*B*a^3*b*
c*d^3*n + 4*B*a^4*d^4*n + 5*B*a^4*d^4*log(e) + 5*A*a^4*d^4)*x/d^4 - 1/5*(B
*b^4*c^5*n - 5*B*a*b^3*c^4*d*n + 10*B*a^2*b^2*c^3*d^2*n - 10*B*a^3*b*c^2*d
^3*n + 5*B*a^4*c*d^4*n)*log(-d*x - c)/d^5
```

3.147.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 936, normalized size of antiderivative = 5.47

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$

$$= x^4\left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{20\,d}-\frac{A\,b^3\,(5\,a\,d+5\,b\,c)}{20\,d}\right)$$

$$-x^3\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{15\,b\,d}\right.$$

$$\left.-\frac{a\,b^2\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}+\frac{A\,a\,b^3\,c}{3\,d}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(B\,a^4\,x+2\,B\,a^3\,b\,x^2+2\,B\,a^2\,b^2\,x^3+B\,a\,b^3\,x^4+\frac{B\,b^4\,x^5}{5}\right)$$

$$+x\left(\frac{a^3\,(5\,A\,a\,d+10\,A\,b\,c+2\,B\,a\,d\,n-2\,B\,b\,c\,n)}{d}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{2\,a^2\,b\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,(5\,a\,d+5}{5\,d}\right.}{5\,b\,d}}{5\,b\,d}\right.}{5\,b\,d}$$

$$\left.+\frac{a\,c\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{b^3\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{5\,b\,d}-\frac{a\,b^2\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b^3\,c}{d}\right)}{b\,d}\right)$$

$$+x^2\left(\frac{a^2\,b\,(5\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}\right.$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^4,x)
```

output
```
x^4*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(20*d) - (A*b^3*(5*a*d
 + 5*b*c))/(20*d)) - x^3*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a
*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(15*b*d) - (a*b^2
*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) + (A*a*b^3*c)/(3*d)) + lo
g((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^4*x^5)/5 + B*a^4*x + 2*B*a^3*b*x^2 +
B*a*b^3*x^4 + 2*B*a^2*b^2*x^3) + x*((a^3*(5*A*a*d + 10*A*b*c + 2*B*a*d*n -
 2*B*b*c*n))/d - ((5*a*d + 5*b*c)*((2*a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n -
 B*b*c*n))/d + ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*
c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) -
(a*b^2*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(5*b*
d) - (a*c*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(
5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*(((5*a*d + 5*b*c)*((b^3*(25
*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*
d)))/(5*b*d) - (a*b^2*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b
^3*c)/d))/(b*d)) + x^2*((a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d
+ ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n -
 B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) - (a*b^2*(10*A*
a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(10*b*d) - (a*c*((
b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*
c))/(5*d)))/(2*b*d)) + (A*b^4*x^5)/5 - (log(c + d*x)*(B*b^4*c^5*n + 5*B...
```

3.147. $\int (a+bx)^4 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

3.148 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

3.148.1 Optimal result

Integrand size = 31, antiderivative size = 142

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = -\frac{B(bc-ad)^3nx}{4d^3}+\frac{B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)n(a+bx)^3}{12bd}+\frac{B(bc-ad)^4n\log(c+dx)}{4bd^4}+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4b}$$

output
```
-1/4*B*(-a*d+b*c)^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*n*(b*x+a)^2/b/d^2-1/12*B*(-
a*d+b*c)*n*(b*x+a)^3/b/d+1/4*B*(-a*d+b*c)^4*n*ln(d*x+c)/b/d^4+1/4*(b*x+a)^
4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

3.148.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.92

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{bdx(6a^3d^3(4A+3Bn)+9a^2bd^2(-4Bcn+4Adx+Bdnx)+b^3(6Ad^3x^3+Bcn(-6c^2+3cdx-2d^2x^2))+}{}$$

input
```
Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(6*a^3*d^3*(4*A + 3*B*n) + 9*a^2*b*d^2*(-4*B*c*n + 4*A*d*x + B*d*n*
x) + b^3*(6*A*d^3*x^3 + B*c*n*(-6*c^2 + 3*c*d*x - 2*d^2*x^2)) + 2*a*b^2*d*
(12*A*d^2*x^2 + B*n*(12*c^2 - 6*c*d*x + d^2*x^2))) - 18*a^4*B*d^4*n*Log[a
+ b*x] + 6*B*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ 4*a^4*d^4)*n*Log[c + d*x] + 6*B*d^4*(4*a^4 + 4*a^3*b*x + 6*a^2*b^2*x^2 +
 4*a*b^3*x^3 + b^4*x^4)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(24*b*d^4)
```

3.148.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$

$$\Big\downarrow \ 2948$$

$$\frac{(a+bx)^4 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4b}-\frac{Bn(bc-ad) \int \frac{(a+bx)^3}{c+dx}dx}{4b}$$

$$\Big\downarrow \ 49$$

$$\frac{(a+bx)^4 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4b}-\frac{Bn(bc-ad) \int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}$$

$$\Big\downarrow \ 2009$$

$$\frac{(a+bx)^4 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4b}-\frac{Bn(bc-ad) \left(-\frac{(bc-ad)^3 \log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}$$

input
```
Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/4*(B*(b*c - a*d)*n*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)
/(2*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/b + ((a
+ b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*b)
```

3.148.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.148.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(132) = 264$.

Time = 52.80 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.68

method	result
parallelrisch	$\frac{-6B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{4}c^{4}n+3B\,x^{2}b^{4}c^{2}d^{2}n^{2}+18Bx\,a^{3}b\,d^{4}n^{2}-6Bx\,b^{4}c^{3}d\,n^{2}+24Ax\,a^{3}b\,d^{4}n+36A\,x^{2}a^{2}b^{2}d^{4}n+9B\,a^{3}b}{}$
risch	Expression too large to display

input
```
int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*n+3*B*x^2*b^4*c^2*d^2*n^
2+18*B*x*a^3*b*d^4*n^2-6*B*x*b^4*c^3*d*n^2+24*A*x*a^3*b*d^4*n+36*A*x^2*a^2
*b^2*d^4*n+9*B*a^3*b*c*d^3*n^2+24*B*a^2*b^2*c^2*d^2*n^2-21*B*a*b^3*c^3*d*n
^2-60*A*a^3*b*c*d^3*n+2*B*x^3*a*b^3*d^4*n^2-2*B*x^3*b^4*c*d^3*n^2+24*A*x^3
*a*b^3*d^4*n+9*B*x^2*a^2*b^2*d^4*n^2-18*B*a^4*d^4*n^2+6*B*b^4*c^4*n^2-24*A
*a^4*d^4*n+24*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*d^4*n+36*B*x^2*ln(e*
(b*x+a)^n/((d*x+c)^n))*a^2*b^2*d^4*n-12*B*x^2*a*b^3*c*d^3*n^2+24*B*x*ln(e*
(b*x+a)^n/((d*x+c)^n))*a^3*b*d^4*n-36*B*x*a^2*b^2*c*d^3*n^2+24*B*x*a*b^3*c
^2*d^2*n^2+24*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b*c*d^3*n-36*B*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^2*b^2*c^2*d^2*n+24*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*
c^3*d*n-24*B*ln(b*x+a)*a^3*b*c*d^3*n^2+36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*n^2-
24*B*ln(b*x+a)*a*b^3*c^3*d*n^2+6*A*x^4*b^4*d^4*n-6*B*ln(e*(b*x+a)^n/((d*x+
c)^n))*b^4*c^4*n+6*B*ln(b*x+a)*a^4*d^4*n^2+6*B*ln(b*x+a)*b^4*c^4*n^2)/d^4/
n/b
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(132) = 264$.

Time = 0.28 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.94

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4x^4 + 2\,(12\,Aab^3d^4 - (Bb^4cd^3 - Bab^3d^4)n)x^3 + 3\,(12\,Aa^2b^2d^4 + (Bb^4c^2d^2 - 4\,Bab^3cd^3 + 3\,Ba^2b^2d^4}{}$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/24*(6*A*b^4*d^4*x^4 + 2*(12*A*a*b^3*d^4 - (B*b^4*c*d^3 - B*a*b^3*d^4)*n)
*x^3 + 3*(12*A*a^2*b^2*d^4 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^
2*d^4)*n)*x^2 + 6*(4*A*a^3*b*d^4 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*
a^2*b^2*c*d^3 - 3*B*a^3*b*d^4)*n)*x + 6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n
*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x + B*a^4*d^4*n)*log(b*x +
a) - 6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*
B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B
*a^3*b*c*d^3)*n)*log(d*x + c) + 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B
*a^2*b^2*d^4*x^2 + 4*B*a^3*b*d^4*x)*log(e))/(b*d^4)
```

3.148.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(132) = 264.

Time = 0.21 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.29

$$\begin{aligned}
&\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx \\
&= \frac{1}{4}\,Bb^3x^4\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{4}\,Ab^3x^4 + Bab^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aab^2x^3 \\
&\quad + \frac{3}{2}\,Ba^2bx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{3}{2}\,Aa^2bx^2 + Ba^3x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aa^3x \\
&\quad + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Ba^3}{e} - \frac{3\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Ba^2b}{2\,e} \\
&\quad + \frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3} - \frac{2\,c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bab^2}{2\,e} \\
&\quad - \frac{\left(\frac{6\,a^4en\log(bx+a)}{b^4} - \frac{6\,c^4en\log(dx+c)}{d^4} + \frac{2\,(b^3cd^2en-ab^2d^3en)x^3-3\,(b^3c^2den-a^2bd^3en)x^2+6\,(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bb^3}{24\,e}
\end{aligned}$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/4*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*b^3*x^4 + B*a*b^2*x^3
*log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^2*x^3 + 3/2*B*a^2*b*x^2*log((b*x +
 a)^n*e/(d*x + c)^n) + 3/2*A*a^2*b*x^2 + B*a^3*x*log((b*x + a)^n*e/(d*x +
c)^n) + A*a^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^3/e -
3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*
e*n)*x/(b*d))*B*a^2*b/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(
d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2
*e*n)*x)/(b^2*d^2))*B*a*b^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e
*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*
d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B
*b^3/e
```

3.148.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(132) = 264$.

Time = 2.07 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.56

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{Ba^4 n\log(bx+a)}{4\,b} + \frac{1}{4}\left(Bb^3\log(e)+Ab^3\right)x^4 - \frac{(Bb^3cn - Bab^2dn - 12\,Bab^2d\log(e) - 12\,Aab^2d)x^3}{12\,d}$$
$$+ \frac{1}{4}\left(Bb^3nx^4 + 4\,Bab^2nx^3 + 6\,Ba^2bnx^2 + 4\,Ba^3nx\right)\log(bx+a)$$
$$- \frac{1}{4}\left(Bb^3nx^4 + 4\,Bab^2nx^3 + 6\,Ba^2bnx^2 + 4\,Ba^3nx\right)\log(dx+c)$$
$$+ \frac{(Bb^3c^2n - 4\,Bab^2cdn + 3\,Ba^2bd^2n + 12\,Ba^2bd^2\log(e) + 12\,Aa^2bd^2)x^2}{8\,d^2}$$
$$- \frac{(Bb^3c^3n - 4\,Bab^2c^2dn + 6\,Ba^2bcd^2n - 3\,Ba^3d^3n - 4\,Ba^3d^3\log(e) - 4\,Aa^3d^3)x}{4\,d^3}$$
$$+ \frac{(Bb^3c^4n - 4\,Bab^2c^3dn + 6\,Ba^2bc^2d^2n - 4\,Ba^3cd^3n)\log(dx+c)}{4\,d^4}$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
1/4*B*a^4*n*log(b*x + a)/b + 1/4*(B*b^3*log(e) + A*b^3)*x^4 - 1/12*(B*b^3*
c*n - B*a*b^2*d*n - 12*B*a*b^2*d*log(e) - 12*A*a*b^2*d)*x^3/d + 1/4*(B*b^3
*n*x^4 + 4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(b*x + a) - 1
/4*(B*b^3*n*x^4 + 4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(d*x
 + c) + 1/8*(B*b^3*c^2*n - 4*B*a*b^2*c*d*n + 3*B*a^2*b*d^2*n + 12*B*a^2*b*
d^2*log(e) + 12*A*a^2*b*d^2)*x^2/d^2 - 1/4*(B*b^3*c^3*n - 4*B*a*b^2*c^2*d*
n + 6*B*a^2*b*c*d^2*n - 3*B*a^3*d^3*n - 4*B*a^3*d^3*log(e) - 4*A*a^3*d^3)*
x/d^3 + 1/4*(B*b^3*c^4*n - 4*B*a*b^2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n - 4*B*a
^3*c*d^3*n)*log(d*x + c)/d^4
```

3.148. $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

3.148.9 Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.66

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= x^3 \left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{12\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{12\,d}\right)$$
$$+\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(B\,a^3\,x+\frac{3\,B\,a^2\,b\,x^2}{2}+B\,a\,b^2\,x^3+\frac{B\,b^3\,x^4}{4}\right)$$
$$+x\left(\frac{a^2\,\left(8\,A\,a\,d+12\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n\right)}{2\,d}\right.$$
$$+\frac{\left(4\,a\,d+4\,b\,c\right)\,\left(\frac{\left(4\,a\,d+4\,b\,c\right)\,\left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{4\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{4\,d}\right)}{4\,b\,d}-\frac{a\,b\,\left(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{d}+\frac{A\,a\,b^2}{d}\cdots}{4\,b\,d}$$
$$\left.-\frac{a\,c\,\left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{4\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{4\,d}\right)}{b\,d}\right)$$
$$-x^2\left(\frac{\left(4\,a\,d+4\,b\,c\right)\,\left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{4\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{4\,d}\right)}{8\,b\,d}\right.$$
$$\left.-\frac{a\,b\,\left(6\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{2\,d}+\frac{A\,a\,b^2\,c}{2\,d}\right)$$
$$+\frac{A\,b^3\,x^4}{4}+\frac{\ln\left(c+d\,x\right)\,\left(-4\,B\,n\,a^3\,c\,d^3+6\,B\,n\,a^2\,b\,c^2\,d^2-4\,B\,n\,a\,b^2\,c^3\,d+B\,n\,b^3\,c^4\right)}{4\,d^4}$$
$$+\frac{B\,a^4\,n\,\ln\left(a+b\,x\right)}{4\,b}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^3,x)
```

output
```
x^3*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*(4*a*d
 + 4*b*c))/(12*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^3*x^4)/4 + B*a
^3*x + (3*B*a^2*b*x^2)/2 + B*a*b^2*x^3) + x*((a^2*(8*A*a*d + 12*A*b*c + 3*
B*a*d*n - 3*B*b*c*n))/(2*d) + ((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((b^2*(16
*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(4*
d)))/(4*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^2*
c)/d))/(4*b*d) - (a*c*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d
) - (A*b^2*(4*a*d + 4*b*c))/(4*d)))/(b*d)) - x^2*(((4*a*d + 4*b*c)*((b^2*(
16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(
4*d)))/(8*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*
a*b^2*c)/(2*d)) + (A*b^3*x^4)/4 + (log(c + d*x)*(B*b^3*c^4*n - 4*B*a^3*c*d
^3*n - 4*B*a*b^2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n))/(4*d^4) + (B*a^4*n*log(a
+ b*x))/(4*b)
```

3.148. $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

3.149 $\int (a+bx)^2 (A+B\log (e(a+bx)^n(c+dx)^{-n}))\,dx$

3.149.1 Optimal result

Integrand size = 31, antiderivative size = 113

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{B(bc-ad)^2nx}{3d^2}-\frac{B(bc-ad)n(a+bx)^2}{6bd}-\frac{B(bc-ad)^3n\log(c+dx)}{3bd^3}+\frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b}$$

output
```
1/3*B*(-a*d+b*c)^2*n*x/d^2-1/6*B*(-a*d+b*c)*n*(b*x+a)^2/b/d-1/3*B*(-a*d+b*
c)^3*n*ln(d*x+c)/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

3.149.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.72

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{bdx(2a^2d^2(3A+2Bn)+abd(-6Bcn+6Adx+Bdnx)+b^2(2Ad^2x^2+Bcn(2c-dx)))-4a^3Bd^3n\log(a}{}$$

input
```
Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(2*a^2*d^2*(3*A + 2*B*n) + a*b*d*(-6*B*c*n + 6*A*d*x + B*d*n*x) + b
^2*(2*A*d^2*x^2 + B*c*n*(2*c - d*x))) - 4*a^3*B*d^3*n*Log[a + b*x] - 2*B*(
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - 3*a^3*d^3)*n*Log[c + d*x] + 2*B*
d^3*(3*a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)*Log[(e*(a + b*x)^n)/(c + d
*x)^n])/(6*b*d^3)
```

3.149.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{(a+bx)^3 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{(a+bx)^3 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)^3 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/3*(B*(b*c - a*d)*n*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*
c - a*d)^2*Log[c + d*x])/d^3))/b + ((a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n]))/(3*b)
```

3.149.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.149.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(105) = 210$.

Time = 18.61 (sec) , antiderivative size = 462, normalized size of antiderivative = 4.09

method	result
parallelrisch	$\frac{6B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)a^{2}bc\,d^{2}n-6B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)a\,b^{2}c^{2}dn-6B\ln(bx+a)a^{2}bc\,d^{2}n^{2}+6B\ln(bx+a)a\,b^{2}c^{2}d\,n^{2}+\cdots}{}$
risch	Expression too large to display

input
```
int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b*c*d^2*n-6*B*ln(e*(b*x+a)^n/((d*
x+c)^n))*a*b^2*c^2*d*n-6*B*ln(b*x+a)*a^2*b*c*d^2*n^2+6*B*ln(b*x+a)*a*b^2*c
^2*d*n^2+6*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^2*d^3*n+6*B*x*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^2*b*d^3*n-6*B*x*a*b^2*c*d^2*n^2+B*a^2*b*c*d^2*n^2+5*B*a
*b^2*c^2*d*n^2-12*A*a^2*b*c*d^2*n+6*A*x^2*a*b^2*d^3*n+4*B*x*a^2*b*d^3*n^2+
2*B*x*b^3*c^2*d*n^2+6*A*x*a^2*b*d^3*n+2*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*
b^3*d^3*n+B*x^2*a*b^2*d^3*n^2-B*x^2*b^3*c*d^2*n^2+2*B*ln(b*x+a)*a^3*d^3*n^
2-2*B*ln(b*x+a)*b^3*c^3*n^2+2*A*x^3*b^3*d^3*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^
n))*b^3*c^3*n-4*B*a^3*d^3*n^2-2*B*b^3*c^3*n^2-6*A*a^3*d^3*n)/b/d^3/n
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.50

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{2\,Ab^3d^3x^3 + (6\,Aab^2d^3 - (Bb^3cd^2 - Bab^2d^3)n)x^2 + 2\,(3\,Aa^2bd^3 + (Bb^3c^2d - 3\,Bab^2cd^2 + 2\,Ba^2bd^3)n)x +}{}$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/6*(2*A*b^3*d^3*x^3 + (6*A*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2
 + 2*(3*A*a^2*b*d^3 + (B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*n)*x
 + 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + B*a^3*d^
3*n)*log(b*x + a) - 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d
^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) +
 2*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x)*log(e))/(b*d^3)
```

3.149.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(105) = 210$.

Time = 0.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.60

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= \frac{1}{3} Bb^2x^3 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{3} Ab^2x^3 + Babx^2 \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aabx^2 + Ba^2x \log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aa^2x + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Ba^2}{e} - \frac{\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Bab}{e} + \frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3} - \frac{2\,c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bb^2}{6\,e}$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/3*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*b^2*x^3 + B*a*b*x^2*l
og((b*x + a)^n*e/(d*x + c)^n) + A*a*b*x^2 + B*a^2*x*log((b*x + a)^n*e/(d*x
 + c)^n) + A*a^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^2/e
 - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e
*n)*x/(b*d))*B*a*b/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x
 + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*
n)*x)/(b^2*d^2))*B*b^2/e
```

3.149.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(105) = 210$.

Time = 0.83 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.13

$$\begin{aligned}
&\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx \\
&= \frac{Ba^3n\log(bx+a)}{3b} + \frac{1}{3}\left(Bb^2\log(e)+Ab^2\right)x^3 \\
&\quad - \frac{(Bb^2cn - Babdn - 6\,Babd\log(e) - 6\,Aabd)x^2}{6\,d} \\
&\quad + \frac{1}{3}\left(Bb^2nx^3 + 3\,Babnx^2 + 3\,Ba^2nx\right)\log(bx+a) \\
&\quad - \frac{1}{3}\left(Bb^2nx^3 + 3\,Babnx^2 + 3\,Ba^2nx\right)\log(dx+c) \\
&\quad + \frac{(Bb^2c^2n - 3\,Babcdn + 2\,Ba^2d^2n + 3\,Ba^2d^2\log(e) + 3\,Aa^2d^2)x}{3\,d^2} \\
&\quad - \frac{(Bb^2c^3n - 3\,Babc^2dn + 3\,Ba^2cd^2n)\log(-dx-c)}{3\,d^3}
\end{aligned}$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
1/3*B*a^3*n*log(b*x + a)/b + 1/3*(B*b^2*log(e) + A*b^2)*x^3 - 1/6*(B*b^2*c
*n - B*a*b*d*n - 6*B*a*b*d*log(e) - 6*A*a*b*d)*x^2/d + 1/3*(B*b^2*n*x^3 +
3*B*a*b*n*x^2 + 3*B*a^2*n*x)*log(b*x + a) - 1/3*(B*b^2*n*x^3 + 3*B*a*b*n*x
^2 + 3*B*a^2*n*x)*log(d*x + c) + 1/3*(B*b^2*c^2*n - 3*B*a*b*c*d*n + 2*B*a^
2*d^2*n + 3*B*a^2*d^2*log(e) + 3*A*a^2*d^2)*x/d^2 - 1/3*(B*b^2*c^3*n - 3*B
*a*b*c^2*d*n + 3*B*a^2*c*d^2*n)*log(-d*x - c)/d^3
```

3.149.9 Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.32

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(B\,a^2\,x+B\,a\,b\,x^2+\frac{B\,b^2\,x^3}{3}\right)$$
$$+x^2\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{3\,d}\right)(3\,a\,d+3\,b\,c)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b\,c}{d}\right)+\frac{A\,b^2\,x^3}{3}$$
$$-\frac{\ln(c+d\,x)\,(3\,B\,n\,a^2\,c\,d^2-3\,B\,n\,a\,b\,c^2\,d+B\,n\,b^2\,c^3)}{3\,d^3}+\frac{B\,a^3\,n\,\ln(a+b\,x)}{3\,b}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^2,x)
```

output
```
log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^2*x^3)/3 + B*a^2*x + B*a*b*x^2) + x
^2*((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*(3*a*d + 3*b*
c))/(6*d)) - x*((((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(3*d) - (A*b
*(3*a*d + 3*b*c))/(3*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*(3*A*a*d + 3*A*b*c
+ B*a*d*n - B*b*c*n))/d + (A*a*b*c)/d) + (A*b^2*x^3)/3 - (log(c + d*x)*(B*
b^2*c^3*n + 3*B*a^2*c*d^2*n - 3*B*a*b*c^2*d*n))/(3*d^3) + (B*a^3*n*log(a +
 b*x))/(3*b)
```

3.150 $\int (a+bx) (A + B \log (e(a + bx)^n (c + dx)^{-n})) \, dx$

3.150.1 Optimal result

Integrand size = 29, antiderivative size = 84

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = -\frac{B(bc-ad)nx}{2d}+\frac{B(bc-ad)^2n\log(c+dx)}{2bd^2} + \frac{(a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2b}$$

output
```
-1/2*B*(-a*d+b*c)*n*x/d+1/2*B*(-a*d+b*c)^2*n*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

3.150.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{-a^2Bd^2n\log(a+bx)+B(b^2c^2-2abcd+2a^2d^2)\,n\log(c+dx)+d(bx(2aAd-bBcn+aBdn+Abdx)+}{2bd^2}$$

input
```
Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(-(a^2*B*d^2*n*Log[a + b*x]) + B*(b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*n*Log[c
 + d*x] + d*(b*x*(2*a*A*d - b*B*c*n + a*B*d*n + A*b*d*x) + B*d*(2*a^2 + 2*
a*b*x + b^2*x^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b*d^2)
```

3.150.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx) \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(a+bx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2b}-\frac{Bn(bc-ad) \int \frac{a+bx}{c+dx}dx}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2b}-\frac{Bn(bc-ad) \int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2b}-\frac{Bn(bc-ad) \left(\frac{bx}{d}-\frac{(bc-ad) \log(c+dx)}{d^2}\right)}{2b}$$

input
```
Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*n*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/b + ((a
+ b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b)
```

3.150.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.150.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(78) = 156$.

Time = 5.40 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.15

method	result
parallelrisch	$\frac{B x^2 \ln\left(e(bx+a)^n (dx+c)^{-n}\right) b^2 d^2 n + A x^2 b^2 d^2 n + B \ln(bx+a) a^2 d^2 n^2 - 2B \ln(bx+a) abcd n^2 + B \ln(bx+a) b^2 c^2 n^2 + 2Bx \ln\left(e(bx\right.}{}$
risch	Expression too large to display

input
```
int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*d^2*n+A*x^2*b^2*d^2*n+B*ln(b*x+
a)*a^2*d^2*n^2-2*B*ln(b*x+a)*a*b*c*d*n^2+B*ln(b*x+a)*b^2*c^2*n^2+2*B*x*ln(
e*(b*x+a)^n/((d*x+c)^n))*a*b*d^2*n+B*x*a*b*d^2*n^2-B*x*b^2*c*d*n^2+2*A*x*a
*b*d^2*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b*c*d*n-B*ln(e*(b*x+a)^n/((d*x+
c)^n))*b^2*c^2*n-B*a^2*d^2*n^2+B*b^2*c^2*n^2-2*A*a^2*d^2*n-3*A*a*b*c*d*n)/
b/d^2/n
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(78) = 156$.

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.94

$$\int (a+bx) \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\, dx$$
$$= \frac{Ab^2d^2x^2 + (2\,Aabd^2 - (Bb^2cd - Babd^2)n)x + (Bb^2d^2nx^2 + 2\,Babd^2nx + Ba^2d^2n)\log(bx+a) - (Bb^2d^2n}{2\,bd^2}$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas"
)
```

output
```
1/2*(A*b^2*d^2*x^2 + (2*A*a*b*d^2 - (B*b^2*c*d - B*a*b*d^2)*n)*x + (B*b^2*
d^2*n*x^2 + 2*B*a*b*d^2*n*x + B*a^2*d^2*n)*log(b*x + a) - (B*b^2*d^2*n*x^2
 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c) + (B*b^2*d^
2*x^2 + 2*B*a*b*d^2*x)*log(e))/(b*d^2)
```

3.150.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83

$$\begin{aligned}&\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx\\ &= \frac{1}{2}\,Bbx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}\,Abx^2+Bax\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aax\\ &\quad+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Ba}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bb}{2\,e}\end{aligned}$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima"
)
```

output
```
1/2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*b*x^2 + B*a*x*log((b*x
+ a)^n*e/(d*x + c)^n) + A*a*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)
/d)*B*a/e - 1/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*
c*e*n - a*d*e*n)*x/(b*d))*B*b/e
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.56

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{Ba^2n\log(bx+a)}{2\,b}+\frac{1}{2}\,(Bb\log(e)+Ab)x^2 + \frac{1}{2}\left(Bbnx^2+2\,Banx\right)\log(bx+a)-\frac{1}{2}\left(Bbnx^2+2\,Banx\right)\log(dx+c) - \frac{(Bbcn-Badn-2\,Bad\log(e)-2\,Aad)x}{2\,d}+\frac{(Bbc^2n-2\,Bacdn)\log(dx+c)}{2\,d^2}$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

output
```
1/2*B*a^2*n*log(b*x + a)/b + 1/2*(B*b*log(e) + A*b)*x^2 + 1/2*(B*b*n*x^2 +
 2*B*a*n*x)*log(b*x + a) - 1/2*(B*b*n*x^2 + 2*B*a*n*x)*log(d*x + c) - 1/2*
(B*b*c*n - B*a*d*n - 2*B*a*d*log(e) - 2*A*a*d)*x/d + 1/2*(B*b*c^2*n - 2*B*
a*c*d*n)*log(d*x + c)/d^2
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.51

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{B\,b\,x^2}{2}+B\,a\,x\right) + x\left(\frac{4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n}{2\,d}-\frac{A\,(2\,a\,d+2\,b\,c)}{2\,d}\right) + \frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,n-2\,B\,a\,c\,d\,n\right)}{2\,d^2}+\frac{A\,b\,x^2}{2}+\frac{B\,a^2\,n\,\ln(a+b\,x)}{2\,b}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x),x)
```

output
```
log((e*(a + b*x)^n)/(c + d*x)^n)*(B*a*x + (B*b*x^2)/2) + x*((4*A*a*d + 2*A
*b*c + B*a*d*n - B*b*c*n)/(2*d) - (A*(2*a*d + 2*b*c))/(2*d)) + (log(c + d*
x)*(B*b*c^2*n - 2*B*a*c*d*n))/(2*d^2) + (A*b*x^2)/2 + (B*a^2*n*log(a + b*x
))/(2*b)
```

3.151 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx$

3.151.1 Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b}+\frac{Bn\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b+B*n*polylog(2
,1+(-a*d+b*c)/d/(b*x+a))/b
```

3.151.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \frac{-Bn\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+2A\log(a+bx)-2B\log\left(\frac{-bc+ad}{d(a+bx)}\right)\left(n\log\left(\frac{b(c+dx)}{bc-ad}\right)+\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) -}{2b}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
```

output
```
(-(B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2) + 2*A*Log[a + b*x] - 2*B*Log[(
-(b*c) + a*d)/(d*(a + b*x))]*(n*Log[(b*(c + d*x))/(b*c - a*d)] + Log[(e*(a
 + b*x)^n)/(c + d*x)^n]) + 2*B*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])
/(2*b)
```

3.151.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{a+bx}dx$$

$$\Big\downarrow \text{2942}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{2858}$$

$$\frac{Bn(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{2778}$$

$$-\frac{Bn(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\Big\downarrow \text{2005}$$

$$-\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

$$\downarrow \text{2752}$$

$$\frac{Bn\,\text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)}+1\right)}{b} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A)}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n]))/b) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b
```

3.151.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.151. $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx$

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

3.151.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.59 (sec) , antiderivative size = 523, normalized size of antiderivative = 6.62

method	result
risch	$-\frac{B\ln(bx+a)\ln((dx+c)^n)}{b}+\frac{Bn\,\mathrm{dilog}\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{b}+\frac{Bn\ln(bx+a)\ln\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{b}+\frac{iB\pi\,\mathrm{csgn}(ie)\,\mathrm{csgn}\left(ie(dx+c\right.}{2b}$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-B/b*ln(b*x+a)*ln((d*x+c)^n)+1/b*B*n*dilog((-a*d+c*b+d*(b*x+a))/(-a*d+b*c)
)+1/b*B*n*ln(b*x+a)*ln((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))+1/2*I/b*B*Pi*csgn(
I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*ln(b*x+a)+1/2*I/b*B*Pi*csgn(I*(b*x+
a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*ln(b*x+a)+1/2*I/b*B*Pi*csgn(I/((d*x+
c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*ln(b*x+a)+1/2*I/b*B*Pi*csgn(I*(b*x+
a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*ln(b*x+a)+A*ln(b*x+a)/
b+1/b*B*ln(e)*ln(b*x+a)+1/2/b*B/n*ln((b*x+a)^n)^2-1/2*I/b*B*Pi*csgn(I*(b*x
+a)^n/((d*x+c)^n))^3*ln(b*x+a)-1/2*I/b*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^3*ln(b*x+a)-1/2*I/b*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*
e/((d*x+c)^n)*(b*x+a)^n)*ln(b*x+a)-1/2*I/b*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/(
(d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*ln(b*x+a)
```

3.151.5 Fricas [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="fricas"
)
```

output
```
integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

3.151.6 Sympy [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int \frac{A+B\log\left(e(a+bx)^n\left(c+dx\right)^{-n}\right)}{a+bx}\,dx$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a),x)
```

output
```
Integral((A + B*log(e*(a + b*x)**n/(c + d*x)**n))/(a + b*x), x)
```

3.151.7 Maxima [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="maxima"
)
```

output
```
B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/b + int
egrate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d*x
^2 + a*b*c + (b^2*c + a*b*d)*x), x)) + A*log(b*x + a)/b
```

3.151.8 Giac [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{a+b\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x), x)
```

3.152 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx$

3.152.1 Optimal result

Integrand size = 31, antiderivative size = 97

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{Bn}{b(a+bx)}-\frac{Bdn\log(a+bx)}{b(bc-ad)}+\frac{Bdn\log(c+dx)}{b(bc-ad)} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)}$$

output
```
-B*n/b/(b*x+a)-B*d*n*ln(b*x+a)/b/(-a*d+b*c)+B*d*n*ln(d*x+c)/b/(-a*d+b*c)+(
-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)
```

3.152.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = \frac{-Bdn(a+bx)\log(a+bx)+Bdn(a+bx)\log(c+dx)-(bc-ad)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-\right.}\right.}{b(bc-ad)(a+bx)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

output
```
(-(B*d*n*(a + b*x)*Log[a + b*x]) + B*d*n*(a + b*x)*Log[c + d*x] - (b*c - a
*d)*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)*(a + b*
x))
```

3.152.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2948, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)^2}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^2(c+dx)}dx}{b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int \left(\frac{d^2}{(bc-ad)^2(c+dx)}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{b}{(bc-ad)(a+bx)^2}\right)dx}{b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\right)}{b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

output
```
(B*(b*c - a*d)*n*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a
*d)^2 + (d*Log[c + d*x])/(b*c - a*d)^2))/b - (A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])/(b*(a + b*x))
```

3.152.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.152.4 Maple [A] (verified)

Time = 5.72 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

method	result
parallelrisch	$-\frac{-Bx\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{3}d^{2}n-B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{3}cdn+Ba\,b^{2}d^{2}n^{2}-B\,b^{3}cd\,n^{2}+Aa\,b^{2}d^{2}n-A\,b^{3}cdn}{(bx+a)b^{3}dn(ad-cb)}$
risch	$\frac{B\ln((dx+c)^{n})}{b(bx+a)}-\frac{2Abc-2Badn+2Bbcn-2Aad-2Bad\ln((bx+a)^{n})-2B\ln(e)ad-2B\ln(dx+c)adn+2B\ln(-bx-a)adn+iB\pi$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n-B*ln(e*(b*x+a)^n/((d*x+c)^n))
*b^3*c*d*n+B*a*b^2*d^2*n^2-B*b^3*c*d*n^2+A*a*b^2*d^2*n-A*b^3*c*d*n)/(b*x+a
)/b^3/d/n/(a*d-b*c)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)}{(a+bx)^{2}}\,dx = -\frac{Abc-Aad+(Bbc-Bad)n+(Bbdnx+Bbcn)\log\left(bx+a\right)-(Bbdnx+Bbcn)\log\left(dx+c\right)+(Bbc-}{ab^{2}c-a^{2}bd+(b^{3}c-ab^{2}d)x}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="frica
s")
```

output
```
-(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*d*n*x + B*b*c*n)*log(b*x + a) -
 (B*b*d*n*x + B*b*c*n)*log(d*x + c) + (B*b*c - B*a*d)*log(e))/(a*b^2*c - a
^2*b*d + (b^3*c - a*b^2*d)*x)
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**2,x)
```

output
```
Timed out
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx=-\frac{\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)B}{e}-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^2x+ab}-\frac{A}{b^2x+ab}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="maxim
a")
```

output
```
-(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d)
+ e*n/(b^2*x + a*b))*B/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b)
- A/(b^2*x + a*b)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx=-\frac{Bdn\log(bx+a)}{b^2c-abd}+\frac{Bdn\log(dx+c)}{b^2c-abd}-\frac{Bn\log(bx+a)}{b^2x+ab}+\frac{Bn\log(dx+c)}{b^2x+ab}-\frac{Bn+B\log(e)+A}{b^2x+ab}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="giac"
)
```

output
```
-B*d*n*log(b*x + a)/(b^2*c - a*b*d) + B*d*n*log(d*x + c)/(b^2*c - a*b*d) -
 B*n*log(b*x + a)/(b^2*x + a*b) + B*n*log(d*x + c)/(b^2*x + a*b) - (B*n +
B*log(e) + A)/(b^2*x + a*b)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = -\frac{A+B\,n}{x\,b^2+a\,b}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{b\,(a+b\,x)} - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^2,x)
```

output
```
- (A + B*n)/(a*b + b^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(b*(a + b
*x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c
))
```

3.153 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$

3.153.1 Optimal result

Integrand size = 31, antiderivative size = 137

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{Bn}{4b(a+bx)^2}+\frac{Bdn}{2b(bc-ad)(a+bx)} + \frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}$$

output
```
-1/4*B*n/b/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/(b*x+a)+1/2*B*d^2*n*ln(b*x+a)/
b/(-a*d+b*c)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2+1/2*(-A-B*ln(e*(b*x+a)
^n/((d*x+c)^n)))/b/(b*x+a)^2
```

3.153.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{\frac{2A}{(a+bx)^2}+Bn\left(\frac{1+\frac{2d(a+bx)}{-bc+ad}}{(a+bx)^2}-\frac{2d^2\log(a+bx)}{(bc-ad)^2}+\frac{2d^2\log(c+dx)}{(bc-ad)^2}\right)+\frac{2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}}{4b}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

output
```
-1/4*((2*A)/(a + b*x)^2 + B*n*((1 + (2*d*(a + b*x))/(-(b*c) + a*d))/(a + b
*x)^2 - (2*d^2*Log[a + b*x])/(b*c - a*d)^2 + (2*d^2*Log[c + d*x])/(b*c - a
*d)^2) + (2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2)/b
```

3.153.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)^3}dx$$

$$\Big\downarrow \ 2948$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{2b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}$$

$$\Big\downarrow \ 54$$

$$\frac{Bn(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{2b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{Bn(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{2b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

output
```
(B*(b*c - a*d)*n*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a +
 b*x)) + (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)
^3))/(2*b) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*(a + b*x)^2)
```

3.153.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.153.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(130) = 260$.

Time = 19.18 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.44

method	result
parallelrisch	$-\frac{-4Bac\,d^{2}n\,b^{4}+2B\ln(dx+c)x^{2}b^{5}d^{3}n-2B\ln(bx+a)a^{2}b^{3}d^{3}n+2B\ln(dx+c)a^{2}b^{3}d^{3}n+2Bxa\,b^{4}d^{3}n-2Bx\,b^{5}c\,d^{2}n-4B\ln\Big(e(bx}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*B*a*c*d^2*n*b^4+2*B*ln(d*x+c)*x^2*b^5*d^3*n-2*B*ln(b*x+a)*a^2*b^3
*d^3*n+2*B*ln(d*x+c)*a^2*b^3*d^3*n+2*B*x*a*b^4*d^3*n-2*B*x*b^5*c*d^2*n-4*B
*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2+2*A*a^2*b^3*d^3+2*A*b^5*c^2*d-4*A
*a*b^4*c*d^2-2*B*ln(b*x+a)*x^2*b^5*d^3*n+3*B*a^2*b^3*d^3*n+2*B*ln(e*(b*x+a
)^n/((d*x+c)^n))*a^2*b^3*d^3+2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d+B*b
^5*c^2*n*d-4*B*ln(b*x+a)*x*a*b^4*d^3*n+4*B*ln(d*x+c)*x*a*b^4*d^3*n)/(a^2*d
^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2/b^4/d
```

3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(127) = 254$.

Time = 0.27 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.16

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{2\,Ab^2c^2 - 4\,Aabcd + 2\,Aa^2d^2 - 2\,(Bb^2cd - Babd^2)nx + (Bb^2c^2 - 4\,Babcd + 3\,Ba^2d^2)n - 2\,(Bb^2d^2nx^2}{4\,(a^2b^3c^2 - 2\,a^3b^2cd}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="frica
s")
```

output
```
-1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*
n*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n - 2*(B*b^2*d^2*n*x^2 + 2*B
*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(b*x + a) + 2*(B*b^2*d^2*n*
x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c) + 2*(B*b
^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a
^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^
2*b^3*c*d + a^3*b^2*d^2)*x)
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**3,x)
```

output
```
Timed out
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.68

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$$
$$= \frac{\left(\frac{2\,d^2en\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2} - \frac{2\,d^2en\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2} + \frac{2\,bdenx-bcen+3\,aden}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)B}{4\,e}$$
$$- \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\,(b^3x^2+2\,ab^2x+a^2b)} - \frac{A}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="maxim
a")
```

output
```
1/4*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*
n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*
n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c -
 a^2*b^2*d)*x))*B/e - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^3*x^2 + 2*a*
b^2*x + a^2*b) - 1/2*A/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$$
$$= \frac{Bd^2n\log(bx+a)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)} - \frac{Bd^2n\log(dx+c)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)}$$
$$- \frac{Bn\log(bx+a)}{2\,(b^3x^2+2\,ab^2x+a^2b)} + \frac{Bn\log(dx+c)}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$
$$+ \frac{2\,Bbdnx-Bbcn+3\,Badn-2\,Bbc\log(e)+2\,Bad\log(e)-2\,Abc+2\,Aad}{4\,(b^4cx^2-ab^3dx^2+2\,ab^3cx-2\,a^2b^2dx+a^2b^2c-a^3bd)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="giac"
)
```

output
```
1/2*B*d^2*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*d^2*n
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*n*log(b*x + a)/(
b^3*x^2 + 2*a*b^2*x + a^2*b) + 1/2*B*n*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x +
 a^2*b) + 1/4*(2*B*b*d*n*x - B*b*c*n + 3*B*a*d*n - 2*B*b*c*log(e) + 2*B*a*
d*log(e) - 2*A*b*c + 2*A*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2*a
^2*b^2*d*x + a^2*b^2*c - a^3*b*d)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.40

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = -\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)} - \frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,{\left(a\,d-b\,c\right)}^2}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^3,x)
```

output
```
- ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)
/(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (B*log((e*(a + b*x)^n)/(
c + d*x)^n))/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) - (B*d^2*n*atanh((2*b^3*c^2 -
 2*a^2*b*d^2)/(2*b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)
^2)
```

3.154 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$

3.154.1 Optimal result

Integrand size = 31, antiderivative size = 166

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{Bn}{9b(a+bx)^3}+\frac{Bdn}{6b(bc-ad)(a+bx)^2} - \frac{Bd^2n}{3b(bc-ad)^2(a+bx)} - \frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}$$

output
```
-1/9*B*n/b/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/(b*x+a)^2-1/3*B*d^2*n/b/(-a*d+
b*c)^2/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3+1/3*B*d^3*n*ln(d*x+c)/
b/(-a*d+b*c)^3+1/3*(-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)^3
```

3.154.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{\frac{6A}{(a+bx)^3}+Bn\left(\frac{2+\frac{3d(a+bx)}{-bc+ad}+\frac{6d^2(a+bx)^2}{(bc-ad)^2}}{(a+bx)^3}+\frac{6d^3\log(a+bx)}{(bc-ad)^3}-\frac{6d^3\log(c+dx)}{(bc-ad)^3}\right)+\frac{6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}}{18b}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

output
```
-1/18*((6*A)/(a + b*x)^3 + B*n*((2 + (3*d*(a + b*x))/(-(b*c) + a*d) + (6*d
^2*(a + b*x)^2)/(b*c - a*d)^2)/(a + b*x)^3 + (6*d^3*Log[a + b*x])/(b*c - a
*d)^3 - (6*d^3*Log[c + d*x])/(b*c - a*d)^3) + (6*B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])/(a + b*x)^3)/b
```

3.154.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2948, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)^4}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3b}-\frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3b}-\frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3b}-\frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

output
```
(B*(b*c - a*d)*n*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(a
 + b*x)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*d
)^4 + (d^3*Log[c + d*x])/(b*c - a*d)^4))/(3*b) - (A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])/(3*b*(a + b*x)^3)
```

3.154.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.154.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(157) = 314$.

Time = 54.43 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.04

method	result
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-6B\ln(bx+a)x^3b^7d^4n+6B\,x^2a\,b^6d^4n-6B\,x^2b^7c\,d^3n+15Bx\,a^2b^5d^4n+3Bx\,b^7c^2d^2n+6B\ln(d}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(-18*A*a^2*b^5*c*d^3+18*A*a*b^6*c^2*d^2+6*B*ln(e*(b*x+a)^n/((d*x+c)^
n))*a^3*b^4*d^4-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c^3*d-6*B*ln(b*x+a)*x^
3*b^7*d^4*n+6*B*x^2*a*b^6*d^4*n-6*B*x^2*b^7*c*d^3*n+15*B*x*a^2*b^5*d^4*n+3
*B*x*b^7*c^2*d^2*n-18*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3+18*B*ln(
e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2+6*B*ln(d*x+c)*x^3*b^7*d^4*n-6*B*ln(
b*x+a)*a^3*b^4*d^4*n+6*B*ln(d*x+c)*a^3*b^4*d^4*n-18*B*x*a*b^6*c*d^3*n-18*B
*ln(b*x+a)*x^2*a*b^6*d^4*n+18*B*ln(d*x+c)*x^2*a*b^6*d^4*n-18*B*ln(b*x+a)*x
*a^2*b^5*d^4*n+18*B*ln(d*x+c)*x*a^2*b^5*d^4*n-18*B*a^2*b^5*c*d^3*n+9*B*a*b
^6*c^2*d^2*n+6*A*a^3*b^4*d^4-6*A*b^7*c^3*d+11*B*a^3*b^4*d^4*n-2*B*b^7*c^3*
d*n)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^3/b^5/d
```

3.154.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(154) = 308$.

Time = 0.29 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.25

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = -\frac{6\,Ab^3c^3 - 18\,Aab^2c^2d + 18\,Aa^2bcd^2 - 6\,Aa^3d^3 + 6\,(Bb^3cd^2 - Bab^2d^3)nx^2 - 3\,(Bb^3c^2d - 6\,Bab^2cd^2 + 5}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="frica
s")
```

output
```
-1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6
*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*
B*a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*
B*a^3*d^3)*n + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*
x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(b*x + a) - 6*(B
*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*
B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) + 6*(B*b^3*c^3 - 3*B*a*b^
2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^
2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c
*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2
 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 -
 a^5*b^2*d^3)*x)
```

3.154.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**4,x)
```

output
```
Timed out
```

3.154.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(154) = 308$.

Time = 0.20 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.41

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx=$$
$$-\frac{\left(\frac{6\,d^3en\log(bx+a)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{6\,d^3en\log(dx+c)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}+\frac{6\,b^2d^2enx^2+2\,b^2c^2en-7\,abcden+11\,a\cdots}{a^3b^3c^2-2\,a^4b^2cd+a^5bd^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^3+3\,(ab^5c^2\cdots}\cdots\right.}{18\,e}$$
$$-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}-\frac{A}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="maxim
a")
```

output
```
-1/18*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*
c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n +
11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b
^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2)*x))*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a
*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/3*A/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2
*x + a^3*b)
```

3.154.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(154) = 308$.

Time = 0.29 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.73

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$$
$$= -\frac{Bd^3n\log(bx+a)}{3\,(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3)} + \frac{Bd^3n\log(dx+c)}{3\,(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3)}$$
$$- \frac{Bn\log(bx+a)}{3\,(b^4x^3 + 3\,ab^3x^2 + 3\,a^2b^2x + a^3b)} + \frac{Bn\log(dx+c)}{3\,(b^4x^3 + 3\,ab^3x^2 + 3\,a^2b^2x + a^3b)}$$
$$- \frac{6\,Bb^2d^2nx^2 - 3\,Bb^2cdnx + 15\,Babd^2nx + 2\,Bb^2c^2n - 7\,Babcdn + 11\,Ba^2d^2n + 6\,Bb^2c^2\log(e) - 12\,Ba}{18\,(b^6c^2x^3 - 2\,ab^5cdx^3 + a^2b^4d^2x^3 + 3\,ab^5c^2x^2 - 6\,a^2b^4cdx^2 + 3\,a^3b^3d^2x^2 + 3\,a^2b^4c^2x - 6\,a}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="giac"
)
```

output
```
-1/3*B*d^3*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3
*b*d^3) + 1/3*B*d^3*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3) - 1/3*B*n*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2
*x + a^3*b) + 1/3*B*n*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x +
a^3*b) - 1/18*(6*B*b^2*d^2*n*x^2 - 3*B*b^2*c*d*n*x + 15*B*a*b*d^2*n*x + 2*
B*b^2*c^2*n - 7*B*a*b*c*d*n + 11*B*a^2*d^2*n + 6*B*b^2*c^2*log(e) - 12*B*a
*b*c*d*log(e) + 6*B*a^2*d^2*log(e) + 6*A*b^2*c^2 - 12*A*a*b*c*d + 6*A*a^2*
d^2)/(b^6*c^2*x^3 - 2*a*b^5*c*d*x^3 + a^2*b^4*d^2*x^3 + 3*a*b^5*c^2*x^2 -
6*a^2*b^4*c*d*x^2 + 3*a^3*b^3*d^2*x^2 + 3*a^2*b^4*c^2*x - 6*a^3*b^3*c*d*x
+ 3*a^4*b^2*d^2*x + a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = \frac{2\,A\,a\,c\,d}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{3\,b\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2\,n}{9\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,n\,x}{6\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,d^2\,n\,x^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{7\,B\,a\,c\,d\,n}{18\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{11\,B\,a^2\,d^2\,n}{18\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,n\,x}{6\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,{\left(a\,d-b\,c\right)}^3}$$

input

```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^4,x)
```

output

```
(2*A*a*c*d)/(3*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*(a*d - b*c)^2*(a
+ b*x)^3) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(3*b*(a + b*x)^3) - (A*a^
2*d^2)/(3*b*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2*n)/(9*(a*d - b*c)^2*(a +
 b*x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*
b*(a*d - b*c)^3) - (5*B*a*d^2*n*x)/(6*(a*d - b*c)^2*(a + b*x)^3) - (B*b*d^
2*n*x^2)/(3*(a*d - b*c)^2*(a + b*x)^3) + (7*B*a*c*d*n)/(18*(a*d - b*c)^2*(
a + b*x)^3) - (11*B*a^2*d^2*n)/(18*b*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c*d
*n*x)/(6*(a*d - b*c)^2*(a + b*x)^3)
```

3.155 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$

3.155.1 Optimal result

Integrand size = 31, antiderivative size = 195

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = -\frac{Bn}{16b(a+bx)^4}+\frac{Bdn}{12b(bc-ad)(a+bx)^3} - \frac{Bd^2n}{8b(bc-ad)^2(a+bx)^2}+\frac{Bd^3n}{4b(bc-ad)^3(a+bx)} + \frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4}$$

output
```
-1/16*B*n/b/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/(b*x+a)^3-1/8*B*d^2*n/b/(-a*
d+b*c)^2/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/(b*x+a)+1/4*B*d^4*n*ln(b*x+a
)/b/(-a*d+b*c)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4+1/4*(-A-B*ln(e*(b*x+
a)^n/((d*x+c)^n)))/b/(b*x+a)^4
```

3.155.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = -\frac{\frac{12A}{(a+bx)^4}+Bn\left(\frac{3+\frac{4d(a+bx)}{-bc+ad}+\frac{6d^2(a+bx)^2}{(bc-ad)^2}-\frac{12d^3(a+bx)^3}{(bc-ad)^3}}{(a+bx)^4}-\frac{12d^4\log(a+bx)}{(bc-ad)^4}+\frac{12d^4\log(c+dx)}{(bc-ad)^4}\right)+\frac{12B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}}{48b}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

output
```
-1/48*((12*A)/(a + b*x)^4 + B*n*((3 + (4*d*(a + b*x))/(-(b*c) + a*d) + (6*
d^2*(a + b*x)^2)/(b*c - a*d)^2 - (12*d^3*(a + b*x)^3)/(b*c - a*d)^3)/(a +
b*x)^4 - (12*d^4*Log[a + b*x])/(b*c - a*d)^4 + (12*d^4*Log[c + d*x])/(b*c
- a*d)^4) + (12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4)/b
```

3.155.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)^5}\,dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{4b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{Bn(bc-ad)\int \left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+bx}\right.}{4b}$$
$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{4b}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

output
```
(B*(b*c - a*d)*n*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2*(a
 + b*x)^3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a + b
*x)) + (d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d)^5
))/(4*b) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*(a + b*x)^4)
```

3.155.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.155.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2308 vs. $2(184) = 368$.

Time = 135.60 (sec) , antiderivative size = 2309, normalized size of antiderivative = 11.84

method	result	size
parallelrisch	Expression too large to display	2309
risch	Expression too large to display	2583

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(48*B*ln(b*x+a)*x^4*a^5*b^4*c^2*d^3*n-288*B*ln(b*x+a)*x^3*a^5*b^4*c^3
*d^2*n+192*B*ln(b*x+a)*x^3*a^4*b^5*c^4*d*n-192*B*ln(d*x+c)*x^3*a^6*b^3*c^2
*d^3*n+288*B*ln(d*x+c)*x^3*a^5*b^4*c^3*d^2*n-192*B*ln(d*x+c)*x^3*a^4*b^5*c
^4*d*n+288*B*ln(b*x+a)*x^2*a^7*b^2*c^2*d^3*n-432*B*ln(b*x+a)*x^2*a^6*b^3*c
^3*d^2*n+288*B*ln(b*x+a)*x^2*a^5*b^4*c^4*d*n-288*B*ln(d*x+c)*x^2*a^7*b^2*c
^2*d^3*n+432*B*ln(d*x+c)*x^2*a^6*b^3*c^3*d^2*n-288*B*ln(d*x+c)*x^2*a^5*b^4
*c^4*d*n+192*B*ln(b*x+a)*x*a^8*b*c^2*d^3*n-288*B*ln(b*x+a)*x*a^7*b^2*c^3*d
^2*n+192*B*ln(b*x+a)*x*a^6*b^3*c^4*d*n-192*B*ln(d*x+c)*x*a^8*b*c^2*d^3*n+2
88*B*ln(d*x+c)*x*a^7*b^2*c^3*d^2*n-192*B*ln(d*x+c)*x*a^6*b^3*c^4*d*n+12*A*
x^4*a^2*b^7*c^5+48*A*x^3*a^3*b^6*c^5+72*A*x^2*a^4*b^5*c^5+48*A*x*a^9*c*d^4
+48*A*x*a^5*b^4*c^5+48*B*ln(b*x+a)*a^9*c^2*d^3*n-12*B*ln(b*x+a)*a^6*b^3*c^
5*n-48*B*ln(d*x+c)*a^9*c^2*d^3*n+12*B*ln(d*x+c)*a^6*b^3*c^5*n+12*B*x^4*ln(
e*(b*x+a)^n/((d*x+c)^n))*a^2*b^7*c^5+3*B*x^4*a^2*b^7*c^5*n+12*B*x^4*ln(e*(
b*x+a)^n/((d*x+c)^n))*a^6*b^3*c*d^4-48*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a
^5*b^4*c^2*d^3+72*B*x^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^4*b^5*c^3*d^2-48*B*x
^4*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^6*c^4*d+25*B*x^4*a^6*b^3*c*d^4*n-48*B
*x^4*a^5*b^4*c^2*d^3*n+36*B*x^4*a^4*b^5*c^3*d^2*n-16*B*x^4*a^3*b^6*c^4*d*n
-192*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^8*b*c^2*d^3+288*B*x*ln(e*(b*x+a)^n/
((d*x+c)^n))*a^7*b^2*c^3*d^2-192*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^6*b^3*c
^4*d-120*B*x*a^8*b*c^2*d^3*n+120*B*x*a^7*b^2*c^3*d^2*n-60*B*x*a^6*b^3*c...
```

3.155.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(181) = 362$.

Time = 0.29 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.21

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = -\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\,(Bb^4cd^3 - Bab^3d^4)nx^3 + 6\,(Bb^4c}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="frica
s")
```

output
```
-1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b
*c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^
2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*
b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16
*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n
 - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B
*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*
a^3*b*c*d^3)*n)*log(b*x + a) + 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 +
 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d
+ 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(d*x + c) + 12*(B*b^4*c^4 -
 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a^4*d^4)*log(
e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 +
 a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2
- 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 -
4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

3.155.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**5,x)
```

output
```
Timed out
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(181) = 362$.

Time = 0.21 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.17

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$$
$$= \frac{\left(\frac{12\,d^4en\log(bx+a)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} - \frac{12\,d^4en\log(dx+c)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} + \frac{}{a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3+(b\ldots}\right.}{}$$
$$- \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)}$$
$$- \frac{A}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="maxim
a")
```

output
```
1/48*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b
^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*
e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3
*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*
a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*
a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a
^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b
^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3
*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*x))*B/e - 1/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3
+ 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A/(b^5*x^4 + 4*a*b^4*x^3 + 6*
a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

3.155.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(181) = 362$.

Time = 0.31 (sec) , antiderivative size = 718, normalized size of antiderivative = 3.68

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$$
$$= \frac{Bd^4n\log(bx+a)}{4\left(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4\right)} - \frac{Bd^4n\log(dx+c)}{4\left(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4\right)} - \frac{Bn\log(bx+a)}{4\left(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b\right)} + \frac{Bn\log(dx+c)}{4\left(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b\right)} + \frac{12\,Bb^3d^3nx^3 - 6\,Bb^3cd^2nx^2 + 42\,Bab^2d^3nx^2 + 4\,Bb^3c^2dnx - 20\,Bab^2cd^2nx + 52\,Ba^2bd^3nx - 3\,Bb^3c^3n}{48\left(b^8c^3x^4 - 3\,ab^7c^2dx^4 + 3\,a^2b^6cd^2x^4 - a^3b^5d^3x^4 + 4\,ab^7c^3x^3 - 12\,a^2b^6c^2dx^3 + 12\,a^3b^5cd^2x^3\right.}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="giac"
)
```

output
```
1/4*B*d^4*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*d^4*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c
^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*n*log(b*x
+ a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/4*B
*n*log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4
*b) + 1/48*(12*B*b^3*d^3*n*x^3 - 6*B*b^3*c*d^2*n*x^2 + 42*B*a*b^2*d^3*n*x^
2 + 4*B*b^3*c^2*d*n*x - 20*B*a*b^2*c*d^2*n*x + 52*B*a^2*b*d^3*n*x - 3*B*b^
3*c^3*n + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n + 25*B*a^3*d^3*n - 12*B*
b^3*c^3*log(e) + 36*B*a*b^2*c^2*d*log(e) - 36*B*a^2*b*c*d^2*log(e) + 12*B*
a^3*d^3*log(e) - 12*A*b^3*c^3 + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 + 12*A
*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2*d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5
*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 -
 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2 - 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4
*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b^5*c^3*x - 12*a^4*b^4*c^2*d*x + 12
*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6
*b^2*c*d^2 - a^7*b*d^3)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.85

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{d\,x\,(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\,c}{3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,c}}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x}$$

$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{4\,b\,\left(a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4\right)}$$

$$-\frac{B\,d^4\,n\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4+8\,a^3\,b^2\,c\,d^3-8\,a\,b^4\,c^3\,d+4\,b^5\,c^4}{4\,b\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^4}\right)}{2\,b\,(a\,d-b\,c)^4}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^5,x)
```

output
```
- ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*
b^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(1
2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n
 + 13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b
^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2) - (B*log((e*(a + b*x)^n)/(c + d*x)^n
))/(4*b*(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B*d^
4*n*atanh((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^2*c*d^3 - 8*a*b^4*c^3*d)/(4*b
*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2))/(a*d - b*c)^4))/(2*b*(a*d - b*c)^4)
```

3.156 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

3.156.1 Optimal result

Integrand size = 33, antiderivative size = 322

$$\begin{aligned}\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx &= -\frac{B(bc-ad)n(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{6bd}\\ &+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b}\\ &+\frac{B(bc-ad)^2n(a+bx)^2\left(3A+Bn+3B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^2}\\ &-\frac{B(bc-ad)^3n(a+bx)\left(6A+5Bn+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^3}\\ &-\frac{B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11Bn+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{12bd^4}\\ &-\frac{B^2(bc-ad)^4n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)*n*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/4*(b
*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/12*B*(-a*d+b*c)^2*n*(b*x+a
)^2*(3*A+B*n+3*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-1/12*B*(-a*d+b*c)^3*n*
(b*x+a)*(6*A+5*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3-1/12*B*(-a*d+b*c
)^4*n*ln((-a*d+b*c)/b/(d*x+c))*(6*A+11*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n))
)/b/d^4-1/2*B^2*(-a*d+b*c)^4*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

3.156.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1709 vs. $2(322) = 644$.

Time = 0.99 (sec) , antiderivative size = 1709, normalized size of antiderivative = 5.31

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= \frac{-24a^4ABd^4n + 6ab^3B^2c^3dn^2 - 24a^2b^2B^2c^2d^2n^2 + 36a^3bB^2cd^3n^2 - 24a^4B^2d^4n^2 + 12a^3A^2bd^4x - 6Ab^4Bc}{}$$

input
```
Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(-24*a^4*A*B*d^4*n + 6*a*b^3*B^2*c^3*d*n^2 - 24*a^2*b^2*B^2*c^2*d^2*n^2 +
36*a^3*b*B^2*c*d^3*n^2 - 24*a^4*B^2*d^4*n^2 + 12*a^3*A^2*b*d^4*x - 6*A*b^4
*B*c^3*d*n*x + 24*a*A*b^3*B*c^2*d^2*n*x - 36*a^2*A*b^2*B*c*d^3*n*x + 18*a^
3*A*b*B*d^4*n*x - 5*b^4*B^2*c^3*d*n^2*x + 17*a*b^3*B^2*c^2*d^2*n^2*x - 19*
a^2*b^2*B^2*c*d^3*n^2*x + 7*a^3*b*B^2*d^4*n^2*x + 18*a^2*A^2*b^2*d^4*x^2 +
 3*A*b^4*B*c^2*d^2*n*x^2 - 12*a*A*b^3*B*c*d^3*n*x^2 + 9*a^2*A*b^2*B*d^4*n*
x^2 + b^4*B^2*c^2*d^2*n^2*x^2 - 2*a*b^3*B^2*c*d^3*n^2*x^2 + a^2*b^2*B^2*d^
4*n^2*x^2 + 12*a*A^2*b^3*d^4*x^3 - 2*A*b^4*B*c*d^3*n*x^3 + 2*a*A*b^3*B*d^4
*n*x^3 + 3*A^2*b^4*d^4*x^4 - 3*a^4*B^2*d^4*n^2*Log[a + b*x]^2 + 6*A*b^4*B*
c^4*n*Log[c + d*x] - 24*a*A*b^3*B*c^3*d*n*Log[c + d*x] + 36*a^2*A*b^2*B*c^
2*d^2*n*Log[c + d*x] - 24*a^3*A*b*B*c*d^3*n*Log[c + d*x] + 11*b^4*B^2*c^4*
n^2*Log[c + d*x] - 38*a*b^3*B^2*c^3*d*n^2*Log[c + d*x] + 45*a^2*b^2*B^2*c^
2*d^2*n^2*Log[c + d*x] - 18*a^3*b*B^2*c*d^3*n^2*Log[c + d*x] - 24*a^4*B^2*
d^4*n^2*Log[c + d*x] + 3*b^4*B^2*c^4*n^2*Log[c + d*x]^2 - 12*a*b^3*B^2*c^3
*d*n^2*Log[c + d*x]^2 + 18*a^2*b^2*B^2*c^2*d^2*n^2*Log[c + d*x]^2 - 12*a^3
*b*B^2*c*d^3*n^2*Log[c + d*x]^2 - 24*a^4*B^2*d^4*n*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] + 24*a^3*A*b*B*d^4*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^4*B^
2*c^3*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a*b^3*B^2*c^2*d^2*n*x*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n] - 36*a^2*b^2*B^2*c*d^3*n*x*Log[(e*(a + b*x)
^n)/(c + d*x)^n] + 18*a^3*b*B^2*d^4*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n...
```

3.156.3 Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2973, 2949, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \; 2973$$

$$\int (a+bx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \; 2949$$

$$(bc-ad)^4\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \; 2781$$

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}\right)$$

$$\Big\downarrow \; 2784$$

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int \frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d}{3d}\right.}{2b}$$

$$\Big\downarrow \; 2784$$

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots}{\cdots}\right.}{2b}\cdots\right.$$

$\downarrow$ 2784

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots}{\cdots}\right.}{2b}\cdots\right.$$

$\downarrow$ 2754

3.156. $\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots}{\cdots}\right)}{\cdots}\right)$$

$\downarrow$ 2838

$$(bc-ad)^4\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots}{\cdots}\right)}{\cdots}\right)$$

input
```
Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

3.156. $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

output
```
(b*c - a*d)^4*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b
*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c +
d*x))^3) - (((a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))
/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*
B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))
/(c + d*x))) - (-(((6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log
[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6*B*n*PolyLog[2, (d*(a + b*x))/(b
*(c + d*x))])/d)/d)/(2*d))/(3*d)))/(2*b))
```

3.156.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.156.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 269.19 (sec) , antiderivative size = 10586, normalized size of antiderivative = 32.88

method	result	size
risch	Expression too large to display	10586

input
```
int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.156.5 Fricas [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int (bx+a)^3 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b^3*x^3 + 3*A^2*a*b^2*x^2 + 3*A^2*a^2*b*x + A^2*a^3 + (B^2*b^
3*x^3 + 3*B^2*a*b^2*x^2 + 3*B^2*a^2*b*x + B^2*a^3)*log((b*x + a)^n*e/(d*x
+ c)^n)^2 + 2*(A*B*b^3*x^3 + 3*A*B*a*b^2*x^2 + 3*A*B*a^2*b*x + A*B*a^3)*lo
g((b*x + a)^n*e/(d*x + c)^n), x)
```

3.156.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(309) = 618$.

Time = 0.73 (sec) , antiderivative size = 1871, normalized size of antiderivative = 5.81

$$\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="max
ima")
```

output
```
1/2*A*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^2*b^3*x^4 + 2*A*B*a
*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b^2*x^3 + 3*A*B*a^2*b*x^2*
log((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^2*a^2*b*x^2 + 2*A*B*a^3*x*log((b*x
+ a)^n*e/(d*x + c)^n) + A^2*a^3*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A*B*a^3/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d
^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a^2*b/e + (2*a^3*e*n*log(b*x + a)/b^
3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2
*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*a*b^2/e - 1/12*(6*a^4*e*n*log(b*
x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*
n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*
e*n)*x)/(b^3*d^3))*A*B*b^3/e + 1/12*((11*n^2 + 6*n*log(e))*b^3*c^4 - 2*(19
*n^2 + 12*n*log(e))*a*b^2*c^3*d + 9*(5*n^2 + 4*n*log(e))*a^2*b*c^2*d^2 - 6
*(3*n^2 + 4*n*log(e))*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*n^2 -
 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2 + a^4*d^4*n
^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d
)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*x^4*log(e)^2 - 3*B^2*a^4
*d^4*n^2*log(b*x + a)^2 - 2*(b^4*c*d^3*n*log(e) - (n*log(e) + 6*log(e)^2)*
a*b^3*d^4)*B^2*x^3 + ((n^2 + 3*n*log(e))*b^4*c^2*d^2 - 2*(n^2 + 6*n*log(e)
)*a*b^3*c*d^3 + (n^2 + 9*n*log(e) + 18*log(e)^2)*a^2*b^2*d^4)*B^2*x^2 - 6*
(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^...
```

3.156.8 Giac [F]

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \int (bx+a)^3 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 (a+b\,x)^3 \, dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3, x)
```

3.157 $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

3.157.1 Optimal result

Integrand size = 33, antiderivative size = 263

$$\begin{aligned}
&\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)n(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd} \\
&\quad+\frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b} \\
&\quad+\frac{B(bc-ad)^2n(a+bx)\left(2A+Bn+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^2} \\
&\quad+\frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3Bn+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3} \\
&\quad+\frac{2B^2(bc-ad)^3n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)*n*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/3*(b
*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/3*B*(-a*d+b*c)^2*n*(b*x+a)
*(2*A+B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+1/3*B*(-a*d+b*c)^3*n*ln((
-a*d+b*c)/b/(d*x+c))*(2*A+3*B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2/3
*B^2*(-a*d+b*c)^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

3.157.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1149 vs. $2(263) = 526$.

Time = 0.64 (sec) , antiderivative size = 1149, normalized size of antiderivative = 4.37

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \frac{-6a^3ABd^3n - 2ab^2B^2c^2dn^2 + 6a^2bB^2cd^2n^2 - 6a^3B^2d^3n^2 + 3a^2A^2bd^3x + 2Ab^3Bc^2dnx - 6aAb^2Bcd^2nx \cdots}{}$$

input
```
Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(-6*a^3*A*B*d^3*n - 2*a*b^2*B^2*c^2*d*n^2 + 6*a^2*b*B^2*c*d^2*n^2 - 6*a^3*
B^2*d^3*n^2 + 3*a^2*A^2*b*d^3*x + 2*A*b^3*B*c^2*d*n*x - 6*a*A*b^2*B*c*d^2*
n*x + 4*a^2*A*b*B*d^3*n*x + b^3*B^2*c^2*d*n^2*x - 2*a*b^2*B^2*c*d^2*n^2*x
+ a^2*b*B^2*d^3*n^2*x + 3*a*A^2*b^2*d^3*x^2 - A*b^3*B*c*d^2*n*x^2 + a*A*b^
2*B*d^3*n*x^2 + A^2*b^3*d^3*x^3 - a^3*B^2*d^3*n^2*Log[a + b*x]^2 - 2*A*b^3
*B*c^3*n*Log[c + d*x] + 6*a*A*b^2*B*c^2*d*n*Log[c + d*x] - 6*a^2*A*b*B*c*d
^2*n*Log[c + d*x] - 3*b^3*B^2*c^3*n^2*Log[c + d*x] + 7*a*b^2*B^2*c^2*d*n^2
*Log[c + d*x] - 4*a^2*b*B^2*c*d^2*n^2*Log[c + d*x] - 6*a^3*B^2*d^3*n^2*Log
[c + d*x] - b^3*B^2*c^3*n^2*Log[c + d*x]^2 + 3*a*b^2*B^2*c^2*d*n^2*Log[c +
 d*x]^2 - 3*a^2*b*B^2*c*d^2*n^2*Log[c + d*x]^2 - 6*a^3*B^2*d^3*n*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 6*a^2*A*b*B*d^3*x*Log[(e*(a + b*x)^n)/(c + d*x)^
n] + 2*b^3*B^2*c^2*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b^2*B^2*c*
d^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*a^2*b*B^2*d^3*n*x*Log[(e*(a +
 b*x)^n)/(c + d*x)^n] + 6*a*A*b^2*B*d^3*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^
n] - b^3*B^2*c*d^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + a*b^2*B^2*d^3*
n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*A*b^3*B*d^3*x^3*Log[(e*(a + b*x
)^n)/(c + d*x)^n] - 2*b^3*B^2*c^3*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c +
d*x)^n] + 6*a*b^2*B^2*c^2*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n
] - 6*a^2*b*B^2*c*d^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*
a^2*b*B^2*d^3*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*a*b^2*B^2*d^3*x^...
```

3.157.3 Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2973, 2949, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^2 \, dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int (a+bx)^2 \left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2949}$$

$$(bc-ad)^3 \int \frac{(a+bx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2781}$$

$$(bc-ad)^3 \left(\frac{(a+bx)^3 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn \int \frac{(a+bx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{3b}\right)$$

$$\Big\downarrow \text{ 2784}$$

$$(bc-ad)^3 \left(\frac{(a+bx)^3 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn \left(\frac{(a+bx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{(a+bx)\left(2A+Bn+2B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2} d}{2d}\right.}{3b}\right.$$

$$\Big\downarrow \text{ 2784}$$

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\cdots}{2d}\cdots\right)}{3b}\cdots\right.$$

$\downarrow$ 2754

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\cdots}{\cdots}\cdots\right)}{3\cdots}\cdots\right.$$

$\downarrow$ 2838

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\cdots}{\cdots}\cdots\right)}{3b}\cdots\right.$$

input
```
Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

3.157. $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

output
```
(b*c - a*d)^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b
*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c
+ d*x))^2) - (((a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n]))
/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B*n + 2*B*Log[
e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*
B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))
```

3.157.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.157.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 107.03 (sec) , antiderivative size = 7208, normalized size of antiderivative = 27.41

method	result	size
risch	Expression too large to display	7208

input
```
int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.157.5 Fricas [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$$
$$= \int (bx+a)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b^2*x^2 + 2*A^2*a*b*x + A^2*a^2 + (B^2*b^2*x^2 + 2*B^2*a*b*x
+ B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*b^2*x^2 + 2*A*B*a*b*x
 + A*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.157.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. 2(252) = 504.

Time = 0.69 (sec) , antiderivative size = 1284, normalized size of antiderivative = 4.88

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="max
ima")
```

output
```
2/3*A*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*b^2*x^3 + 2*A*B*a
*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b*x^2 + 2*A*B*a^2*x*log((b*x
 + a)^n*e/(d*x + c)^n) + A^2*a^2*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d
*x + c)/d)*A*B*a^2/e - 2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/
d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a*b/e + 1/3*(2*a^3*e*n*log(b*x + a)
/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(
b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*b^2/e - 1/3*((3*n^2 + 2*n*log
(e))*b^2*c^3 - (7*n^2 + 6*n*log(e))*a*b*c^2*d + 2*(2*n^2 + 3*n*log(e))*a^2
*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^
2*b*c*d^2*n^2 - a^3*d^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*x^
3*log(e)^2 - B^2*a^3*d^3*n^2*log(b*x + a)^2 - (b^3*c*d^2*n*log(e) - (n*log
(e) + 3*log(e)^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2
+ 3*a^2*b*c*d^2*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^3*c^3*n^2 - 3*a*b^
2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2)*B^2*log(d*x + c)^2 + ((n^2 + 2*n*log(e))*
b^3*c^2*d - 2*(n^2 + 3*n*log(e))*a*b^2*c*d^2 + (n^2 + 4*n*log(e) + 3*log(e
)^2)*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*n^2 - 5*a^2*b*c*d^2*n^2 + (3*n^2 +
2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3
*x^2 + 3*B^2*a^2*b*d^3*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*x^3 + 3*B^2*a*
b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*x^...
```

3.157.8 Giac [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$$
$$= \int (bx+a)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$$
$$= \int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2\,{\left(a+b\,x\right)}^2 \,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2, x)
```

3.158 $\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

3.158.1 Optimal result

Integrand size = 31, antiderivative size = 195

$$\begin{aligned}
&\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)n(a+bx)\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd} \\
&\quad + \frac{(a+bx)^2\left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b} \\
&\quad - \frac{B(bc-ad)^2 n\log \left(\frac{bc-ad}{b(c+dx)}\right)\left(A+Bn+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} \\
&\quad - \frac{B^2(bc-ad)^2 n^2 \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}
\end{aligned}$$

output
```
-B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/2*(b*x+a)^
2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b-B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(
d*x+c))*(A+B*n+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-B^2*(-a*d+b*c)^2*n^2*p
olylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

3.158.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 656 vs. $2(195) = 390$.

Time = 0.52 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.36

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = -\frac{2a^2ABn}{b}-\frac{2a^2B^2n^2}{b}+\frac{aB^2cn^2}{d}+aA^2x+aABnx-\frac{AbBcnx}{d}+\frac{1}{2}A^2bx^2-\frac{a^2B^2n^2\log^2(a+bx)}{2b}+\frac{AbBc^2n\log(c+dx)}{d^2}-\frac{2aABcn\log(c+dx)}{d}-\frac{2a^2B^2n^2\log(c+dx)}{b}+\frac{bB^2c^2n^2\log(c+dx)}{d^2}-\frac{aB^2cn^2\log(c+dx)}{d}+\frac{bB^2c^2n^2\log^2(c+dx)}{2d^2}-\frac{aB^2cn^2\log^2(c+dx)}{d}-\frac{2a^2B^2n\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+2aABx\log\left(e(a+bx)^n(c+dx)^{-n}\right)+aB^2nx\log\left(e(a+bx)^n(c+dx)^{-n}\right)-\frac{bB^2cnx\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d}+AbBx^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{bB^2c^2n\log(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d^2}-\frac{2aB^2cn\log(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{d}+aB^2x\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{1}{2}bB^2x^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+\frac{Bn\log(a+bx)\left(bBc(-bc+2ad)n\log(c+dx)+B(bc-ad)^2n\log\left(\frac{b(c+dx)}{bc-ad}\right)+ad(-bBcn+ad(A+3B\right.}{bd^2}+\frac{B^2(bc-ad)^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bd^2}$$

input
```
Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(-2*a^2*A*B*n)/b - (2*a^2*B^2*n^2)/b + (a*B^2*c*n^2)/d + a*A^2*x + a*A*B*n
*x - (A*b*B*c*n*x)/d + (A^2*b*x^2)/2 - (a^2*B^2*n^2*Log[a + b*x]^2)/(2*b)
+ (A*b*B*c^2*n*Log[c + d*x])/d^2 - (2*a*A*B*c*n*Log[c + d*x])/d - (2*a^2*B
^2*n^2*Log[c + d*x])/b + (b*B^2*c^2*n^2*Log[c + d*x])/d^2 - (a*B^2*c*n^2*L
og[c + d*x])/d + (b*B^2*c^2*n^2*Log[c + d*x]^2)/(2*d^2) - (a*B^2*c*n^2*Log
[c + d*x]^2)/d - (2*a^2*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + 2*a*A*
B*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + a*B^2*n*x*Log[(e*(a + b*x)^n)/(c +
d*x)^n] - (b*B^2*c*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + A*b*B*x^2*Log
[(e*(a + b*x)^n)/(c + d*x)^n] + (b*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/d^2 - (2*a*B^2*c*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])/d + a*B^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (b*B^2*x^2*Log[
(e*(a + b*x)^n)/(c + d*x)^n]^2)/2 + (B*n*Log[a + b*x]*(b*B*c*(-(b*c) + 2*a
*d)*n*Log[c + d*x] + B*(b*c - a*d)^2*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*
d*(-(b*B*c*n) + a*d*(A + 3*B*n) + a*B*d*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
)/(b*d^2) + (B^2*(b*c - a*d)^2*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
])/(b*d^2)
```

3.158.3 Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2973, 2949, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2\,dx$$

$$\Big\downarrow \ 2973$$

$$\int (a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2dx$$

$$\Big\downarrow \ 2949$$

$$(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2781$$

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)$$

$\Big\downarrow$ 2784

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}\right)$$

$\Big\downarrow$ 2754

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(\cdots\right.}{\cdots}}{d}\right.}{b}\right.$$

$\Big\downarrow$ 2838

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+\cdots\right.}{d}}{d}\right.}{b}\right.$$

input

```
Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

3.158. $\int(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

output
```
(b*c - a*d)^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b
*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(((a + b*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))
 - (-(((A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/
(b*(c + d*x))])/d) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d))/
b)
```

3.158.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.158.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 37.57 (sec) , antiderivative size = 4394, normalized size of antiderivative = 22.53

method	result	size
risch	Expression too large to display	4394

input
```
int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B^2*x*(b*x+2*a)*ln((d*x+c)^n)^2+5/4/d^2*n^2*B^2*b*c^2-3/2/d*n^2*B^2*a*
c+1/4*B^2*a^2*n^2/b+B^2*n*ln((b*x+a)^n)*x*a-1/2*B^2/b*n^2*a^2*ln(b*x+a)^2+
(-B^2*x*(b*x+2*a)*ln((b*x+a)^n)-1/2*B*(2*A*b^2*d^2*x^2+2*B*a*b*d^2*n*x-2*B
*b^2*c*d*n*x+2*B*ln(d*x+c)*b^2*c^2*n+4*B*ln(e)*a*b*d^2*x+4*A*a*b*d^2*x+2*B
*a^2*n*ln(b*x+a)*d^2+2*B*ln(e)*b^2*d^2*x^2-4*B*ln(d*x+c)*a*b*c*d*n+I*B*Pi*
b^2*d^2*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2
*d^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2
*I*B*Pi*a*b*d^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)-2*I*B*Pi*a*b*d^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*c
sgn(I*(b*x+a)^n/((d*x+c)^n))-I*B*Pi*b^2*d^2*x^2*csgn(I*(b*x+a)^n)*csgn(I/(
(d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*I*B*Pi*a*b*d^2*x*csgn(I*e)*csg
n(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b^2*d^2*x^2*csgn(I*e)*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^2+I*B*Pi*b^2*d^2*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/
((d*x+c)^n))^2-2*I*B*Pi*a*b*d^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi
*a*b*d^2*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*b^2*d^2*x^2*csgn(I*(b*
x+a)^n/((d*x+c)^n))^3-I*B*Pi*b^2*d^2*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3
+2*I*B*Pi*a*b*d^2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*I*
B*Pi*a*b*d^2*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*I*B*P
i*a*b*d^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^
2-I*B*Pi*b^2*d^2*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/(...
```

3.158.5 Fricas [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b*x + A^2*a + (B^2*b*x + B^2*a)*log((b*x + a)^n*e/(d*x + c)^n
)^2 + 2*(A*B*b*x + A*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.158.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(192) = 384$.

Time = 0.69 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.99

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= ABbx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}A^2bx^2+2\,ABax\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^2ax$$
$$+\frac{2\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)ABa}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)ABb}{e}$$
$$+\frac{((n^2+n\log(e))bc^2-(n^2+2\,n\log(e))acd)B^2\log(dx+c)}{d^2}$$
$$+\frac{(b^2c^2n^2-2\,abcdn^2+a^2d^2n^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}$$
$$-\frac{B^2a^2d^2n^2\log(bx+a)^2-B^2b^2d^2x^2\log(e)^2+2\,(b^2c^2n^2-2\,abcdn^2)B^2\log(bx+a)\log(dx+c)-(b^2c^2n}{}$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxim
a")
```

output
```
A*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*b*x^2 + 2*A*B*a*x*log((
b*x + a)^n*e/(d*x + c)^n) + A^2*a*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(
d*x + c)/d)*A*B*a/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2
 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*b/e + ((n^2 + n*log(e))*b*c^2 - (n^2 +
 2*n*log(e))*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*n^2 - 2*a*b*c*d*n^2 +
a^2*d^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*
d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*n^2*log(b*x + a)^2
 - B^2*b^2*d^2*x^2*log(e)^2 + 2*(b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(b*x
+ a)*log(d*x + c) - (b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(d*x + c)^2 + 2*(
b^2*c*d*n*log(e) - (n*log(e) + log(e)^2)*a*b*d^2)*B^2*x + 2*(a*b*c*d*n^2 -
 (n^2 + n*log(e))*a^2*d^2)*B^2*log(b*x + a) - (B^2*b^2*d^2*x^2 + 2*B^2*a*b
*d^2*x)*log((b*x + a)^n)^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((d*x
+ c)^n)^2 - 2*(B^2*b^2*d^2*x^2*log(e) + B^2*a^2*d^2*n*log(b*x + a) + (a*b*
d^2*(n + 2*log(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*log(
d*x + c))*log((b*x + a)^n) + 2*(B^2*b^2*d^2*x^2*log(e) + B^2*a^2*d^2*n*log
(b*x + a) + (a*b*d^2*(n + 2*log(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*
b*c*d*n)*B^2*log(d*x + c) + (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((b*x +
 a)^n))*log((d*x + c)^n))/(b*d^2)
```

3.158.8 Giac [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 (a+b\,x)\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x), x)
```

3.159 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{a+bx}\,dx$

3.159.1 Optimal result

Integrand size = 33, antiderivative size = 131

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx = -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b+2*B*n*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+2*B^2*n^2*
polylog(3,b*(d*x+c)/d/(b*x+a))/b
```

3.159.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 269 vs. 2(131) = 262.

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.05

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx$$
$$= \frac{-ABn\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+A^2\log(a+bx)-2ABn\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log(e(a-}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

output
```
(-(A*B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2) + A^2*Log[a + b*x] - 2*A*B*n
*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*
Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - B^2*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*A*
B*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*B^2*n*Log[(e*(a + b*x)^n)
/(c + d*x)^n]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*n^2*PolyLog[
3, (b*(c + d*x))/(d*(a + b*x))])/b
```

3.159.3 Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2949, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{a+bx}\,dx$$
$$\Big\downarrow \text{ 2973}$$
$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{a+bx}dx$$
$$\Big\downarrow \text{ 2949}$$
$$\int \frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2779}$$

$$\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}$$

$$\Big\downarrow \text{2821}$$

$$\frac{2Bn\left(\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2Bn\left(\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*
(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/
b
```

3.159.3.1 Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.159.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{bx+a}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

3.159.5 Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="frica
s")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(b*x + a), x)
```

3.159.6 Sympy [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx=\int \frac{\left(A+B\log\left(e(a+bx)^n\,(c+dx)^{-n}\right)\right)^2}{a+bx}\,dx$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a),x)
```

output
```
Integral((A + B*log(e*(a + b*x)**n/(c + d*x)**n))**2/(a + b*x), x)
```

3.159.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="maxim
a")
```

output
```
B^2*log(b*x + a)*log((d*x + c)^n)^2/b + A^2*log(b*x + a)/b - integrate(-(B
^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n
)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e) + A*B*b*
c + (B^2*b*d*log(e) + A*B*b*d)*x)*log((b*x + a)^n) - 2*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d*n)*log(b*x
+ a) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*x^
2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

3.159.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{a+b\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x), x)
```

3.160 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx$

3.160.1 Optimal result

Integrand size = 33, antiderivative size = 129

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx = -\frac{2B^2n^2(c+dx)}{(bc-ad)(a+bx)}-\frac{2Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)} - \frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)}$$

output
```
-2*B^2*n^2*(d*x+c)/(-a*d+b*c)/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((
d*x+c)^n)))/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
/(-a*d+b*c)/(b*x+a)
```

3.160.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.83

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx = \frac{B^2dn^2(a+bx)\log^2(a+bx)+B^2dn^2(a+bx)\log^2(c+dx)+2Bdn(a+bx)\log(c+dx)\left(A+Bn+B\log\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

output
```
(B^2*d*n^2*(a + b*x)*Log[a + b*x]^2 + B^2*d*n^2*(a + b*x)*Log[c + d*x]^2 +
 2*B*d*n*(a + b*x)*Log[c + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)
^n]) - 2*B*d*n*(a + b*x)*Log[a + b*x]*(A + B*n + B*n*Log[c + d*x] + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]) - (b*c - a*d)*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2
*B*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c
 + d*x)^n]^2))/(b*(b*c - a*d)*(a + b*x))
```

3.160.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2973, 2949, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^2}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2742}$$

$$\frac{2Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{bc-ad}$$

$$\Big\downarrow \text{2741}$$

$$\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{bc-ad}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2*B*n
*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(a + b*x)))/(b*c - a*d)
```

3.160.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.160.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(129) = 258.

Time = 7.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.32

method	result
parallelrisch	$-\frac{-A^2b^3cdn+2B^2a\,b^2d^2n^3-2B^2b^3cd\,n^3+A^2a\,b^2d^2n+2ABa\,b^2d^2n^2-2AB\,b^3cd\,n^2-B^2x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2b^3d^2n-2}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-A^2*b^3*c*d*n+2*B^2*a*b^2*d^2*n^3-2*B^2*b^3*c*d*n^3+A^2*a*b^2*d^2*n+2*A
*B*a*b^2*d^2*n^2-2*A*B*b^3*c*d*n^2-B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^3
*d^2*n-2*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n^2-B^2*ln(e*(b*x+a)^n/
((d*x+c)^n))^2*b^3*c*d*n-2*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n^2-2*A
*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n-2*A*B*ln(e*(b*x+a)^n/((d*x+c)^n
))*b^3*c*d*n)/(b*x+a)/b^3/d/n/(a*d-b*c)
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(129) = 258.

Time = 0.28 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.63

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = -\frac{A^2bc-A^2ad+2\,(B^2bc-B^2ad)n^2+(B^2bdn^2x+B^2bcn^2)\log(bx+a)^2+(B^2bdn^2x+B^2bcn^2)\log(dx}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="fri
cas")
```

output
```
-(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*d*n^2*x + B^2*b*c
*n^2)*log(b*x + a)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log(d*x + c)^2 + (B^2
*b*c - B^2*a*d)*log(e)^2 + 2*(A*B*b*c - A*B*a*d)*n + 2*(B^2*b*c*n^2 + A*B*
b*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B^2*b*d*n*x + B^2*b*c*n)*log(e))*lo
g(b*x + a) - 2*(B^2*b*c*n^2 + A*B*b*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B
^2*b*d*n^2*x + B^2*b*c*n^2)*log(b*x + a) + (B^2*b*d*n*x + B^2*b*c*n)*log(e
))*log(d*x + c) + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n)*log(e))/(a
*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**2,x)
```

output
```
Timed out
```

3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(129) = 258.

Time = 0.22 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.48

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx =$$
$$-B^2\left(\frac{2\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{e}+\frac{2\,bce^2n^2-2\,ade^2n^2-(bde^2n^2x+ade^2n^2)}{}\right.$$
$$-\frac{B^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2}{b^2x+ab}-\frac{2\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)AB}{e}$$
$$-\frac{2\,AB\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^2x+ab}-\frac{A^2}{b^2x+ab}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="max
ima")
```

output
```
-B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c -
a*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x + c)^n)/e + (2*b*c*e^2*
n^2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*
e^2*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*
log(b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n
^2)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
*e^2)) - B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 2*(d*e*n*log
(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*
x + a*b))*A*B/e - 2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) - A^2
/(b^2*x + a*b)
```

3.160.8 Giac [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bx+a)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^2, x)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.55

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{2\,A\,B}{x\,b^2+a\,b} + \frac{2\,B^2\,n}{x\,b^2+a\,b}\right) - \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{b\,(a+b\,x)} - \frac{B^2\,d}{b\,(a\,d-b\,c)}\right) - \frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{x\,b^2+a\,b} - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{\left(\frac{c\,b^2+a\,d\,b}{b}+2\,b\,d\,x\right)1\mathrm{i}}{a\,d-b\,c}\right)(A+B\,n)\,4\mathrm{i}}{b\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^2,x)
```

output
```
- log((e*(a + b*x)^n)/(c + d*x)^n)*((2*A*B)/(a*b + b^2*x) + (2*B^2*n)/(a*b
 + b^2*x)) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(b*(a + b*x)) - (B^2*
d)/(b*(a*d - b*c))) - (A^2 + 2*B^2*n^2 + 2*A*B*n)/(a*b + b^2*x) - (B*d*n*a
tan((((b^2*c + a*b*d)/b + 2*b*d*x)*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*(a*d
- b*c))
```

3.161 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx$

3.161.1 Optimal result

Integrand size = 33, antiderivative size = 274

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx &= \frac{2B^2dn^2(c+dx)}{(bc-ad)^2(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2(a+bx)^2} \\ &+ \frac{2Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)} \\ &- \frac{bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^2(a+bx)^2} \\ &+ \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} \\ &- \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^2(a+bx)^2}\end{aligned}$$

output
```
2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d+b*c
)^2/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)
^2/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c
)^2/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(
b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(b
*x+a)^2
```

3.161.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.21

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx = -\frac{2B^2d^2n^2(a+bx)^2\log^2(a+bx)+2B^2d^2n^2(a+bx)^2\log^2(c+dx)+2Bd^2n(a+bx)^2\log(c+dx)\,(2A+3}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

output
```
-1/4*(2*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*x]^2 + 2*B^2*d^2*n^2*(a + b*x)^2
*Log[c + d*x]^2 + 2*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A + 3*B*n + 2*B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]) - 2*B*d^2*n*(a + b*x)^2*Log[a + b*x]*(2*A
+ 3*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + (b*
c - a*d)*(2*A^2*(b*c - a*d) + B^2*n^2*(b*c - 7*a*d - 6*b*d*x) + 2*A*B*n*(b
*c - 3*a*d - 2*b*d*x) + 2*B*(2*A*(b*c - a*d) + B*n*(b*c - 3*a*d - 2*b*d*x)
)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*(b*c - a*d)*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

3.161.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{2Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{b(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(e\left(\right.\right.\right.}{a+b}}{(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

output
```
((2*B^2*d*n^2*(c + d*x))/(a + b*x) - (b*B^2*n^2*(c + d*x)^2)/(4*(a + b*x)^
2) + (2*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)
- (b*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^
2) + (d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (b
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2))/(b
*c - a*d)^2
```

3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

3.161. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx$

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.161.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(268) = 536$.

Time = 21.60 (sec) , antiderivative size = 871, normalized size of antiderivative = 3.18

method	result
parallelrisch	$-\frac{-6B^2\ln(bx+a)x^2b^5d^3n^2+6B^2\ln(dx+c)x^2b^5d^3n^2-6B^2\ln(bx+a)a^2b^3d^3n^2+6B^2\ln(dx+c)a^2b^3d^3n^2+6B^2xa\,b^4d^3n^2-4A^2}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-6*B^2*ln(b*x+a)*x^2*b^5*d^3*n^2+6*B^2*ln(d*x+c)*x^2*b^5*d^3*n^2-6*B
^2*ln(b*x+a)*a^2*b^3*d^3*n^2+6*B^2*ln(d*x+c)*a^2*b^3*d^3*n^2-4*B^2*x*ln(e*
(b*x+a)^n/((d*x+c)^n))^2*a*b^4*d^3+6*B^2*x*a*b^4*d^3*n^2-4*A^2*a*b^4*c*d^2
-6*B^2*x*b^5*c*d^2*n^2-4*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^4*c*d^2+6*B
^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^3*d^3*n+2*B^2*ln(e*(b*x+a)^n/((d*x+c)
^n))*b^5*c^2*d*n+4*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^3*d^3+4*A*B*ln(e*
(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d-8*B^2*a*b^4*c*d^2*n^2+6*A*B*a^2*b^3*d^3*n
+2*A*B*b^5*c^2*d*n-8*A*B*a*b^4*c*d^2*n-8*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a
*b^4*c*d^2*n-8*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2+4*B^2*x*ln(e*(b
*x+a)^n/((d*x+c)^n))*a*b^4*d^3*n-4*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c
*d^2*n+4*A*B*x*a*b^4*d^3*n-4*A*B*x*b^5*c*d^2*n-4*A*B*ln(b*x+a)*x^2*b^5*d^3
*n+4*A*B*ln(d*x+c)*x^2*b^5*d^3*n-12*B^2*ln(b*x+a)*x*a*b^4*d^3*n^2+12*B^2*l
n(d*x+c)*x*a*b^4*d^3*n^2-4*A*B*ln(b*x+a)*a^2*b^3*d^3*n+4*A*B*ln(d*x+c)*a^2
*b^3*d^3*n+2*A^2*a^2*b^3*d^3+2*A^2*b^5*c^2*d+7*B^2*a^2*b^3*d^3*n^2+B^2*b^5
*c^2*d*n^2-8*A*B*ln(b*x+a)*x*a*b^4*d^3*n+8*A*B*ln(d*x+c)*x*a*b^4*d^3*n-2*B
^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^5*d^3+2*B^2*ln(e*(b*x+a)^n/((d*x+c)
^n))^2*b^5*c^2*d)/(b*x+a)^2/b^4/d/(a^2*d^2-2*a*b*c*d+b^2*c^2)
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(268) = 536$.

Time = 0.30 (sec) , antiderivative size = 919, normalized size of antiderivative = 3.35

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx = -\frac{2A^2b^2c^2 - 4A^2abcd + 2A^2a^2d^2 + (B^2b^2c^2 - 8B^2abcd + 7B^2a^2d^2)n^2 - 2(B^2b^2d^2n^2x^2 + 2B^2abd^2n^2x}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="fri
cas")
```

output
```
-1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2
*a*b*c*d + 7*B^2*a^2*d^2)*n^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2
*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x + a)^2 - 2*(B^2*b^2*d^2*n^
2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(d*x +
 c)^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2)*log(e)^2 + 2*(A*B*b^
2*c^2 - 4*A*B*a*b*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2
)*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*((B^2*b^2*c^2 - 4*B^2*a*b*c
*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A
*B*a*b*c*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x
- 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)
*n)*log(e))*log(b*x + a) - 2*((B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 - (3*B^2*b
^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A*B*a*b*c*d)*n - 2*
(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x - 2*(B^2*b^2*d^2*n
^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x
+ a) - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b
*c*d)*n)*log(e))*log(d*x + c) + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a
^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d
+ 3*B^2*a^2*d^2)*n)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^
3*b^2*d^2)*x)
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**3,x)
```

output
```
Timed out
```

3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(268) = 536$.

Time = 0.24 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.28

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx$$
$$= \frac{1}{4}B^2\left(\frac{2\left(\frac{2\,d^2en\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{2\,d^2en\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2}+\frac{2\,bdenx-bcen+3\,aden}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{e}-\frac{b^2c^2e\right.$$
$$-\frac{B^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$
$$+\frac{\left(\frac{2\,d^2en\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{2\,d^2en\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2}+\frac{2\,bdenx-bcen+3\,aden}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)AB}{2\,e}$$
$$-\frac{AB\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^3x^2+2\,ab^2x+a^2b}-\frac{A^2}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="max
ima")
```

output
```
1/4*B^2*(2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2
*d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x -
 b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*
b^3*c - a^2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (b^2*c^2*e^2*n^2
 - 8*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*
d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 +
 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^
2 - a*b*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^
2*d^2*e^2*n^2)*log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2
*x + 3*a^2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^
2*d^2*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d +
 a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*
a^2*b^3*c*d + a^3*b^2*d^2)*x)*e^2)) - 1/2*B^2*log((b*x + a)^n*e/(d*x + c)^
n)^2/(b^3*x^2 + 2*a*b^2*x + a^2*b) + 1/2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2
- 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d +
 (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*A*B/e - A*B*log((b*x
+ a)^n*e/(d*x + c)^n)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^2/(b^3*x^2 + 2
*a*b^2*x + a^2*b)
```

3.161.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^3, x)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.62

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx$$
$$= -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}-\frac{B^2\,d^2}{2\,b\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$
$$-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+6\,A\,B\,a\,d\,n-2\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,\left(3\,b\,B^2\,n^2+2\,A\,b\,B\,n\right)}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$
$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{A\,B}{a^2\,b+2\,a\,b^2\,x+b^3\,x^2}+\frac{B^2\,d^2\left(\frac{b\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{b^2\,n\,x\,(a\,d-b\,c)}{d}+\frac{a\,b\,n\,(a\,d-b\,c)}{2\,d}\right)}{b\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^2\,b+2\,a\,b^2\,x+b^3\,x^2\right)}\right)$$
$$-\frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x-\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A+3\,B\,n)\,1\mathrm{i}}{b\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^3,x)
```

output
```
- log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(2*b*(a^2 + b^2*x^2 + 2*a*b*x))
- (B^2*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b
*c + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n)/(2*(a*d - b*
c)) + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 +
4*a*b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)*((A*B)/(a^2*b + b^3*x^2 + 2*
a*b^2*x) + (B^2*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2) + (b^2*n*x*(a
*d - b*c))/d + (a*b*n*(a*d - b*c))/(2*d)))/(b*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)*(a^2*b + b^3*x^2 + 2*a*b^2*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2
 - 2*a^2*b*d^2)/(2*b*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b*(
a*d - b*c)^2)
```

3.162 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx$

3.162.1 Optimal result

Integrand size = 33, antiderivative size = 427

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx &= -\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3(a+bx)}+\frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3(a+bx)^2}-\frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3(a+bx)^3}\\
&-\frac{2Bd^2n(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)^3(a+bx)}\\
&+\frac{bBdn(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)^3(a+bx)^2}\\
&-\frac{2b^2Bn(c+dx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{9(bc-ad)^3(a+bx)^3}\\
&-\frac{d^2(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^3(a+bx)}\\
&+\frac{bd(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^3(a+bx)^2}\\
&-\frac{b^2(c+dx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{3(bc-ad)^3(a+bx)^3}\end{aligned}$$

output
```
-2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/(-a*
d+b*c)^3/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-2*B*d
^2*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+b*B*d*
n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^2-2/9*b
^2*B*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^3-
d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)+b*d*(
d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^
2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^3
```

3.162.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.01

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx = \frac{18B^2d^3n^2(a+bx)^3\log^2(a+bx)+18B^2d^3n^2(a+bx)^3\log^2(c+dx)+6Bd^3n(a+bx)^3\log(c+dx)\,(6A+1}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

output
```
(18*B^2*d^3*n^2*(a + b*x)^3*Log[a + b*x]^2 + 18*B^2*d^3*n^2*(a + b*x)^3*Lo
g[c + d*x]^2 + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(6*A + 11*B*n + 6*B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]) - 6*B*d^3*n*(a + b*x)^3*Log[a + b*x]*(6*A +
11*B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) - (b*c
 - a*d)*(18*A^2*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x
) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*
c + 147*d*x) + b^2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)) + 6*B*(6*A*(b*c - a*d)
^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^
2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*(b*c - a*d)^2*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]^2))/(54*b*(b*c - a*d)^3*(a + b*x)^3)
```

3.162.3 Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^2}{(a+bx)^4} \, dx$$

$$\Big\downarrow \; 2973$$

$$\int \frac{\left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^2}{(a+bx)^4}dx$$

$$\Big\downarrow \; 2949$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{(bc-ad)^3}$$

$$\Big\downarrow \; 2795$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{(bc-ad)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}-\frac{d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}-\frac{2Bd^2n(c+dx)\left(B\right.}{}}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

output
```
((-2*B^2*d^2*n^2*(c + d*x))/(a + b*x) + (b*B^2*d*n^2*(c + d*x)^2)/(2*(a +
b*x)^2) - (2*b^2*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B*d^2*n*(c + d
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (b*B*d*n*(c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (2*b^2*B*n*(c + d
*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^3) - (d^2*(c +
d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) + (b*d*(c + d*x)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^2 - (b^2*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(a + b*x)^3))/(b*c - a*d)^3
```

3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.162.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1399 vs. $2(419) = 838$.

Time = 57.75 (sec) , antiderivative size = 1400, normalized size of antiderivative = 3.28

method	result	size
parallelrisch	Expression too large to display	1400
risch	Expression too large to display	25057

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/54*(-66*B^2*ln(b*x+a)*x^3*b^7*d^4*n^2+66*B^2*ln(d*x+c)*x^3*b^7*d^4*n^2-
66*B^2*ln(b*x+a)*a^3*b^4*d^4*n^2+66*B^2*ln(d*x+c)*a^3*b^4*d^4*n^2-36*A*B*l
n(b*x+a)*x^3*b^7*d^4*n+36*A*B*ln(d*x+c)*x^3*b^7*d^4*n-198*B^2*ln(b*x+a)*x^
2*a*b^6*d^4*n^2+198*B^2*ln(d*x+c)*x^2*a*b^6*d^4*n^2-198*B^2*ln(b*x+a)*x*a^
2*b^5*d^4*n^2+198*B^2*ln(d*x+c)*x*a^2*b^5*d^4*n^2-36*A*B*ln(b*x+a)*a^3*b^4
*d^4*n+36*A*B*ln(d*x+c)*a^3*b^4*d^4*n-108*A*B*x*a*b^6*c*d^3*n-66*B^2*x^2*b
^7*c*d^3*n^2-54*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a^2*b^5*d^4+147*B^2*x*
a^2*b^5*d^4*n^2+15*B^2*x*b^7*c^2*d^2*n^2-54*B^2*ln(e*(b*x+a)^n/((d*x+c)^n)
)^2*a^2*b^5*c*d^3+54*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*c^2*d^2+66*B^
2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^4*d^4*n-12*B^2*ln(e*(b*x+a)^n/((d*x+c)
^n))*b^7*c^3*d*n+36*A*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b^4*d^4-36*A*B*ln(
e*(b*x+a)^n/((d*x+c)^n))*b^7*c^3*d-54*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^
2*a*b^6*d^4+66*B^2*x^2*a*b^6*d^4*n^2-108*A*B*ln(b*x+a)*x^2*a*b^6*d^4*n+108
*A*B*ln(d*x+c)*x^2*a*b^6*d^4*n-108*A*B*ln(b*x+a)*x*a^2*b^5*d^4*n+108*A*B*l
n(d*x+c)*x*a^2*b^5*d^4*n-18*B^2*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*d^4-
18*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*c^3*d+27*B^2*a*b^6*c^2*d^2*n^2+66
*A*B*a^3*b^4*d^4*n-12*A*B*b^7*c^3*d*n-54*A^2*a^2*b^5*c*d^3+54*A^2*a*b^6*c^
2*d^2+36*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*d^4*n-36*B^2*x^2*ln(e*(
b*x+a)^n/((d*x+c)^n))*b^7*c*d^3*n+36*A*B*x^2*a*b^6*d^4*n-36*A*B*x^2*b^7*c*
d^3*n+90*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*d^4*n+18*B^2*x*ln(e*...
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1635 vs. $2(419) = 838$.

Time = 0.33 (sec) , antiderivative size = 1635, normalized size of antiderivative = 3.83

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^2}{(a + bx)^4} dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="fri
cas")
```

output
```
-1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a
^3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^
2*a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*(A*B*b^3*c*
d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^
2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a
^2*b*c*d^2)*n^2)*log(b*x + a)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^
3*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B
^2*a^2*b*c*d^2)*n^2)*log(d*x + c)^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d
+ 3*B^2*a^2*b*c*d^2 - B^2*a^3*d^3)*log(e)^2 + 6*(2*A*B*b^3*c^3 - 9*A*B*a*b
^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3)*n - 3*((5*B^2*b^3*c^2*d -
54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*(A*B*b^3*c^2*d - 6*A*B*a*b^
2*c*d^2 + 5*A*B*a^2*b*d^3)*n)*x + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n
)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(
6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*(A*B*
b^3*c^3 - 3*A*B*a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3*n
- (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x + 6*(B^2*b^
3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n)*log(e))*log(b*x + a) - 6*((11*B
^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d
 + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + ...
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n (c + dx)^{-n}))^2}{(a + bx)^4} dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**4,x)
```

output
```
Timed out
```

3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. 2(419) = 838.

Time = 0.29 (sec) , antiderivative size = 1500, normalized size of antiderivative = 3.51

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="max
ima")
```

output
```
-1/54*B^2*(6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*
c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a
^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d
*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 -
2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*
(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3
*c*d + a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e + (4*b^3*c^3*e^2*
n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d^3*e^2*n^
2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e^2*n^2*x
^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*lo
g(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b
*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2*n^2
- 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n^2*x
^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*lo
g(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a^2
*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d
^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a))*lo
g(d*x + c))/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3
+ (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6
*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^...
```

3.162.8 Giac [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^4}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bx + a)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^4, x)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 911, normalized size of antiderivative = 2.13

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^4}\,dx$$
$$= \frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2\,n-42\,A\,B\,a\,b\,c\,d\,n+12\,A\,B\,b^2\,c^2\,n+85\,B^2\,a^2\,d^2\,n^2-23\,B^2\,a\,b\,c\,d\,n^2+4\,B^2\,b^2\,c^2\,n^2}{6\,(a\,d-b\,c)} + \frac{x\,(-5\,c \cdots}{\cdots}}{x^3\,\left(9\,b^5\,c-9\,a\,b^4\,d\right)+x\,\left(27\,a^2\,b^3\,c-27\,a^3\,b^2\,d\right)-x^2\,\left(27\,a^2\,b^3\,d - \cdots\right.}$$
$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{3\,b\,\left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)} - \frac{B^2\,d^3}{3\,b\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)$$
$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{2\,A\,B}{3\,\left(a^3\,b+3\,a^2\,b^2\,x+3\,a\,b^3\,x^2+b^4\,x^3\right)}\right.$$
$$+\frac{2\,B^2\,d^3\,\left(a\,\left(\frac{b\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}+\frac{a\,b\,n\,(a\,d-b\,c)}{d}\right)+x\,\left(b\,\left(\frac{b\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}+\frac{a\,b\,n\,(a\,d-b\,c)}{d}\right)+\frac{2\,a\,b^2\,n\,(a\,d-b\,c)}{d}+\cdots\right.\right.}{9\,b\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)\,\left(a^3\,b+3\,a^2\,b^2 \cdots\right.}$$
$$-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{B\,d^3\,n\,(6\,A+11\,B\,n)\,\left(\frac{a^3\,b\,d^3-a^2\,b^2\,c\,d^2-a\,b^3\,c^2\,d+b^4\,c^3}{a^2\,b\,d^2-2\,a\,b^2\,c\,d+b^3\,c^2}+2\,b\,d\,x\right)\,\left(a^2\,b\,d^2-2\,a\,b^2\,c\,d+b^3\,c^2\right)\,1\mathrm{i}}{b\,\left(11\,B^2\,d^3\,n^2+6\,A\,B\,d^3\,n\right)\,{(a\,d-b\,c)}^3}\right)\,(6\,A+11\,B\,n)\,2\mathrm{i}}{9\,b\,{(a\,d-b\,c)}^3}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^4,x)
```

output
```
((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
 - 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n
^2 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^
2*c*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*
(11*B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x^3*(9*b^5*c - 9*a*b^4*d
) + x*(27*a^2*b^3*c - 27*a^3*b^2*d) - x^2*(27*a^2*b^3*d - 27*a*b^4*c) + 9*
a^3*b^2*c - 9*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(3*b*(a^3
 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^2*d^3)/(3*b*(a^3*d^3 - b^3*c^3
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((2
*A*B)/(3*(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2)) + (2*B^2*d^3*(a*((
b*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + x*(b*((b
*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + (2*a*b^2*
n*(a*d - b*c))/d + (b^2*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + (b*n*(a*d - b*
c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3 + (3*b^3*n*x^2*(a*d - b*c))/d))/
(9*b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*b + b^4*x^3
+ 3*a^2*b^2*x + 3*a*b^3*x^2))) - (B*d^3*n*atan((B*d^3*n*(6*A + 11*B*n)*((b
^4*c^3 + a^3*b*d^3 - a^2*b^2*c*d^2 - a*b^3*c^2*d)/(b^3*c^2 + a^2*b*d^2 - 2
*a*b^2*c*d) + 2*b*d*x)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*1i)/(b*(11*B^2*
d^3*n^2 + 6*A*B*d^3*n)*(a*d - b*c)^3))*(6*A + 11*B*n)*2i)/(9*b*(a*d - b*c)
^3)
```

3.162. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx$

3.163 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx$

3.163.1 Optimal result

Integrand size = 33, antiderivative size = 587

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx \\
&= \frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4(a+bx)} - \frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4(a+bx)^2} + \frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4(a+bx)^3} \\
&\quad - \frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4(a+bx)^4} + \frac{2Bd^3n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^4(a+bx)} \\
&\quad - \frac{3bBd^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2(bc-ad)^4(a+bx)^2} \\
&\quad + \frac{2b^2Bdn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3(bc-ad)^4(a+bx)^3} \\
&\quad - \frac{b^3Bn(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{8(bc-ad)^4(a+bx)^4} \\
&\quad + \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)} \\
&\quad - \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bc-ad)^4(a+bx)^2} \\
&\quad + \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)^3} \\
&\quad - \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^4(a+bx)^4}
\end{aligned}$$

output
```
2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2/(-a
*d+b*c)^4/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-1/3
2*b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+2*B*d^3*n*(d*x+c)*(A+B*ln(e
*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-3/2*b*B*d^2*n*(d*x+c)^2*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B*d*n*(d*x+c)
^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-1/8*b^3*B*n*(d
*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+
c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)-3/2*b*d^2*(d*x
+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^2+b^2*d*(d*
x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^3-1/4*b^3*
(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^4
```

3.163.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 1011, normalized size of antiderivative = 1.72

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx =$$
$$-\frac{72bB^2n^2(-4a^3d^3(c+dx)+6a^2bd^2(c^2-d^2x^2)-4ab^2d(c^3+d^3x^3)+b^3(c^4-d^4x^4))\log^2(a+bx)+72bI}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

output
```
-1/288*(72*b*B^2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) -
 4*a*b^2*d*(c^3 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[a + b*x]^2 + 72*b*B^
2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) - 4*a*b^2*d*(c^3
 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[c + d*x]^2 - 4*B*d*(b*c - a*d)^3*n*
(a + b*x)*(12*A + 7*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(
e*(a + b*x)^n)/(c + d*x)^n])) + 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2*(12*A
+ 13*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/
(c + d*x)^n])) - 12*B*d^3*(b*c - a*d)*n*(a + b*x)^3*(12*A + 25*B*n + 12*B*
(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) -
 12*B*d^4*n*(a + b*x)^4*Log[a + b*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*
x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 12*B*d^4*n*(a
+ b*x)^4*Log[c + d*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c +
 d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 9*(b*c - a*d)^4*(8*A^2 + 4*A*
B*n + B^2*n^2 + 16*A*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b
*x)^n)/(c + d*x)^n]) + 4*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(
e*(a + b*x)^n)/(c + d*x)^n]) + 8*B^2*(-(n*Log[a + b*x]) + n*Log[c + d*x] +
 Log[(e*(a + b*x)^n)/(c + d*x)^n])^2) - 12*B*(b*c - a*d)*n*Log[a + b*x]*(4
*B*d*(b*c - a*d)^2*n*(a + b*x) + 6*B*d^2*(-(b*c) + a*d)*n*(a + b*x)^2 + 12
*B*d^3*n*(a + b*x)^3 - 3*(b*c - a*d)^3*(4*A + B*n + 4*B*(-(n*Log[a + b*x])
 + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))) + 12*B*n*Log[c ...
```

3.163.3 Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\log\right.}{}\right.}{(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^4}-\frac{b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8(a+bx)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(a+bx)^3}+\frac{2b^2Bdn(c+dx)^3}{}}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

output
```
((2*B^2*d^3*n^2*(c + d*x))/(a + b*x) - (3*b*B^2*d^2*n^2*(c + d*x)^2)/(4*(a
 + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d*x)^3)/(9*(a + b*x)^3) - (b^3*B^2*n^2*
(c + d*x)^4)/(32*(a + b*x)^4) + (2*B*d^3*n*(c + d*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(a + b*x) - (3*b*B*d^2*n*(c + d*x)^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (2*b^2*B*d*n*(c + d*x)^3*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - (b^3*B*n*(c + d*x)^4*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*(a + b*x)^4) + (d^3*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^3 - (b^3*(c + d*x)^4*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*(a + b*x)^4))/(b*c - a*d)^4
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.163. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx$

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.163.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4945 vs. 2(571) = 1142.

Time = 142.59 (sec) , antiderivative size = 4946, normalized size of antiderivative = 8.43

method	result	size
parallelrisch	Expression too large to display	4946
risch	Expression too large to display	33370

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/288*(144*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^5*b^4*c^5*n-1008*B^2*x*a^8*
b*c^2*d^3*n^2+624*B^2*x*a^7*b^2*c^3*d^2*n^2-228*B^2*x*a^6*b^3*c^4*d*n^2+28
8*A^2*x*a^5*b^4*c^5-36*B^2*ln(b*x+a)*x^4*a^2*b^7*c^5*n^2+36*B^2*ln(d*x+c)*
x^4*a^2*b^7*c^5*n^2-144*B^2*ln(b*x+a)*x^3*a^3*b^6*c^5*n^2+144*B^2*ln(d*x+c
)*x^3*a^3*b^6*c^5*n^2-216*B^2*ln(b*x+a)*x^2*a^4*b^5*c^5*n^2+216*B^2*ln(d*x
+c)*x^2*a^4*b^5*c^5*n^2-144*B^2*ln(b*x+a)*x*a^5*b^4*c^5*n^2+3456*A*B*ln(b*
x+a)*x^2*a^5*b^4*c^4*d*n-3456*A*B*ln(d*x+c)*x^2*a^7*b^2*c^2*d^3*n+5184*A*B
*ln(d*x+c)*x^2*a^6*b^3*c^3*d^2*n-3456*A*B*ln(d*x+c)*x^2*a^5*b^4*c^4*d*n-57
6*A*B*ln(d*x+c)*x^4*a^3*b^6*c^4*d*n+576*A*B*ln(b*x+a)*x^4*a^5*b^4*c^2*d^3*
n-864*A*B*ln(b*x+a)*x^4*a^4*b^5*c^3*d^2*n+576*A*B*ln(b*x+a)*x^4*a^3*b^6*c^
4*d*n-576*A*B*ln(d*x+c)*x^4*a^5*b^4*c^2*d^3*n+864*A*B*ln(d*x+c)*x^4*a^4*b^
5*c^3*d^2*n+3456*A*B*ln(d*x+c)*x*a^7*b^2*c^3*d^2*n-2304*A*B*ln(d*x+c)*x*a^
6*b^3*c^4*d*n+2304*A*B*ln(b*x+a)*x*a^8*b*c^2*d^3*n-3456*A*B*ln(b*x+a)*x*a^
7*b^2*c^3*d^2*n+2304*A*B*ln(b*x+a)*x^3*a^6*b^3*c^2*d^3*n-3456*A*B*ln(b*x+a
)*x^3*a^5*b^4*c^3*d^2*n+2304*A*B*ln(b*x+a)*x^3*a^4*b^5*c^4*d*n-2304*A*B*ln
(d*x+c)*x^3*a^6*b^3*c^2*d^3*n+3456*A*B*ln(d*x+c)*x^3*a^5*b^4*c^3*d^2*n-230
4*A*B*ln(d*x+c)*x^3*a^4*b^5*c^4*d*n+3456*A*B*ln(b*x+a)*x^2*a^7*b^2*c^2*d^3
*n-5184*A*B*ln(b*x+a)*x^2*a^6*b^3*c^3*d^2*n+2304*A*B*ln(b*x+a)*x*a^6*b^3*c
^4*d*n-2304*A*B*ln(d*x+c)*x*a^8*b*c^2*d^3*n-2304*B^2*ln(d*x+c)*x*a^8*b*c^2
*d^3*n^2+1728*B^2*ln(d*x+c)*x*a^7*b^2*c^3*d^2*n^2-768*B^2*ln(d*x+c)*x*a...
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2458 vs. $2(571) = 1142$.

Time = 0.37 (sec) , antiderivative size = 2458, normalized size of antiderivative = 4.19

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="fri
cas")
```

output
```
-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 2
88*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^
4)*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B
^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a
^4*d^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b
^2*d^4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)
*n)*x^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^
2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d +
 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(b*x + a)^2 - 72*(B^2*
b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*
B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2
*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(d*x + c)^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*
b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e
)^2 + 12*(3*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 36*A*B*a^2*b^2*c^2*d^2 - 48
*A*B*a^3*b*c*d^3 + 25*A*B*a^4*d^4)*n - 4*((7*B^2*b^4*c^3*d - 60*B^2*a*b^3*
c^2*d^2 + 324*B^2*a^2*b^2*c*d^3 - 271*B^2*a^3*b*d^4)*n^2 + 12*(A*B*b^4*c^3
*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a^3*b*d^4)*n)*x -
 12*((25*B^2*b^4*d^4*n^2 + 12*A*B*b^4*d^4*n)*x^4 + 4*(12*A*B*a*b^3*d^4*n +
 (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a
*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3)*n^2 + 6*(12*A...
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right))^2}{(a + bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**5,x)
```

output
```
Timed out
```

3.163.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2238 vs. $2(571) = 1142$.

Time = 0.34 (sec) , antiderivative size = 2238, normalized size of antiderivative = 3.81

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="max
ima")
```

output
```
1/288*B^2*(12*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^
4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12
*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n
 + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d
*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c
^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*
c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^
2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2
- a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
a^6*b^2*d^3)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (9*b^4*c^4*e^2*n^2 - 6
4*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*
n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^
3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*
e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b
^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a)
^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2
*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(d*x + c)^2 - 4*(7*
b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2*b^2*c*d^3*e^2*n^2 -
 271*a^3*b*d^4*e^2*n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*...
```

3.163.8 Giac [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^5}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bx + a)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^5, x)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 1579, normalized size of antiderivative = 2.69

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^5,x)
```

output
```
(B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*((b^5*c^4 - a^4*b*d^4 + 2*a^3*b^2*c
*d^3 - 2*a*b^4*c^3*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2
*d) + 2*b*d*x)*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*1i)
/(b*(25*B^2*d^4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(1
2*b*(a*d - b*c)^4) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(4*b*(a^4 + b
^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B^2*d^4)/(4*b*(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - ((72
*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9*B^2*b^3*c^3*n^2 +
216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3*d^3*n - 36*A*B*b^3
*c^3*n + 55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n^2 + 156*A*B*a*b^2*
c^2*d*n - 276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^
3*n^2 - 13*B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*A*B*b^3*c*d^2*n))/(
2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 - 53*B^2*
a*b^2*c*d^2*n^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*d*n - 60*A*B*a*b^2*
c*d^2*n))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2 + 12*A*B*b^3*d^2*n)
)/(a*d - b*c))/(x*(96*a^3*b^4*c^2 + 96*a^5*b^2*d^2 - 192*a^4*b^3*c*d) + x^
3*(96*a*b^6*c^2 + 96*a^3*b^4*d^2 - 192*a^2*b^5*c*d) + x^4*(24*b^7*c^2 + 24
*a^2*b^5*d^2 - 48*a*b^6*c*d) + x^2*(144*a^2*b^5*c^2 + 144*a^4*b^3*d^2 - 28
8*a^3*b^4*c*d) + 24*a^6*b*d^2 + 24*a^4*b^3*c^2 - 48*a^5*b^2*c*d) - log((e*
(a + b*x)^n)/(c + d*x)^n)*((A*B)/(2*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*...
```

3.164 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

3.164.1 Optimal result

Integrand size = 33, antiderivative size = 809

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$= -\frac{B^3(bc-ad)^3n^3x}{4d^3} - \frac{B^3(bc-ad)^4n^3\log\left(\frac{a+bx}{c+dx}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\log(c+dx)}{2bd^4}$$
$$- \frac{7B^2(bc-ad)^3n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4bd^3}$$
$$+ \frac{bB^2(bc-ad)^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4d^4}$$
$$- \frac{9B^2(bc-ad)^4n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2bd^4}$$
$$- \frac{9B(bc-ad)^3n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^3}$$
$$+ \frac{9bB(bc-ad)^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{8d^4}$$
$$- \frac{b^2B(bc-ad)n(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4d^4}$$
$$- \frac{3B(bc-ad)^4n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4bd^4}$$
$$+ \frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4b}$$
$$+ \frac{7B^2(bc-ad)^4n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4}$$
$$- \frac{9B^3(bc-ad)^4n^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{3B^2(bc-ad)^4n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$
$$- \frac{7B^3(bc-ad)^4n^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{4bd^4} + \frac{3B^3(bc-ad)^4n^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}$$

output
```
-1/4*B^3*(-a*d+b*c)^3*n^3*x/d^3-1/4*B^3*(-a*d+b*c)^4*n^3*ln((b*x+a)/(d*x+c
))/b/d^4+3/2*B^3*(-a*d+b*c)^4*n^3*ln(d*x+c)/b/d^4-7/4*B^2*(-a*d+b*c)^3*n^2
*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+1/4*b*B^2*(-a*d+b*c)^2*n^
2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/d^4-9/2*B^2*(-a*d+b*c)^4*n^2
*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^4-9/4*B*(-
a*d+b*c)^3*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+9/8*b*B*(-a
*d+b*c)^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-1/4*b^2*B*(-
a*d+b*c)*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-3/4*B*(-a*d+b
*c)^4*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^4
+1/4*(b*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+7/4*B^2*(-a*d+b*c)^4*
n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^4-9/2*
B^3*(-a*d+b*c)^4*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4-3/2*B^2*(-a*d+b*
c)^4*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/
b/d^4-7/4*B^3*(-a*d+b*c)^4*n^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b/d^4+3/2*B^
3*(-a*d+b*c)^4*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^4
```

3.164.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6885 vs. 2(809) = 1618.

Time = 2.32 (sec) , antiderivative size = 6885, normalized size of antiderivative = 8.51

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
Result too large to show
```

3.164.3 Rubi [A] (warning: unable to verify)

Time = 1.21 (sec) , antiderivative size = 747, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2949, 2781, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 \, dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int (a+bx)^3 \left(B \log \left(e(a+bx)^n (c+dx)^{-n}\right)+A\right)^3dx$$

$$\Big\downarrow \text{ 2949}$$

$$(bc-ad)^4 \int \frac{(a+bx)^3 \left(A+B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2781}$$

$$(bc-ad)^4 \left(\frac{(a+bx)^4 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3Bn \int \frac{(a+bx)^3 \left(A+B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right)$$

$$\Big\downarrow \text{ 2795}$$

$$(bc-ad)^4 \left(\frac{(a+bx)^4 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3Bn \int \left(\frac{\left(A+B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3\left(A+B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^2}{d^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3\left(A+B\right.}{d^3}\right.}{4b}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$(bc-ad)^4 \left(\frac{(a+bx)^4 \left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3Bn\left(\frac{b^3\left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2\left(B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{b^2Bn\left(B\log\right.}{3d^4\left(\right.}\right.}{}\right.$$

input
```
Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

3.164. $\int (a+bx)^3 \left(A+B \log \left(e(a+bx)^n (c+dx)^{-n}\right)\right)^3\,dx$

output
```
(b*c - a*d)^4*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(4*b
*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (3*B*n*((b*B^2*n^2)/(3*d^4
*(b - (d*(a + b*x))/(c + d*x))) - (b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (7*B*n*(a + b*x)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^3*(c + d*x)*(b - (d*(a + b*x))/(c +
 d*x))) + (b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*d^4*(b - (d*(a
 + b*x))/(c + d*x))^3) - (3*b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*(a + b*x)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B^
2*n^2*Log[(a + b*x)/(c + d*x)])/(3*d^4) + (2*B^2*n^2*Log[b - (d*(a + b*x))
/(c + d*x)])/d^4 + (6*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (
d*(a + b*x))/(b*(c + d*x))])/d^4 + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 - (7*B*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*d^4) + (6*B^2*
n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (2*B*n*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (7*B
^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(3*d^4) - (2*B^2*n^2*PolyL
og[3, (d*(a + b*x))/(b*(c + d*x))])/d^4))/(4*b))
```

3.164.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.164.4 Maple [F]

$$\int (bx+a)^3 \left(A + B\ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

input
```
int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

3.164.5 Fricas [F]

$$\int (a+bx)^3 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$= \int (bx+a)^3 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3 dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fri
cas")
```

output
```
integral(A^3*b^3*x^3 + 3*A^3*a*b^2*x^2 + 3*A^3*a^2*b*x + A^3*a^3 + (B^3*b^
3*x^3 + 3*B^3*a*b^2*x^2 + 3*B^3*a^2*b*x + B^3*a^3)*log((b*x + a)^n*e/(d*x
+ c)^n)^3 + 3*(A*B^2*b^3*x^3 + 3*A*B^2*a*b^2*x^2 + 3*A*B^2*a^2*b*x + A*B^2
*a^3)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^3*x^3 + 3*A^2*B*a*b^2*
x^2 + 3*A^2*B*a^2*b*x + A^2*B*a^3)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.164.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.164.7 Maxima [F]

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$$
$$= \int (bx+a)^3 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 \, dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="max
ima")
```

output
```
3/4*A^2*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^3*b^3*x^4 + 3*A^2
*B*a*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b^2*x^3 + 9/2*A^2*B*a^
2*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^3*a^2*b*x^2 + 3*A^2*B*a^3*x
*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a^3*x + 3*(a*e*n*log(b*x + a)/b - c*
e*n*log(d*x + c)/d)*A^2*B*a^3/e - 9/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*
log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a^2*b/e + 3/2*(2*a^3
*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d
^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*a*b^2/e -
1/8*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*(b^3*c*d
^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b
^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*A^2*B*b^3/e - 1/8*(2*(B^3*b^4*d^4*
x^4 + 4*B^3*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log
((d*x + c)^n)^3 - (6*B^3*a^4*d^4*n*log(b*x + a) + 6*(B^3*b^4*d^4*log(e) +
A*B^2*b^4*d^4)*x^4 + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n
- 4*a^3*b*c*d^3*n)*B^3*log(d*x + c) + 2*(12*A*B^2*a*b^3*d^4 + (a*b^3*d^4*(
n + 12*log(e)) - b^4*c*d^3*n)*B^3)*x^3 + 3*(12*A*B^2*a^2*b^2*d^4 + (3*a^2*
b^2*d^4*(n + 4*log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^3)*x^2 + 6*(4*
A*B^2*a^3*b*d^4 + (a^3*b*d^4*(3*n + 4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*
d^2*n - 6*a^2*b^2*c*d^3*n)*B^3)*x + 6*(B^3*b^4*d^4*x^4 + 4*B^3*a*b^3*d^4*x
^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log((b*x + a)^n))*log((...
```

3.164.8 Giac [F]

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (bx+a)^3 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$$
$$= \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3 (a+b\,x)^3 \, dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3, x)
```

3.165 $\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

3.165.1 Optimal result

Integrand size = 33, antiderivative size = 614

$$
\begin{aligned}
&\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx \\
&= -\frac{B^3(bc-ad)^3n^3\log(c+dx)}{bd^3} \\
&+\frac{B^2(bc-ad)^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} \\
&+\frac{4B^2(bc-ad)^3n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^3} \\
&+\frac{2B(bc-ad)^2n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^2} \\
&-\frac{bB(bc-ad)n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2d^3} \\
&+\frac{B(bc-ad)^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{bd^3} \\
&+\frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b} \\
&-\frac{B^2(bc-ad)^3n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3} \\
&+\frac{4B^3(bc-ad)^3n^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} \\
&+\frac{2B^2(bc-ad)^3n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} \\
&+\frac{B^3(bc-ad)^3n^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bd^3}-\frac{2B^3(bc-ad)^3n^3\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^3}
\end{aligned}
$$

output
```
-B^3*(-a*d+b*c)^3*n^3*ln(d*x+c)/b/d^3+B^2*(-a*d+b*c)^2*n^2*(b*x+a)*(A+B*ln
(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+4*B^2*(-a*d+b*c)^3*n^2*ln((-a*d+b*c)/b/(d
*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2*B*(-a*d+b*c)^2*n*(b*x+a)*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^2-1/2*b*B*(-a*d+b*c)*n*(d*x+c)^2*(
A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^3+B*(-a*d+b*c)^3*n*ln((-a*d+b*c)/b/(d
*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^3/b-B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^3+4*B^3*(-a*d+b*c)^3*n^3*polylog(2,d
*(b*x+a)/b/(d*x+c))/b/d^3+2*B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3+B^3*(-a*d+b*c)^3*n^3*polylog
(2,b*(d*x+c)/d/(b*x+a))/b/d^3-2*B^3*(-a*d+b*c)^3*n^3*polylog(3,d*(b*x+a)/b
/(d*x+c))/b/d^3
```

3.165.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4802 vs. 2(614) = 1228.

Time = 1.38 (sec) , antiderivative size = 4802, normalized size of antiderivative = 7.82

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(-6*a^3*A*B^2*n^2)/b - (2*a*A*b*B^2*c^2*n^2)/d^2 + (4*a^2*A*B^2*c*n^2)/d -
 (4*a^3*B^3*n^3)/b - (3*a*b*B^3*c^2*n^3)/d^2 + (7*a^2*B^3*c*n^3)/d + a^2*A
^3*x + 2*a^2*A^2*B*n*x + (A^2*b^2*B*c^2*n*x)/d^2 - (3*a*A^2*b*B*c*n*x)/d +
 a^2*A*B^2*n^2*x + (A*b^2*B^2*c^2*n^2*x)/d^2 - (2*a*A*b*B^2*c*n^2*x)/d + a
*A^3*b*x^2 + (a*A^2*b*B*n*x^2)/2 - (A^2*b^2*B*c*n*x^2)/(2*d) + (A^3*b^2*x^
3)/3 + (a^3*A^2*B*n*Log[a + b*x])/b + (3*a^3*A*B^2*n^2*Log[a + b*x])/b + (
2*a*A*b*B^2*c^2*n^2*Log[a + b*x])/d^2 - (5*a^2*A*B^2*c*n^2*Log[a + b*x])/d
 + (7*a^3*B^3*n^3*Log[a + b*x])/b + (3*a*b*B^3*c^2*n^3*Log[a + b*x])/d^2 -
 (6*a^2*B^3*c*n^3*Log[a + b*x])/d - (a^3*A*B^2*n^2*Log[a + b*x]^2)/b - (3*
a^3*B^3*n^3*Log[a + b*x]^2)/(2*b) - (a*b*B^3*c^2*n^3*Log[a + b*x]^2)/d^2 +
 (5*a^2*B^3*c*n^3*Log[a + b*x]^2)/(2*d) + (a^3*B^3*n^3*Log[a + b*x]^3)/(3*
b) - (A^2*b^2*B*c^3*n*Log[c + d*x])/d^3 + (3*a*A^2*b*B*c^2*n*Log[c + d*x])
/d^2 - (3*a^2*A^2*B*c*n*Log[c + d*x])/d - (3*A*b^2*B^2*c^3*n^2*Log[c + d*x
])/d^3 + (7*a*A*b*B^2*c^2*n^2*Log[c + d*x])/d^2 - (4*a^2*A*B^2*c*n^2*Log[c
 + d*x])/d - (6*a^3*B^3*n^3*Log[c + d*x])/b - (b^2*B^3*c^3*n^3*Log[c + d*x
])/d^3 + (3*a^2*B^3*c*n^3*Log[c + d*x])/d + (2*a^3*A*B^2*n^2*Log[a + b*x]*
Log[c + d*x])/b + (2*A*b^2*B^2*c^3*n^2*Log[a + b*x]*Log[c + d*x])/d^3 - (6
*a*A*b*B^2*c^2*n^2*Log[a + b*x]*Log[c + d*x])/d^2 + (6*a^2*A*B^2*c*n^2*Log
[a + b*x]*Log[c + d*x])/d + (3*b^2*B^3*c^3*n^3*Log[a + b*x]*Log[c + d*x])/
d^3 - (7*a*b*B^3*c^2*n^3*Log[a + b*x]*Log[c + d*x])/d^2 + (4*a^2*B^3*c*...
```

3.165.3 Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2949, 2781, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$$\Big\downarrow \ 2973$$

$$\int (a+bx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3 dx$$

$$\Big\downarrow \ 2949$$

$$(bc-ad)^3\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)$$

$$\Big\downarrow \text{2795}$$

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b^2\left(A+B\log\ldots\right.}{d^2\left(b-\ldots\right.}\right.}{b}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\ldots\right.}{d^3}\right.}{}\right.$$

input
```
Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(b*c - a*d)^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*b
*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(-((B*n*(a + b*x)*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(d^2*(c + d*x)*(b - (d*(a + b*x))/(c
+ d*x)))) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d^3*(b - (d*
(a + b*x))/(c + d*x))^2) - (2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2)/(d^2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (B^2*n^2*Log[b - (
d*(a + b*x))/(c + d*x)])/d^3 - (4*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - ((A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 + (B*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/d^3 - (
4*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 - (2*B*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3
- (B^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/d^3 + (2*B^2*n^2*PolyL
og[3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/b
```

3.165. $\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.165.4 Maple [F]

$$\int (bx+a)^2 \left(A+B\ln \left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

input
```
int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

3.165.5 Fricas [F]

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (bx+a)^2 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fri
cas")
```

output
```
integral(A^3*b^2*x^2 + 2*A^3*a*b*x + A^3*a^2 + (B^3*b^2*x^2 + 2*B^3*a*b*x
+ B^3*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*b^2*x^2 + 2*A*B^2*a
*b*x + A*B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^2*x^2 + 2*
A^2*B*a*b*x + A^2*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.165.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.165.7 Maxima [F]

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (bx+a)^2 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="max
ima")
```

output
```
A^2*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*b^2*x^3 + 3*A^2*B*a
*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b*x^2 + 3*A^2*B*a^2*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^3*a^2*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log
(d*x + c)/d)*A^2*B*a^2/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x +
 c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a*b/e + 1/2*(2*a^3*e*n*log(b*
x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2
 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*b^2/e - 1/6*(2*(B^3*b
^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*log((d*x + c)^n)^3 -
 3*(2*B^3*a^3*d^3*n*log(b*x + a) - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*
b*c*d^2*n)*B^3*log(d*x + c) + 2*(B^3*b^3*d^3*log(e) + A*B^2*b^3*d^3)*x^3 +
 (6*A*B^2*a*b^2*d^3 + (a*b^2*d^3*(n + 6*log(e)) - b^3*c*d^2*n)*B^3)*x^2 +
2*(3*A*B^2*a^2*b*d^3 + (a^2*b*d^3*(2*n + 3*log(e)) + b^3*c^2*d*n - 3*a*b^2
*c*d^2*n)*B^3)*x + 2*(B^3*b^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*
d^3*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b*d^3) - integrate(-(B^3*a^2
*b*c*d^2*log(e)^3 + 3*A*B^2*a^2*b*c*d^2*log(e)^2 + (B^3*b^3*d^3*log(e)^3 +
 3*A*B^2*b^3*d^3*log(e)^2)*x^3 + (B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2 + (b^3
*c*d^2 + 2*a*b^2*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*log((b*
x + a)^n)^3 + (3*(b^3*c*d^2*log(e)^2 + 2*a*b^2*d^3*log(e)^2)*A*B^2 + (b^3*
c*d^2*log(e)^3 + 2*a*b^2*d^3*log(e)^3)*B^3)*x^2 + 3*(B^3*a^2*b*c*d^2*log(e
) + A*B^2*a^2*b*c*d^2 + (B^3*b^3*d^3*log(e) + A*B^2*b^3*d^3)*x^3 + ((b^...
```

3.165.8 Giac [F]

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (bx+a)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$= \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3 (a+b\,x)^2\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2, x)
```

3.166 $\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

3.166.1 Optimal result

Integrand size = 31, antiderivative size = 376

$$\begin{aligned}
&\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx \\
&= -\frac{3B^2(bc-ad)^2n^2\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2} \\
&\quad -\frac{3B(bc-ad)n(a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd} \\
&\quad -\frac{3B(bc-ad)^2n\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^2} \\
&\quad +\frac{(a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b} \\
&\quad -\frac{3B^3(bc-ad)^2n^3 \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} \\
&\quad -\frac{3B^2(bc-ad)^2n^2(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)) \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} \\
&\quad +\frac{3B^3(bc-ad)^2n^3 \operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}
\end{aligned}$$

output
```
-3*B^2*(-a*d+b*c)^2*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))/b/d^2-3/2*B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))
)^2/b/d-3/2*B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/
((d*x+c)^n)))^2/b/d^2+1/2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b-
3*B^3*(-a*d+b*c)^2*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2-3*B^2*(-a*d+b*
c)^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/
b/d^2+3*B^3*(-a*d+b*c)^2*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^2
```

3.166.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2984 vs. $2(376) = 752$.

Time = 0.74 (sec) , antiderivative size = 2984, normalized size of antiderivative = 7.94

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(-12*a^2*A*B^2*d^2*n^2 + 6*a*b*B^3*c*d*n^3 - 6*a^2*B^3*d^2*n^3 + 2*a*A^3*b
*d^2*x - 3*A^2*b^2*B*c*d*n*x + 3*a*A^2*b*B*d^2*n*x + A^3*b^2*d^2*x^2 + 3*a
^2*A^2*B*d^2*n*Log[a + b*x] - 6*a*A*b*B^2*c*d*n^2*Log[a + b*x] + 6*a^2*A*B
^2*d^2*n^2*Log[a + b*x] + 12*a^2*B^3*d^2*n^3*Log[a + b*x] - 3*a^2*A*B^2*d^
2*n^2*Log[a + b*x]^2 + 3*a*b*B^3*c*d*n^3*Log[a + b*x]^2 - 3*a^2*B^3*d^2*n^
3*Log[a + b*x]^2 + a^2*B^3*d^2*n^3*Log[a + b*x]^3 + 3*A^2*b^2*B*c^2*n*Log[
c + d*x] - 6*a*A^2*b*B*c*d*n*Log[c + d*x] + 6*A*b^2*B^2*c^2*n^2*Log[c + d*
x] - 6*a*A*b*B^2*c*d*n^2*Log[c + d*x] - 12*a^2*B^3*d^2*n^3*Log[c + d*x] -
6*A*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[c + d*x] + 12*a*A*b*B^2*c*d*n^2*Log[a
 + b*x]*Log[c + d*x] + 6*a^2*A*B^2*d^2*n^2*Log[a + b*x]*Log[c + d*x] - 6*b
^2*B^3*c^2*n^3*Log[a + b*x]*Log[c + d*x] + 6*a*b*B^3*c*d*n^3*Log[a + b*x]*
Log[c + d*x] + 3*b^2*B^3*c^2*n^3*Log[a + b*x]^2*Log[c + d*x] - 6*a*b*B^3*c
*d*n^3*Log[a + b*x]^2*Log[c + d*x] - 6*a^2*B^3*d^2*n^3*Log[a + b*x]^2*Log[
c + d*x] - 6*a^2*A*B^2*d^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d
*x] + 6*a^2*B^3*d^2*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log
[c + d*x] + 3*A*b^2*B^2*c^2*n^2*Log[c + d*x]^2 - 6*a*A*b*B^2*c*d*n^2*Log[c
 + d*x]^2 + 3*b^2*B^3*c^2*n^3*Log[c + d*x]^2 - 3*a*b*B^3*c*d*n^3*Log[c + d
*x]^2 - 6*b^2*B^3*c^2*n^3*Log[a + b*x]*Log[c + d*x]^2 + 12*a*b*B^3*c*d*n^3
*Log[a + b*x]*Log[c + d*x]^2 + 3*a^2*B^3*d^2*n^3*Log[a + b*x]*Log[c + d*x]
^2 + 3*b^2*B^3*c^2*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2...
```

3.166.3 Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2973, 2949, 2781, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3\,dx$$

$$\Big\downarrow \text{2973}$$

$$\int (a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3\,dx$$

$$\Big\downarrow \text{2949}$$

$$(bc-ad)^2\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2781}$$

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)$$

$$\Big\downarrow \text{2795}$$

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{2b}\right)$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\left(\frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}+\frac{2Bn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{}\right.}{}\right.$$

input
```
Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(b*c - a*d)^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(2*b
*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (3*B*n*(((a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2)/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*
x))) + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))
/(b*(c + d*x))])/d^2 + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (
d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/d^2 + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2
, (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/
(b*(c + d*x))])/d^2))/(2*b))
```

3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
 + b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.166.4 Maple [F]

$$\int (bx+a)\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}dx$$

input
```
int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

3.166.5 Fricas [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)^{3}\,dx = \int (bx+a)\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{3}dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="frica
s")
```

output
```
integral(A^3*b*x + A^3*a + (B^3*b*x + B^3*a)*log((b*x + a)^n*e/(d*x + c)^n
)^3 + 3*(A*B^2*b*x + A*B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*
b*x + A^2*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.166.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.166.7 Maxima [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxim
a")
```

output
```
3/2*A^2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*b*x^2 + 3*A^2*B*a
*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*x + 3*(a*e*n*log(b*x + a)/b - c*
e*n*log(d*x + c)/d)*A^2*B*a/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*lo
g(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*b/e - 1/2*((B^3*b^2*d^
2*x^2 + 2*B^3*a*b*d^2*x)*log((d*x + c)^n)^3 - 3*(B^3*a^2*d^2*n*log(b*x + a
) + (b^2*c^2*n - 2*a*b*c*d*n)*B^3*log(d*x + c) + (B^3*b^2*d^2*log(e) + A*B
^2*b^2*d^2)*x^2 + (2*A*B^2*a*b*d^2 + (a*b*d^2*(n + 2*log(e)) - b^2*c*d*n)*
B^3)*x + (B^3*b^2*d^2*x^2 + 2*B^3*a*b*d^2*x)*log((b*x + a)^n))*log((d*x +
c)^n)^2)/(b*d^2) - integrate(-(B^3*a*b*c*d*log(e)^3 + 3*A*B^2*a*b*c*d*log(
e)^2 + (B^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*
x + a)^n)^3 + (B^3*b^2*d^2*log(e)^3 + 3*A*B^2*b^2*d^2*log(e)^2)*x^2 + 3*(B
^3*a*b*c*d*log(e) + A*B^2*a*b*c*d + (B^3*b^2*d^2*log(e) + A*B^2*b^2*d^2)*x
^2 + ((b^2*c*d + a*b*d^2)*A*B^2 + (b^2*c*d*log(e) + a*b*d^2*log(e))*B^3)*x
)*log((b*x + a)^n)^2 + (3*(b^2*c*d*log(e)^2 + a*b*d^2*log(e)^2)*A*B^2 + (b
^2*c*d*log(e)^3 + a*b*d^2*log(e)^3)*B^3)*x + 3*(B^3*a*b*c*d*log(e)^2 + 2*A
*B^2*a*b*c*d*log(e) + (B^3*b^2*d^2*log(e)^2 + 2*A*B^2*b^2*d^2*log(e))*x^2
+ (2*(b^2*c*d*log(e) + a*b*d^2*log(e))*A*B^2 + (b^2*c*d*log(e)^2 + a*b*d^2
*log(e)^2)*B^3)*x)*log((b*x + a)^n) - 3*(B^3*a^2*d^2*n^2*log(b*x + a) + B^
3*a*b*c*d*log(e)^2 + 2*A*B^2*a*b*c*d*log(e) + (b^2*c^2*n^2 - 2*a*b*c*d*n^2
)*B^3*log(d*x + c) + ((n*log(e) + log(e)^2)*B^3*b^2*d^2 + A*B^2*b^2*d^2...
```

3.166.8 Giac [F]

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac"
)
```

output
```
integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3(a+b\,x)\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x), x)
```

3.167 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{a+bx}\,dx$

3.167.1 Optimal result

Integrand size = 33, antiderivative size = 186

$$\begin{aligned}\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{a+bx}\,dx &= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b}\\ &\quad+\frac{3Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\\ &\quad+\frac{6B^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\\ &\quad+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{b(c+dx)}{d(a+bx)}\right)}{b}\end{aligned}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-b*(d*x+c)/d/(b*x+a))/b+3*B*n*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b+6*B^2*n^
2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,b*(d*x+c)/d/(b*x+a))/b+6*B^3
*n^3*polylog(4,b*(d*x+c)/d/(b*x+a))/b
```

3.167.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2513 vs. $2(186) = 372$.

Time = 0.57 (sec) , antiderivative size = 2513, normalized size of antiderivative = 13.51

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{a + bx}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

output
```
(4*A^3*Log[a + b*x] - 6*A^2*B*n*Log[a + b*x]^2 + 4*A*B^2*n^2*Log[a + b*x]^
3 - B^3*n^3*Log[a + b*x]^4 + B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^4 -
 4*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^3*Log[-((d*(a + b*x))/(b*(c +
 d*x)))] + 6*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x
))/(b*(c + d*x)))]^2 - 4*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[-((
d*(a + b*x))/(b*(c + d*x)))]^3 + B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x))
)]^4 - 12*A*B^2*n^2*Log[a + b*x]*Log[c + d*x]^2 + 12*B^3*n^3*Log[a + b*x]^
2*Log[c + d*x]^2 + 12*A*B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c +
d*x]^2 - 12*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c +
 d*x]^2 - 8*B^3*n^3*Log[a + b*x]*Log[c + d*x]^3 + 8*B^3*n^3*Log[(d*(a + b*
x))/(-(b*c) + a*d)]*Log[c + d*x]^3 + 12*A^2*B*n*Log[a + b*x]*Log[(b*(c + d
*x))/(b*c - a*d)] - 12*A*B^2*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a
*d)] + 4*B^3*n^3*Log[a + b*x]^3*Log[(b*(c + d*x))/(b*c - a*d)] + 8*B^3*n^3
*Log[(d*(a + b*x))/(-(b*c) + a*d)]^3*Log[(b*(c + d*x))/(b*c - a*d)] - 12*B
^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x))/(b*(c + d*x
)))]*Log[(b*(c + d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[a + b*x]*Log[c + d*
x]*Log[(b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]^2*Log[c + d*x]
*Log[(b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[a + b*x]*Log[c + d*x]^2*L
og[(b*(c + d*x))/(b*c - a*d)] + 6*B^3*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))
/(b*c - a*d)]^2 + 12*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a...
```

3.167.3 Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2973, 2949, 2779, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{a+bx}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{a+bx}dx$$

$$\Big\downarrow \text{2949}$$

$$\int \frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2779}$$

$$\frac{3Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{b}$$

$$\Big\downarrow \text{2821}$$

$$\frac{3Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{b}$$

$$\Big\downarrow \text{2830}$$

$$\frac{3Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(Bn\int \frac{(c+dx)\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(3,\right.\right.\right.}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(-\left(\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right.\right.\right.}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{b}$$

3.167. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}dx$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^3*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b) + (3*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*PolyLog[2, (
b*(c + d*x))/(d*(a + b*x))] - 2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^
n])*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]) - B*n*PolyLog[4, (b*(c + d*x)
)/(d*(a + b*x))])))/b
```

3.167.3.1 Defintions of rubi rules used

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.167.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{bx+a}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

3.167.5 Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="frica
s")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*x + a),
 x)
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a),x)
```

output
```
Timed out
```

3.167.7 Maxima [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="maxim
a")
```

output
```
-B^3*log(b*x + a)*log((d*x + c)^n)^3/b + A^3*log(b*x + a)/b + integrate((B
^3*b*c*log(e)^3 + 3*A*B^2*b*c*log(e)^2 + 3*A^2*B*b*c*log(e) + (B^3*b*d*x +
 B^3*b*c)*log((b*x + a)^n)^3 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*lo
g(e) + A*B^2*b*d)*x)*log((b*x + a)^n)^2 + 3*(B^3*b*c*log(e) + A*B^2*b*c +
(B^3*b*d*log(e) + A*B^2*b*d)*x + (B^3*b*d*n*x + B^3*a*d*n)*log(b*x + a) +
(B^3*b*d*x + B^3*b*c)*log((b*x + a)^n))*log((d*x + c)^n)^2 + (B^3*b*d*log(
e)^3 + 3*A*B^2*b*d*log(e)^2 + 3*A^2*B*b*d*log(e))*x + 3*(B^3*b*c*log(e)^2
+ 2*A*B^2*b*c*log(e) + A^2*B*b*c + (B^3*b*d*log(e)^2 + 2*A*B^2*b*d*log(e)
+ A^2*B*b*d)*x)*log((b*x + a)^n) - 3*(B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e
) + A^2*B*b*c + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^2 + (B^3*b*d*log(e)
^2 + 2*A*B^2*b*d*log(e) + A^2*B*b*d)*x + 2*(B^3*b*c*log(e) + A*B^2*b*c + (
B^3*b*d*log(e) + A*B^2*b*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*
x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

3.167.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3}{bx+a}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a), x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{a+b\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x), x)
```

3.168 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^2}\,dx$

3.168.1 Optimal result

Integrand size = 33, antiderivative size = 184

$$\begin{aligned}\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^2}\,dx &= -\frac{6B^3n^3(c+dx)}{(bc-ad)(a+bx)} - \frac{6B^2n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)}\\ &\quad - \frac{3Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)(a+bx)}\\ &\quad - \frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)(a+bx)}\end{aligned}$$

output
```
-6*B^3*n^3*(d*x+c)/(-a*d+b*c)/(b*x+a)-6*B^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/(-a*d+b*c)/(b*x+a)-3*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))^2/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/
(-a*d+b*c)/(b*x+a)
```

3.168.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 524 vs. $2(184) = 368$.

Time = 0.50 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.85

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx$$
$$= \frac{-B^3dn^3(a+bx)\log^3(a+bx)+B^3dn^3(a+bx)\log^3(c+dx)+3B^2dn^2(a+bx)\log^2(c+dx)\left(A+Bn+B\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

output
```
(-(B^3*d*n^3*(a + b*x)*Log[a + b*x]^3) + B^3*d*n^3*(a + b*x)*Log[c + d*x]^
3 + 3*B^2*d*n^2*(a + b*x)*Log[c + d*x]^2*(A + B*n + B*Log[(e*(a + b*x)^n)/
(c + d*x)^n]) + 3*B^2*d*n^2*(a + b*x)*Log[a + b*x]^2*(A + B*n + B*n*Log[c
+ d*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 3*B*d*n*(a + b*x)*Log[c + d
*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*
x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(A^3 + 3*A^2
*B*n + 6*A*B^2*n^2 + 6*B^3*n^3 + 3*B*(A^2 + 2*A*B*n + 2*B^2*n^2)*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 3*B^2*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]
^2 + B^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 3*B*d*n*(a + b*x)*Log[a + b
*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*n^2*Log[c + d*x]^2 + 2*B*(A + B*n)*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 +
2*B*n*Log[c + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*
c - a*d)*(a + b*x))
```

3.168.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2949, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^2}\,dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^2}dx$$

$$\Big\downarrow \text{ 2949}$$

$$\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^2}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{ 2742}$$

$$\frac{3Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{a+bx}}{bc-ad}$$

$$\Big\downarrow \text{ 2742}$$

$$\frac{3Bn\left(2Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{a+bx}}{bc-ad}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{3Bn\left(2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}}{bc-ad}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

output
```
(-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(a + b*x)) + 3*B*n
*(-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2*B*
n*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(a + b*x))))/(b*c - a*d)
```

3.168.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeqQ[m, -1]
```

3.168. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}dx$

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.168.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(184) = 368$.

Time = 38.58 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.95

method	result
parallelrisch	$-\frac{-3A\,B^2x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)^2 b^3d^2n-6A\,B^2x\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^3d^2n^2-3A^2Bx\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^3d^2n}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-3*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^3*d^2*n-6*A*B^2*x*ln(e*(b*x+a
)^n/((d*x+c)^n))*b^3*d^2*n^2-3*A^2*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2
*n-3*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^3*c*d*n-6*A*B^2*ln(e*(b*x+a)^n/
((d*x+c)^n))*b^3*c*d*n^2-3*A^2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n+6*A
*B^2*a*b^2*d^2*n^3-6*A*B^2*b^3*c*d*n^3+3*A^2*B*a*b^2*d^2*n^2-3*A^2*B*b^3*c
*d*n^2-B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^3*b^3*d^2*n-3*B^3*x*ln(e*(b*x+a)^
n/((d*x+c)^n))^2*b^3*d^2*n^2-6*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^2*n
^3-B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^3*b^3*c*d*n-3*B^3*ln(e*(b*x+a)^n/((d*x+
c)^n))^2*b^3*c*d*n^2+6*B^3*a*b^2*d^2*n^4-6*B^3*b^3*c*d*n^4+A^3*a*b^2*d^2*n
-A^3*b^3*c*d*n-6*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d*n^3)/(b*x+a)/b^3/
d/n/(a*d-b*c)
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(184) = 368$.

Time = 0.29 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.48

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx =$$
$$-\frac{A^3bc - A^3ad + 6\,(B^3bc - B^3ad)n^3 + (B^3bdn^3x + B^3bcn^3)\log(bx+a)^3 - (B^3bdn^3x + B^3bcn^3)\log(dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="fri
cas")
```

output
```
-(A^3*b*c - A^3*a*d + 6*(B^3*b*c - B^3*a*d)*n^3 + (B^3*b*d*n^3*x + B^3*b*c
*n^3)*log(b*x + a)^3 - (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(d*x + c)^3 + (B^3
*b*c - B^3*a*d)*log(e)^3 + 6*(A*B^2*b*c - A*B^2*a*d)*n^2 + 3*(B^3*b*c*n^3
+ A*B^2*b*c*n^2 + (B^3*b*d*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b
*c*n^2)*log(e))*log(b*x + a)^2 + 3*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3*b*d
*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x + a) + (B^
3*b*d*n^2*x + B^3*b*c*n^2)*log(e))*log(d*x + c)^2 + 3*(A*B^2*b*c - A*B^2*a
*d + (B^3*b*c - B^3*a*d)*n)*log(e)^2 + 3*(A^2*B*b*c - A^2*B*a*d)*n + 3*(2*
B^3*b*c*n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n*x + B^3*b*c*n)*lo
g(e)^2 + (2*B^3*b*d*n^3 + 2*A*B^2*b*d*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^
2 + A*B^2*b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log(e))*log(b*x + a) - 3*
(2*B^3*b*c*n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n^3*x + B^3*b*c*
n^3)*log(b*x + a)^2 + (B^3*b*d*n*x + B^3*b*c*n)*log(e)^2 + (2*B^3*b*d*n^3
+ 2*A*B^2*b*d*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3
*b*d*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b*c*n^2)*log(e))*log(b*
x + a) + 2*(B^3*b*c*n^2 + A*B^2*b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log
(e))*log(d*x + c) + 3*(A^2*B*b*c - A^2*B*a*d + 2*(B^3*b*c - B^3*a*d)*n^2 +
 2*(A*B^2*b*c - A*B^2*a*d)*n)*log(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*
d)*x)
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**2,x)
```

output
```
Timed out
```

3.168.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. 2(184) = 368.

Time = 0.26 (sec) , antiderivative size = 1129, normalized size of antiderivative = 6.14

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="max
ima")
```

output
```
-B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^2*x + a*b) - (3*(d*e*n*log(b*x +
a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b
))*log((b*x + a)^n*e/(d*x + c)^n)^2/e + (3*(2*b*c*e^2*n^2 - 2*a*d*e^2*n^2
- (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*e^2*n^2*x + a*d*e^2*
n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a) - 2*(b*
d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a))*lo
g(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a*b^2*c - a^2*b*d + (b^3*c -
a*b^2*d)*x)*e) + (6*b*c*e^3*n^3 - 6*a*d*e^3*n^3 + (b*d*e^3*n^3*x + a*d*e^3
*n^3)*log(b*x + a)^3 - (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(d*x + c)^3 - 3*(b
*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 3*(b*d*e^3*n^3*x + a*d*e^3*n^
3 - (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x + a))*log(d*x + c)^2 + 6*(b*d*e^
3*n^3*x + a*d*e^3*n^3)*log(b*x + a) - 3*(2*b*d*e^3*n^3*x + 2*a*d*e^3*n^3 +
 (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 2*(b*d*e^3*n^3*x + a*d*e^3
*n^3)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*
x)*e^2))/e)*B^3 - 3*A*B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*l
og(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x +
c)^n)/e + (2*b*c*e^2*n^2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*l
og(b*x + a)^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*
n^2*x + a*d*e^2*n^2)*log(b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*
e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*...
```

3.168.8 Giac [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx+a)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^2, x)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.58

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx$$
$$= -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{3\,B\,b\,d\,A^2\,x^2+3\,B\,(a\,d+b\,c)\,A^2\,x+3\,B\,a\,c\,A^2}{b\,(a+b\,x)^2\,(c+d\,x)}+\frac{6\,d\,(n\,B^3+A\,B^2)\,\left(b^2\,n\,x^2\,(a\,d-b\,c)+\frac{a\,b\,c\,n\,(a\,d-b\,c)}{d}+\frac{b\,n\,x\,(a\,d+b\,c)\,(a\,d-b\,c)}{d}\right)}{b^2\,(a\,d-b\,c)\,(a+b\,x)^2\,(c+d\,x)}\right) - \frac{A^3+3\,A^2\,B\,n+6\,A\,B^2\,n^2+6\,B^3\,n^3}{x\,b^2+a\,b} - \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{3\,A\,B^2}{x\,b^2+a\,b}+\frac{3\,B^3\,n}{x\,b^2+a\,b}-\frac{3\,d\,(n\,B^3+A\,B^2)}{b\,(a\,d-b\,c)}\right) - \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3\left(\frac{B^3}{b\,(a+b\,x)}-\frac{B^3\,d}{b\,(a\,d-b\,c)}\right) - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{B\,d\,n\,\left(\frac{c\,b^2+a\,d\,b}{b}+2\,b\,d\,x\right)\,(A^2+2\,A\,B\,n+2\,B^2\,n^2)\,3\mathrm{i}}{(a\,d-b\,c)\,(3\,d\,A^2\,B\,n+6\,d\,A\,B^2\,n^2+6\,d\,B^3\,n^3)}\right)\,(A^2+2\,A\,B\,n+2\,B^2\,n^2)\,6\mathrm{i}}{b\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^2,x)
```

output
```
- log((e*(a + b*x)^n)/(c + d*x)^n)*((3*A^2*B*a*c + 3*A^2*B*x*(a*d + b*c) +
 3*A^2*B*b*d*x^2)/(b*(a + b*x)^2*(c + d*x)) + (6*d*(A*B^2 + B^3*n)*(b^2*n*
x^2*(a*d - b*c) + (a*b*c*n*(a*d - b*c))/d + (b*n*x*(a*d + b*c)*(a*d - b*c)
)/d))/(b^2*(a*d - b*c)*(a + b*x)^2*(c + d*x))) - (A^3 + 6*B^3*n^3 + 6*A*B^
2*n^2 + 3*A^2*B*n)/(a*b + b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*
A*B^2)/(a*b + b^2*x) + (3*B^3*n)/(a*b + b^2*x) - (3*d*(A*B^2 + B^3*n))/(b*
(a*d - b*c))) - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(b*(a + b*x)) - (B
^3*d)/(b*(a*d - b*c))) - (B*d*n*atan((B*d*n*((b^2*c + a*b*d)/b + 2*b*d*x)*
(A^2 + 2*B^2*n^2 + 2*A*B*n)*3i)/((a*d - b*c)*(6*B^3*d*n^3 + 3*A^2*B*d*n +
6*A*B^2*d*n^2)))*(A^2 + 2*B^2*n^2 + 2*A*B*n)*6i)/(b*(a*d - b*c))
```

3.169 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx$

3.169.1 Optimal result

Integrand size = 33, antiderivative size = 390

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx \\
&= \frac{6B^3dn^3(c+dx)}{(bc-ad)^2(a+bx)} - \frac{3bB^3n^3(c+dx)^2}{8(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{6B^2dn^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{3bB^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{3Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{3bBn(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^2(a+bx)^2} \\
&\quad + \frac{d(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^2(a+bx)} \\
&\quad - \frac{b(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^2(a+bx)^2}
\end{aligned}$$

output
```
6*B^3*d*n^3*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-3/8*b*B^3*n^3*(d*x+c)^2/(-a*d+b*c
)^2/(b*x+a)^2+6*B^2*d*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+
b*c)^2/(b*x+a)-3/4*b*B^2*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(
-a*d+b*c)^2/(b*x+a)^2+3*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/
(-a*d+b*c)^2/(b*x+a)-3/4*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
^2/(-a*d+b*c)^2/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-
a*d+b*c)^2/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a
*d+b*c)^2/(b*x+a)^2
```

3.169.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.78

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = -\frac{-4B^3d^2n^3(a+bx)^2\log^3(a+bx)+4B^3d^2n^3(a+bx)^2\log^3(c+dx)+6B^2d^2n^2(a+bx)^2\log^2(c+dx)\,(2A}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

output
```
-1/8*(-4*B^3*d^2*n^3*(a + b*x)^2*Log[a + b*x]^3 + 4*B^3*d^2*n^3*(a + b*x)^
2*Log[c + d*x]^3 + 6*B^2*d^2*n^2*(a + b*x)^2*Log[c + d*x]^2*(2*A + 3*B*n +
 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 6*B^2*d^2*n^2*(a + b*x)^2*Log[a +
 b*x]^2*(2*A + 3*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d
*x)^n]) + 6*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2
+ 2*B*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b
*x)^n)/(c + d*x)^n]^2) + (b*c - a*d)*(4*A^3*(b*c - a*d) + 3*B^3*n^3*(-15*a
*d + b*(c - 14*d*x)) + 6*A*B^2*n^2*(-7*a*d + b*(c - 6*d*x)) + 6*A^2*B*n*(-
3*a*d + b*(c - 2*d*x)) + 6*B*(2*A^2*(b*c - a*d) + B^2*n^2*(-7*a*d + b*(c -
 6*d*x)) + 2*A*B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b*x)^n)/(c + d*x)
^n] + 6*B^2*(2*A*(b*c - a*d) + B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b
*x)^n)/(c + d*x)^n]^2 + 4*B^3*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d*x)^n]
^3) - 6*B*d^2*n*(a + b*x)^2*Log[a + b*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*
B^2*n^2*Log[c + d*x]^2 + 2*B*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n
] + 2*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Log[c + d*x]*(2*A + 3
*B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*c - a*d)^2*(a + b*x)^
2)
```

3.169.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^3}d\frac{a+bx}{c+dx}}{(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^3}-\frac{d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3bB^2n^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^2}+\frac{6B^2dn^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{3bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^2}+\frac{3Bdn(c-\cdots}{(bc\cdots}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

output
```
((6*B^3*d*n^3*(c + d*x))/(a + b*x) - (3*b*B^3*n^3*(c + d*x)^2)/(8*(a + b*x
)^2) + (6*B^2*d*n^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a +
 b*x) - (3*b*B^2*n^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
4*(a + b*x)^2) + (3*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(a + b*x) - (3*b*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(4*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^3)/(a + b*x) - (b*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(
2*(a + b*x)^2))/(b*c - a*d)^2
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.169.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1621 vs. $2(382) = 764$.

Time = 48.69 (sec) , antiderivative size = 1622, normalized size of antiderivative = 4.16

method	result	size
parallelrisch	Expression too large to display	1622
risch	Expression too large to display	120138

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(24*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*d^3*n-24*A*B^2*ln(e*(b*
x+a)^n/((d*x+c)^n))^2*a*b^4*c*d^2+36*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2
*b^3*d^3*n+12*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d*n-24*A^2*B*ln(e*
(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2-48*A*B^2*a*b^4*c*d^2*n^2-24*A^2*B*a*b^4
*c*d^2*n+12*A^2*B*ln(d*x+c)*a^2*b^3*d^3*n-24*B^3*ln(e*(b*x+a)^n/((d*x+c)^n
))^2*a*b^4*c*d^2*n-36*A*B^2*ln(b*x+a)*a^2*b^3*d^3*n^2+36*A*B^2*ln(d*x+c)*a
^2*b^3*d^3*n^2-12*A^2*B*ln(b*x+a)*a^2*b^3*d^3*n+36*A*B^2*x*a*b^4*d^3*n^2-3
6*A*B^2*x*b^5*c*d^2*n^2-48*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*c*d^2*n^2
+12*A^2*B*x*a*b^4*d^3*n-12*A^2*B*x*b^5*c*d^2*n+84*B^3*ln(d*x+c)*x*a*b^4*d^
3*n^3-84*B^3*ln(b*x+a)*x*a*b^4*d^3*n^3-36*A*B^2*ln(b*x+a)*x^2*b^5*d^3*n^2-
24*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^4*d^3*n-12*B^3*x*ln(e*(b*x+a)^n
/((d*x+c)^n))^2*b^5*c*d^2*n+36*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^4*d^3
*n^2-36*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c*d^2*n^2-24*A*B^2*x*ln(e*(b
*x+a)^n/((d*x+c)^n))^2*a*b^4*d^3-12*A^2*B*ln(b*x+a)*x^2*b^5*d^3*n+12*A^2*B
*ln(d*x+c)*x^2*b^5*d^3*n+36*A*B^2*ln(d*x+c)*x^2*b^5*d^3*n^2+4*A^3*a^2*b^3*
d^3+4*A^3*b^5*c^2*d-8*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))^3*a*b^4*c*d^2+6*B^3*
ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^5*c^2*d*n+42*B^3*ln(e*(b*x+a)^n/((d*x+c)^n
))*a^2*b^3*d^3*n^2+6*B^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d*n^2+12*A*B^
2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^5*c^2*d+12*A^2*B*ln(e*(b*x+a)^n/((d*x+c)
^n))*a^2*b^3*d^3+12*A^2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^5*c^2*d-42*B^3*...
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(382) = 764$.

Time = 0.36 (sec) , antiderivative size = 2244, normalized size of antiderivative = 5.75

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="fri
cas")
```

output
```
-1/8*(4*A^3*b^2*c^2 - 8*A^3*a*b*c*d + 4*A^3*a^2*d^2 + 3*(B^3*b^2*c^2 - 16*
B^3*a*b*c*d + 15*B^3*a^2*d^2)*n^3 - 4*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2
*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a)^3 + 4*(B^3*b^2*d^
2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(d
*x + c)^3 + 4*(B^3*b^2*c^2 - 2*B^3*a*b*c*d + B^3*a^2*d^2)*log(e)^3 + 6*(A*
B^2*b^2*c^2 - 8*A*B^2*a*b*c*d + 7*A*B^2*a^2*d^2)*n^2 + 6*((B^3*b^2*c^2 - 4
*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d
^2*n^3 + 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d
+ 2*B^3*a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (
B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(b*x + a)^2 + 6*((B^3*b^2*c^2
 - 4*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b
^2*d^2*n^3 + 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*
c*d + 2*B^3*a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x
 - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a) - 2*(B^3*b^2*d^2*n^2*x^
2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(d
*x + c)^2 + 6*(2*A*B^2*b^2*c^2 - 4*A*B^2*a*b*c*d + 2*A*B^2*a^2*d^2 - 2*(B^
3*b^2*c*d - B^3*a*b*d^2)*n*x + (B^3*b^2*c^2 - 4*B^3*a*b*c*d + 3*B^3*a^2*d^
2)*n)*log(e)^2 + 6*(A^2*B*b^2*c^2 - 4*A^2*B*a*b*c*d + 3*A^2*B*a^2*d^2)*n -
 6*(7*(B^3*b^2*c*d - B^3*a*b*d^2)*n^3 + 6*(A*B^2*b^2*c*d - A*B^2*a*b*d^2)*
n^2 + 2*(A^2*B*b^2*c*d - A^2*B*a*b*d^2)*n)*x + 6*((B^3*b^2*c^2 - 8*B^3*...
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**3,x)
```

output
```
Timed out
```

3.169.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2246 vs. 2(382) = 764.

Time = 0.33 (sec) , antiderivative size = 2246, normalized size of antiderivative = 5.76

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="max
ima")
```

output
```
-1/2*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^3*x^2 + 2*a*b^2*x + a^2*b) +
1/8*(6*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2
*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c
*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*
c - a^2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e - (6*(b^2*c^2*e^2*n^
2 - 8*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b
*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2
+ 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n
^2 - a*b*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a
^2*d^2*e^2*n^2)*log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^
2*x + 3*a^2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a
^2*d^2*e^2*n^2)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)
/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*
b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e) + (3*b^2*
c^2*e^3*n^3 - 48*a*b*c*d*e^3*n^3 + 45*a^2*d^2*e^3*n^3 - 4*(b^2*d^2*e^3*n^3
*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^3 + 4*(b^2*d^2*
e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(d*x + c)^3 + 18*(
b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^
2 + 6*(3*b^2*d^2*e^3*n^3*x^2 + 6*a*b*d^2*e^3*n^3*x + 3*a^2*d^2*e^3*n^3 - 2
*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x ...
```

3.169.8 Giac [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx + a)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^3, x)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.48

$$\int \frac{(A + B \log(e(a + bx)^n (c + dx)^{-n}))^3}{(a + bx)^3} \, dx$$

$$= -\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3 \left(\frac{B^3}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)} - \frac{B^3\,d^2}{2\,b\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$

$$- \frac{\frac{4\,A^3\,a\,d-4\,A^3\,b\,c+45\,B^3\,a\,d\,n^3-3\,B^3\,b\,c\,n^3+18\,A^2\,B\,a\,d\,n-6\,A^2\,B\,b\,c\,n+42\,A\,B^2\,a\,d\,n^2-6\,A\,B^2\,b\,c\,n^2}{2\,(a\,d-b\,c)} + \frac{3\,x\,\left(2\,b\,d\,A^2\,B\,n+6\,b\,d\,A\,B^2\,n^2+7\,b}{a\,d-b\,c}}{4\,a^2\,b+8\,a\,b^2\,x+4\,b^3\,x^2}$$

$$- \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2 \left(\frac{3\,A\,B^2}{2\,\left(a^2\,b+2\,a\,b^2\,x+b^3\,x^2\right)} - \frac{3\,d^2\,\left(3\,n\,B^3+2\,A\,B^2\right)}{4\,b\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right.$$

$$\left. + \frac{3\,B^3\,d^2\,\left(\frac{b\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{d^2}+\frac{2\,b^2\,n\,x\,(a\,d-b\,c)}{d}+\frac{a\,b\,n\,(a\,d-b\,c)}{d}\right)}{4\,b\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(a^2\,b+2\,a\,b^2\,x+b^3\,x^2\right)}\right)$$

$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right) \left(\frac{3\,B\,b\,d\,\left(A^2-B^2\,n^2\right)\,x^2+3\,B\,(a\,d+b\,c)\,\left(A^2-B^2\,n^2\right)\,x+3\,B\,a\,c\,\left(A^2-B^2\,n^2\right)}{2\,b\,(a+b\,x)^3\,(c+d\,x)}\right.$$

$$+ \frac{3\,d^2\,\left(3\,n\,B^3+2\,A\,B^2\right)\,\left(x\,\left(\left(\frac{b\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{d^2}+\frac{a\,b\,n\,(a\,d-b\,c)}{d}\right)\,(a\,d+b\,c)+\frac{2\,a\,b^2\,c\,n\,(a\,d-b\,c)}{d}\right)+x^2\,\left(b\,d\,\left(\frac{b}{}\right.\right.\right.}{4\,b^2\,(a+b\,x)^3\,(c-}$$

$$- \frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{B\,d^2\,n\,\left(2\,b\,d\,x-\frac{b^3\,c^2-a^2\,b\,d^2}{b\,(a\,d-b\,c)}\right)\,\left(2\,A^2+6\,A\,B\,n+7\,B^2\,n^2\right)\,3{}\mathrm{i}}{(a\,d-b\,c)\,\left(6\,A^2\,B\,d^2\,n+18\,A\,B^2\,d^2\,n^2+21\,B^3\,d^2\,n^3\right)}\right)\,\left(2\,A^2+6\,A\,B\,n+7\,B^2\,n^2\right)\,3{}\mathrm{i}}{2\,b\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^3,x)
```

output
```
- log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(2*b*(a^2 + b^2*x^2 + 2*a*b*x))
- (B^3*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((4*A^3*a*d - 4*A^3*b
*c + 45*B^3*a*d*n^3 - 3*B^3*b*c*n^3 + 18*A^2*B*a*d*n - 6*A^2*B*b*c*n + 42*
A*B^2*a*d*n^2 - 6*A*B^2*b*c*n^2)/(2*(a*d - b*c)) + (3*x*(7*B^3*b*d*n^3 + 2
*A^2*B*b*d*n + 6*A*B^2*b*d*n^2))/(a*d - b*c))/(4*a^2*b + 4*b^3*x^2 + 8*a*b
^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*A*B^2)/(2*(a^2*b + b^3*x^2
+ 2*a*b^2*x)) - (3*d^2*(2*A*B^2 + 3*B^3*n))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d)) + (3*B^3*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (2*b^2*n*x*(a*
d - b*c))/d + (a*b*n*(a*d - b*c))/d))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
*(a^2*b + b^3*x^2 + 2*a*b^2*x))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((3*B*
a*c*(A^2 - B^2*n^2) + 3*B*x*(a*d + b*c)*(A^2 - B^2*n^2) + 3*B*b*d*x^2*(A^2
 - B^2*n^2))/(2*b*(a + b*x)^3*(c + d*x)) + (3*d^2*(2*A*B^2 + 3*B^3*n)*(x*(
((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d)*(a*d + b*c)
+ (2*a*b^2*c*n*(a*d - b*c))/d) + x^2*(b*d*((b*n*(a*d - b*c)*(2*a*d - b*c))
/d^2 + (a*b*n*(a*d - b*c))/d) + (2*b^2*n*(a*d + b*c)*(a*d - b*c))/d) + a*c
*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d) + 2*b^3*n*x
^3*(a*d - b*c)))/(4*b^2*(a + b*x)^3*(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d))) - (B*d^2*n*atan((B*d^2*n*(2*b*d*x - (b^3*c^2 - a^2*b*d^2)/(b*(a*d -
b*c)))*(2*A^2 + 7*B^2*n^2 + 6*A*B*n)*3i)/((a*d - b*c)*(21*B^3*d^2*n^3 + 6*
A^2*B*d^2*n + 18*A*B^2*d^2*n^2)))*(2*A^2 + 7*B^2*n^2 + 6*A*B*n)*3i)/(2*...
```

3.169. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx$

3.170 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx$

3.170.1 Optimal result

Integrand size = 33, antiderivative size = 611

$$
\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx \\
&= -\frac{6B^3d^2n^3(c+dx)}{(bc-ad)^3(a+bx)} + \frac{3bB^3dn^3(c+dx)^2}{4(bc-ad)^3(a+bx)^2} - \frac{2b^2B^3n^3(c+dx)^3}{27(bc-ad)^3(a+bx)^3} \\
&\quad - \frac{6B^2d^2n^2(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{(bc-ad)^3(a+bx)} \\
&\quad + \frac{3bB^2dn^2(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{2(bc-ad)^3(a+bx)^2} \\
&\quad - \frac{2b^2B^2n^2(c+dx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{9(bc-ad)^3(a+bx)^3} \\
&\quad - \frac{3Bd^2n(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(bc-ad)^3(a+bx)} \\
&\quad + \frac{3bBdn(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{2(bc-ad)^3(a+bx)^2} \\
&\quad - \frac{b^2Bn(c+dx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{3(bc-ad)^3(a+bx)^3} \\
&\quad - \frac{d^2(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(bc-ad)^3(a+bx)} \\
&\quad + \frac{bd(c+dx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(bc-ad)^3(a+bx)^2} \\
&\quad - \frac{b^2(c+dx)^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{3(bc-ad)^3(a+bx)^3}
\end{aligned}
$$

output
```
-6*B^3*d^2*n^3*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+3/4*b*B^3*d*n^3*(d*x+c)^2/(-a*
d+b*c)^3/(b*x+a)^2-2/27*b^2*B^3*n^3*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-6*B^2
*d^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+3/
2*b*B^2*d*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*
x+a)^2-2/9*b^2*B^2*n^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b
*c)^3/(b*x+a)^3-3*B*d^2*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*
d+b*c)^3/(b*x+a)+3/2*b*B*d*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^2*B*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))^2/(-a*d+b*c)^3/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))^3/(-a*d+b*c)^3/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^
3/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))^3/(-a*d+b*c)^3/(b*x+a)^3
```

3.170.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 1003, normalized size of antiderivative = 1.64

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx$$
$$= \frac{-36B^3d^3n^3(a+bx)^3\log^3(a+bx)+36B^3d^3n^3(a+bx)^3\log^3(c+dx)+18B^2d^3n^2(a+bx)^3\log^2(c+dx)\,(6}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

output
```
(-36*B^3*d^3*n^3*(a + b*x)^3*Log[a + b*x]^3 + 36*B^3*d^3*n^3*(a + b*x)^3*L
og[c + d*x]^3 + 18*B^2*d^3*n^2*(a + b*x)^3*Log[c + d*x]^2*(6*A + 11*B*n +
6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 18*B^2*d^3*n^2*(a + b*x)^3*Log[a +
 b*x]^2*(6*A + 11*B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c +
d*x)^n]) + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(18*A^2 + 66*A*B*n + 85*B^2*
n^2 + 6*B*(6*A + 11*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*Log[(e*
(a + b*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(36*A^3*b^2*c^2 - 72*a*A^3*b*c*
d + 36*a^2*A^3*d^2 + 36*A^2*b^2*B*c^2*n - 126*a*A^2*b*B*c*d*n + 198*a^2*A^
2*B*d^2*n + 24*A*b^2*B^2*c^2*n^2 - 138*a*A*b*B^2*c*d*n^2 + 510*a^2*A*B^2*d
^2*n^2 + 8*b^2*B^3*c^2*n^3 - 73*a*b*B^3*c*d*n^3 + 575*a^2*B^3*d^2*n^3 - 54
*A^2*b^2*B*c*d*n*x + 270*a*A^2*b*B*d^2*n*x - 90*A*b^2*B^2*c*d*n^2*x + 882*
a*A*b*B^2*d^2*n^2*x - 57*b^2*B^3*c*d*n^3*x + 1077*a*b*B^3*d^2*n^3*x + 108*
A^2*b^2*B*d^2*n*x^2 + 396*A*b^2*B^2*d^2*n^2*x^2 + 510*b^2*B^3*d^2*n^3*x^2
+ 6*B*(18*A^2*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x)
+ b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c
+ 147*d*x) + b^2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] + 18*B^2*(6*A*(b*c - a*d)^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15
*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^
n]^2 + 36*B^3*(b*c - a*d)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 6*B*d^3*
n*(a + b*x)^3*Log[a + b*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*n^2...
```

3.170.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^4}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)^4}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^4}d\frac{a+bx}{c+dx}}{(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^4}{(a+bx)^4}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^3}{(a+bx)^3}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^2B^2n^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}-\frac{b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3(a+bx)^3}-\frac{6B^2d^2n^2(c}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

output
```
((-6*B^3*d^2*n^3*(c + d*x))/(a + b*x) + (3*b*B^3*d*n^3*(c + d*x)^2)/(4*(a
+ b*x)^2) - (2*b^2*B^3*n^3*(c + d*x)^3)/(27*(a + b*x)^3) - (6*B^2*d^2*n^2*
(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (3*b*B^2*d*n
^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) - (
2*b^2*B^2*n^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a +
b*x)^3) - (3*B*d^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(
a + b*x) + (3*b*B*d*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(2*(a + b*x)^2) - (b^2*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2)/(3*(a + b*x)^3) - (d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^3)/(a + b*x) + (b*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^3)/(a + b*x)^2 - (b^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^3)/(3*(a + b*x)^3))/(b*c - a*d)^3
```

3.170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.170. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^4}\,dx$

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.170.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2686 vs. $2(597) = 1194$.

Time = 95.36 (sec) , antiderivative size = 2687, normalized size of antiderivative = 4.40

method	result	size
parallelrisch	Expression too large to display	2687
risch	Expression too large to display	175812

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/108*(-648*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c*d^3*n-216*A*B^2*x
^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c*d^3*n-324*B^3*x*ln(e*(b*x+a)^n/((d*x+
c)^n))^2*a*b^6*c*d^3*n-972*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c*d^3*n
^2+540*A*B^2*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*d^4*n+108*A*B^2*x*ln(e*
(b*x+a)^n/((d*x+c)^n))*b^7*c^2*d^2*n-972*A*B^2*x*a*b^6*c*d^3*n^2-324*A^2*B
*x*a*b^6*c*d^3*n-648*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*c*d^3*n+324
*A*B^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^6*c^2*d^2*n-1188*A*B^2*ln(b*x+a)*x^
2*a*b^6*d^4*n^2+1188*A*B^2*ln(d*x+c)*x^2*a*b^6*d^4*n^2-324*A^2*B*ln(b*x+a)
*x^2*a*b^6*d^4*n+324*A^2*B*ln(d*x+c)*x^2*a*b^6*d^4*n-1188*A*B^2*ln(b*x+a)*
x*a^2*b^5*d^4*n^2+1188*A*B^2*ln(d*x+c)*x*a^2*b^5*d^4*n^2-324*A^2*B*ln(b*x+
a)*x*a^2*b^5*d^4*n+324*A^2*B*ln(d*x+c)*x*a^2*b^5*d^4*n+108*A^2*B*ln(d*x+c)
*a^3*b^4*d^4*n-486*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*d^4*n-108*B
^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*c*d^3*n+396*B^3*x^2*ln(e*(b*x+a)^
n/((d*x+c)^n))*a*b^6*d^4*n^2-396*B^3*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^7*c
*d^3*n^2-324*A*B^2*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))^2*a*b^6*d^4+396*A*B^2*x
^2*a*b^6*d^4*n^2-396*A*B^2*x^2*b^7*c*d^3*n^2-324*B^3*x*ln(e*(b*x+a)^n/((d*
x+c)^n))^2*a^2*b^5*d^4*n-396*A*B^2*ln(b*x+a)*x^3*b^7*d^4*n^2+396*A*B^2*ln(
d*x+c)*x^3*b^7*d^4*n^2+54*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))^2*b^7*c^2*d^2*
n+882*B^3*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^5*d^4*n^2+90*B^3*x*ln(e*(b*x
+a)^n/((d*x+c)^n))*b^7*c^2*d^2*n^2-1134*B^3*x*a*b^6*c*d^3*n^3+108*A^2*B...
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4008 vs. $2(597) = 1194$.

Time = 0.41 (sec) , antiderivative size = 4008, normalized size of antiderivative = 6.56

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="fri
cas")
```

output
```
-1/108*(36*A^3*b^3*c^3 - 108*A^3*a*b^2*c^2*d + 108*A^3*a^2*b*c*d^2 - 36*A^
3*a^3*d^3 + (8*B^3*b^3*c^3 - 81*B^3*a*b^2*c^2*d + 648*B^3*a^2*b*c*d^2 - 57
5*B^3*a^3*d^3)*n^3 + 36*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3
*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^
2)*n^3)*log(b*x + a)^3 - 36*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2
 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*
c*d^2)*n^3)*log(d*x + c)^3 + 36*(B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a
^2*b*c*d^2 - B^3*a^3*d^3)*log(e)^3 + 6*(4*A*B^2*b^3*c^3 - 27*A*B^2*a*b^2*c
^2*d + 108*A*B^2*a^2*b*c*d^2 - 85*A*B^2*a^3*d^3)*n^2 + 6*(85*(B^3*b^3*c*d^
2 - B^3*a*b^2*d^3)*n^3 + 66*(A*B^2*b^3*c*d^2 - A*B^2*a*b^2*d^3)*n^2 + 18*(
A^2*B*b^3*c*d^2 - A^2*B*a*b^2*d^3)*n)*x^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b
^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3
*n^2)*x^3 + 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*
n^2 + 3*(6*A*B^2*a*b^2*d^3*n^2 + (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*
x^2 + 3*(6*A*B^2*a^2*b*d^3*n^2 - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^
3*a^2*b*d^3)*n^3)*x + 6*(B^3*b^3*d^3*n^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3
*B^3*a^2*b*d^3*n^2*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^
2)*n^2)*log(e))*log(b*x + a)^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d +
18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 +
 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*...
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**4,x)
```

output
```
Timed out
```

3.170.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3630 vs. 2(597) = 1194.

Time = 0.43 (sec) , antiderivative size = 3630, normalized size of antiderivative = 5.94

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="max
ima")
```

output
```
-1/3*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b
^2*x + a^3*b) - 1/108*(18*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*
n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a
^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*
d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^
2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e +
(6*(4*b^3*c^3*e^2*n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 -
 85*a^3*d^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*
(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a
^3*d^3*e^2*n^2)*log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2
*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*
b^3*c^2*d*e^2*n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*
(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a
^3*d^3*e^2*n^2)*log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^
2*n^2*x^2 + 33*a^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n
^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2
)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a^3*b^4*c^3
 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c...
```

3.170.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^4, x)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 2069, normalized size of antiderivative = 3.39

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^4,x)
```

output
```
((36*A^3*a^2*d^2 + 36*A^3*b^2*c^2 + 575*B^3*a^2*d^2*n^3 + 8*B^3*b^2*c^2*n^
3 + 198*A^2*B*a^2*d^2*n + 36*A^2*B*b^2*c^2*n - 72*A^3*a*b*c*d + 510*A*B^2*
a^2*d^2*n^2 + 24*A*B^2*b^2*c^2*n^2 - 73*B^3*a*b*c*d*n^3 - 126*A^2*B*a*b*c*
d*n - 138*A*B^2*a*b*c*d*n^2)/(6*(a*d - b*c)) + (x*(359*B^3*a*b*d^2*n^3 - 1
9*B^3*b^2*c*d*n^3 + 90*A^2*B*a*b*d^2*n - 18*A^2*B*b^2*c*d*n + 294*A*B^2*a*
b*d^2*n^2 - 30*A*B^2*b^2*c*d*n^2))/(2*(a*d - b*c)) + (x^2*(85*B^3*b^2*d^2*
n^3 + 18*A^2*B*b^2*d^2*n + 66*A*B^2*b^2*d^2*n^2))/(a*d - b*c))/(x^3*(18*b^
5*c - 18*a*b^4*d) + x*(54*a^2*b^3*c - 54*a^3*b^2*d) - x^2*(54*a^2*b^3*d -
54*a*b^4*c) + 18*a^3*b^2*c - 18*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n
)^3*(B^3/(3*b*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^3*d^3)/(3*b*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n
)/(c + d*x)^n)^2*((A*B^2)/(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2) -
(d^3*(6*A*B^2 + 11*B^3*n))/(6*b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2)) + (B^3*d^3*(a*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*b*n
*(a*d - b*c))/(3*d)) + x*(b*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*
b*n*(a*d - b*c))/(3*d)) + (2*a*b^2*n*(a*d - b*c))/(3*d) + (b^2*n*(a*d - b*
c)*(3*a*d - b*c))/(3*d^2)) + (b*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b
*c*d))/(3*d^3) + (b^3*n*x^2*(a*d - b*c))/d))/(b*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2)))
- log((e*(a + b*x)^n)/(c + d*x)^n)*((x*((a*d + b*c)*(3*A^2*B*a*d - 3*A^...
```

3.171 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx$

3.171.1 Optimal result

Integrand size = 33, antiderivative size = 830

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx$$
$$= \frac{6B^3d^3n^3(c+dx)}{(bc-ad)^4(a+bx)} - \frac{9bB^3d^2n^3(c+dx)^2}{8(bc-ad)^4(a+bx)^2} + \frac{2b^2B^3dn^3(c+dx)^3}{9(bc-ad)^4(a+bx)^3}$$
$$- \frac{3b^3B^3n^3(c+dx)^4}{128(bc-ad)^4(a+bx)^4} + \frac{6B^2d^3n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^4(a+bx)}$$
$$- \frac{9bB^2d^2n^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4(bc-ad)^4(a+bx)^2}$$
$$+ \frac{2b^2B^2dn^2(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3(bc-ad)^4(a+bx)^3}$$
$$- \frac{3b^3B^2n^2(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{32(bc-ad)^4(a+bx)^4}$$
$$+ \frac{3Bd^3n(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)}$$
$$- \frac{9bBd^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4(bc-ad)^4(a+bx)^2}$$
$$+ \frac{b^2Bdn(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bc-ad)^4(a+bx)^3}$$
$$- \frac{3b^3Bn(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{16(bc-ad)^4(a+bx)^4}$$
$$+ \frac{d^3(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^4(a+bx)}$$
$$- \frac{3bd^2(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2(bc-ad)^4(a+bx)^2}$$
$$+ \frac{b^2d(c+dx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bc-ad)^4(a+bx)^3}$$
$$- \frac{b^3(c+dx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{4(bc-ad)^4(a+bx)^4}$$

output
```
6*B^3*d^3*n^3*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-9/8*b*B^3*d^2*n^3*(d*x+c)^2/(-a
*d+b*c)^4/(b*x+a)^2+2/9*b^2*B^3*d*n^3*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-3/1
28*b^3*B^3*n^3*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+6*B^2*d^3*n^2*(d*x+c)*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-9/4*b*B^2*d^2*n^2*(d*x+
c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B^2*
d*n^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-3
/32*b^3*B^2*n^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(
b*x+a)^4+3*B*d^3*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^
4/(b*x+a)-9/4*b*B*d^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*
d+b*c)^4/(b*x+a)^2+b^2*B*d*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
/(-a*d+b*c)^4/(b*x+a)^3-3/16*b^3*B*n*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^2/(-a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n
)))^3/(-a*d+b*c)^4/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^3/(-a*d+b*c)^4/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))^3/(-a*d+b*c)^4/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))^3/(-a*d+b*c)^4/(b*x+a)^4
```

3.171.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 1370, normalized size of antiderivative = 1.65

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx =$$
$$-\frac{-288B^3d^4n^3(a+bx)^4\log^3(a+bx)+288B^3d^4n^3(a+bx)^4\log^3(c+dx)+72B^2d^4n^2(a+bx)^4\log^2(c+d}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

output
```
-1/1152*(-288*B^3*d^4*n^3*(a + b*x)^4*Log[a + b*x]^3 + 288*B^3*d^4*n^3*(a
+ b*x)^4*Log[c + d*x]^3 + 72*B^2*d^4*n^2*(a + b*x)^4*Log[c + d*x]^2*(12*A
+ 25*B*n + 12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 72*B^2*d^4*n^2*(a + b*
x)^4*Log[a + b*x]^2*(12*A + 25*B*n + 12*B*n*Log[c + d*x] + 12*B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]) + 12*B*d^4*n*(a + b*x)^4*Log[c + d*x]*(72*A^2 + 30
0*A*B*n + 415*B^2*n^2 + 12*B*(12*A + 25*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)
^n] + 72*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) + (b*c - a*d)*(288*A^3*b^
3*c^3 - 864*a*A^3*b^2*c^2*d + 864*a^2*A^3*b*c*d^2 - 288*a^3*A^3*d^3 + 216*
A^2*b^3*B*c^3*n - 936*a*A^2*b^2*B*c^2*d*n + 1656*a^2*A^2*b*B*c*d^2*n - 180
0*a^3*A^2*B*d^3*n + 108*A*b^3*B^2*c^3*n^2 - 660*a*A*b^2*B^2*c^2*d*n^2 + 19
32*a^2*A*b*B^2*c*d^2*n^2 - 4980*a^3*A*B^2*d^3*n^2 + 27*b^3*B^3*c^3*n^3 - 2
29*a*b^2*B^3*c^2*d*n^3 + 1067*a^2*b*B^3*c*d^2*n^3 - 5845*a^3*B^3*d^3*n^3 -
 288*A^2*b^3*B*c^2*d*n*x + 1440*a*A^2*b^2*B*c*d^2*n*x - 3744*a^2*A^2*b*B*d
^3*n*x - 336*A*b^3*B^2*c^2*d*n^2*x + 2544*a*A*b^2*B^2*c*d^2*n^2*x - 13008*
a^2*A*b*B^2*d^3*n^2*x - 148*b^3*B^3*c^2*d*n^3*x + 1676*a*b^2*B^3*c*d^2*n^3
*x - 16468*a^2*b*B^3*d^3*n^3*x + 432*A^2*b^3*B*c*d^2*n*x^2 - 3024*a*A^2*b^
2*B*d^3*n*x^2 + 936*A*b^3*B^2*c*d^2*n^2*x^2 - 11736*a*A*b^2*B^2*d^3*n^2*x^
2 + 690*b^3*B^3*c*d^2*n^3*x^2 - 15630*a*b^2*B^3*d^3*n^3*x^2 - 864*A^2*b^3*
B*d^3*n*x^3 - 3600*A*b^3*B^2*d^3*n^2*x^3 - 4980*b^3*B^3*d^3*n^3*x^3 + 12*B
*(72*A^2*(b*c - a*d)^3 + B^2*n^2*(-415*a^3*d^3 + a^2*b*d^2*(161*c - 108...
```

3.171.3 Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 669, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2973, 2949, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(a+bx)^5} \, dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{\left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(a+bx)^5}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)^5}d\frac{a+bx}{c+dx}}{(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^5}{(a+bx)^5}-\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^4}{(a+bx)^4}+\frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3(c+dx)^3}{(a+bx)^3}-\frac{d^3\left(A+B\log\right.}{}\right.}{(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3b^3B^2n^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{32(a+bx)^4}-\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{4(a+bx)^4}-\frac{3b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{16(a+bx)^4}+\frac{2b^2B^2dn^2}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

output
```
((6*B^3*d^3*n^3*(c + d*x))/(a + b*x) - (9*b*B^3*d^2*n^3*(c + d*x)^2)/(8*(a
 + b*x)^2) + (2*b^2*B^3*d*n^3*(c + d*x)^3)/(9*(a + b*x)^3) - (3*b^3*B^3*n^
3*(c + d*x)^4)/(128*(a + b*x)^4) + (6*B^2*d^3*n^2*(c + d*x)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(a + b*x) - (9*b*B^2*d^2*n^2*(c + d*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(4*(a + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d
*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - (3*b^3*B^2
*n^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(32*(a + b*x)^4)
 + (3*B*d^3*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)
 - (9*b*B*d^2*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*(
a + b*x)^2) + (b^2*B*d*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(a + b*x)^3 - (3*b^3*B*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2)/(16*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^3)/(a + b*x) - (3*b*d^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^3)/(2*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^3)/(a + b*x)^3 - (b^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^3)/(4*(a + b*x)^4))/(b*c - a*d)^4
```

3.171. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx$

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.171.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8291 vs. $2(810) = 1620$.

Time = 191.85 (sec) , antiderivative size = 8292, normalized size of antiderivative = 9.99

method	result	size
parallelrisch	Expression too large to display	8292
risch	Expression too large to display	236754

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.171.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6057 vs. 2(810) = 1620.

Time = 0.53 (sec) , antiderivative size = 6057, normalized size of antiderivative = 7.30

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^5} dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="fri
cas")
```

output
```
Too large to include
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^5} dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**5,x)
```

output
```
Timed out
```

3.171.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5280 vs. 2(810) = 1620.

Time = 0.57 (sec) , antiderivative size = 5280, normalized size of antiderivative = 6.36

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^5} dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="max
ima")
```

output
```
-1/4*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b
^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/1152*(72*(12*d^4*e*n*log(b*x + a)/(b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12
*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c
^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^
2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*
x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3
 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a
^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3
*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b
^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*log((b*x + a)^n*e/(d*x + c)^
n)^2/e - (12*(9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2
*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*
d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b
^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4
+ 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^
2*x + a^4*d^4*e^2*n^2)*log(b*x + a)^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*
d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*
d^4*e^2*n^2)*log(d*x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2...
```

3.171.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^5, x)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 4257, normalized size of antiderivative = 5.13

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5}\,dx = \text{Too large to display}$$

input

```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^5,x)
```

output

```
log((e*(a + b*x)^n)/(c + d*x)^n)*((x*((a*d + b*c)*(a*((9*B^3*a*d^2*n^2)/2
- (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A
^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d
) + a*c*(b*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n
^2)/2 - (9*B^3*b^2*c*d*n^2)/2)) + x^2*((a*d + b*c)*(b*((9*B^3*a*d^2*n^2)/2
 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n^2)/2)
+ b*d*(a*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2
+ (11*B^3*b^2*c^2*n^2)/2 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b
*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) + 6*B^3*a*b^2*c*d^2*n^2) + x^3*(b*d*(b*((9
*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3
*b^2*c*d*n^2)/2) + 6*B^3*b^2*d^2*n^2*(a*d + b*c)) + a*c*(a*((9*B^3*a*d^2*n
^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2
 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a
*b*c*d) + 6*B^3*b^3*d^3*n^2*x^4)/(8*b*(a*d - b*c)^2*(a + b*x)^5*(c + d*x))
 - (d^4*(12*A*B^2 + 25*B^3*n)*(x^3*((a*d + b*c)*(b*(b*((2*a*b*n*(a*d - b*c
)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*
c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^3*n*(a*d
 - b*c)^3*(4*a*d - b*c))/d^2 + (6*a*b^3*n*(a*d - b*c)^3)/d) + b*d*(b*(a*((
2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) +
(2*b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + a*(b...
```

3.172 $\int \frac{1}{(ag+bgx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

3.172.1 Optimal result

Integrand size = 36, antiderivative size = 96

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

output
```
exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei((-A-B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

3.172.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]
```

3.172.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2973, 2949, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \ 2973$$

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \ 2949$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$

$$\Big\downarrow \ 2747$$

$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2n(a+bx)(bc-ad)}$$

$$\Big\downarrow \ 2609$$

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a +
b*x))
```

3.172.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.172.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln \left(e\left(bx+a\right)^n \left(dx+c\right)^{-n}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"fricas")
```

output
```
e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/
(b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

3.172.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"maxima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

3.172.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\,(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\,(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

3.173 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

3.173.1 Optimal result

Integrand size = 30, antiderivative size = 180

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = -\frac{B(bc-ad)^4g^4x}{5d^4} + \frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} - \frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4}{20bd} + \frac{B(bc-ad)^5g^4\log(c+dx)}{5bd^5} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b}$$

output
```
-1/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-1/15
*B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/
5*B*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(
b*x+a)))/b
```

3.173.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$= \frac{g^4\left(-\frac{B(-bc+ad)\left(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx)\right)}{12d^5}+(a+bx)^5\left(A\right.\right.}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(g^4*(-1/12*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)
^2*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12
*(b*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x))/(
a + b*x)])))/(5*b)
```

3.173.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)dx$$
$$\Big\downarrow \text{ 2948}$$
$$\frac{B(bc-ad)\int \frac{g^5(a+bx)^4}{c+dx}dx}{5bg}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{Bg^4(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}$$
$$\Big\downarrow \text{ 49}$$

$$\frac{Bg^4(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}+\frac{Bg^4(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(B*(b*c - a*d)*g^4*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^
2)/(2*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c
 - a*d)^4*Log[c + d*x])/d^5))/(5*b) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c +
d*x))/(a + b*x)]))/(5*b)
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.173.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(168) = 336$.

Time = 1.16 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.46

method	result
risch	$\frac{g^4b^4Ax^5}{5}+g^4b^3Aa\,x^4-\frac{g^4b^3Ba\,x^4}{20}+\frac{g^4b^4Bc\,x^4}{20d}+2g^4b^2A\,a^2x^3-\frac{4g^4b^2B\,a^2x^3}{15}-\frac{g^4b^4B\,c^2x^3}{15d^2}+2g$
parts	$\frac{A\,g^4(bx+a)^5}{5b}+B\,g^4e^5(ad-cb)^5\left(-\frac{1}{20deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^4}+\frac{1}{15d^2e^2b\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^3}+\right.$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{Ab\,e^4g^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{5\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^5}+B\,b^2e^4g^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)\right.}{}$
default	$\frac{e(ad-cb)\left(-\frac{Ab\,e^4g^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{5\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^5}+B\,b^2e^4g^4\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)\right.}{}$
parallelrisch	$\frac{120Bx\,a^3b^2c\,d^4g^4-120Bx\,a^2b^3c^2d^3g^4+60Bxa\,b^4c^3d^2g^4+60B\ln\left(\frac{e(dx+c)}{bx+a}\right)a^4bc\,d^4g^4-120B\ln\left(\frac{e(dx+c)}{bx+a}\right)a^3b^2c^2d^3g^4+}{}$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/5*g^4*b^4*A*x^5+g^4*b^3*A*a*x^4-1/20*g^4*b^3*B*a*x^4+1/20*g^4/d*b^4*B*c*
x^4+2*g^4*b^2*A*a^2*x^3-4/15*g^4*b^2*B*a^2*x^3-1/15*g^4/d^2*b^4*B*c^2*x^3+
2*g^4*b*A*a^3*x^2-3/5*g^4*b*B*a^3*x^2+1/10*g^4/d^3*b^4*B*c^3*x^2+g^4*A*a^4
*x-4/5*g^4*B*a^4*x-1/5*g^4/d^4*b^4*B*c^4*x+1/5*g^4/d^5*b^4*B*ln(d*x+c)*c^5
+g^4/d*B*ln(d*x+c)*a^4*c+2*g^4/d*b*B*a^3*c*x-2*g^4/d^2*b^2*B*a^2*c^2*x+g^4
/d^3*b^3*B*a*c^3*x-2*g^4/d^2*b*B*ln(d*x+c)*a^3*c^2+2*g^4/d^3*b^2*B*ln(d*x+
c)*a^2*c^3-g^4/d^4*b^3*B*ln(d*x+c)*a*c^4+1/3*g^4/d*b^3*B*a*c*x^3-1/5*g^4/b
*B*ln(d*x+c)*a^5+g^4/d*b^2*B*a^2*c*x^2-1/2*g^4/d^2*b^3*B*a*c^2*x^2+1/5*(b*
x+a)^5*g^4*B/b*ln(e*(d*x+c)/(b*x+a))
```

3.173.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(168) = 336$.

Time = 0.35 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.41

$$\int (ag + bgx)^4 \left(A + B\log\left(\frac{e(c + dx)}{a + bx}\right)\right) dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 - 12\,Ba^5d^5g^4\log(bx + a) + 3\,(Bb^5cd^4 + (20\,A - B)ab^4d^5)g^4x^4 - 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4 -}{}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^
4 + (20*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - 2
*(15*A - 2*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3
+ 10*B*a^2*b^3*c*d^4 + 2*(10*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 12*(B*b^5*c^4
*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 - (5*A
- 4*B)*a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c
^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*
b^5*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*
a^3*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d*e*x + c*e)/(b*x + a)))/(
b*d^5)
```

3.173.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(155) = 310$.

Time = 3.78 (sec) , antiderivative size = 969, normalized size of antiderivative = 5.38

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx$$

$$= \frac{Ab^4g^4x^5}{5} - \frac{Ba^5g^4\log\left(x+\frac{\frac{Ba^6d^5g^4}{b}+5Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5Ba^2b^3c^4dg^4+Bab^4c^5g^4}{Ba^5d^5g^4+5Ba^4bcd^4g^4-10Ba^3b^2c^2d^3g^4+10Ba^2b^3c^3d^2g^4-5Bab^4c^4dg^4+Bb^5c^5g^4}\right)}{5b}$$

$$+ \frac{Bcg^4\cdot(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4)\log\left(x+\frac{6Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5B}{}\right.}{5d^5}$$

$$+ x^4\left(Aab^3g^4-\frac{Bab^3g^4}{20}+\frac{Bb^4cg^4}{20d}\right)+x^3\cdot\left(2Aa^2b^2g^4-\frac{4Ba^2b^2g^4}{15}+\frac{Bab^3cg^4}{3d}-\frac{Bb^4c^2g^4}{15d^2}\right)$$

$$+ x^2\cdot\left(2Aa^3bg^4-\frac{3Ba^3bg^4}{5}+\frac{Ba^2b^2cg^4}{d}-\frac{Bab^3c^2g^4}{2d^2}+\frac{Bb^4c^3g^4}{10d^3}\right)$$

$$+ x\left(Aa^4g^4-\frac{4Ba^4g^4}{5}+\frac{2Ba^3bcg^4}{d}-\frac{2Ba^2b^2c^2g^4}{d^2}+\frac{Bab^3c^3g^4}{d^3}-\frac{Bb^4c^4g^4}{5d^4}\right)$$

$$+ \left(Ba^4g^4x+2Ba^3bg^4x^2+2Ba^2b^2g^4x^3+Bab^3g^4x^4+\frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
A*b**4*g**4*x**5/5 - B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*
d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5
*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**
4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2
*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) + B*c*g**4*(5*a*
*4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b*
*4*c**4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B
*a**3*b**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4
 - B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5
*a*b**3*c**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**
3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*
g**4 + 5*B*a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*
b**3*c**3*d**2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5)
 + x**4*(A*a*b**3*g**4 - B*a*b**3*g**4/20 + B*b**4*c*g**4/(20*d)) + x**3*(
2*A*a**2*b**2*g**4 - 4*B*a**2*b**2*g**4/15 + B*a*b**3*c*g**4/(3*d) - B*b**
4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 - 3*B*a**3*b*g**4/5 + B*a**
2*b**2*c*g**4/d - B*a*b**3*c**2*g**4/(2*d**2) + B*b**4*c**3*g**4/(10*d**3)
) + x*(A*a**4*g**4 - 4*B*a**4*g**4/5 + 2*B*a**3*b*c*g**4/d - 2*B*a**2*b**2
*c**2*g**4/d**2 + B*a*b**3*c**3*g**4/d**3 - B*b**4*c**4*g**4/(5*d**4)) + (
B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b*...
```

3.173. $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx$

3.173.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(168) = 336$.

Time = 0.23 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.44

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = \frac{1}{5}\,Ab^4g^4x^5 + Aab^3g^4x^4 + 2\,Aa^2b^2g^4x^3$$
$$+ 2\,Aa^3bg^4x^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Ba^4g^4$$
$$+2\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Ba^3bg^4$$
$$+\left(2\,x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{2\,a^3\log(bx+a)}{b^3} + \frac{2\,c^3\log(dx+c)}{d^3} + \frac{(b^2cd-abd^2)x^2 - 2\,(b^2c^2-a^2d}{b^2d^2}\right.$$
$$+\frac{1}{6}\left(6\,x^4\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) + \frac{6\,a^4\log(bx+a)}{b^4} - \frac{6\,c^4\log(dx+c)}{d^4} + \frac{2\,(b^3cd^2-ab^2d^3)x^3 - 3\,(b^3c^2c}{b^3}\right.$$
$$+\frac{1}{60}\left(12\,x^5\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right) - \frac{12\,a^5\log(bx+a)}{b^5} + \frac{12\,c^5\log(dx+c)}{d^5} + \frac{3\,(b^4cd^3-ab^3d^4)x^4 - 4\,(}{}\right.$$
$$+ Aa^4g^4x$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*
x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d
*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^3*b*g^4
 + (2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2
*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x
)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a
)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3))*B*a*b^3*g^4 + 1/60*(12*x^5*log(d*e*x/(b*x + a) + c*e/(b*x + a)) -
12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*
d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x
^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

3.173.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(168) = 336$.

Time = 0.47 (sec) , antiderivative size = 2030, normalized size of antiderivative = 11.28

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
-1/60*(12*(B*b^6*c^6*e^6*g^4 - 6*B*a*b^5*c^5*d*e^6*g^4 + 15*B*a^2*b^4*c^4*
d^2*e^6*g^4 - 20*B*a^3*b^3*c^3*d^3*e^6*g^4 + 15*B*a^4*b^2*c^2*d^4*e^6*g^4
- 6*B*a^5*b*c*d^5*e^6*g^4 + B*a^6*d^6*e^6*g^4)*log((d*e*x + c*e)/(b*x + a)
)/(b*d^5*e^5 - 5*(d*e*x + c*e)*b^2*d^4*e^4/(b*x + a) + 10*(d*e*x + c*e)^2*
b^3*d^3*e^3/(b*x + a)^2 - 10*(d*e*x + c*e)^3*b^4*d^2*e^2/(b*x + a)^3 + 5*(
d*e*x + c*e)^4*b^5*d*e/(b*x + a)^4 - (d*e*x + c*e)^5*b^6/(b*x + a)^5) + (1
2*A*b^6*c^6*d^4*e^6*g^4 - 25*B*b^6*c^6*d^4*e^6*g^4 - 72*A*a*b^5*c^5*d^5*e^
6*g^4 + 150*B*a*b^5*c^5*d^5*e^6*g^4 + 180*A*a^2*b^4*c^4*d^6*e^6*g^4 - 375*
B*a^2*b^4*c^4*d^6*e^6*g^4 - 240*A*a^3*b^3*c^3*d^7*e^6*g^4 + 500*B*a^3*b^3*
c^3*d^7*e^6*g^4 + 180*A*a^4*b^2*c^2*d^8*e^6*g^4 - 375*B*a^4*b^2*c^2*d^8*e^
6*g^4 - 72*A*a^5*b*c*d^9*e^6*g^4 + 150*B*a^5*b*c*d^9*e^6*g^4 + 12*A*a^6*d^
10*e^6*g^4 - 25*B*a^6*d^10*e^6*g^4 + 77*(d*e*x + c*e)*B*b^7*c^6*d^3*e^5*g^
4/(b*x + a) - 462*(d*e*x + c*e)*B*a*b^6*c^5*d^4*e^5*g^4/(b*x + a) + 1155*(
d*e*x + c*e)*B*a^2*b^5*c^4*d^5*e^5*g^4/(b*x + a) - 1540*(d*e*x + c*e)*B*a^
3*b^4*c^3*d^6*e^5*g^4/(b*x + a) + 1155*(d*e*x + c*e)*B*a^4*b^3*c^2*d^7*e^5
*g^4/(b*x + a) - 462*(d*e*x + c*e)*B*a^5*b^2*c*d^8*e^5*g^4/(b*x + a) + 77*
(d*e*x + c*e)*B*a^6*b*d^9*e^5*g^4/(b*x + a) - 94*(d*e*x + c*e)^2*B*b^8*c^6
*d^2*e^4*g^4/(b*x + a)^2 + 564*(d*e*x + c*e)^2*B*a*b^7*c^5*d^3*e^4*g^4/(b*
x + a)^2 - 1410*(d*e*x + c*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^4/(b*x + a)^2 + 18
80*(d*e*x + c*e)^2*B*a^3*b^5*c^3*d^5*e^4*g^4/(b*x + a)^2 - 1410*(d*e*x ...
```

3.173.9 Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 1008, normalized size of antiderivative = 5.60

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(B\,a^4\,g^4\,x+2\,B\,a^3\,b\,g^4\,x^2+2\,B\,a^2\,b^2\,g^4\,x^3+B\,a\,b^3\,g^4\,x^4+\frac{B\,b^4\,g^4\,x^5}{5}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{15\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+x^2\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}\right.}{10\,b\,d}+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}-\frac{a\,c\,\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}\right)$$

$$+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}-\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}\right.}{5\,b\,d}\right.}{5\,b\,d}\right.$$

$$a\,c\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)\,(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
log((e*(c + d*x))/(a + b*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*
g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a
*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b*c
))/(3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*
A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d
))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - B*
a*d + B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*
d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d +
 10*A*b*c - 2*B*a*d + 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*(((
(b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c
- B*a*d + B*b*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d
+ 5*A*b*c - B*a*d + B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d
 + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (
a*c*((((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(
5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5
*A*b*c - B*a*d + B*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(2
5*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c)...
```

3.173. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

3.174 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

3.174.1 Optimal result

Integrand size = 30, antiderivative size = 149

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = \frac{B(bc-ad)^3g^3x}{4d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2} + \frac{B(bc-ad)g^3(a+bx)^3}{12bd} - \frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}$$

output
```
1/4*B*(-a*d+b*c)^3*g^3*x/d^3-1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/12*B
*(-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g
^3*(b*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

3.174.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = \frac{g^3\left(\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4} + (a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*
x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4) + (a + b
*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(4*b)
```

3.174.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{B(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{4bg}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bg^3(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{4b}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{Bg^3(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(B*(b*c - a*d)*g^3*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2
*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(4*b) + (g^
3*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(4*b)
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.174.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(139) = 278$.

Time = 0.93 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.11

method	result
risch	$\frac{g^3(bx+a)^4 B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{4b}+\frac{g^3b^3A\,x^4}{4}+g^3b^2Aa\,x^3-\frac{g^3b^2Ba\,x^3}{12}+\frac{g^3b^3Bc\,x^3}{12d}+\frac{3g^3bA\,a^2x^2}{2}-\frac{3g^3bB\,a^2x}{8}$
parts	$\frac{A\,g^3(bx+a)^4}{4b}-B\,g^3e^4(ad-cb)^4\left(-\frac{1}{4d^3e^3b\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{4d^4e^4b}-\frac{}{12deb}\right.$
derivativedivides	$\frac{e(ad-cb)\left(\frac{Ab\,e^3g^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{4\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^4}-B\,b^2e^3g^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(-\frac{1}{4d^3e^3b\left(\left(\frac{de}{b}-\frac{e(ad-c}{b(bx+c}\right.\right.}\right.\right.}{}$
default	$\frac{e(ad-cb)\left(\frac{Ab\,e^3g^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{4\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^4}-B\,b^2e^3g^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(-\frac{1}{4d^3e^3b\left(\left(\frac{de}{b}-\frac{e(ad-c}{b(bx+c}\right.\right.}\right.\right.}{}$
parallelrisch	$\frac{24B\ln(bx+a)a^3bc\,d^3g^3-9B\,a^3bc\,d^3g^3-24B\,a^2b^2c^2d^2g^3+21Ba\,b^3c^3d\,g^3+36Bx\,a^2b^2c\,d^3g^3-24Bxa\,b^3c^2d^2g^3+24Bx\ln}{}$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/4*g^3*(b*x+a)^4*B/b*ln(e*(d*x+c)/(b*x+a))+1/4*g^3*b^3*A*x^4+g^3*b^2*A*a*
x^3-1/12*g^3*b^2*B*a*x^3+1/12*g^3*b^3/d*B*c*x^3+3/2*g^3*b*A*a^2*x^2-3/8*g^
3*b*B*a^2*x^2+1/2*g^3*b^2/d*B*a*c*x^2-1/8*g^3*b^3/d^2*B*c^2*x^2+g^3*A*a^3*
x-1/4*g^3/b*B*ln(d*x+c)*a^4+g^3/d*B*ln(d*x+c)*a^3*c-3/2*g^3*b/d^2*B*ln(d*x
+c)*a^2*c^2+g^3*b^2/d^3*B*ln(d*x+c)*a*c^3-1/4*g^3*b^3/d^4*B*ln(d*x+c)*c^4-
3/4*g^3*B*a^3*x+3/2*g^3*b/d*B*a^2*c*x-g^3*b^2/d^2*B*a*c^2*x+1/4*g^3*b^3/d^
3*B*c^3*x
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(139) = 278$.

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.15

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$=\frac{6\,Ab^4d^4g^3x^4-6\,Ba^4d^4g^3\log{(bx+a)}+2\,(Bb^4cd^3+(12\,A-B)ab^3d^4)g^3x^3-3\,(Bb^4c^2d^2-4\,Bab^3cd^3-3}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output

```
1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3
+ (12*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(
4*A - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a
^2*b^2*c*d^3 + (4*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3
*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^
4*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^
4*g^3*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^4)
```

3.174.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.

Time = 2.22 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx$$

$$= \frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{4b}$$

$$+ \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-bc)\cdots}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g\cdots}\right.}{4d^4}$$

$$+ x^3\left(Aab^2g^3-\frac{Bab^2g^3}{12}+\frac{Bb^3cg^3}{12d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}-\frac{3Ba^2bg^3}{8}+\frac{Bab^2cg^3}{2d}-\frac{Bb^3c^2g^3}{8d^2}\right)$$

$$+ x\left(Aa^3g^3-\frac{3Ba^3g^3}{4}+\frac{3Ba^2bcg^3}{2d}-\frac{Bab^2c^2g^3}{d^2}+\frac{Bb^3c^3g^3}{4d^3}\right)$$

$$+ \left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

input

```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*
d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b*
*3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c
**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) + B*c*g*
*3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c
*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b
**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d
)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**
3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*
g**3 - B*a*b**2*g**3/12 + B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2
- 3*B*a**2*b*g**3/8 + B*a*b**2*c*g**3/(2*d) - B*b**3*c**2*g**3/(8*d**2)) +
 x*(A*a**3*g**3 - 3*B*a**3*g**3/4 + 3*B*a**2*b*c*g**3/(2*d) - B*a*b**2*c**
2*g**3/d**2 + B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**
3*x**2/2 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)/(a + b
*x))
```

3.174.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(139) = 278.

Time = 0.20 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.93

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{1}{4}Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}Aa^2bg^3x^2$$
$$+\left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)Ba^3g^3$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)Ba^2bg^3$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{2a^3\log(bx+a)}{b^3}+\frac{2c^3\log(dx+c)}{d^3}+\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log(dx+c)}{d^4}+\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2}{}\right.$$
$$+Aa^3g^3x$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d*e*x/(
b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^3*g^3
 + 3/2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 -
c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(d
*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x +
 c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B
*a*b^2*g^3 + 1/24*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log(
b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g
^3 + A*a^3*g^3*x
```

3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(139) = 278$.

Time = 0.45 (sec) , antiderivative size = 1506, normalized size of antiderivative = 10.11

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
1/24*(6*(B*b^5*c^5*e^5*g^3 - 5*B*a*b^4*c^4*d*e^5*g^3 + 10*B*a^2*b^3*c^3*d^
2*e^5*g^3 - 10*B*a^3*b^2*c^2*d^3*e^5*g^3 + 5*B*a^4*b*c*d^4*e^5*g^3 - B*a^5
*d^5*e^5*g^3)*log((d*e*x + c*e)/(b*x + a))/(b*d^4*e^4 - 4*(d*e*x + c*e)*b^
2*d^3*e^3/(b*x + a) + 6*(d*e*x + c*e)^2*b^3*d^2*e^2/(b*x + a)^2 - 4*(d*e*x
 + c*e)^3*b^4*d*e/(b*x + a)^3 + (d*e*x + c*e)^4*b^5/(b*x + a)^4) + (6*A*b^
5*c^5*d^3*e^5*g^3 - 11*B*b^5*c^5*d^3*e^5*g^3 - 30*A*a*b^4*c^4*d^4*e^5*g^3
+ 55*B*a*b^4*c^4*d^4*e^5*g^3 + 60*A*a^2*b^3*c^3*d^5*e^5*g^3 - 110*B*a^2*b^
3*c^3*d^5*e^5*g^3 - 60*A*a^3*b^2*c^2*d^6*e^5*g^3 + 110*B*a^3*b^2*c^2*d^6*e
^5*g^3 + 30*A*a^4*b*c*d^7*e^5*g^3 - 55*B*a^4*b*c*d^7*e^5*g^3 - 6*A*a^5*d^8
*e^5*g^3 + 11*B*a^5*d^8*e^5*g^3 + 26*(d*e*x + c*e)*B*b^6*c^5*d^2*e^4*g^3/(
b*x + a) - 130*(d*e*x + c*e)*B*a*b^5*c^4*d^3*e^4*g^3/(b*x + a) + 260*(d*e*
x + c*e)*B*a^2*b^4*c^3*d^4*e^4*g^3/(b*x + a) - 260*(d*e*x + c*e)*B*a^3*b^3
*c^2*d^5*e^4*g^3/(b*x + a) + 130*(d*e*x + c*e)*B*a^4*b^2*c*d^6*e^4*g^3/(b*
x + a) - 26*(d*e*x + c*e)*B*a^5*b*d^7*e^4*g^3/(b*x + a) - 21*(d*e*x + c*e)
^2*B*b^7*c^5*d*e^3*g^3/(b*x + a)^2 + 105*(d*e*x + c*e)^2*B*a*b^6*c^4*d^2*e
^3*g^3/(b*x + a)^2 - 210*(d*e*x + c*e)^2*B*a^2*b^5*c^3*d^3*e^3*g^3/(b*x +
a)^2 + 210*(d*e*x + c*e)^2*B*a^3*b^4*c^2*d^4*e^3*g^3/(b*x + a)^2 - 105*(d*
e*x + c*e)^2*B*a^4*b^3*c*d^5*e^3*g^3/(b*x + a)^2 + 21*(d*e*x + c*e)^2*B*a^
5*b^2*d^6*e^3*g^3/(b*x + a)^2 + 6*(d*e*x + c*e)^3*B*b^8*c^5*e^2*g^3/(b*x +
 a)^3 - 30*(d*e*x + c*e)^3*B*a*b^7*c^4*d*e^2*g^3/(b*x + a)^3 + 60*(d*e*...
```

3.174. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

3.174.9 Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$

$$= x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+A\,a}{4\,b\,d}\right.$$

$$\left.+\frac{a^2\,g^3\,(8\,A\,a\,d+12\,A\,b\,c-3\,B\,a\,d+3\,B\,b\,c)}{2\,d}-\frac{a\,c\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$\left.-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)$$

$$+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)$$

$$+x^3\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{12\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{12\,d}\right)$$

$$-\frac{\ln(c+d\,x)\,(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3)}{4\,d^4}$$

$$+\frac{A\,b^3\,g^3\,x^4}{4}-\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{4\,b}$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d
) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3
*(6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a
^2*g^3*(8*A*a*d + 12*A*b*c - 3*B*a*d + 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(1
6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4
*d)))/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*
(6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log(
(e*(c + d*x))/(a + b*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3
*x^2)/2 + B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B
*b*c))/(12*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) - (log(c + d*x)*(B*b^3
*c^4*g^3 - 4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3
))/(4*d^4) + (A*b^3*g^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(4*b)
```

3.174. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

3.175 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

3.175.1 Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = -\frac{B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{6bd}+\frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}$$

output
```
-1/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*B*(-a
*d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a))
)/b
```

3.175.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{g^2\left(\frac{B(bc-ad)(d(a^2d+4abdx+b^2x(-2c+dx))+2(bc-ad)^2\log(c+dx))}{2d^3}+(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{3b}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c
- a*d)^2*Log[c + d*x]))/(2*d^3) + (a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a
+ b*x)])))/(3*b)
```

3.175.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bg^2(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{Bg^2(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}+\frac{Bg^2(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(B*(b*c - a*d)*g^2*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c -
 a*d)^2*Log[c + d*x])/d^3))/(3*b) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*
x))/(a + b*x)]))/(3*b)
```

3.175. $\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx$

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.175.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(bx+a)^3 g^2 B \ln\left(\frac{e(dx+c)}{bx+a}\right)}{3b} + \frac{g^2 b^2 A x^3}{3} + g^2 bAa\, x^2 - \frac{g^2 bBa\, x^2}{6} + \frac{g^2 b^2 Bc\, x^2}{6d} + g^2 A\, a^2 x - \frac{g^2 B \ln(dx+c) c}{3b}$
parts	$\frac{A\, g^2 (bx+a)^3}{3b} + B\, g^2 e^3 (ad-cb)^3 \left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2} + \frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{3d^3e^3b} + \frac{}{3d^2e^2b}\right.$
derivativedivides	$e(ad-cb)\left(-\frac{Ab\, e^2 g^2 \left(a^2d^2-2abcd+b^2c^2\right)}{3\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^3} + B\, b^2 e^2 g^2 \left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2} + \frac{\ln\left(\left(\frac{de}{b}-\frac{e(a}{b(b}\right.\right.}{3d^3}\right.\right.$
default	$e(ad-cb)\left(-\frac{Ab\, e^2 g^2 \left(a^2d^2-2abcd+b^2c^2\right)}{3\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^3} + B\, b^2 e^2 g^2 \left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{1}{6deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2} + \frac{\ln\left(\left(\frac{de}{b}-\frac{e(a}{b(b}\right.\right.}{3d^3}\right.\right.$
parallelrisch	$2B\, x^3 \ln\left(\frac{e(dx+c)}{bx+a}\right) b^3 d^3 g^2 + 4B\, a^3 d^3 g^2 + 6A\, x^2 a\, b^2 d^3 g^2 - B\, x^2 a\, b^2 d^3 g^2 + B\, x^2 b^3 c\, d^2 g^2 + 6Ax\, a^2 b\, d^3 g^2 - 4Bx\, a^2 b\, d^3 g^2 - 2B$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3*g^2*B/b*ln(e*(d*x+c)/(b*x+a))+1/3*g^2*b^2*A*x^3+g^2*b*A*a*x^
2-1/6*g^2*b*B*a*x^2+1/6*g^2*b^2/d*B*c*x^2+g^2*A*a^2*x-1/3*g^2/b*B*ln(d*x+c
)*a^3+g^2/d*B*ln(d*x+c)*a^2*c-g^2*b/d^2*B*ln(d*x+c)*a*c^2+1/3*g^2*b^2/d^3*
B*ln(d*x+c)*c^3-2/3*g^2*B*a^2*x+g^2*b/d*B*a*c*x-1/3*g^2*b^2/d^2*B*c^2*x
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3g^2x^3 - 2\,Ba^3d^3g^2\log(bx+a) + (Bb^3cd^2 + (6\,A-B)ab^2d^3)g^2x^2 - 2\,(Bb^3c^2d - 3\,Bab^2cd^2 - (3\,A}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (
6*A - B)*a*b^2*d^3)*g^2*x^2 - 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 - (3*A - 2*
B)*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^
2*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*
d^3*g^2*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^3)
```

3.175.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.48 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$

$$= \frac{Ab^2g^2x^3}{3} - \frac{Ba^3g^2\log\left(x+\frac{\frac{Ba^4d^3g^2}{b}+3Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3b}$$
$$+ \frac{Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{4Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2-Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{Bbc^2g^2\cdot(3a^2d^2-3}{d}}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right.}{3d^3}$$
$$+ x^2\left(Aabg^2-\frac{Babg^2}{6}+\frac{Bb^2cg^2}{6d}\right) + x\left(Aa^2g^2-\frac{2Ba^2g^2}{3}+\frac{Babcg^2}{d}-\frac{Bb^2c^2g^2}{3d^2}\right)$$
$$+ \left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
A*b**2*g**2*x**3/3 - B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*
d**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2
 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3
*b) + B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**
2*d**2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d +
 b**2*c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**
2*d*g**2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 - B*a*b*g**2/6 +
 B*b**2*c*g**2/(6*d)) + x*(A*a**2*g**2 - 2*B*a**2*g**2/3 + B*a*b*c*g**2/d
- B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g
**2*x**3/3)*log(e*(c + d*x)/(a + b*x))
```

3.175.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(110) = 220$.

Time = 0.22 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.36

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{1}{3}Ab^2g^2x^3+Aabg^2x^2 + \left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)Ba^2g^2 + \left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)Babg^2 + \frac{1}{6}\left(2x^3\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{2a^3\log(bx+a)}{b^3}+\frac{2c^3\log(dx+c)}{d^3}+\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a}{b^2d^2}\right. + Aa^2g^2x$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)
) - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d*e*x/(b*x +
 a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c
- a*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)
) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

3.175.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(110) = 220$.

Time = 0.40 (sec) , antiderivative size = 1056, normalized size of antiderivative = 8.95

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = -\frac{1}{6}\left(\frac{2\left(Bb^4c^4e^4g^2-4Bab^3c^3de^4g^2+6Ba^2b^2c^2d^2e^4g^2-4Ba^3bcd^3e^4g^2+Ba^4d^4e^4g^2\right)\log\left(\frac{dex+ce}{bx+a}\right)}{bd^3e^3-\frac{3(dex+ce)b^2d^2e^2}{bx+a}+\frac{3(dex+ce)^2b^3de}{(bx+a)^2}-\frac{(dex+ce)^3b^4}{(bx+a)^3}}+\frac{2A}{\quad}\right.$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
-1/6*(2*(B*b^4*c^4*e^4*g^2 - 4*B*a*b^3*c^3*d*e^4*g^2 + 6*B*a^2*b^2*c^2*d^2
*e^4*g^2 - 4*B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2)*log((d*e*x + c*e)/
(b*x + a))/(b*d^3*e^3 - 3*(d*e*x + c*e)*b^2*d^2*e^2/(b*x + a) + 3*(d*e*x +
 c*e)^2*b^3*d*e/(b*x + a)^2 - (d*e*x + c*e)^3*b^4/(b*x + a)^3) + (2*A*b^4*
c^4*d^2*e^4*g^2 - 3*B*b^4*c^4*d^2*e^4*g^2 - 8*A*a*b^3*c^3*d^3*e^4*g^2 + 12
*B*a*b^3*c^3*d^3*e^4*g^2 + 12*A*a^2*b^2*c^2*d^4*e^4*g^2 - 18*B*a^2*b^2*c^2
*d^4*e^4*g^2 - 8*A*a^3*b*c*d^5*e^4*g^2 + 12*B*a^3*b*c*d^5*e^4*g^2 + 2*A*a^
4*d^6*e^4*g^2 - 3*B*a^4*d^6*e^4*g^2 + 5*(d*e*x + c*e)*B*b^5*c^4*d*e^3*g^2/
(b*x + a) - 20*(d*e*x + c*e)*B*a*b^4*c^3*d^2*e^3*g^2/(b*x + a) + 30*(d*e*x
 + c*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(b*x + a) - 20*(d*e*x + c*e)*B*a^3*b^2*c
*d^4*e^3*g^2/(b*x + a) + 5*(d*e*x + c*e)*B*a^4*b*d^5*e^3*g^2/(b*x + a) - 2
*(d*e*x + c*e)^2*B*b^6*c^4*e^2*g^2/(b*x + a)^2 + 8*(d*e*x + c*e)^2*B*a*b^5
*c^3*d*e^2*g^2/(b*x + a)^2 - 12*(d*e*x + c*e)^2*B*a^2*b^4*c^2*d^2*e^2*g^2/
(b*x + a)^2 + 8*(d*e*x + c*e)^2*B*a^3*b^3*c*d^3*e^2*g^2/(b*x + a)^2 - 2*(d
*e*x + c*e)^2*B*a^4*b^2*d^4*e^2*g^2/(b*x + a)^2)/(b*d^5*e^3 - 3*(d*e*x + c
*e)*b^2*d^4*e^2/(b*x + a) + 3*(d*e*x + c*e)^2*b^3*d^3*e/(b*x + a)^2 - (d*e
*x + c*e)^3*b^4*d^2/(b*x + a)^3) + 2*(B*b^4*c^4*e*g^2 - 4*B*a*b^3*c^3*d*e*
g^2 + 6*B*a^2*b^2*c^2*d^2*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)
*log(-d*e + (d*e*x + c*e)*b/(b*x + a))/(b*d^3) - 2*(B*b^4*c^4*e*g^2 - 4*B*
a*b^3*c^3*d*e*g^2 + 6*B*a^2*b^2*c^2*d^2*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 +...
```

3.175.9 Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$$
$$= x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$+\frac{\ln(c+d\,x)\,(3\,B\,a^2\,c\,d^2\,g^2-3\,B\,a\,b\,c^2\,d\,g^2+B\,b^2\,c^3\,g^2)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}-\frac{B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
x^2*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d + B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d +
 B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a
*d + 3*A*b*c - B*a*d + B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(c + d*x))/(a
 + b*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) + (log(c + d*x)
*(B*b^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2
*g^2*x^3)/3 - (B*a^3*g^2*log(a + b*x))/(3*b)
```

3.175. $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

3.176 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

3.176.1 Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx=\frac{B(bc-ad)gx}{2d}-\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b}$$

output
```
1/2*B*(-a*d+b*c)*g*x/d-1/2*B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^
2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

3.176.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx=\frac{g\left(\frac{B(bc-ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}+(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(g*((B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a + b*x)^
2*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(2*b)
```

3.176.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{g^2(a+bx)}{c+dx}dx}{2bg}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bg(bc-ad)\int \frac{a+bx}{c+dx}dx}{2b}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{Bg(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}+\frac{Bg(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
(B*(b*c - a*d)*g*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(2*b) + (g*(a
 + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*b)
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.176.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{B\,a^2g\ln(bx+a)}{2b}+\frac{gB\ln(-dx-c)ac}{d}-\frac{gbB\ln(-dx-c)c^2}{2d^2}-g$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)ab\,d^2g+2Axab\,d^2g-B\ln(bx+a)a^2d^2g+2B\ln(bx+a)abcdg-B\ln}{2}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-Bg\,e^2(ad-cb)^2\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\frac{\ln}{}\right.$
derivativedivides	$\frac{e(ad-cb)\left(\frac{Abeg(ad-cb)}{2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2}-B\,b^2eg(ad-cb)\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\frac{\ln(}{}\right.\right.}{b^2}$
default	$\frac{e(ad-cb)\left(\frac{Abeg(ad-cb)}{2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)^2}-B\,b^2eg(ad-cb)\left(-\frac{1}{2deb\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}-\frac{\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{2d^2e^2b}+\frac{\ln(}{}\right.\right.}{b^2}$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*B*x*(b*x+2*a)*ln(e*(d*x+c)/(b*x+a))+1/2*g*b*A*x^2+g*A*a*x-1/2*B*a^2*
g/b*ln(b*x+a)+g/d*B*ln(-d*x-c)*a*c-1/2*g*b/d^2*B*ln(-d*x-c)*c^2-1/2*g*B*a*
x+1/2*g*b/d*B*c*x
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = \frac{Ab^2d^2gx^2 - Ba^2d^2g\log(bx+a) + (Bb^2cd + (2A-B)abd^2)gx - (Bb^2c^2 - 2\,Babcd)g\log(dx+c) + (Bb}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 - B*a^2*d^2*g*log(b*x + a) + (B*b^2*c*d + (2*A - B)*a
*b*d^2)*g*x - (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2
+ 2*B*a*b*d^2*g*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^2)
```

3.176.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(68) = 136.

Time = 0.94 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\begin{aligned}\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx &= \frac{Abgx^2}{2} - \frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b}\\ &+ \frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2}\\ &+ x\left(Aag-\frac{Bag}{2}+\frac{Bbcg}{2d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)\end{aligned}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*
c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) + B*c*g*(2*
a*d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c)
+ B*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2
*g))/(2*d**2) + x*(A*a*g - B*a*g/2 + B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**
2/2)*log(e*(c + d*x)/(a + b*x))
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.77

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx$$
$$=\frac{1}{2}Abgx^2+\left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)Bag$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)Bbg$$
$$+Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

output
```
1/2*A*b*g*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b
 + c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))
 + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*b*
g + A*a*g*x
```

3.176.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(75) = 150.

Time = 0.46 (sec) , antiderivative size = 627, normalized size of antiderivative = 7.74

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx$$
$$=\frac{1}{2}\left(\frac{(Bb^3c^3e^3g-3Bab^2c^2de^3g+3Ba^2bcd^2e^3g-Ba^3d^3e^3g)\log\left(\frac{dex+ce}{bx+a}\right)}{bd^2e^2-\frac{2(dex+ce)b^2de}{bx+a}+\frac{(dex+ce)^2b^3}{(bx+a)^2}}+\frac{Ab^3c^3de^3g-Bb^3c^3de^3g-3A\cdots}{\cdots}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
1/2*((B*b^3*c^3*e^3*g - 3*B*a*b^2*c^2*d*e^3*g + 3*B*a^2*b*c*d^2*e^3*g - B*
a^3*d^3*e^3*g)*log((d*e*x + c*e)/(b*x + a))/(b*d^2*e^2 - 2*(d*e*x + c*e)*b
^2*d*e/(b*x + a) + (d*e*x + c*e)^2*b^3/(b*x + a)^2) + (A*b^3*c^3*d*e^3*g -
 B*b^3*c^3*d*e^3*g - 3*A*a*b^2*c^2*d^2*e^3*g + 3*B*a*b^2*c^2*d^2*e^3*g + 3
*A*a^2*b*c*d^3*e^3*g - 3*B*a^2*b*c*d^3*e^3*g - A*a^3*d^4*e^3*g + B*a^3*d^4
*e^3*g + (d*e*x + c*e)*B*b^4*c^3*e^2*g/(b*x + a) - 3*(d*e*x + c*e)*B*a*b^3
*c^2*d*e^2*g/(b*x + a) + 3*(d*e*x + c*e)*B*a^2*b^2*c*d^2*e^2*g/(b*x + a) -
 (d*e*x + c*e)*B*a^3*b*d^3*e^2*g/(b*x + a))/(b*d^3*e^2 - 2*(d*e*x + c*e)*b
^2*d^2*e/(b*x + a) + (d*e*x + c*e)^2*b^3*d/(b*x + a)^2) + (B*b^3*c^3*e*g -
 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log(-d*e + (d*
e*x + c*e)*b/(b*x + a))/(b*d^2) - (B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3
*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log((d*e*x + c*e)/(b*x + a))/(b*d^2))*
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.176.9 Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx = x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)-\frac{\ln\left(c+d\,x\right)\,\left(B\,b\,c^2\,g-2\,B\,a\,c\,d\,g\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}-\frac{B\,a^2\,g\,\ln\left(a+b\,x\right)}{2\,b}$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
x*((g*(4*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(
2*d)) + log((e*(c + d*x))/(a + b*x))*((B*b*g*x^2)/2 + B*a*g*x) - (log(c +
d*x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 - (B*a^2*g*log(a +
 b*x))/(2*b)
```

3.177 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx$

3.177.1 Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg} - \frac{B\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/g-B*polylog(2,1+(-a
*d+b*c)/d/(b*x+a))/b/g
```

3.177.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = \frac{\log(g(a+bx))\left(B\log(g(a+bx))+2\left(A-B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)-2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right.}{2bg}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

output
```
(Log[g*(a + b*x)]*(B*Log[g*(a + b*x)] + 2*(A - B*Log[(b*(c + d*x))/(b*c -
a*d)] + B*Log[(e*(c + d*x))/(a + b*x)])) - 2*B*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)])/(2*b*g)
```

3.177.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2944, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{ag+bgx}dx$$

$$\Big\downarrow \text{2944}$$

$$-\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$-\frac{B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$-\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$\frac{B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}-\frac{B\,\mathrm{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))
/(b*g)) - (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

3.177.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2944
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
- a*d)/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x
] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && EqQ[d*f - c*g, 0]
```

3.177.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.84

method	result
parts	$\frac{A\ln(bx+a)}{gb}+\frac{B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{b}-\frac{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{b}\right)}{g}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{gb}-\frac{B\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{gb}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{bA\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{ge(ad-cb)}-\frac{b^2B\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{b}+\frac{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\cdots}{de}\right)}{b}\right)}{ge(ad-cb)}\right)}{b^2}$
default	$\frac{e(ad-cb)\left(-\frac{bA\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de\right)}{ge(ad-cb)}-\frac{b^2B\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-de}{de}\right)}{b}+\frac{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\cdots}{de}\right)}{b}\right)}{ge(ad-cb)}\right)}{b^2}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
A/g*ln(b*x+a)/b+B/g*(-dilog(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/d/e)/b-
ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/
d/e)/b)
```

3.177.5 Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="fricas")
```

output
```
integral((B*log((d*e*x + c*e)/(b*x + a)) + A)/(b*g*x + a*g), x)
```

3.177.6 Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(c*e/(a + b*x) + d*e*x/(a + b*x)
)/(a + b*x), x))/g
```

3.177.7 Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="maxima")
```

output
```
B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x*log(e) + b*c*log(e)
 - (2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g +
 a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

3.177.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(80) = 160$.

Time = 37.79 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.62

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx =$$
$$-\frac{1}{2}\left(\frac{\left(Bb^3c^3e^3 - 3\,Bab^2c^2de^3 + 3\,Ba^2bcd^2e^3 - Ba^3d^3e^3\right)\log\left(\frac{dex+ce}{bx+a}\right)}{bd^2e^2g - \frac{2\,(dex+ce)b^2deg}{bx+a} + \frac{(dex+ce)^2b^3g}{(bx+a)^2}} + \frac{Ab^3c^3de^3 - Bb^3c^3de^3 - 3\,Aab^2c^2}{}\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="giac")
```

output
```
-1/2*((B*b^3*c^3*e^3 - 3*B*a*b^2*c^2*d*e^3 + 3*B*a^2*b*c*d^2*e^3 - B*a^3*d
^3*e^3)*log((d*e*x + c*e)/(b*x + a))/(b*d^2*e^2*g - 2*(d*e*x + c*e)*b^2*d*
e*g/(b*x + a) + (d*e*x + c*e)^2*b^3*g/(b*x + a)^2) + (A*b^3*c^3*d*e^3 - B*
b^3*c^3*d*e^3 - 3*A*a*b^2*c^2*d^2*e^3 + 3*B*a*b^2*c^2*d^2*e^3 + 3*A*a^2*b*
c*d^3*e^3 - 3*B*a^2*b*c*d^3*e^3 - A*a^3*d^4*e^3 + B*a^3*d^4*e^3 + (d*e*x +
 c*e)*B*b^4*c^3*e^2/(b*x + a) - 3*(d*e*x + c*e)*B*a*b^3*c^2*d*e^2/(b*x + a
) + 3*(d*e*x + c*e)*B*a^2*b^2*c*d^2*e^2/(b*x + a) - (d*e*x + c*e)*B*a^3*b*
d^3*e^2/(b*x + a))/(b*d^3*e^2*g - 2*(d*e*x + c*e)*b^2*d^2*e*g/(b*x + a) +
(d*e*x + c*e)^2*b^3*d*g/(b*x + a)^2) + (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e +
3*B*a^2*b*c*d^2*e - B*a^3*d^3*e)*log(-d*e + (d*e*x + c*e)*b/(b*x + a))/(b*
d^2*g) - (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3*B*a^2*b*c*d^2*e - B*a^3*d^3*
e)*log((d*e*x + c*e)/(b*x + a))/(b*d^2*g))*(b*c/((b*c*e - a*d*e)*(b*c - a*
d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{a\,g + b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x), x)
```

3.178 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx$

3.178.1 Optimal result

Integrand size = 30, antiderivative size = 64

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx=-\frac{A-B}{bg^2(a+bx)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)}$$

output
```
(-A+B)/b/g^2/(b*x+a)-B*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a
)
```

3.178.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.34

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx =\frac{Bd(a+bx)\log(a+bx)-Bd(a+bx)\log(c+dx)-(bc-ad)\left(A-B+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2(a+bx)}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

output
```
(B*d*(a + b*x)*Log[a + b*x] - B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)*(A
- B + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2*(a + b*x))
```

3.178.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2952, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{A(c+dx)}{a+bx}+\frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

output
```
-(((A*(c + d*x))/(a + b*x) - (B*(c + d*x))/(a + b*x) + (B*(c + d*x)*Log[(e
*(c + d*x))/(a + b*x)])/(a + b*x))/((b*c - a*d)*g^2))
```

3.178.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.178.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

method	result	si
parts	$-\frac{A}{g^{2}(bx+a)b}+\frac{B\left(\frac{e(dx+c)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{bx+a}-\frac{e(dx+c)}{bx+a}\right)}{g^{2}e(ad-cb)}$	81
norman	$\frac{\frac{(A-B)x}{ga}+\frac{Bc\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}+\frac{Bdx\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}}{g(bx+a)}$	89
parallelrisch	$-\frac{Aa\,b^{2}d^{2}-A\,b^{3}cd-Ba\,b^{2}d^{2}+B\,b^{3}cd-Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)b^{3}d^{2}-B\ln\left(\frac{e(dx+c)}{bx+a}\right)b^{3}cd}{g^{2}(bx+a)b^{3}d(ad-cb)}$	11
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{b\,g^{2}(bx+a)}-\frac{B\ln(bx+a)bdx-B\ln(-dx-c)bdx+B\ln(bx+a)ad-B\ln(-dx-c)ad+Aad-Abc-Bad+Bbc}{g^{2}(bx+a)b(ad-cb)}$	12
derivativedivides	$\frac{e(ad-cb)\left(\frac{b^{2}A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^{2}e^{2}g^{2}}+\frac{b^{2}B\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^{2}e^{2}g^{2}}\right)}{b^{2}}$	17
default	$\frac{e(ad-cb)\left(\frac{b^{2}A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^{2}e^{2}g^{2}}+\frac{b^{2}B\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^{2}e^{2}g^{2}}\right)}{b^{2}}$	17

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/g^2/(b*x+a)/b+B/g^2/e/(a*d-b*c)*(e*(d*x+c)/(b*x+a)*ln(e*(d*x+c)/(b*x+a)
)-e*(d*x+c)/(b*x+a))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

$$\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^{2}}\,dx=-\frac{(A-B)bc-(A-B)ad+(Bbdx+Bbc)\log\left(\frac{dex+ce}{bx+a}\right)}{(b^{3}c-ab^{2}d)g^{2}x+(ab^{2}c-a^{2}bd)g^{2}}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="fricas"
)
```

output
```
-((A - B)*b*c - (A - B)*a*d + (B*b*d*x + B*b*c)*log((d*e*x + c*e)/(b*x + a
)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

3.178.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(48) = 96$.

Time = 0.65 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.61

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x} + \frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2(ad-bc)} - \frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2(ad-bc)} + \frac{-A+B}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**2,x)
```

output
```
-B*log(e*(c + d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + B*d*log(x + (-B*a
**2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2
*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) - B*d*log(x +
 (B*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2
*c**2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A +
B)/(a*b*g**2 + b**2*g**2*x)
```

3.178.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.09

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^2g^2x+abg^2} - \frac{1}{b^2g^2x+abg^2} - \frac{d\log(bx+a)}{(b^2c-abd)g^2} + \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{A}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="maxima"
)
```

output
```
-B*(log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2) - 1/(b^2*g^
2*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log(d*x + c)/((b
^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{(dex+ce)B\log\left(\frac{dex+ce}{bx+a}\right)}{(bx+a)g^2}+\frac{(dex+ce)(A-B)}{(bx+a)g^2}\right)$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

output
```
-(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*(
(d*e*x + c*e)*B*log((d*e*x + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*e*x + c*
e)*(A - B)/((b*x + a)*g^2))
```

3.178.9 Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{A-B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}+\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^2,x)
```

output
```
(B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*(a*d - b*c)) -
(B*log((e*(c + d*x))/(a + b*x)))/(b^2*g^2*(x + a/b)) - (A - B)/(b^2*g^2*x
+ a*b*g^2)
```

3.179 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx$

3.179.1 Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = \frac{B}{4bg^3(a+bx)^2}-\frac{Bd}{2b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}$$

output
```
1/4*B/b/g^3/(b*x+a)^2-1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)-1/2*B*d^2*ln(b*x+a)
/b/(-a*d+b*c)^2/g^3+1/2*B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*
(d*x+c)/(b*x+a)))/b/g^3/(b*x+a)^2
```

3.179.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = -\frac{2Bd^2(a+bx)^2\log(a+bx)-2Bd^2(a+bx)^2\log(c+dx)+(bc-ad)\left(2Abc-bBc-2aAd+3aBd+2\right.}{4b(bc-ad)^2g^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

output
```
-1/4*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x]
+ (b*c - a*d)*(2*A*b*c - b*B*c - 2*a*A*d + 3*a*B*d + 2*b*B*d*x + 2*B*(b*c
- a*d)*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

3.179.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{(ag+bgx)^3} \, dx$$

$$\Big\downarrow \text{ 2948}$$

$$-\frac{B(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{2bg}-\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{2bg^3}-\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{B(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{2bg^3}-\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}-\frac{B(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{2bg^3}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(B*(b*c - a*d)*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(
a + b*x)) + (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a
*d)^3))/(b*g^3) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(2*b*g^3*(a + b*x)^
2)
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.179.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.56

method	result
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{Bb\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{4}-\frac{de\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right.\right.}{b}\right.}{g^3e^2(ad-cb)^2}$
norman	$\frac{\frac{Ba\,d^2x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{(a^2d^2-2abcd+b^2c^2)g}-\frac{2Aabd-2A\,b^2c-3Babd+B\,b^2c}{4g\,b^2(ad-cb)}+\frac{Bdx}{2g(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2g(a^2d^2-2abcd+b^2c^2)}+\frac{d^2Bb\,x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2(a^2d^2-2abcd+b^2c^2)g}}{g^2(bx+a)^2}$
parallelrisch	$-\frac{-2Bxa\,b^4d^3+2Bx\,b^5c\,d^2+2B\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5c^2d-4Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)a\,b^4d^3-4B\ln\left(\frac{e(dx+c)}{bx+a}\right)a\,b^4c\,d^2+2A\,a^2b^3d^3+2A}{4g^3(bx+a)^2(a^2d^2-2abcd+b^2c^2)b^4d}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2b\,g^3(bx+a)^2}-\frac{2B\ln(bx+a)b^2d^2x^2-2B\ln(-dx-c)b^2d^2x^2+4B\ln(bx+a)ab\,d^2x-4B\ln(-dx-c)ab\,d^2x+2B\,a^2\ln}{4(a^2d^2-2a}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{b^3A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{b^3B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{4}\right.}{(ad-cb)^3e^3g^3}\right.}{b^2}$
default	$\frac{e(ad-cb)\left(-\frac{b^3A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{b^3B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{4}\right.}{(ad-cb)^3e^3g^3}\right.}{b^2}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/g^3/(b*x+a)^2/b-B/g^3*b/e^2/(a*d-b*c)^2*(1/2*(d*e/b-e*(a*d-b*c)/b/(
b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a)
)^2-d*e/b*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e
*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{(2\,A-B)b^2c^2-4\,(A-B)abcd+(2\,A-3\,B)a^2d^2+2\,(Bb^2cd-Babd^2)x-2\,(Bb^2d^2x^2+2\,Babd^2x-}{4\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2c}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="fricas"
)
```

output
```
-1/4*((2*A - B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - 3*B)*a^2*d^2 + 2*(B*b
^2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B
*a*b*c*d)*log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(124) = 248$.

Time = 1.17 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx$$
$$= -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^2bg^3 + 4ab^2g^3x + 2b^3g^3x^2} + \frac{Bd^2\log\left(x + \frac{-\frac{Ba^3d^5}{(ad-bc)^2} + \frac{3Ba^2bcd^4}{(ad-bc)^2} - \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 + \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2} - \frac{Bd^2\log\left(x + \frac{\frac{Ba^3d^5}{(ad-bc)^2} - \frac{3Ba^2bcd^4}{(ad-bc)^2} + \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 - \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2} + \frac{-2Aad + 2Abc + 3Bad - Bbc + 2Bbdx}{4a^3bdg^3 - 4a^2b^2cg^3 + x^2\cdot(4ab^3dg^3 - 4b^4cg^3) + x\left(8a^2b^2dg^3 - 8ab^3cg^3\right)}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**3,x)
```

output
```
-B*log(e*(c + d*x)/(a + b*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g*
*3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4
/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*
c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)
**2) - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
 - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*
d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
+ (-2*A*a*d + 2*A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*
a**2*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2
*d*g**3 - 8*a*b**3*c*g**3))
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{2}{(b^3c^2}\right.$$

$$-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="maxima"
)
```

output
```
-1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*
e/(b*x + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a
)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a
^2*b*g^3)
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.62

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\left(2\left(\frac{(dex+ce)^2Bb}{(bceg^3-adeg^3)(bx+a)^2}-\frac{2\,(dex+ce)Bd}{(bcg^3-adg^3)(bx+a)}\right)\log\left(\frac{dex+ce}{bx+a}\right)+\frac{(dex+ce)^2(2\,Ab-Bb}{(bceg^3-adeg^3)(bx+a}\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

output
```
-1/4*(2*((d*e*x + c*e)^2*B*b/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 2*(d*
e*x + c*e)*B*d/((b*c*g^3 - a*d*g^3)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a
)) + (d*e*x + c*e)^2*(2*A*b - B*b)/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) -
 4*(d*e*x + c*e)*(A*d - B*d)/((b*c*g^3 - a*d*g^3)*(b*x + a)))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.179.9 Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.44

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = \frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2} - \frac{B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{\frac{2\,A\,a\,d-2\,A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,\left(a\,d-b\,c\right)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^3,x)
```

output
```
(B*d^2*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (
2*b*d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x))/(a + b
*x)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((2*A*a*d - 2*A*b*c - 3*B*a*d
+ B*b*c)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3
*x^2 + 4*a*b^2*g^3*x)
```

3.180 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx$

3.180.1 Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx=\frac{B}{9bg^4(a+bx)^3}-\frac{Bd}{6b(bc-ad)g^4(a+bx)^2}+\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}+\frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4}-\frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}$$

output
```
1/9*B/b/g^4/(b*x+a)^3-1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+1/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)+1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-1/3*B*d^3*ln(d*
x+c)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(d*x+c)/(b*x+a)))/b/g^4/(b*x+a)^3
```

3.180.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx=\frac{\frac{B\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}-6\left(A+B\log\left(\frac{e(c+dx}{a+bx}\right.\right.}{18bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

output
```
((B*((b*c - a*d)*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*
x + 6*d^2*x^2)) + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c
 + d*x]))/(b*c - a*d)^3 - 6*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(18*b*g^
4*(a + b*x)^3)
```

3.180.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{(ag+bgx)^4} \, dx$$

$$\Big\downarrow \text{ 2948}$$

$$-\frac{B(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{B(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{B \log \left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}-\frac{B(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}$$

---

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

output
```
-1/3*(B*(b*c - a*d)*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2
*(a + b*x)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c -
a*d)^4 + (d^3*Log[c + d*x])/(b*c - a*d)^4))/(b*g^4) - (A + B*Log[(e*(c + d
*x))/(a + b*x)])/(3*b*g^4*(a + b*x)^3)
```

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.180. $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx$

3.180.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.89

method	result
parts	$-\frac{A}{3g^4(bx+a)^3 b}+\frac{B b^2\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{3}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{9}-\frac{2de\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad}{b(bx}\right.\right.}{2}\right.}{b}\right.}{g^4e^3(ad-cb)}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{3b g^4(bx+a)^3}-\frac{6B\ln(bx+a)b^3d^3x^3-6B\ln(-dx-c)b^3d^3x^3+18B\ln(bx+a)a b^2d^3x^2-18B\ln(-dx-c)a b^2d^3x^2+1}{}$
parallelrisch	$-\frac{-18A a^2b^5c d^3+18Aa b^6c^2d^2+18Bxa b^6c d^3-15Bx a^2b^5d^4-3Bx b^7c^2d^2-6B x^2a b^6d^4+6B x^2b^7c d^3+6A a^3b^4d^4-6A}{}$
norman	$\frac{B a^2d^3x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^3d^3-3a^2bc d^2+3a b^2c^2d-b^3c^3\right)g}+\frac{Bab d^3x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^3d^3-3a^2bc d^2+3a b^2c^2d-b^3c^3\right)g}-\frac{6A a^2b d^2-12Aa b^2cd+6A c^2b^3-9B a^2b d^2+7Ba b^2cd-}{18g b^2\left(a^2d^2-2abcd+b^2c^2\right)}$
derivativedivides	$\frac{e(ad-cb)\left(\frac{b^4A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{3(ad-cb)^4e^4g^4}-\frac{b^3Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{(ad-cb)^4e^3g^4}+\frac{b^2A d^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^4e^2g^4}+\frac{b^4B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(ad}{b(bx}\right.}{3}\right.}{(ad-cb)^4e^4}\right.}{}$
default	$\frac{e(ad-cb)\left(\frac{b^4A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{3(ad-cb)^4e^4g^4}-\frac{b^3Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{(ad-cb)^4e^3g^4}+\frac{b^2A d^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^4e^2g^4}+\frac{b^4B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3\ln\left(\frac{de}{b}-\frac{e(ad}{b(bx}\right.}{3}\right.}{(ad-cb)^4e^4}\right.}{}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/g^4/(b*x+a)^3/b+B/g^4*b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+
a))^3-2*d*e/b*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/
(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+d^2*e^2/b^2*((d*e/b-e*(a*d-b
*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b
))
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(163) = 326$.

Time = 0.28 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.35

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = -\frac{2\,(3\,A-B)b^3c^3 - 9\,(2\,A-B)ab^2c^2d + 18\,(A-B)a^2bcd^2 - (6\,A-11\,B)a^3d^3 - 6\,(Bb^3cd^2 - Bab^2d^3)x^2}{18\,((b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3\,(ab^6c^3 - 3\,a^2b^5c^2d + 3\,a^3b^4cd^2 -}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="fricas"
)
```

output
```
-1/18*(2*(3*A - B)*b^3*c^3 - 9*(2*A - B)*a*b^2*c^2*d + 18*(A - B)*a^2*b*c*
d^2 - (6*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*
c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*
d^3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)
*log((d*e*x + c*e)/(b*x + a)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2
 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2
 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d
^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2
 - a^6*b*d^3)*g^4)
```

3.180.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(151) = 302$.

Time = 1.77 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.75

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3}$$
$$+ \frac{Bd^3\log\left(x + \frac{-\frac{Ba^4d^7}{(ad-bc)^3} + \frac{4Ba^3bcd^6}{(ad-bc)^3} - \frac{6Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{4Bab^3c^3d^4}{(ad-bc)^3} + Bad^4 - \frac{Bb^4c^4d^3}{(ad-bc)^3} + Bbcd^3}{2Bbd^4}\right)}{3bg^4\,(ad-bc)^3}$$
$$- \frac{Bd^3\log\left(x + \frac{\frac{Ba^4d^7}{(ad-bc)^3} - \frac{4Ba^3bcd^6}{(ad-bc)^3} + \frac{6Ba^2b^2c^2d^5}{(ad-bc)^3} - \frac{4Bab^3c^3d^4}{(ad-bc)^3} + Bad^4 + \frac{Bb^4c^4d^3}{(ad-bc)^3} + Bbcd^3}{2Bbd^4}\right)}{3bg^4\,(ad-bc)^3}$$
$$+ \frac{-6Aa^2d^2 + 12Aabcd - 6Ab^2c^2 + 11Ba^2d^2 - 7Babcd + 2Bb^2c^2 +}{18a^5bd^2g^4 - 36a^4b^2cdg^4 + 18a^3b^3c^2g^4 + x^3\cdot(18a^2b^4d^2g^4 - 36ab^5cdg^4 + 18b^6c^2g^4) + x^2\cdot(54a^3b^3d^2g^4 -}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**4,x)
```

output
```
-B*log(e*(c + d*x)/(a + b*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b*
*3*g**4*x**2 + 3*b**4*g**4*x**3) + B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c
)**3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b
*c)**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3
/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - B*
d**3*log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**
3 + 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b
*c)**3 + B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d
**4))/(3*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**
2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2
 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c
*d*g**4 + 18*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**
5*c*d*g**4 + 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*
b**4*c*d*g**4 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**
3*b**3*c*d*g**4 + 54*a**2*b**4*c**2*g**4))
```

3.180.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 0.21 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.45

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^4}\,dx$$
$$= \frac{1}{18}B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2)\right.$$
$$- \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="maxima"
)
```

output
```
1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
 + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 6
*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a
^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^
3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

3.180.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(163) = 326$.

Time = 0.44 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.55

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^4}\,dx =$$
$$-\frac{1}{18}\left(6\left(\frac{(dex + ce)^3 Bb^2}{(b^2c^2e^2g^4 - 2\,abcde^2g^4 + a^2d^2e^2g^4)(bx + a)^3} - \frac{3\,(dex + ce)^2 Bbd}{(b^2c^2eg^4 - 2\,abcdeg^4 + a^2d^2eg^4)(bx + a)^2} + \frac{}{(b}\right.\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

output
```
-1/18*(6*((d*e*x + c*e)^3*B*b^2/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*e^2*g^4 + a^
2*d^2*e^2*g^4)*(b*x + a)^3) - 3*(d*e*x + c*e)^2*B*b*d/((b^2*c^2*e*g^4 - 2*
a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 3*(d*e*x + c*e)*B*d^2/((b^2*
c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e)/(b*x
+ a)) + 2*(3*A*b^2 - B*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*
e^2*g^4 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 9*(2*A*b*d - B*b*d)*(d*e*x + c*e
)^2/((b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 18*(
A*d^2 - B*d^2)*(d*e*x + c*e)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*
(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))
```

3.180.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.94

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = \frac{B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{3\,b\,g^4\,(a+b\,x)^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{11\,B\,a^2\,d^2}{18\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}+\frac{5\,B\,a\,d^2\,x}{6\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}+\frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{7\,B\,a\,c\,d}{18\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{B\,b\,c\,d\,x}{6\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^4,x)
```

output
```
(B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d -
b*c)^3) - (B*log((e*(c + d*x))/(a + b*x)))/(3*b*g^4*(a + b*x)^3) - (A*b*c^
2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a +
 b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a^2*d^2
)/(18*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^
2*(a + b*x)^3) + (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*A*a*
c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2
*(a + b*x)^3) - (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.181 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$

3.181.1 Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = \frac{B}{16bg^5(a+bx)^4} - \frac{Bd}{12b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)} - \frac{Bd^4\log(a+bx)}{4b(bc-ad)^4g^5} + \frac{Bd^4\log(c+dx)}{4b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}$$

output
```
1/16*B/b/g^5/(b*x+a)^4-1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/8*B*d^2/b/(-a
*d+b*c)^2/g^5/(b*x+a)^2-1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/4*B*d^4*ln(
b*x+a)/b/(-a*d+b*c)^4/g^5+1/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B
*ln(e*(d*x+c)/(b*x+a)))/b/g^5/(b*x+a)^4
```

3.181.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = \frac{\frac{B(-bc+ad)\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^5}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4}}{4bg^5}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

output
```
((B*(-(b*c) + a*d)*((-3*(b*c - a*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(
a + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a +
 b*x) + 12*d^4*Log[a + b*x] - 12*d^4*Log[c + d*x]))/(12*(b*c - a*d)^5) - (
A + B*Log[(e*(c + d*x))/(a + b*x)])/(a + b*x)^4)/(4*b*g^5)
```

3.181.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{ 2948}$$

$$-\frac{B(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{4bg}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{4bg^5}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{B(bc-ad)\int \left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+b}\right.}{4bg^5}$$

$$\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}-\frac{B(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{4bg^5}$$

3.181. $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}dx$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

output
```
-1/4*(B*(b*c - a*d)*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2
*(a + b*x)^3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a
+ b*x)) + (d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d
)^5))/(b*g^5) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(4*b*g^5*(a + b*x)^4)
```

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.181. $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$

3.181.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(195) = 390$.

Time = 2.20 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.12

method	result
parts	$-\frac{A}{4g^5(bx+a)^4 b}-\frac{B b^3\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^4 \ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{4}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^4}{16}-\frac{3de\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3 \ln\left(\frac{de}{b}-\frac{e(ad}{b(bx}\right.}{3}\right.}{b}\right.}{}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{4b g^5(bx+a)^4}-\frac{48Bab^3cd^3x^2+72Ba^2b^2cd^3x-24Bab^3c^2d^2x-48Aa^3bcd^3+72Aa^2b^2c^2d^2-48Aab^3c^3d-12Bab}{}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{b^5A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^4}{4(ad-cb)^5e^5g^5}+\frac{b^4Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{(ad-cb)^5e^4g^5}-\frac{3b^3Ad^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^5e^3g^5}+\frac{b^2Ad^3\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^5e^2g^5}-\frac{b^5B(}{}\right.}{}$
default	$\frac{e(ad-cb)\left(-\frac{b^5A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^4}{4(ad-cb)^5e^5g^5}+\frac{b^4Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3}{(ad-cb)^5e^4g^5}-\frac{3b^3Ad^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^5e^3g^5}+\frac{b^2Ad^3\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^5e^2g^5}-\frac{b^5B(}{}\right.}{}$
parallelrisch	$\frac{48Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)a^9cd^4-72B\ln\left(\frac{e(dx+c)}{bx+a}\right)a^8bc^3d^2+48B\ln\left(\frac{e(dx+c)}{bx+a}\right)a^7b^2c^4d+12Ax^4a^2b^7c^5-3Bx^4a^2b^7c^5+48Ax^3c}{}$
norman	$\frac{Ba^3d^4x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)g}+\frac{ad^4Bb^2x^3\ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)g}+\frac{\left(4Aa^3d^3-12Aa^2bcd^2+12Aa\right.}{4ga}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/g^5/(b*x+a)^4/b-B/g^5*b^3/e^4/(a*d-b*c)^4*(1/4*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/16*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b
/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3*d^2*e^2/b^2*(1/2*(d*e/b-e
*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-
b*c)/b/(b*x+a))^2)-d^3*e^3/b^3*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(
a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

3.181.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(192) = 384$.

Time = 0.27 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.09

$$\int \frac{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{3\,(4\,A-B)b^4c^4 - 16\,(3\,A-B)ab^3c^3d + 36\,(2\,A-B)a^2b^2c^2d^2 - 48\,(A-B)a^3bcd^3 + (12\,A-25\,B)a^4d}{48\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4\,(ab^8c^4 - 4\,a^2b^7c^3}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="fricas"
)
```

output
```
-1/48*(3*(4*A - B)*b^4*c^4 - 16*(3*A - B)*a*b^3*c^3*d + 36*(2*A - B)*a^2*b
^2*c^2*d^2 - 48*(A - B)*a^3*b*c*d^3 + (12*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*
d^3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*
d^4)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*
a^3*b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2
 + 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4
*B*a^3*b*c*d^3)*log((d*e*x + c*e)/(b*x + a)))/((b^9*c^4 - 4*a*b^8*c^3*d +
6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4
- 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5
*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*
d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*
c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*
c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

3.181.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(178) = 356$.

Time = 2.67 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.58

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4} + \frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4} - \frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4} + \frac{-12Aa^3d^3+36Aa^2bcd^2-36Aab^2c}{48a^7bd^3g^5-144a^6b^2cd^2g^5+144a^5b^3c^2dg^5-48a^4b^4c^3g^5+x^4\cdot(48a^3b^5d^3g^5-144a^2b^6cd^2g^5+144ab^7c^2c}$$

input

```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**5,x)
```

output

```
-B*log(e*(c + d*x)/(a + b*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a*
*2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*log(x
 + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*
a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)*
*4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*
d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) - B*d**4
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 +
10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d -
b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**
4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (
-12*A*a**3*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3
 + 25*B*a**3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**
3 + 12*B*b**3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(5
2*B*a**2*b*d**3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(48*a**7*b*d**3*g
**5 - 144*a**6*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b**4
*c**3*g**5 + x**4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 + 14
4*a*b**7*c**2*d*g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**5
- 576*a**3*b**5*c*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**3*
g**5) + x**2*(288*a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*a*
*3*b**5*c**2*d*g**5 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**3*...
```

3.181.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(192) = 384$.

Time = 0.22 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.14

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{1}{48}B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-2}{(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)g^5x^4+4\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)g^5x^3+6}\right. - \frac{A}{4\,(b^5g^5x^4+4\,ab^4g^5x^3+6\,a^2b^3g^5x^2+4\,a^3b^2g^5x+a^4bg^5)}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="maxima"
)
```

output
```
-1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*
d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^
3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*
b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*
d^3)*g^5) + 12*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4
*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b
*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*
b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

3.181.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(192) = 384$.

Time = 0.83 (sec) , antiderivative size = 751, normalized size of antiderivative = 3.65

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{1}{48}\left(12\left(\frac{(dex+ce)^4Bb^3}{(b^3c^3e^3g^5-3\,ab^2c^2de^3g^5+3\,a^2bcd^2e^3g^5-a^3d^3e^3g^5)(bx+a)^4} - \frac{4\,(dex}{(b^3c^3e^2g^5-3\,ab^2c^2de^2g^5+3}\right.\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

output
```
-1/48*(12*((d*e*x + c*e)^4*B*b^3/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5
 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 4*(d*e*x + c*e)
^3*B*b^2*d/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g
^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 6*(d*e*x + c*e)^2*B*b*d^2/((b^3*c^3*e
*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(b*x + a
)^2) - 4*(d*e*x + c*e)*B*d^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c
*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 3*(4*A*
b^3 - B*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*g^5 + 3
*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 16*(3*A*b^2*d - B*b
^2*d)*(d*e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*
c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 36*(2*A*b*d^2 - B*b*d^2)*(
d*e*x + c*e)^2/((b^3*c^3*e*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5
 - a^3*d^3*e*g^5)*(b*x + a)^2) - 48*(A*d^3 - B*d^3)*(d*e*x + c*e)/((b^3*c^
3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*(
b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.181.9 Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.81

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$$

$$= \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{2\,b\,g^5\,{(a\,d-b\,c)}^4}$$

$$-\frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}$$

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3-25\,B\,a^3\,d^3+3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2-13\,B\,a\,b^2\,c^2\,d+23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b\cdots\right.}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x^3\cdots}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*
b^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4) - (B*
log((e*(c + d*x))/(a + b*x)))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^
2*b*x^2 + 4*a*b^2*x^3)) - ((12*A*a^3*d^3 - 12*A*b^3*c^3 - 25*B*a^3*d^3 + 3
*B*b^3*c^3 + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B
*a^2*b*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (
d^2*x^2*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^
3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^
4 + 16*a^3*b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2)
```

3.181. $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$

3.182 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

3.182.1 Optimal result

Integrand size = 32, antiderivative size = 503

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx \\
&= \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{60bd^3} + \frac{B^2(bc-ad)^2g^4(a+bx)^3}{30bd^2} \\
&\quad - \frac{5B^2(bc-ad)^5g^4\log(a+bx)}{6bd^5} - \frac{13B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{30bd^5} \\
&\quad + \frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5bd^3} \\
&\quad - \frac{2B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{15bd^2} \\
&\quad + \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{10bd} \\
&\quad - \frac{2B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5d^5} + \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} \\
&\quad - \frac{2B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} \\
&\quad + \frac{2B^2(bc-ad)^5g^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}$$

output
```
13/30*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/60*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3
+1/30*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-5/6*B^2*(-a*d+b*c)^5*g^4*ln(b*x
+a)/b/d^5-13/30*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+1/5*B*(-a*d
+b*c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^3-2/15*B*(-a*d+b*c)^
2*g^4*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b
*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/5*B*(-a*d+b*c)^4*g^4*(d*x+c)*(A+
B*ln(e*(d*x+c)/(b*x+a)))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(b*x+a)))
^2/b-2/5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(
d*x+c))/b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^
5
```

3.182.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.02

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 - \frac{B(bc-ad)\left(24Abd(bc-ad)^3x+24B(bc-ad)^4\log(a+bx)-4B(bc-ad)^2(2bd(bc-ad)x-d^2(a\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*
(24*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[a + b*x] - 4*B*(b*c - a
*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x
]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a*d)^
3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 24*b*B*(b*c - a*d)^3*(c + d*x)*L
og[(e*(c + d*x))/(a + b*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[
(e*(c + d*x))/(a + b*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(c
 + d*x))/(a + b*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x
)]) - 24*(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) -
 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

3.182.3 Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.39, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.594, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2952}$$

$$g^4\left(-(bc-ad)^5\right)\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^6}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{2756}$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}}{5b}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}\right)}{5b}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{4b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\cdots\right.}{(c+dx)\left(d-\cdots\right.}}{\cdots}\right.}{5b}\right.$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int\left(\frac{b}{d^4\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d^2\left(d-\frac{b(c+dx}{a+bx}\right.}\cdots\right.}{4b}\right.}{d}\cdots\right.}{5\cdots}\right.$$

$\downarrow$ 2009

3.182. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^4}\right.}{}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log}{4b(}\right.}{}\right.}{}\right.$$

$\downarrow$ 2756

3.182. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\cdots\right.\right.}{(c+dx)\left(d-\frac{b(c+\cdots}{a+\cdots}\right.}}{d}}{d}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\int\left(\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right.}\cdots\right.}{3b}\cdots\right.}{d}\cdots}{d}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2009

3.182. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^3}-\cdots\right.}{\cdots}\right.}{d}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\cdots\right.}{3b\left(d-\cdots\right.}\right.}{\cdots}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2756

3.182. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\cdots\right.\right.}{(c+dx)\left(d-\frac{b(c+\cdots}{a+\cdots}\right.}}{d}}{d}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 54$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d\frac{c+\cdots}{a+\cdots}}{2b}\cdots\right.}{d}\cdots}{d}\cdots\right.}{\cdots}\right.$$

3.182. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$\downarrow \text{2009}$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\right.}{}\right.}{d}}{d}\right.}{}\right.$$

$$\downarrow \text{2789}$$

3.182. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\right.}{2b\left(d-\right.}\right.}{d}}{}\right.}{}\right)$$

$\downarrow$ 2751

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e}{}\right.\right.}{(c+dx)\left(d-\frac{b(c+}{a+}\right.}}{d}}{d}}{}\right.}{}\right)$$

3.182. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$\Big\downarrow \; 16$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{B\log\left(d\cdots\right.}{\cdots}\right.}{d}}{d}}{\cdots}}{\cdots}\cdots\right.\right)$$

$$\Big\downarrow \; 2779$$

3.182. $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{\frac{\frac{B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}}{d}+\frac{b\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2838

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{2b}\right)}{d}+\cdots\right.}{\cdots}\right.$$

3.182. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
-((b*c - a*d)^5*g^4*((A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(5*b*(d - (b*(
c + d*x))/(a + b*x))^5) - (2*B*((b*((A + B*Log[(e*(c + d*x))/(a + b*x)])/(
4*b*(d - (b*(c + d*x))/(a + b*x))^4) - (B*(1/(3*d*(d - (b*(c + d*x))/(a +
b*x))^3) + 1/(2*d^2*(d - (b*(c + d*x))/(a + b*x))^2) + 1/(d^3*(d - (b*(c +
 d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^4 - Log[d - (b*(c + d*x))/
(a + b*x)]/d^4))/(4*b)))/d + ((b*((A + B*Log[(e*(c + d*x))/(a + b*x)])/(3*
b*(d - (b*(c + d*x))/(a + b*x))^3) - (B*(1/(2*d*(d - (b*(c + d*x))/(a + b*
x))^2) + 1/(d^2*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/
d^3 - Log[d - (b*(c + d*x))/(a + b*x)]/d^3))/(3*b)))/d + ((b*((A + B*Log[(
e*(c + d*x))/(a + b*x)])/(2*b*(d - (b*(c + d*x))/(a + b*x))^2) - (B*(1/(d*
(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^2 - Log[d - (b
*(c + d*x))/(a + b*x)]/d^2))/(2*b)))/d + ((b*(((c + d*x)*(A + B*Log[(e*(c
+ d*x))/(a + b*x)]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (B*Log[
d - (b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x))/(a
+ b*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (B*PolyLog[2, (d*(a +
b*x))/(b*(c + d*x))])/d)/d)/d)/d)/d))/(5*b)))
```

3.182.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.182.4 Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.182.5 Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d*e*x + c*e)
/(b*x + a))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g
^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((d*e*x + c*e)/(b*x + a)), x)
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

3.182.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2395 vs. 2(478) = 956.

Time = 0.31 (sec) , antiderivative size = 2395, normalized size of antiderivative = 4.76

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^
3*b*g^4*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b
 + c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x +
 a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*
A*B*a^3*b*g^4 + 2*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(
b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(d*e*x/(b*x +
 a) + c*e/(b*x + a)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(d*e*x/(b*x + a)
 + c*e/(b*x + a)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3
*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4
+ A^2*a^4*g^4*x + 1/30*((12*g^4*log(e) - 25*g^4)*b^4*c^5 - (60*g^4*log(e)
- 113*g^4)*a*b^3*c^4*d + 4*(30*g^4*log(e) - 49*g^4)*a^2*b^2*c^3*d^2 - 12*(
10*g^4*log(e) - 13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) - 4*g^4)*a^4*c*d^
4)*B^2*log(d*x + c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^
3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*
(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b
*c - a*d)))*B^2/(b*d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 + 6*(b^...
```

3.182.8 Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^4*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^4 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.183 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

3.183.1 Optimal result

Integrand size = 32, antiderivative size = 420

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx \\
&= -\frac{5B^2(bc-ad)^3g^3x}{12d^3}+\frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2}+\frac{11B^2(bc-ad)^4g^3\log(a+bx)}{12bd^4} \\
&\quad+\frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{12bd^4}-\frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4bd^2} \\
&\quad+\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6bd} \\
&\quad+\frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2d^4}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} \\
&\quad+\frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} \\
&\quad-\frac{B^2(bc-ad)^4g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}
\end{aligned}$$

output
```
-5/12*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2
+11/12*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/12*B^2*(-a*d+b*c)^4*g^3*ln((
d*x+c)/(b*x+a))/b/d^4-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(
b*x+a)))/b/d^2+1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/
b/d+1/2*B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/d^4+1/4*g^3
*(b*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+1/2*B*(-a*d+b*c)^4*g^3*(A+B*ln(
e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g
^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

3.183.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.93

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 + \frac{B(bc-ad)\left(6Abd(bc-ad)^2x+6B(bc-ad)^3\log(a+bx)-B(bc-ad)\left(2bd(bc-ad)x-d^2(a+bx\right.\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*
(6*A*b*d*(b*c - a*d)^2*x + 6*B*(b*c - a*d)^3*Log[a + b*x] - B*(b*c - a*d)*
(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) - 3
*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*b*B*(b*c - a*d)
^2*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*
(c + d*x))/(a + b*x)]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B*Log[(e*(c + d
*x))/(a + b*x)]) - 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
/(3*d^4)))/(4*b)
```

3.183.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2952}$$

$$g^3(bc-ad)^4 \int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^5} d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4 \left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4} d\frac{c+dx}{a+bx}}{2b}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^3(bc-ad)^4 \left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4} d\frac{c+dx}{a+bx}}{d} + \frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3} d\frac{c+dx}{a+bx}}{d}\right)}{2b}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+}{a+}\cdots}{d}\cdots}{2b}\cdots\right.$$

$\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\int\left(\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^3}+\frac{a+bx}{d^3(c+dx)}\right)d\cdots}{3b}\cdots\right.}{d}\cdots}{2b}\cdots\right.$$

$\downarrow$ 2009

3.183. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^3}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^3}\right.}{}\right.}{d}\right.}{2b}\right.$$

$\downarrow$ 2789

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\right.}{}\right.}{2b}\right.$$

$\downarrow$ 2756

3.183. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d}\right.}{}\right.$$

$\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int\frac{(a+bx)(\cdots}{(c+dx\cdots}}{\cdots}}{d}\right.}{}\right.$$

$\downarrow$ 2009

3.183. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\cdots\right.}{2b}\cdots\right.}{d}\cdots}{d}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2789

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\cdots\right.}{\cdots}\cdots\right.}{d}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2751

3.183. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^3(bc-$$

$$ad)^4 \left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}}{d}\right.\right.$$

$\downarrow$ 16

$$g^3(bc-$$

$$ad)^4 \left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4} - \frac{B\left(\frac{\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{B\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{bd}\right)}{d}}{d}+\cdots}{d}\right.\right.$$

$\downarrow$ 2779

3.183. $\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

$$g^3(bc-$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\frac{B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c}{a-}\right.\right.}{d(a+bx)\left(d-\frac{b(}{c}\right.}\right.}{}}{d}\right.}{}\right.$$

$$\downarrow \text{2838}$$

$$g^3(bc-$$

$$ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{2b}\right)}{d}+\frac{\frac{B\,\text{PolyLog}\left(2,\frac{d(a+}{b(c+}\right.}{d}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

3.183. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

output
```
(b*c - a*d)^4*g^3*((A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(4*b*(d - (b*(c
+ d*x))/(a + b*x))^4) - (B*((b*((A + B*Log[(e*(c + d*x))/(a + b*x)])/(3*b*
(d - (b*(c + d*x))/(a + b*x))^3) - (B*(1/(2*d*(d - (b*(c + d*x))/(a + b*x)
)^2) + 1/(d^2*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^
3 - Log[d - (b*(c + d*x))/(a + b*x)]/d^3))/(3*b)))/d + ((b*((A + B*Log[(e*
(c + d*x))/(a + b*x)])/(2*b*(d - (b*(c + d*x))/(a + b*x))^2) - (B*(1/(d*(d
 - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^2 - Log[d - (b*(
c + d*x))/(a + b*x)]/d^2))/(2*b)))/d + ((b*(((c + d*x)*(A + B*Log[(e*(c +
d*x))/(a + b*x)]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (B*Log[d
- (b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x))/(a +
b*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (B*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/d)/d)/d)/d))/(2*b))
```

3.183.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.183. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.183.4 Maple [F]

$$\int (bgx+ag)^{3} \left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^{2} dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.183.5 Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^3 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2
*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((d*e*x + c*e)/(b*x + a)),
x)
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

3.183.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(399) = 798$.

Time = 0.31 (sec) , antiderivative size = 1735, normalized size of antiderivative = 4.13

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log
(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A
*B*a^3*g^3 + 3*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a
)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3
*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log
(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d
^2))*A*B*a*b^2*g^3 + 1/12*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*
a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
*A*B*b^3*g^3 + A^2*a^3*g^3*x - 1/12*((6*g^3*log(e) - 11*g^3)*b^3*c^4 - 2*(
12*g^3*log(e) - 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) - 5*g^3)*a^2*b*c^2*d
^2 - 6*(4*g^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c
^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a
^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d
*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2
 + 2*(b^4*c*d^3*g^3*log(e) + (6*g^3*log(e)^2 - g^3*log(e))*a*b^3*d^4)*B^2*
x^3 - ((3*g^3*log(e) - g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) - g^3)*a*b^3*c*d
^3 - (18*g^3*log(e)^2 - 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 + ((6*g^3
*log(e) - 5*g^3)*b^4*c^3*d - (24*g^3*log(e) - 17*g^3)*a*b^3*c^2*d^2 + (36*
g^3*log(e) - 19*g^3)*a^2*b^2*c*d^3 + (12*g^3*log(e)^2 - 18*g^3*log(e) +...
```

3.183.8 Giac [F]

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx=\int (bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.184 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

3.184.1 Optimal result

Integrand size = 32, antiderivative size = 335

$$\begin{aligned}\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx &= \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2\log(a+bx)}{bd^3} - \frac{B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} \\ &+ \frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3bd} \\ &- \frac{2B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3d^3} + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} \\ &- \frac{2B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} \\ &+ \frac{2B^2(bc-ad)^3g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-1/3*B^
2*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^
2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+B*ln(e
*(d*x+c)/(b*x+a)))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b-2
/3*B*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c)
)/b/d^3+2/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

3.184.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.87

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2-\frac{B(bc-ad)\left(2Abd(bc-ad)x+2B(bc-ad)^2\log(a+bx)-B(bc-ad)(bdx+(-bc+ad)\log(c+dx\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*
(2*A*b*d*(b*c - a*d)*x + 2*B*(b*c - a*d)^2*Log[a + b*x] - B*(b*c - a*d)*(b
*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*b*B*(b*c - a*d)*(c + d*x)*Log[(e*(
c + d*x))/(a + b*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]
) - 2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - B*
(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c
 + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

3.184.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2dx$$
$$\Big\downarrow \; 2952$$
$$g^2\left(-(bc-ad)^3\right)\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}$$
$$\Big\downarrow \; 2756$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)$$

$$\downarrow 2789$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}\right)}{3b}\right)$$

$$\downarrow 2756$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\cdots\right.}{(c+dx)\left(d-\cdots\right.}\cdots}{\cdots}\right.}{3b}\right.$$

$$\downarrow 54$$

3.184. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d\frac{c+\cdots}{a+\cdots}}{2b}\cdots\right)}{d}\cdots\right)}{3b\cdots}\cdots\right)$$

$\downarrow$ 2009

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{3b\cdots}\cdots\right)$$

$\downarrow$ 2789

3.184. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\cdots}{2b(\cdots}\cdots\right.}{\cdots}\cdots\right)}{3b\cdots}\cdots\right.$$

$\downarrow$ 2751

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(\cdots}{\cdots}\right.\right.}{(c+dx)\left(d-\frac{b(c+\cdots}{a+\cdots}\right.}\cdots}{d}}{d}\cdots\right.}{\cdots}\cdots\right.$$

$\downarrow$ 16

3.184. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{B\log\left(d-\right.}{}\right.}{d}}{d}\right.}{}\right.$$

$\downarrow$ 2779

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\frac{\frac{B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\left(\right.}{}}{d}\right.}{}\right.$$

$\downarrow$ 2838

3.184. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2dx$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{2b}\right)}{d}-\cdots\right.}{\cdots}\right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
-((b*c - a*d)^3*g^2*((A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(3*b*(d - (b*(
c + d*x))/(a + b*x))^3) - (2*B*((b*((A + B*Log[(e*(c + d*x))/(a + b*x)])/(
2*b*(d - (b*(c + d*x))/(a + b*x))^2) - (B*(1/(d*(d - (b*(c + d*x))/(a + b*
x))) + Log[(c + d*x)/(a + b*x)]/d^2 - Log[d - (b*(c + d*x))/(a + b*x)]/d^2
))/(2*b)))/d + ((b*(((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(d*(a
 + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (B*Log[d - (b*(c + d*x))/(a + b*x
)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/d) + (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d
)/d))/(3*b)))
```

3.184.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.184. $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

3.184.4 Maple [F]

$$\int (bgx+ag)^2 \left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.184.5 Fricas [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^2 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B
*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((d*e*x + c*e)/(b*x + a))
, x)
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

3.184.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1172 vs. $2(320) = 640$.

Time = 0.31 (sec) , antiderivative size = 1172, normalized size of antiderivative = 3.50

$$\int (ag + bgx)^2 \left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*
x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(d*
e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d
^2 + (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(d*e*x/(b*x + a) + c
*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a
^2*g^2*x + 1/3*((2*g^2*log(e) - 3*g^2)*b^2*c^3 - (6*g^2*log(e) - 7*g^2)*a*
b*c^2*d + 2*(3*g^2*log(e) - 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(
b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*
x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (b^3*c*d^2*g^2*log(
e) + (3*g^2*log(e)^2 - g^2*log(e))*a*b^2*d^3)*B^2*x^2 - ((2*g^2*log(e) - g
^2)*b^3*c^2*d - 2*(3*g^2*log(e) - g^2)*a*b^2*c*d^2 - (3*g^2*log(e)^2 - 4*g
^2*log(e) + g^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3
*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*
b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c
^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - (2*B
^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*d^2*g^2 + (6*g^2*log(e) - g^2)*a*b^2*d^
3)*B^2*x^2 - 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - (3*g^2*log(e) - 2*g^2)
*a^2*b*d^3)*B^2*x - (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 - (2*g^2*log...
```

3.184.8 Giac [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \int (bgx+ag)^2 \left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.185 $\int (ag + bgx) \left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

3.185.1 Optimal result

Integrand size = 30, antiderivative size = 202

$$\begin{aligned}\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx &= \frac{B^2(bc-ad)^2 g\log(a+bx)}{bd^2} + \frac{B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{d^2} \\ &+ \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} \\ &+ \frac{B(bc-ad)^2 g\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} \\ &- \frac{B^2(bc-ad)^2 g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}\end{aligned}$$

output
```
B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d*x+c
)/(b*x+a)))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+B*(-a*d+b*
c)^2*g*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^2-B^2*(-a
*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

3.185.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$$
$$=\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B(bc-ad)\left(2Abdx+2B(bc-ad)\log(a+bx)+2bB(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)-2(bc-ad)\log(c+dx\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*(2
*A*b*d*x + 2*B*(b*c - a*d)*Log[a + b*x] + 2*b*B*(c + d*x)*Log[(e*(c + d*x)
)/(a + b*x)] - 2*(b*c - a*d)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*
x)]) - B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

3.185.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2952, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2\,dx$$
$$\Big\downarrow \; 2952$$
$$g(bc-ad)^2\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}$$
$$\Big\downarrow \; 2756$$
$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)$$

$\downarrow$ 2789

$$g(bc- ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}\right)}{b}\right)$$

$\downarrow$ 2751

$$g(bc- ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+d}{a+b}}{d}}{b}$$

$\downarrow$ 16

$$g(bc- ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{B\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{bd}\right)}{d}\right)}{b}$$

$\downarrow$ 2779

3.185. $\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\frac{B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e}{}\right.\right.}{d(a+bx)\left(d-\right.}\right.}{}\right)}{b}\right.$$

$\downarrow$ 2838

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\frac{B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right.}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}\right.}{d}\right.}{b}\right.$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
(b*c - a*d)^2*g*((A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(2*b*(d - (b*(c +
d*x))/(a + b*x))^2) - (B*((b*(((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x
)]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (B*Log[d - (b*(c + d*x)
)/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 -
 (d*(a + b*x))/(b*(c + d*x))])/d) + (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/d)/d))/b)
```

3.185. $\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

3.185.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.185. $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

3.185.4 Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.185.5 Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx=\int (bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/(b*
x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a)), x)
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

3.185.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(199) = 398$.

Time = 0.31 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.06

$$\int (ag + bgx)\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$$
$$= \frac{1}{2}A^2bgx^2 + 2\left(x\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)ABag$$
$$+ \left(x^2\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)ABbg$$
$$+ A^2agx - \frac{((g\log(e) - g)bc^2 - (2g\log(e) - g)acd)B^2\log(dx+c)}{d^2}$$
$$+ \frac{(b^2c^2g - 2abcdg + a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad} + 1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}$$
$$+ \frac{B^2b^2d^2gx^2\log(e)^2 + 2\left(b^2cdg\log(e) + \left(g\log(e)^2 - g\log(e)\right)abd^2\right)B^2x + (B^2b^2d^2gx^2 + 2B^2abd^2gx + \ldots}{}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima"
)
```

output
```
1/2*A^2*b*g*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x +
a)/b + c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a
)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*
B*b*g + A^2*a*g*x - ((g*log(e) - g)*b*c^2 - (2*g*log(e) - g)*a*c*d)*B^2*lo
g(d*x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((
b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*
d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c*d*g*log(e) + (g*log(e)^2
 - g*log(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2
*a^2*d^2*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2
*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 2*(B^2*b^2*d^2*g*x^2*log(e) +
((2*g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x + ((g*log(e) - g)*a^2*d^2 + a
*b*c*d*g)*B^2)*log(b*x + a) + 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) -
 g)*a*b*d^2 + b^2*c*d*g)*B^2*x - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x +
B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

3.185.8 Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx=\int (bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.186 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$

3.186.1 Optimal result

Integrand size = 32, antiderivative size = 128

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg} - \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{2B^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g-2*B*(A+B*ln(e*(
d*x+c)/(b*x+a)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d*x
+c)/d/(b*x+a))/b/g
```

3.186.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.97

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{AB\log^2\left(\frac{-bc+ad}{d(a+bx)}\right) + A^2\log(a+bx) + 2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right) - 2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right) \cdots}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

output
```
(A*B*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + A^2*Log[a + b*x] + 2*A*B*Log[(-
(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*Log[(-(
b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)] - B^2*Log[(-(b*c)
+ a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)]^2 - 2*A*B*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)] - 2*B^2*Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*
x))])/(b*g)
```

3.186.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2952, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{ag+bgx} \, dx$$

$$\Big\downarrow \text{ 2952}$$

$$\frac{\int \frac{\left(A+B \log \left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{g}$$

$$\Big\downarrow \text{ 2754}$$

$$\frac{\frac{2B \int \frac{(a+bx)\left(A+B \log \left(\frac{e(c+dx)}{a+bx}\right)\right) \log \left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{b}-\frac{\log \left(1-\frac{b(c+dx)}{d(a+bx)}\right) \left(B \log \left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{\frac{2B \left(B \int \frac{(a+bx) \operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}-\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right) \left(B \log \left(\frac{e(c+dx)}{a+bx}\right)+A\right)\right)}{b}-\frac{\log \left(1-\frac{b(c+dx)}{d(a+bx)}\right) \left(B \log \left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\frac{2B\left(B\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)-\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[(e*(c + d*x))/(a + b*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b
*x))])/b) + (2*B*(-((A + B*Log[(e*(c + d*x))/(a + b*x)])*PolyLog[2, (b*(c
+ d*x))/(d*(a + b*x))]) + B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/g
```

3.186.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.186.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(127) = 254$.

Time = 1.42 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.70

method	result
parts	$\frac{A^2\ln(bx+a)}{gb} - \frac{B^2 (\ln(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})^2 \ln(1-\frac{b(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})}{de}) + 2\ln(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}) \mathrm{Li}_2(\frac{b(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})}{de}) - 2}{gb}$
risch	$\frac{A^2\ln(bx+a)}{gb} - \frac{B^2\ln(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})^2 \ln(1-\frac{b(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})}{de})}{bg} - \frac{2B^2\ln(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}) \mathrm{Li}_2(\frac{b(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})}{de}}{bg}$
derivativedivides	$\frac{e(ad-cb) (-\frac{bA^2\ln((\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})b-de)}{ge(ad-cb)} - \frac{bB^2(\ln(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})^2\ln(1-\frac{b(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})}{de})+2\ln(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})\mathrm{Li}_2}{ge(ad-cb)}}{}$
default	$\frac{e(ad-cb) (-\frac{bA^2\ln((\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})b-de)}{ge(ad-cb)} - \frac{bB^2(\ln(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})^2\ln(1-\frac{b(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})}{de})+2\ln(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)})\mathrm{Li}_2}{ge(ad-cb)}}{}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
A^2/g*ln(b*x+a)/b-B^2/g/b*(ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(1-b/d/e*(d
*e/b-e*(a*d-b*c)/b/(b*x+a)))+2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*polylog(2,b
/d/e*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))-2*polylog(3,b/d/e*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))))+2*B*A/g*(-dilog(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/d/e)/b-
ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-d*e)/
d/e)/b)
```

3.186.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="fricas"
)
```

output
```
integral((B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*e*x + c*e)/(b*
x + a)) + A^2)/(b*g*x + a*g), x)
```

3.186.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(c*e/(a + b*x) + d*e*x/(a
+ b*x))**2/(a + b*x), x) + Integral(2*A*B*log(c*e/(a + b*x) + d*e*x/(a + b
*x))/(a + b*x), x))/g
```

3.186.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="maxima"
)
```

output
```
B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integ
rate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log(b*x
 + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x - 2*(B^2*b*c*log(e) + A*
B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) + 2*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x - (2*B^2*b*d*x + (b*c + a*d)*B^2)*lo
g(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x),
 x)
```

3.186.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="giac")
```

output
```
integrate((B*log((d*x + c)*e/(b*x + a)) + A)^2/(b*g*x + a*g), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x), x)
```

3.187 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$

3.187.1 Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{2AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)} + \frac{2B^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
2*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+
2*B^2*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln
(e*(d*x+c)/(b*x+a)))^2/(-a*d+b*c)/g^2/(b*x+a)
```

3.187.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-2(bc-ad)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-2d(a+bx)\log\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(2*B*(b*c - a*d + d*(a + b*
x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 2*(b*c - a*d)*(A + B*Log[(e*
(c + d*x))/(a + b*x)]) - 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c + d*x
))/(a + b*x)]) + 2*d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a +
b*x)]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(
b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x))
```

3.187.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2952, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{(ag+bgx)^2}dx$$
$$\Big\downarrow \text{ 2952}$$
$$-\frac{\int \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$
$$\Big\downarrow \text{ 2733}$$

$$-\frac{\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{a+bx}-2B\int\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{a+bx}-2B\left(\frac{A(c+dx)}{a+bx}+\frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx}-\frac{B(c+dx)}{a+bx}\right)}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

output
```
-((((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(a + b*x) - 2*B*((A*
(c + d*x))/(a + b*x) - (B*(c + d*x))/(a + b*x) + (B*(c + d*x)*Log[(e*(c +
d*x))/(a + b*x)])/(a + b*x)))/((b*c - a*d)*g^2))
```

3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.187.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.18

method	result
norman	$\frac{\frac{\left(A^2-2BA+2B^2\right)x}{ga}+\frac{B^2 c \ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g(ad-cb)}+\frac{B^2 dx \ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g(ad-cb)}+\frac{2(-B+A)cB\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}+\frac{2d(-B+A)Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}}{g(bx+a)}$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2\left(\frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)^2 e(dx+c)}{bx+a}-\frac{2e(dx+c)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{bx+a}+\frac{2e(dx+c)}{bx+a}\right)}{g^2e(ad-cb)}+\frac{2BA\left(\frac{e(dx+c)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{bx+a}-\frac{e(d}{bx}\right.}{g^2e(ad-cb)}$
parallelrisch	$-\frac{-2ABx\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3d^2-2AB\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3cd+A^2ab^2d^2-A^2b^3cd+2B^2ab^2d^2-2B^2b^3cd+2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^3cd}{g^2(bx+a)b^3d(ad-cb}$
derivativedivides	$\frac{e(ad-cb)\left(\frac{b^2A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^2e^2g^2}+\frac{2b^2AB\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^2e^2g^2}+\frac{b^2B^2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\right.}{}\right.}{b^2}$
default	$\frac{e(ad-cb)\left(\frac{b^2A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^2e^2g^2}+\frac{2b^2AB\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^2e^2g^2}+\frac{b^2B^2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\right.}{}\right.}{b^2}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)^2 dx}{g^2(ad-cb)(bx+a)}+\frac{B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)^2 c}{g^2(ad-cb)(bx+a)}-\frac{2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)dx}{g^2(ad-cb)(bx+a)}-\frac{2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)c}{g^2(ad-cb)(bx+a)}+\frac{}{g^2(ad}$

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A^2-2*A*B+2*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(d*x+c)/(b*x+a))^2+B^2*d/g
/(a*d-b*c)*x*ln(e*(d*x+c)/(b*x+a))^2+2*(-B+A)*c*B/g/(a*d-b*c)*ln(e*(d*x+c)
/(b*x+a))+2*d*(-B+A)*B/g/(a*d-b*c)*x*ln(e*(d*x+c)/(b*x+a)))/g/(b*x+a)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\frac{(A^2-2\,AB+2\,B^2)bc-(A^2-2\,AB+2\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{dex+ce}{bx+a}\right)^2+2\left((AB-B^2)bdx+\right.}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="frica
s")
```

output
```
-((A^2 - 2*A*B + 2*B^2)*b*c - (A^2 - 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2
*b*c)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((A*B - B^2)*b*d*x + (A*B - B^2)*
b*c)*log((d*e*x + c*e)/(b*x + a)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a
^2*b*d)*g^2)
```

3.187.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(128) = 256$.

Time = 1.21 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.81

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{2Bd(A-B)\log\left(x+\frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd-\frac{2Ba^2d^3(A-B)}{ad-bc}+\frac{4Babcd^2(A-B)}{ad-bc}-\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2(ad-bc)} - \frac{2Bd(A-B)\log\left(x+\frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd+\frac{2Ba^2d^3(A-B)}{ad-bc}-\frac{4Babcd^2(A-B)}{ad-bc}+\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{(-2AB+2B^2)\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x} + \frac{(B^2c+B^2dx)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x} + \frac{-A^2+2AB-2B^2}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**2,x)
```

output
```
2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d - 2*B**2*a*d**2 - 2*B**2
*b*c*d - 2*B*a**2*d**3*(A - B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A - B)/(a*d -
 b*c) - 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*A*B*b*d**2 - 4*B**2*b*d**2
))/(b*g**2*(a*d - b*c)) - 2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*
d - 2*B**2*a*d**2 - 2*B**2*b*c*d + 2*B*a**2*d**3*(A - B)/(a*d - b*c) - 4*B
*a*b*c*d**2*(A - B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*
A*B*b*d**2 - 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B + 2*B**2)*log(
e*(c + d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log(
e*(c + d*x)/(a + b*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2*
c*g**2*x) + (-A**2 + 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

3.187.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(153) = 306$.

Time = 0.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \left(2\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{(bdx+ad)\log(bx+a)^2}{\ldots}\right.$$
$$-2\,AB\left(\frac{\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^2g^2x+abg^2}-\frac{1}{b^2g^2x+abg^2}-\frac{d\log(bx+a)}{(b^2c-abd)g^2}+\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$-\frac{B^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="maxim
a")
```

output
```
(2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + ((
b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d
 - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x +
 a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*
x))*B^2 - 2*A*B*(log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2
) - 1/(b^2*g^2*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log
(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)
)^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(\frac{(dex+ce)B^2\log\left(\frac{dex+ce}{bx+a}\right)^2}{(bx+a)g^2}+\frac{2\,(dex+ce)(AB-B^2)\log\left(\frac{dex+ce}{bx+a}\right)}{(bx+a)g^2}+\frac{(dex+ce)(A^2-2\,AB+2\,B^2)}{(bx+a)g^2}\right)$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="giac"
)
```

output
```
-((d*e*x + c*e)*B^2*log((d*e*x + c*e)/(b*x + a))^2/((b*x + a)*g^2) + 2*(d*
e*x + c*e)*(A*B - B^2)*log((d*e*x + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*e
*x + c*e)*(A^2 - 2*A*B + 2*B^2)/((b*x + a)*g^2))*(b*c/((b*c*e - a*d*e)*(b*
c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.187.9 Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) - \frac{A^2-2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2} + \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-B)\,4\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^2,x)
```

output
```
(log((e*(c + d*x))/(a + b*x))*((2*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^2)))
/(x/d + a/(b*d)) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(b^2*g^2*(x + a/b))
 - (B^2*d)/(b*g^2*(a*d - b*c))) - (A^2 + 2*B^2 - 2*A*B)/(b^2*g^2*x + a*b*g
^2) + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d - b*
c))*(A - B)*4i)/(b*g^2*(a*d - b*c))
```

3.188 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$

3.188.1 Optimal result

Integrand size = 32, antiderivative size = 296

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx &= -\frac{2ABd(c+dx)}{(bc-ad)^2g^3(a+bx)}+\frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} \\ &\quad -\frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2}-\frac{2B^2d(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)^2g^3(a+bx)} \\ &\quad +\frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^2g^3(a+bx)^2} \\ &\quad +\frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} \\ &\quad -\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}\end{aligned}$$

output
```
-2*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3
/(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-2*B^2*d*(d*x+c)*ln
(e*(d*x+c)/(b*x+a))/(-a*d+b*c)^2/g^3/(b*x+a)+1/2*b*B*(d*x+c)^2*(A+B*ln(e*(
d*x+c)/(b*x+a)))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)/(b
*x+a)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+
a)))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

3.188.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.50

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{-2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-2\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

output
```
(-2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(4*B*d*(a + b*x)*(b*c - a*
d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c - a*d)^
2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*
(a + b*x)^2*Log[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[(e*(c + d*x))/(a +
b*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])
 - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) + 4
*d^2*(a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 2*B*d
^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*
d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d^2*(a + b*x)^2*(
(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*b*g^3*(a + b*x)^2)
```

3.188.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{(ag+bgx)^3}\,dx$$
$$\Big\downarrow \; 2952$$

$$\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(d\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2-\frac{b(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{a+bx}\right)d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2(a+bx)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{a+bx}-\frac{2ABd(c+dx)}{a+bx}-\frac{2B^2d(c+dx)\log}{a+bx}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

output
```
((-2*A*B*d*(c + d*x))/(a + b*x) + (2*B^2*d*(c + d*x))/(a + b*x) - (b*B^2*(
c + d*x)^2)/(4*(a + b*x)^2) - (2*B^2*d*(c + d*x)*Log[(e*(c + d*x))/(a + b*
x)])/(a + b*x) + (b*B*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2
*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(a +
b*x) - (b*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(2*(a + b*x)
^2))/((b*c - a*d)^2*g^3)
```

3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

3.188. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.188.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.64

method	result
norman	$\frac{Bd(2Aad-2Bad-Bbc)x\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2 a\, d^2 x \ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(2A^2ad-2A^2bc-4ABad+2ABbc+4B^2ad-B^2bc\right)x}{2ag(ad-cb)}+\frac{Bc(4Aad-\cdots}{\cdots}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}-\frac{B^2b\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}+\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\cdots}{4}\cdots\right.}{g^3e^2(\cdots}$
parallelrisch	$-\frac{-4A^2a\,b^4c\,d^2-6AB\,a^2b^3d^3-2AB\,b^5c^2d-8B^2a\,b^4c\,d^2+8ABa\,b^4c\,d^2-2B^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5c^2d+6B^2x^2\ln\left(\frac{e(dx+c)}{bx+a}\right)b^5\cdots}{\cdots}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{b^3A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2A^2d\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{2b^3AB\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-\cdots}{b(bx-\cdots}\right)\cdots}{4}\right.}{(ad-cb)^3e^3g^3}\cdots\right.}{\cdots}$
default	$\frac{e(ad-cb)\left(-\frac{b^3A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2A^2d\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{2b^3AB\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-\cdots}{b(bx-\cdots}\right)\cdots}{4}\right.}{(ad-cb)^3e^3g^3}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
(B/g*d*(2*A*a*d-2*B*a*d-B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)/
(b*x+a))+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)/(b*x+a))^2
+1/2*(2*A^2*a*d-2*A^2*b*c-4*A*B*a*d+2*A*B*b*c+4*B^2*a*d-B^2*b*c)/a/g/(a*d-
b*c)*x+1/2*B*c*(4*A*a*d-2*A*b*c-4*B*a*d+B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(d*x+c)/(b*x+a))+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(d*x+c)/(b*x+a))^2+1/4*(2*A^2*a*d-2*A^2*b*c-6*A*B*a*d+2*A*B*b*c+7*
B^2*a*d-B^2*b*c)/a^2*b/g/(a*d-b*c)*x^2+1/2*b*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^
2*c^2)/g*x^2*ln(e*(d*x+c)/(b*x+a))^2+1/2*b*B/g*d^2*(2*A-3*B)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)*x^2*ln(e*(d*x+c)/(b*x+a)))/g^2/(b*x+a)^2
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{(2A^2-2AB+B^2)b^2c^2-4(A^2-2AB+2B^2)abcd+(2A^2-6AB+7B^2)a^2d^2-2(B^2b^2d^2x^2+2B$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="frica
s")
```

output
```
-1/4*((2*A^2 - 2*A*B + B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d + (2
*A^2 - 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2
*b^2*c^2 + 2*B^2*a*b*c*d)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((2*A*B - 3*B
^2)*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2
- (2*A*B - B^2)*b^2*c^2 + 4*(A*B - B^2)*a*b*c*d - 2*(B^2*b^2*c*d - 2*(A*B
- B^2)*a*b*d^2)*x)*log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d +
 a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x
+ (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.188.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(269) = 538$.

Time = 2.27 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.01

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$$

$$= \frac{Bd^2\cdot(2A-3B)\log\left(x + \frac{2ABad^3+2ABbcd^2-3B^2ad^3-3B^2bcd^2-\frac{Ba^3d^5\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Ba^2bcd^4\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^3\cdot(2A-3B)}{(ad-bc)^2}+\frac{Bb^3c^3\cdots}{(a\cdots}\cdots}{4ABbd^3-6B^2bd^3}\cdots\right.}{2bg^3(ad-bc)^2\cdots}$$

$$- \frac{Bd^2\cdot(2A-3B)\log\left(x + \frac{2ABad^3+2ABbcd^2-3B^2ad^3-3B^2bcd^2+\frac{Ba^3d^5\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Ba^2bcd^4\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Bab^2c^2d^3\cdot(2A-3B)}{(ad-bc)^2}-\frac{Bb^3\cdots}{\cdots}\cdots}{4ABbd^3-6B^2bd^3}\cdots\right.}{2bg^3(ad-bc)^2\cdots}$$

$$+ \frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+\cdots}$$

$$+ \frac{(-2ABad+2ABbc+3B^2ad-B^2bc+2B^2bdx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^3bdg^3-2a^2b^2cg^3+4a^2b^2dg^3x-4ab^3cg^3x+2ab^3dg^3x^2-2b^4cg^3x^2}$$

$$+ \frac{-2A^2ad+2A^2bc+6ABad-2ABbc-7B^2ad+B^2bc+x(4ABbd-6B^2bd)}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x(8a^2b^2dg^3-8ab^3cg^3)}$$

input

```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**3,x)
```

output
```
B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3
- 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*
d**4*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A - 3*B)/(a*d -
b*c)**2 + B*b**3*c**3*d**2*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 6*B
**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) - B*d**2*(2*A - 3*B)*log(x + (2*A*B
*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 - 3*B**2*b*c*d**2 + B*a**3*d**5*(
2*A - 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A - 3*B)/(a*d - b*c)**2 +
 3*B*a*b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A -
 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c
)**2) + (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*
log(e*(c + d*x)/(a + b*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a*
*3*b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2
*b**2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b
**4*c**2*g**3*x**2) + (-2*A*B*a*d + 2*A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*
B**2*b*d*x)*log(e*(c + d*x)/(a + b*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g*
*3 + 4*a**2*b**2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b
**4*c*g**3*x**2) + (-2*A**2*a*d + 2*A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 7*B
**2*a*d + B**2*b*c + x*(4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2
*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g
**3 - 8*a*b**3*c*g**3))
```

3.188.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(290) = 580$.

Time = 0.24 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.86

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\left(2\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,d^2\log(bx+a)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{}{(b^3c}\right.\right.$$

$$-\frac{1}{2}\,AB\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3} + \frac{}{(b^3c}\right.$$

$$-\frac{B^2\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)} - \frac{A^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="maxim
a")
```

output
```
-1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
 a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)*g^3))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + (b^2*c^2
- 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x
+ a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c
*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2
*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x +
 a^2*d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3
 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 +
 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - 1/2*A*B
*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^
2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*e/(b*x
 + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))
^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*
b^2*g^3*x + a^2*b*g^3)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.25

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\left(2\left(\frac{(dex+ce)^2B^2b}{(bceg^3-adeg^3)(bx+a)^2}-\frac{2\,(dex+ce)B^2d}{(bcg^3-adg^3)(bx+a)}\right)\log\left(\frac{dex+ce}{bx+a}\right)^2+2\left(\frac{(2\,ABb-B^2b)(de}{(bceg^3-adeg^3)(b}\right.\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="giac"
)
```

output
```
-1/4*(2*((d*e*x + c*e)^2*B^2*b/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 2*(
d*e*x + c*e)*B^2*d/((b*c*g^3 - a*d*g^3)*(b*x + a)))*log((d*e*x + c*e)/(b*x
 + a))^2 + 2*((2*A*B*b - B^2*b)*(d*e*x + c*e)^2/((b*c*e*g^3 - a*d*e*g^3)*(
b*x + a)^2) - 4*(A*B*d - B^2*d)*(d*e*x + c*e)/((b*c*g^3 - a*d*g^3)*(b*x +
a)))*log((d*e*x + c*e)/(b*x + a)) + (2*A^2*b - 2*A*B*b + B^2*b)*(d*e*x + c
*e)^2/((b*c*e*g^3 - a*d*e*g^3)*(b*x + a)^2) - 4*(A^2*d - 2*A*B*d + 2*B^2*d
)*(d*e*x + c*e)/((b*c*g^3 - a*d*g^3)*(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b*
c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.188.9 Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.71

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\left(\frac{B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$-\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)$$
$$-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d-B^2\,b\,c-6\,A\,B\,a\,d+2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{x\,(3\,B^2\,b\,d-2\,A\,B\,b\,d)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}$$
$$-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)(2\,A-3\,B)\,1\mathrm{i}}{(a\,d-b\,c)\,(3\,B^2\,d^2-2\,A\,B\,d^2)}\right)(2\,A-3\,B)\,1\mathrm{i}}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^3,x)
```

output
```
(log((e*(c + d*x))/(a + b*x))*((B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*
a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c^2
 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x))/(
a + b*x))^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*
(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d -
B^2*b*c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (x*(3*B^2*b*d - 2*A*B*b
*d))/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*d^2*a
tan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(
2*A - 3*B)*1i)/((a*d - b*c)*(3*B^2*d^2 - 2*A*B*d^2)))*(2*A - 3*B)*1i)/(b*g
^3*(a*d - b*c)^2)
```

3.189 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.189.1 Optimal result

Integrand size = 32, antiderivative size = 399

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx =& -\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2}\\ &-\frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}+\frac{B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}\\ &+\frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)}\\ &-\frac{bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3g^4(a+bx)^2}\\ &+\frac{2b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3}\\ &-\frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^3g^4}\\ &-\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3}\end{aligned}$$

output
```
-2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*
c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+1/3*B
^2*d^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+2*B*d^2*(d*x+c)*(A+B*ln(e*
(d*x+c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)-b*B*d*(d*x+c)^2*(A+B*ln(e*(d*x+
c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^2+2/9*b^2*B*(d*x+c)^3*(A+B*ln(e*(d*x
+c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-2/3*B*d^3*ln((d*x+c)/(b*x+a))*(A+
B*ln(e*(d*x+c)/(b*x+a)))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)/(b*x+a))
)^2/b/g^4/(b*x+a)^3
```

3.189.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$=\frac{-18\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(12A(bc-ad)^3-4B(bc-ad)^3-18Ad(bc-ad)^2(a+bx)+15Bd(bc-ad)^2(a+bx)+36Ad^2(bc-ad)(a+bx)\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

output
```
(-18*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(12*A*(b*c - a*d)^3 - 4*B
*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) + 15*B*d*(b*c - a*d)^2*(a
+ b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(-(b*c) + a*d)*(a + b
*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] - 66*B*d^3*(a + b*x)^3*Log[a + b
*x] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 - 36*A*d^3*(a + b*x)^3*Log[c + d
*x] + 66*B*d^3*(a + b*x)^3*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(d*(a +
 b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2
- 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 12*B*
(b*c - a*d)^3*Log[(e*(c + d*x))/(a + b*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x
)*Log[(e*(c + d*x))/(a + b*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(
c + d*x))/(a + b*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(c + d*x))
/(a + b*x)] - 36*B*d^3*(a + b*x)^3*Log[c + d*x]*Log[(e*(c + d*x))/(a + b*x
)] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 36*B*
d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^3)/(54
*b*g^4*(a + b*x)^3)
```

3.189.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2952, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2d\frac{c+dx}{a+bx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2756}$$

$$-\frac{\frac{2B\int \frac{(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{c+dx}d\frac{c+dx}{a+bx}}{3b}-\frac{\left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$-\frac{\frac{2B\left(-B\int\left(\frac{d^3(a+bx)\log\left(\frac{c+dx}{a+bx}\right)}{c+dx}-\frac{1}{6}b\left(18d^2-\frac{9b(c+dx)d}{a+bx}+\frac{2b^2(c+dx)^2}{(a+bx)^2}\right)\right)d\frac{c+dx}{a+bx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+\right.}{2(a+bx)^2}}{3b}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2B\left(-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2(a+bx)^2}+d^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-\frac{3bd^2(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+\right.}{a+bx}}{3b}}{g^4(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

3.189. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}dx$

output
```
-((-1/3*((d - (b*(c + d*x))/(a + b*x))^3*(A + B*Log[(e*(c + d*x))/(a + b*x
)])^2)/b + (2*B*(-(B*((-3*b*d^2*(c + d*x))/(a + b*x) + (3*b^2*d*(c + d*x)^
2)/(4*(a + b*x)^2) - (b^3*(c + d*x)^3)/(9*(a + b*x)^3) + (d^3*Log[(c + d*x
)/(a + b*x)]^2)/2)) - (3*b*d^2*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x
)]))/(a + b*x) + (3*b^2*d*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])
)/(2*(a + b*x)^2) - (b^3*(c + d*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))
/(3*(a + b*x)^3) + d^3*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x))/(
a + b*x)])))/(3*b))/((b*c - a*d)^3*g^4))
```

3.189.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.189.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(387) = 774$.

Time = 1.41 (sec) , antiderivative size = 836, normalized size of antiderivative = 2.10

method	result
parts	$-\frac{A^2}{3g^4(bx+a)^3 b}+\frac{B^2 b^2\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3 \ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{3}-\frac{2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^3 \ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{9}+\frac{2\left(\frac{de}{b}-\frac{e(ad-c}{b(bx+c}\right.}{27}\right.}{}$
norman	$\frac{B^2 a^2 d^3 x \ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{\left(a^3 d^3-3a^2 bc\, d^2+3a\, b^2 c^2 d-b^3 c^3\right) g}+\frac{B^2 ab\, d^3 x^2 \ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{\left(a^3 d^3-3a^2 bc\, d^2+3a\, b^2 c^2 d-b^3 c^3\right) g}-\frac{18A^2 a^2 b^2 d^2-36A^2 a\, b^3 cd+18A^2 b^4 c^2-66AB\, a^2 b^2 d^2+}{54g}$
parallelrisch	$-\frac{-66AB\, a^3 b^4 d^4+12AB\, b^7 c^3 d-108B^2 a^2 b^5 c\, d^3+27B^2 a\, b^6 c^2 d^2-54B^2 x \ln\left(\frac{e(dx+c)}{bx+a}\right)^2 a^2 b^5 d^4+108B^2 x \ln\left(\frac{e(dx+c)}{bx+a}\right) a^2}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A^2/g^4/(b*x+a)^3/b+B^2/g^4*b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*
c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2/9*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+2/27*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^3-2*d*e/b*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^2-1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))+1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+1/b^2*d^2*e^2*((d*e/b-e*(a*d-b*
c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2*(d*e/b-e*(a*d-b*c)/b/(b*
x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-2*e*(a*d-b*c)/b/(b*x+a)+2*d*e/b))+2*
B*A/g^4*b^2/e^3/(a*d-b*c)^3*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-
e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3-2*d*e/b*(1/2*(d
*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e
*(a*d-b*c)/b/(b*x+a))^2)+d^2*e^2/b^2*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e
/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.70

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2\,(9\,A^2-6\,AB+2\,B^2)b^3c^3-27\,(2\,A^2-2\,AB+B^2)ab^2c^2d+54\,(A^2-2\,AB+2\,B^2)a^2bcd^2-(18\,A^2-}{}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="frica
s")
```

output
```
-1/54*(2*(9*A^2 - 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 - 2*A*B + B^2)*a*b^2*
c^2*d + 54*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 - 66*A*B + 85*B^2)*
a^3*d^3 - 6*((6*A*B - 11*B^2)*b^3*c*d^2 - (6*A*B - 11*B^2)*a*b^2*d^3)*x^2
+ 18*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*
c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d*e*x + c*e)/(b*x + a))^
2 + 3*((6*A*B - 5*B^2)*b^3*c^2*d - 18*(2*A*B - 3*B^2)*a*b^2*c*d^2 + (30*A*
B - 49*B^2)*a^2*b*d^3)*x + 6*((6*A*B - 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B - B^
2)*b^3*c^3 - 9*(2*A*B - B^2)*a*b^2*c^2*d + 18*(A*B - B^2)*a^2*b*c*d^2 - 3*
(2*B^2*b^3*c*d^2 - 3*(2*A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3*(B^2*b^3*c^2*d - 6
*B^2*a*b^2*c*d^2 + 6*(A*B - B^2)*a^2*b*d^3)*x)*log((d*e*x + c*e)/(b*x + a)
))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*
(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a
^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

3.189.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(362) = 724$.

Time = 12.82 (sec) , antiderivative size = 1544, normalized size of antiderivative = 3.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**4,x)
```

output
```
B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 - 11*B**2*a*d**
4 - 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A - 11*B)/(a*d - b*c)**3 + 4*B*a**3*
b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A - 11*B
)/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/(a*d - b*c)**3 - B*b*
*4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 - 22*B**2*b*d**4)
)/(9*b*g**4*(a*d - b*c)**3) - B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 +
6*A*B*b*c*d**3 - 11*B**2*a*d**4 - 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A - 11
*B)/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 + 6*B*a
**2*b**2*c**2*d**5*(6*A - 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A
 - 11*B)/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(1
2*A*B*b*d**4 - 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c
*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B*
*2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)/(a + b*x))**2/(3*a**6*
d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2
*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3
*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x*
*2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c
**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 +
 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 +
12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 11*B**2*a**2*d**2 - 7*B**2*a*b*c*d +...
```

3.189.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. $2(387) = 774$.

Time = 0.29 (sec) , antiderivative size = 1420, normalized size of antiderivative = 3.56

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="maxim
a")
```

output
```
1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3
)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3)*g^4))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - (4*b^3*c^3 - 27*a*
b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2
- 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a
)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x
 + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*
x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*
d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3
+ 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(
a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^
4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g
^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a
^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3
*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 + 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^
2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a...
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.79

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{54}\left(18\left(\frac{(dex+ce)^3B^2b^2}{(b^2c^2e^2g^4-2\,abcde^2g^4+a^2d^2e^2g^4)(bx+a)^3}-\frac{3\,(dex+ce)^2B^2bd}{(b^2c^2eg^4-2\,abcdeg^4+a^2d^2eg^4)(bx+a)^2}+\right.\right.$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="giac"
)
```

output
```
-1/54*(18*((d*e*x + c*e)^3*B^2*b^2/((b^2*c^2*e^2*g^4 - 2*a*b*c*d*e^2*g^4 +
 a^2*d^2*e^2*g^4)*(b*x + a)^3) - 3*(d*e*x + c*e)^2*B^2*b*d/((b^2*c^2*e*g^4
 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)*(b*x + a)^2) + 3*(d*e*x + c*e)*B^2*d^2
/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e
)/(b*x + a))^2 + 6*(2*(3*A*B*b^2 - B^2*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*
g^4 - 2*a*b*c*d*e^2*g^4 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 9*(2*A*B*b*d - B
^2*b*d)*(d*e*x + c*e)^2/((b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)
*(b*x + a)^2) + 18*(A*B*d^2 - B^2*d^2)*(d*e*x + c*e)/((b^2*c^2*g^4 - 2*a*b
*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 2*(9*A^
2*b^2 - 6*A*B*b^2 + 2*B^2*b^2)*(d*e*x + c*e)^3/((b^2*c^2*e^2*g^4 - 2*a*b*c
*d*e^2*g^4 + a^2*d^2*e^2*g^4)*(b*x + a)^3) - 27*(2*A^2*b*d - 2*A*B*b*d + B
^2*b*d)*(d*e*x + c*e)^2/((b^2*c^2*e*g^4 - 2*a*b*c*d*e*g^4 + a^2*d^2*e*g^4)
*(b*x + a)^2) + 54*(A^2*d^2 - 2*A*B*d^2 + 2*B^2*d^2)*(d*e*x + c*e)/((b^2*c
^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(b*x + a)))*(b*c/((b*c*e - a*d*e)*(b
*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.189.9 Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.67

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2-66\,A\,B\,a^2\,d^2+42\,A\,B\,a\,b\,c\,d-12\,A\,B\,b^2\,c^2+85\,B^2\,a^2\,d^2-23\,B^2\,a\,b\,c\,d+4\,B^2\,b^2\,c^2}{6\,(a\,d-b\,c)}+\frac{x\,\left(-5\,c\,B^2\,b^2\,d+49\,a\,\cdots\right.}{\cdots}}{x\,\left(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4\right)-x^2\,\left(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4\right)+x^3\,\left(9\,b^5\,c\,g^4-9\,a\,b^4\cdots\right.}$$
$$-\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\left(\frac{B^2}{3\,b^2\,g^4\,\left(3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2\right)}-\frac{B^2\,d^3}{3\,b\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)$$
$$-\frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\left(\frac{2\,A\,B}{3\,b^2\,d\,g^4}-\frac{2\,B^2\,d^3\,\left(a\,\left(\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2}{6\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{3\,b\,d^2}\right)+\frac{3\,a^3\,d^3-6\,a^2\,b\,c\,d^2+4\,a\,b^2\,c^2\,d-b^3\,c^3}{3\,b\,d^4}\right)}{3\,b\,g^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{2\,B^2\,d^3\,x^2\,\left(\frac{b^2\,c-\cdots}{3\,\cdots}\right.}{3\,b\,g^4\,(a^3\,d^3-3\,a^2\,b\,\cdots}\right.}{\frac{3\,a^2\,x}{d}+\frac{a^3}{b\,d}+\frac{b^2\,x^3}{d}+\frac{3\,a\cdots}{\cdots}}$$
$$-\frac{B\,d^3\,\mathrm{atan}\left(\frac{B\,d^3\,\left(\frac{a^3\,b\,d^3\,g^4-a^2\,b^2\,c\,d^2\,g^4-a\,b^3\,c^2\,d\,g^4+b^4\,c^3\,g^4}{a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4}+2\,b\,d\,x\right)\,(6\,A-11\,B)\,\left(a^2\,b\,d^2\,g^4-2\,a\,b^2\,c\,d\,g^4+b^3\,c^2\,g^4\right)\,1{}\mathrm{i}}{b\,g^4\,(a\,d-b\,c)^3\,(11\,B^2\,d^3-6\,A\,B\,d^3)}\right)\,(6\,A-11\,\cdots}{9\,b\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^4,x)
```

output
```
((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 - 66*A*
B*a^2*d^2 - 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d + 42*A*B*a*b*
c*d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 30*A*B*a*b*d^2
 + 6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d - 6*A*B*b^2*d))/
(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*
g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^
4 - 9*a^4*b*d*g^4) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(3*b^2*g^4*(3*a^2
*x + a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x))/(a + b*x))*((2*A*
B)/(3*b^2*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d
^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
6*a^2*b*c*d^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) + (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c
))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
 - (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d
- b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d
 - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*
atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^
2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A ...
```

3.189. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.190 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.190.1 Optimal result

Integrand size = 32, antiderivative size = 498

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = {} & \frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2} \\
& + \frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4} \\
& - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{4b(bc-ad)^4g^5} - \frac{2Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^4g^5(a+bx)} \\
& + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2(bc-ad)^4g^5(a+bx)^2} \\
& - \frac{2b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} \\
& + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8(bc-ad)^4g^5(a+bx)^4} \\
& + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b(bc-ad)^4g^5} \\
& - \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4}
\end{aligned}
$$

output
```
2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b
*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/3
2*b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-1/4*B^2*d^4*ln((d*x+c)/(b*x
+a))^2/b/(-a*d+b*c)^4/g^5-2*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*
d+b*c)^4/g^5/(b*x+a)+3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a
*d+b*c)^4/g^5/(b*x+a)^2-2/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/
(-a*d+b*c)^4/g^5/(b*x+a)^3+1/8*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))
/(-a*d+b*c)^4/g^5/(b*x+a)^4+1/2*B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*x+c
)/(b*x+a)))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g^5/(b*
x+a)^4
```

3.190.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.34

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$$

$$= \frac{-72\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(36A(bc-ad)^4-9B(bc-ad)^4+28Bd(bc-ad)^3(a+bx)+48Ad(-bc+ad)^3(a+bx)+72Ad^2(bc-ad)^2(a+\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

output
```
(-72*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(36*A*(b*c - a*d)^4 - 9*B
*(b*c - a*d)^4 + 28*B*d*(b*c - a*d)^3*(a + b*x) + 48*A*d*(-(b*c) + a*d)^3*
(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 - 78*B*d^2*(b*c - a*d)^2*(a
 + b*x)^2 + 300*B*d^3*(b*c - a*d)*(a + b*x)^3 + 144*A*d^3*(-(b*c) + a*d)*(
a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] + 300*B*d^4*(a + b*x)^4*Lo
g[a + b*x] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 144*A*d^4*(a + b*x)^4*L
og[c + d*x] - 300*B*d^4*(a + b*x)^4*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*L
og[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 72*B*d^4*(a + b*x)^4*Log[c
 + d*x]^2 + 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*
d)] + 36*B*(b*c - a*d)^4*Log[(e*(c + d*x))/(a + b*x)] + 48*B*d*(-(b*c) + a
*d)^3*(a + b*x)*Log[(e*(c + d*x))/(a + b*x)] + 72*B*d^2*(b*c - a*d)^2*(a +
 b*x)^2*Log[(e*(c + d*x))/(a + b*x)] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^
3*Log[(e*(c + d*x))/(a + b*x)] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e
*(c + d*x))/(a + b*x)] + 144*B*d^4*(a + b*x)^4*Log[c + d*x]*Log[(e*(c + d*
x))/(a + b*x)] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c
 - a*d)^4)/(288*b*g^5*(a + b*x)^4)
```

3.190.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2952, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2d\frac{c+dx}{a+bx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2756}$$

$$\frac{\frac{B\int \frac{(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{c+dx}d\frac{c+dx}{a+bx}}{2b}-\frac{\left(d-\frac{b(c+dx)}{a+bx}\right)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2772}$$

$$B\left(-B\int\left(\frac{(c+dx)^3b^4}{4(a+bx)^3}-\frac{4d(c+dx)^2b^3}{3(a+bx)^2}+\frac{3d^2(c+dx)b^2}{a+bx}-4d^3b+\frac{d^4(a+bx)\log\left(\frac{c+dx}{a+bx}\right)}{c+dx}\right)d\frac{c+dx}{a+bx}+\frac{b^4(c+dx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4(a+bx)^4}-\frac{4b^3d(c+dx)^3\left(B\log\left(\frac{e(c+}{a+b}\right.\right.}{3(a+bx)^3}\right.$$

$$\overline{\qquad 2b \qquad}$$

$\downarrow$ 2009

$$B\left(\frac{b^4(c+dx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4(a+bx)^4}-\frac{4b^3d(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d^2(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{(a+bx)^2}+d^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)\right.$$

$$\overline{\qquad 2b \qquad}$$

g^5

input
```
Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

output
```
(-1/4*((d - (b*(c + d*x))/(a + b*x))^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]
)^2)/b + (B*(-(B*((-4*b*d^3*(c + d*x))/(a + b*x) + (3*b^2*d^2*(c + d*x)^2)
/(2*(a + b*x)^2) - (4*b^3*d*(c + d*x)^3)/(9*(a + b*x)^3) + (b^4*(c + d*x)^
4)/(16*(a + b*x)^4) + (d^4*Log[(c + d*x)/(a + b*x)]^2)/2)) - (4*b*d^3*(c +
 d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(a + b*x) + (3*b^2*d^2*(c + d*
x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(a + b*x)^2 - (4*b^3*d*(c + d*x
)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*(a + b*x)^3) + (b^4*(c + d*x)
^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(4*(a + b*x)^4) + d^4*Log[(c + d*
x)/(a + b*x)]*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(2*b))/((b*c - a*d)^4
*g^5)
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.190. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.190.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(480) = 960$.

Time = 2.38 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.23

method	result	size
parts	Expression too large to display	1112
derivativedivides	Expression too large to display	1422
default	Expression too large to display	1422
norman	Expression too large to display	1796
parallelrisch	Expression too large to display	2035
risch	Expression too large to display	2601

input
```
int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A^2/g^5/(b*x+a)^4/b-B^2/g^5*b^3/e^4/(a*d-b*c)^4*(1/4*(d*e/b-e*(a*d-b*
c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-1/8*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+1/32*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b
/(b*x+a))^2-2/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))+2/27*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3/b^2*d^2*e^2*(1/2*(d*e/b-e*(a
*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-1/2*(d*e/b-e*(a*d-b
*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+1/4*(d*e/b-e*(a*d-b*c)/b/
(b*x+a))^2)-1/b^3*d^3*e^3*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b
*c)/b/(b*x+a))^2-2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b
*x+a))-2*e*(a*d-b*c)/b/(b*x+a)+2*d*e/b))-2*B*A/g^5*b^3/e^4/(a*d-b*c)^4*(1/
4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/16*(d*
e/b-e*(a*d-b*c)/b/(b*x+a))^4-3*d*e/b*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*
ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)+3*d^2
*e^2/b^2*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+
a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)-d^3*e^3/b^3*((d*e/b-e*(a*d-b*c)/b
/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

3.190.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. 2(480) = 960.

Time = 0.29 (sec) , antiderivative size = 1045, normalized size of antiderivative = 2.10

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{9\,(8\,A^2-4\,AB+B^2)b^4c^4-32\,(9\,A^2-6\,AB+2\,B^2)ab^3c^3d+216\,(2\,A^2-2\,AB+B^2)a^2b^2c^2d^2-288\,(\cdots}{\cdots}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="frica
s")
```

output
```
-1/288*(9*(8*A^2 - 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 - 6*A*B + 2*B^2)*a*b^3
*c^3*d + 216*(2*A^2 - 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 - 2*A*B + 2*
B^2)*a^3*b*c*d^3 + (72*A^2 - 300*A*B + 415*B^2)*a^4*d^4 + 12*((12*A*B - 25
*B^2)*b^4*c*d^3 - (12*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((12*A*B - 13*B^2)*
b^4*c^2*d^2 - 16*(6*A*B - 11*B^2)*a*b^3*c*d^3 + (84*A*B - 163*B^2)*a^2*b^2
*d^4)*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*
x^2 + 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*
c^2*d^2 + 4*B^2*a^3*b*c*d^3)*log((d*e*x + c*e)/(b*x + a))^2 + 4*((12*A*B -
 7*B^2)*b^4*c^3*d - 12*(6*A*B - 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B - 3*B^2)
*a^2*b^2*c*d^3 - (156*A*B - 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B - 25*B^2)*
b^4*d^4*x^4 - 3*(4*A*B - B^2)*b^4*c^4 + 16*(3*A*B - B^2)*a*b^3*c^3*d - 36*
(2*A*B - B^2)*a^2*b^2*c^2*d^2 + 48*(A*B - B^2)*a^3*b*c*d^3 - 4*(3*B^2*b^4*
c*d^3 - 2*(6*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b
^3*c*d^3 + 6*(2*A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a
*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 12*(A*B - B^2)*a^3*b*d^4)*x)*log((d*
e*x + c*e)/(b*x + a)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a
^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a
^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 -
 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*
x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*...
```

3.190.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

3.190.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(480) = 960$.

Time = 0.36 (sec) , antiderivative size = 2122, normalized size of antiderivative = 4.26

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="maxim
a")
```

output
```
-1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2
+ 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*
d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b
^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b
^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5
*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a
^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7
*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*
log(d*e*x/(b*x + a) + c*e/(b*x + a)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a
^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^
4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(
b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^
4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2
+ 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*
d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^
4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25
*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x +
 25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4...
```

3.190.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(480) = 960$.

Time = 0.56 (sec) , antiderivative size = 1194, normalized size of antiderivative = 2.40

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="giac"
)
```

output
```
-1/288*(72*((d*e*x + c*e)^4*B^2*b^3/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e^3*
g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 4*(d*e*x + c
*e)^3*B^2*b^2*d/((b^3*c^3*e^2*g^5 - 3*a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*
e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 6*(d*e*x + c*e)^2*B^2*b*d^2/((b^
3*c^3*e*g^5 - 3*a*b^2*c^2*d*e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(
b*x + a)^2) - 4*(d*e*x + c*e)*B^2*d^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 +
3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a))^
2 + 12*(3*(4*A*B*b^3 - B^2*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^
2*c^2*d*e^3*g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) -
16*(3*A*B*b^2*d - B^2*b^2*d)*(d*e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*a*b^2*c
^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3) + 36*
(2*A*B*b*d^2 - B^2*b*d^2)*(d*e*x + c*e)^2/((b^3*c^3*e*g^5 - 3*a*b^2*c^2*d*
e*g^5 + 3*a^2*b*c*d^2*e*g^5 - a^3*d^3*e*g^5)*(b*x + a)^2) - 48*(A*B*d^3 -
B^2*d^3)*(d*e*x + c*e)/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g
^5 - a^3*d^3*g^5)*(b*x + a)))*log((d*e*x + c*e)/(b*x + a)) + 9*(8*A^2*b^3
- 4*A*B*b^3 + B^2*b^3)*(d*e*x + c*e)^4/((b^3*c^3*e^3*g^5 - 3*a*b^2*c^2*d*e
^3*g^5 + 3*a^2*b*c*d^2*e^3*g^5 - a^3*d^3*e^3*g^5)*(b*x + a)^4) - 32*(9*A^2
*b^2*d - 6*A*B*b^2*d + 2*B^2*b^2*d)*(d*e*x + c*e)^3/((b^3*c^3*e^2*g^5 - 3*
a*b^2*c^2*d*e^2*g^5 + 3*a^2*b*c*d^2*e^2*g^5 - a^3*d^3*e^2*g^5)*(b*x + a)^3
) + 216*(2*A^2*b*d^2 - 2*A*B*b*d^2 + B^2*b*d^2)*(d*e*x + c*e)^2/((b^3*c...
```

3.190.9 Mupad [B] (verification not implemented)

Time = 7.81 (sec) , antiderivative size = 1880, normalized size of antiderivative = 3.78

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^5,x)
```

output
```
(log((e*(c + d*x))/(a + b*x))*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*
b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*
a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*
b^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B)/
(2*b^2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*
b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6
*d^3) + (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d -
b*c))/(2*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(4*d^3)))/(2*b*g^5*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) -
 (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^
3*c - a*b^2*d)/(4*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2
- 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^
3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c
^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2
*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) + (6*a^3*d^3 - b^3*c^
3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(4*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4
/(b*d) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - log((e*(c +...
```

3.191 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

3.191.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

3.191.2 Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

3.191.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
$Aborted
```

3.191.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.191.4 Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

3.191.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d*e*x + c*e)/(b*x +
 a)) + A), x)
```

3.191.6 Sympy [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.97

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=g^2\left(\int \frac{a^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{b^2x^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{2abx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Int
egral(b**2*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integra
l(2*a*b*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

3.191.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

3.191.8 Giac [N/A]

Not integrable

Time = 14.77 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

3.191.9 Mupad [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

3.192 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

3.192.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

3.192.2 Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

3.192.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{ag+bgx}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

output
```
$Aborted
```

3.192.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.192.4 Maple [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

3.192.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)/(B*log((d*e*x + c*e)/(b*x + a)) + A), x)
```

3.192.6 Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=g\left(\int \frac{a}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
g*(Integral(a/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(
b*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

3.192.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

3.192.8 Giac [N/A]

Not integrable

Time = 11.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)} \, dx = \int \frac{bgx + ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right) + A} \, dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

3.192.9 Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag + bgx}{A + B\log\left(\frac{e(c+dx)}{a+bx}\right)} \, dx = \int \frac{a\,g + b\,g\,x}{A + B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)} \, dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

3.193 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

3.193.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

3.193.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

3.193.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
$Aborted
```

3.193.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.193.4 Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

3.193.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas"
)
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d*e*x + c*e)/(b*x + a
))), x)
```

3.193.6 Sympy [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bbx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b*x
*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g
```

3.193.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima"
)
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

3.193.8 Giac [N/A]

Not integrable

Time = 9.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

3.193.9 Mupad [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.194 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

3.194.1 Optimal result

Integrand size = 32, antiderivative size = 53

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=-\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)eg^2}$$

output
```
-Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)/e/exp(A/B)/g^2
```

3.194.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B(-bc+ad)eg^2}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(B*(-(b*c) + a*d)*e*E^(A
/B)*g^2)
```

3.194.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2952, 2736, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2736}$$

$$-\frac{\int \frac{e(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}d\log\left(\frac{e(c+dx)}{a+bx}\right)}{eg^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Beg^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
-(ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B]/(B*(b*c - a*d)*e*E
^(A/B)*g^2))
```

3.194.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.194.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{\mathrm{e}^{-\frac{A}{B}} \operatorname{Ei}_1\left(-\ln\left(\frac{e(dx+c)}{bx+a}\right)-\frac{A}{B}\right)}{g^2 e(ad-cb)B}$	55
derivativedivides	$-\frac{\mathrm{e}^{-\frac{A}{B}} \operatorname{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2 B}$	69
default	$-\frac{\mathrm{e}^{-\frac{A}{B}} \operatorname{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2 B}$	69

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/g^2/e/(a*d-b*c)/B*exp(-A/B)*Ei(1,-ln(e*(d*x+c)/(b*x+a))-A/B)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = -\frac{e^{\left(-\frac{A}{B}\right)}\operatorname{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{(Bbc-Bad)eg^2}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="frica
s")
```

output
```
-e^(-A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a))/((B*b*c - B*a*d)*e
*g^2)
```

3.194.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+2Babx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) +
d*e*x/(a + b*x)) + 2*B*a*b*x*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2
*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**2
```

3.194.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

3.194.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.195 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

3.195.1 Optimal result

Integrand size = 32, antiderivative size = 109

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\frac{de^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)^2eg^3}-\frac{be^{-\frac{2A}{B}}\,\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B(bc-ad)^2e^2g^3}$$

output
```
d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3-b*Ei(2*(
A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3
```

3.195.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$
$$=\frac{e^{-\frac{2A}{B}}\left(dee^{A/B}\,\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-b\,\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)\right)}{B(bc-ad)^2e^2g^3}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
(d*e*E^(A/B)*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]] - b*ExpInte
gralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B*(b*c - a*d)^2*e^2*E
^((2*A)/B)*g^3)
```

3.195.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(\frac{d}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}-\frac{b(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\right)d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Be}-\frac{be^{-\frac{2A}{B}}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{Be^2}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

output
```
((d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B*e*E^(A/B)) -
 (b*ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B*e^2*E^((
2*A)/B)))/((b*c - a*d)^2*g^3)
```

3.195.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.195.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$-\frac{-\frac{b\,e^{-\frac{2A}{B}}\,\mathrm{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{B}+\frac{de\,e^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{B}}{e^2(ad-cb)^2g^3}$	126
default	$-\frac{-\frac{b\,e^{-\frac{2A}{B}}\,\mathrm{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{B}+\frac{de\,e^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{B}}{e^2(ad-cb)^2g^3}$	126
risch	$\frac{b\,e^{-\frac{2A}{B}}\,\mathrm{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{g^3(ad-cb)^2e^2B}-\frac{d\,e^{-\frac{A}{B}}\,\mathrm{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{g^3(ad-cb)^2eB}$	139

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/e^2/(a*d-b*c)^2/g^3*(-b/B*exp(-2*A/B)*Ei(1,-2*ln(d*e/b-e*(a*d-b*c)/b/(b
*x+a))-2*A/B)+d*e/B*exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\left(dee^{\frac{A}{B}}\,\text{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)-b\,\text{log\_integral}\left(\frac{(d^2e^2x^2+2\,cde^2x+c^2e^2)e^{\left(\frac{2\,A}{B}\right)}}{b^2x^2+2\,abx+a^2}\right)\right)e^{\left(-\frac{2\,A}{B}\right)}}{(Bb^2c^2-2\,Babcd+Ba^2d^2)e^2g^3}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="frica
s")
```

output
```
(d*e*e^(A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)) - b*log_integra
l((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*e^(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2
)))*e^(-2*A/B)/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*e^2*g^3)
```

3.195.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Ba^2bx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Bab^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^3x^3\log\left(\frac{ce}{a+bx}+\frac{dex}{a+b}\right.}}{g^3}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a**2*b*x*log(c*e/(a + b*x) + d
*e*x/(a + b*x)) + 3*B*a*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B
*b**3*x**3*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**3
```

3.195.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

3.195.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.196 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

3.196.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \mathrm{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.196.2 Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

3.196.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
$Aborted
```

3.196.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.196.4 Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.196.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d*e*x + c*e)/(b*x
 + a))^2 + 2*A*B*log((d*e*x + c*e)/(b*x + a)) + A^2), x)
```

3.196.6 Sympy [N/A]

Not integrable

Time = 11.03 (sec) , antiderivative size = 400, normalized size of antiderivative = 12.50

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$= \frac{-a^3cg^2 - a^3dg^2x - 3a^2bcg^2x - 3a^2bdg^2x^2 - 3ab^2cg^2x^2 - 3ab^2dg^2x^3 - b^3cg^2x^3 - b^3dg^2x^4}{ABad - ABbc + (B^2ad - B^2bc)\log\left(\frac{e(c+dx)}{a+bx}\right)}$$
$$+ \frac{g^2\left(\int \frac{a^3d}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx + \int \frac{3a^2bc}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx + \int \frac{3b^3cx^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx + \int \frac{4b^3dx^3}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\right.}{B\,(ad-bc)}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
(-a**3*c*g**2 - a**3*d*g**2*x - 3*a**2*b*c*g**2*x - 3*a**2*b*d*g**2*x**2 -
 3*a*b**2*c*g**2*x**2 - 3*a*b**2*d*g**2*x**3 - b**3*c*g**2*x**3 - b**3*d*g
**2*x**4)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(c + d*x)/(a +
b*x))) + g**2*(Integral(a**3*d/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))
), x) + Integral(3*a**2*b*c/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
x) + Integral(3*b**3*c*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
x) + Integral(4*b**3*d*x**3/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
x) + Integral(6*a*b**2*c*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x
) + Integral(9*a*b**2*d*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
 x) + Integral(6*a**2*b*d*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
x))/(B*(a*d - b*c))
```

3.196.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 309, normalized size of antiderivative = 9.66

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
-(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c
*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*
log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e
) - a*d*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*
g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)
/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d
)*A*B - (b*c*log(e) - a*d*log(e))*B^2), x)
```

3.196.8 Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

3.196.9 Mupad [N/A]

Not integrable

Time = 5.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.197 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$

3.197.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx = \text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}, x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.197.2 Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx = \int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

3.197.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

output
```
$Aborted
```

3.197.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.197.4 Maple [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.197.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas"
)
```

output
```
integral((b*g*x + a*g)/(B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*
e*x + c*e)/(b*x + a)) + A^2), x)
```

3.197.6 Sympy [N/A]

Not integrable

Time = 6.98 (sec) , antiderivative size = 275, normalized size of antiderivative = 9.17

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\frac{-a^2cg-a^2dgx-2abcgx-2abdgx^2-b^2cgx^2-b^2dgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{2b^2cx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{3b^2dx^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\right.}{B(ad-bc)}$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
(-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b
**2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(c + d*x)/(
a + b*x))) + g*(Integral(a**2*d/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x)
)), x) + Integral(2*a*b*c/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)
 + Integral(2*b**2*c*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) +
Integral(3*b**2*d*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) +
Integral(4*a*b*d*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))/(B*(a
*d - b*c))
```

3.197.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 7.70

$$\int \frac{ag + bgx}{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{bgx + ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima"
)
```

output
```
-(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g
)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c -
 a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*
a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x +
a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e) - a*d*lo
g(e))*B^2), x)
```

3.197.8 Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag + bgx}{\left(A + B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{bgx + ag}{\left(B\log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

3.197.9 Mupad [N/A]

Not integrable

Time = 6.86 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{a\,g+b\,g\,x}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.198 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

3.198.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.198.2 Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]
```

3.198.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
$Aborted
```

3.198.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.198.4 Maple [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

3.198.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.59

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="frica
s")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/
(b*x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a))), x)
```

3.198.6 Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.88

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\frac{-c-dx}{ABadg-ABbcg+(B^2adg-B^2bcg)\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{d\int \frac{1}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg\,(ad-bc)}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
(-c - d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(c + d
*x)/(a + b*x))) + d*Integral(1/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))
), x)/(B*g*(a*d - b*c))
```

3.198.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.19

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxim
a")
```

output
```
d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(
d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2), x) -
(d*x + c)/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x
+ c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

3.198.8 Giac [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2), x)
```

3.198.9 Mupad [N/A]

Not integrable

Time = 8.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.199 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

3.199.1 Optimal result

Integrand size = 32, antiderivative size = 104

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)eg^2} + \frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

output
```
-Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)/e/exp(A/B)/g^2+(d*x+c)/B
/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

3.199.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e} - \frac{B(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}}{B^2(-bc+ad)g^2}$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
(ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(e*E^(A/B)) - (B*(c + d
*x))/((a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(B^2*(-(b*c) + a*d)
*g^2)
```

3.199.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2952, 2734, 2736, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2734}$$

$$-\frac{\frac{\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2736}$$

$$-\frac{\frac{\int \frac{e(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}d\log\left(\frac{e(c+dx)}{a+bx}\right)}{Be}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{\frac{e^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2e}-\frac{c+dx}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
-((ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B]/(B^2*e*E^(A/B)) -
 (c + d*x)/(B*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/((b*c - a*d
)*g^2))
```

3.199.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.199.4 Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{dx+c}{(ad-cb)B(bx+a)g^2\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}-\frac{e^{-\frac{A}{B}}\operatorname{Ei}_1\left(-\ln\left(\frac{e(dx+c)}{bx+a}\right)-\frac{A}{B}\right)}{g^2B^2e(ad-cb)}$	107
derivativedivides	$\frac{-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-e^{-\frac{A}{B}}\operatorname{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2B^2}$	138
default	$\frac{-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-e^{-\frac{A}{B}}\operatorname{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2B^2}$	138

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d-b*c)/B*(d*x+c)/(b*x+a)/g^2/(A+B*ln(e*(d*x+c)/(b*x+a)))-1/g^2/B^2/e
/(a*d-b*c)*exp(-A/B)*Ei(1,-ln(e*(d*x+c)/(b*x+a))-A/B)
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(103) = 206.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\frac{(Bdex+Bce)e^{\frac{A}{B}}-\left(Abx+Aa+(Bbx+Ba)\log\left(\frac{dex+ce}{bx+a}\right)\right)\operatorname{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{((B^3b^2c-B^3abd)eg^2x+(B^3abc-B^3a^2d)eg^2)e^{\frac{A}{B}}\log\left(\frac{dex+ce}{bx+a}\right)+((AB^2b^2c-AB^2abd)eg^2x+(AB^2abc-}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fri
cas")
```

output
```
((B*d*e*x + B*c*e)*e^(A/B) - (A*b*x + A*a + (B*b*x + B*a)*log((d*e*x + c*e
)/(b*x + a)))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)))/(((B^3*b^2*c
- B^3*a*b*d)*e*g^2*x + (B^3*a*b*c - B^3*a^2*d)*e*g^2)*e^(A/B)*log((d*e*x +
 c*e)/(b*x + a)) + ((A*B^2*b^2*c - A*B^2*a*b*d)*e*g^2*x + (A*B^2*a*b*c - A
*B^2*a^2*d)*e*g^2)*e^(A/B))
```

3.199.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$= \frac{-c-dx}{ABa^2dg^2 - ABabcg^2 + ABabdg^2x - ABb^2cg^2x + (B^2a^2dg^2 - B^2abcg^2 + B^2abdg^2x - B^2b^2cg^2x)\log\left(\frac{e(c}{a}\right.}$$
$$+ \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+2Babx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
(-c - d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2
*c*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2
*b**2*c*g**2*x)*log(e*(c + d*x)/(a + b*x))) + Integral(1/(A*a**2 + 2*A*a*b
*x + A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 2*B*a*b*x
*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2*x**2*log(c*e/(a + b*x) + d*
e*x/(a + b*x))), x)/(B*g**2)
```

3.199.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="max
ima")
```

output
```
(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*log
(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*lo
g(e))*B^2)*x - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^
2)*log(b*x + a) + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)
*B^2)*log(d*x + c)) + integrate(1/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 + (B^2
*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g^2)*
x - (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) + (B^2*
b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{dex+ce}{\left(B^2g^2\log\left(\frac{dex+ce}{bx+a}\right)+ABg^2\right)(bx+a)}-\frac{\mathrm{Ei}\left(\frac{A}{B}+\log\left(\frac{de}{b}\right.\right.}{B^2}\right.$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="gia
c")
```

output
```
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((
d*e*x + c*e)/((B^2*g^2*log((d*e*x + c*e)/(b*x + a)) + A*B*g^2)*(b*x + a))
- Ei(A/B + log((d*e*x + c*e)/(b*x + a)))*e^(-A/B)/(B^2*g^2))
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.200 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

3.200.1 Optimal result

Integrand size = 32, antiderivative size = 159

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{de^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)^2eg^3} - \frac{2be^{-\frac{2A}{B}}\,\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2(bc-ad)^2e^2g^3} + \frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

output
```
d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^3-2*b*Ei
(2*(A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3+(d*x
+c)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

3.200.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \frac{\frac{de^{-\frac{A}{B}}\operatorname{ExpIntegralEi}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e}-\frac{2be^{-\frac{2A}{B}}\operatorname{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{e^2}+\frac{B(bc-ad)(c+dx)}{(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}}{B^2(bc-ad)^2g^3}$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output
```
((d*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]])/(e*E^(A/B)) - (2*b*
ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(e^2*E^((2*A)/B
)) + (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a +
b*x)])))/(B^2*(b*c - a*d)^2*g^3)
```

3.200.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2952, 2757, 2736, 2609, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}\,dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2757}$$

$$\frac{-\frac{d\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}+\frac{2\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2736$$

$$\frac{-\frac{d\int \frac{e(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}d\log\left(\frac{e(c+dx)}{a+bx}\right)}{Be}+\frac{2\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2609$$

$$\frac{\frac{2\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2e}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2767$$

$$\frac{\frac{2\int \left(\frac{d}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}-\frac{b(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\right)d\frac{c+dx}{a+bx}}{B}-\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2e}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{B(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2e}+\frac{2\left(\frac{de^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Be}-\frac{be^{-\frac{2A}{B}}\text{ExpIntegralEi}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{Be^2}\right)}{B}}{g^3(bc-ad)^2}$$

input

```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

output

```
(-((d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B^2*e*E^(A/B
))) + (2*((d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B*e*E
^(A/B)) - (b*ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B
*e^2*E^((2*A)/B))))/B - ((c + d*x)*(d - (b*(c + d*x))/(a + b*x)))/(B*(a +
b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/((b*c - a*d)^2*g^3)
```

3.200. $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

3.200.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2757
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
 1), x], x] + Simp[d*(q/(b*n*(p + 1)))   Int[(d + e*x)^(q - 1)*(a + b*Log[c
*x^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && Gt
Q[q, 0]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.200. $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

3.200.4 Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.69

method	result
derivativedivides	$-\frac{\frac{b\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-2\,\mathrm{e}^{-\frac{2A}{B}}\operatorname{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)\right)}{B^2}-\frac{de\left(-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\operatorname{Ei}_1\left(-\ln\left(\cdots\right.\right.\right.}{B^2}}{e^2(ad-cb)^2g^3}$
default	$-\frac{\frac{b\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-2\,\mathrm{e}^{-\frac{2A}{B}}\operatorname{Ei}_1\left(-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)\right)}{B^2}-\frac{de\left(-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\operatorname{Ei}_1\left(-\ln\left(\cdots\right.\right.\right.}{B^2}}{e^2(ad-cb)^2g^3}$
risch	$-\frac{dx+c}{(ad-cb)B(bx+a)^2g^3\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}-\frac{a\,d^2\mathrm{e}^{-\frac{A}{B}}\operatorname{Ei}_1\left(-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e\,g^3B^2(ad-cb)^3}+\frac{bcd\,\mathrm{e}^{-\frac{A}{B}}\operatorname{Ei}_1\left(-\ln\left(\frac{de}{b}-\cdots\right.\right.}{e\,g^3B^2(ad-c\cdots}$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/e^2/(a*d-b*c)^2/g^3*(b/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(ln(d*e/b-
e*(a*d-b*c)/b/(b*x+a))+A/B)-2*exp(-2*A/B)*Ei(1,-2*ln(d*e/b-e*(a*d-b*c)/b/(
b*x+a))-2*A/B))-d*e/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(ln(d*e/b-e*(a*d-b
*c)/b/(b*x+a))+A/B)-exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B)))
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(157) = 314.

Time = 0.25 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.67

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$=\frac{((Bbcd-Bad^2)e^2x+(Bbc^2-Bacd)e^2)e^{\left(\frac{2A}{B}\right)}-2\left(Ab^3x^2+2\,Aab^2x+Aa^2b+(Bb^3x^2+2\,Ba\cdots\right.}{((B^3b^4c^2-2\,B^3ab^3cd+B^3a^2b^2d^2)e^2g^3x^2+2\,(B^3ab^3c^2-2\,B^3a^2b^2cd+B^3a^3bd^2)e^2g^3x+(B^3a^2b^2c^2-2\,B\cdots}$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fri
cas")
```

output
```
(((B*b*c*d - B*a*d^2)*e^2*x + (B*b*c^2 - B*a*c*d)*e^2)*e^(2*A/B) - 2*(A*b^
3*x^2 + 2*A*a*b^2*x + A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log((d
*e*x + c*e)/(b*x + a)))*log_integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)
*e^(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2)) + ((B*b^2*d*e*x^2 + 2*B*a*b*d*e*x +
B*a^2*d*e)*e^(A/B)*log((d*e*x + c*e)/(b*x + a)) + (A*b^2*d*e*x^2 + 2*A*a*b
*d*e*x + A*a^2*d*e)*e^(A/B))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a))
)/(((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*e^2*g^3*x^2 + 2*(B^3
*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*e^2*g^3*x + (B^3*a^2*b^2*c
^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*e^2*g^3)*e^(2*A/B)*log((d*e*x + c*e)/(
b*x + a)) + ((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*e^2*g
^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*e^2*g
^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*e^2*g^3)*e^
(2*A/B))
```

3.200.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

output
```
Timed out
```

3.200.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="max
ima")
```

output
```
(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*g^3
*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^2*
d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3
*log(e) - a^2*b*d*g^3*log(e))*B^2)*x - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2
+ 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log
(b*x + a) + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*
g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate(-(b*
d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e) - a
*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3
*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (
a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 -
a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x - ((
b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2
 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*
log(b*x + a) + ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b
^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 -
 a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.83

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$
$$= \left(\frac{de\mathrm{Ei}\left(\frac{A}{B}+\log\left(\frac{dex+ce}{bx+a}\right)\right)e^{\left(-\frac{A}{B}\right)}}{B^2bceg^3-B^2adeg^3} - \frac{2\,b\mathrm{Ei}\left(\frac{2\,A}{B}+2\,\log\left(\frac{dex+ce}{bx+a}\right)\right)e^{\left(-\frac{2\,A}{B}\right)}}{B^2bceg^3-B^2adeg^3} - \frac{\frac{(dex+ce)de}{bx+a}}{B^2bceg^3\log\left(\frac{dex+ce}{bx+a}\right)-B^2adeg^3}\right.$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="gia
c")
```

output
```
(d*e*Ei(A/B + log((d*e*x + c*e)/(b*x + a)))*e^(-A/B)/(B^2*b*c*e*g^3 - B^2*
a*d*e*g^3) - 2*b*Ei(2*A/B + 2*log((d*e*x + c*e)/(b*x + a)))*e^(-2*A/B)/(B^
2*b*c*e*g^3 - B^2*a*d*e*g^3) - ((d*e*x + c*e)*d*e/(b*x + a) - (d*e*x + c*e
)^2*b/(b*x + a)^2)/(B^2*b*c*e*g^3*log((d*e*x + c*e)/(b*x + a)) - B^2*a*d*e
*g^3*log((d*e*x + c*e)/(b*x + a)) + A*B*b*c*e*g^3 - A*B*a*d*e*g^3))*(b*c/(
(b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.201 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

3.201.1 Optimal result

Integrand size = 32, antiderivative size = 182

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx \\
&= -\frac{2B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3} \\
&\quad -\frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2}+\frac{B(bc-ad)g^4(a+bx)^4}{10bd} \\
&\quad +\frac{2B(bc-ad)^5g^4\log(c+dx)}{5bd^5}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}
\end{aligned}$$

output
```
-2/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-2/15*
B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+2/5
*B*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)^2/
(b*x+a)^2))/b
```

3.201.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx$$
$$=\frac{g^4\left(-\frac{B(-bc+ad)(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx))}{6d^5}+(a+bx)^5\left(A\right.\right.}{5b}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(g^4*(-1/6*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*
(b*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2])))/(5*b)
```

3.201.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{2B(bc-ad)\int \frac{g^5(a+bx)^4}{c+dx}dx}{5bg}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2Bg^4(bc-ad)\int \frac{(a+bx)^4}{c+dx}dx}{5b}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{49}$$

$$\frac{2Bg^4(bc-ad)\int\left(\frac{(ad-bc)^4}{d^4(c+dx)}-\frac{b(bc-ad)^3}{d^4}+\frac{b(a+bx)^3}{d}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(bc-ad)^2(a+bx)}{d^3}\right)dx}{5b}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}+\frac{2Bg^4(bc-ad)\left(\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(2*B*(b*c - a*d)*g^4*(-((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x
)^2)/(2*d^3) - ((b*c - a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b
*c - a*d)^4*Log[c + d*x])/d^5))/(5*b) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c
+ d*x)^2)/(a + b*x)^2]))/(5*b)
```

3.201.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.201.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.63

method	result
derivativedivides	$-\frac{-\frac{g^4A(bx+a)^5}{5}+g^4B\left(-\frac{(bx+a)^5\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2ad}{5}+\frac{2cb}{5}\right)\left(\frac{\left(-a^4d^4+4a^3bc\,d^3-6a^2b^2c^2d^2+4a\,b^3c^3d-b^4\cdots\right.}{d^5}\cdots\right.\right.}{\cdots}$
default	$-\frac{-\frac{g^4A(bx+a)^5}{5}+g^4B\left(-\frac{(bx+a)^5\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2ad}{5}+\frac{2cb}{5}\right)\left(\frac{\left(-a^4d^4+4a^3bc\,d^3-6a^2b^2c^2d^2+4a\,b^3c^3d-b^4\cdots\right.}{d^5}\cdots\right.\right.}{\cdots}$
parts	$\frac{g^4A(bx+a)^5}{5b}-\frac{g^4B\left(-\frac{(bx+a)^5\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{5}-\left(-\frac{2ad}{5}+\frac{2cb}{5}\right)\left(\frac{\left(-a^4d^4+4a^3bc\,d^3-6a^2b^2c^2d^2+4a\,b^3c^3d-b^4c\cdots\right.}{d^5}\cdots\right.\right.}{\cdots}$
risch	$\frac{4g^4b^2B\ln(dx+c)a^2c^3}{d^3}-\frac{2g^4b^3B\ln(dx+c)a\,c^4}{d^4}-\frac{g^4b^3Ba\,x^4}{10}-\frac{8g^4B\,a^4x}{5}-\frac{2g^4B\ln(dx+c)a^5}{5b}-\frac{g^4b^3Ba\,c^2x^2}{d^2}+\cdots$
parallelrisch	$\frac{120Bx\,a^3b^2c\,d^4g^4-120Bx\,a^2b^3c^2d^3g^4+60Bxa\,b^4c^3d^2g^4+60B\ln(bx+a)a^4bc\,d^4g^4-120B\ln(bx+a)a^3b^2c^2d^3g^4+120B\ln\cdots}{\cdots}$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-1/5*g^4*A*(b*x+a)^5+g^4*B*(-1/5*(b*x+a)^5*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(-2/5*a*d+2/5*c*b)*(1/d^5*(-a^4*d^4+4*a^3*b*c*d^3-6*a^2*b^2
*c^2*d^2+4*a*b^3*c^3*d-b^4*c^4)*ln(1/(b*x+a))+1/4/d*(b*x+a)^4-1/3*(-a*d+b*
c)/d^2*(b*x+a)^3-(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/d^4*(b*x+a
)-1/2*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/d^3*(b*x+a)^2+1/d^5*(a*d-b*c)^4*ln(a*d/
(b*x+a)-b*c/(b*x+a)-d))))
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(170) = 340$.

Time = 0.30 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.51

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=\frac{6\,Ab^5d^5g^4x^5-12\,Ba^5d^5g^4\log(bx+a)+3\,(Bb^5cd^4+(10\,A-B)ab^4d^5)g^4x^4-4\,(Bb^5c^2d^3-5\,Bab^4cd^4-\cdots}{\cdots}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
1/30*(6*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^4
 + (10*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - (1
5*A - 4*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 + 2*(5*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 6*(2*B*b^5*c^4*d
- 10*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 - (5*A -
8*B)*a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3
*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5
*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3
*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e
)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^5)
```

3.201.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(163) = 326$.

Time = 4.02 (sec) , antiderivative size = 998, normalized size of antiderivative = 5.48

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx=\frac{Ab^4g^4x^5}{5}$$
$$-\frac{2Ba^5g^4\log\left(x+\frac{\frac{2Ba^6d^5g^4}{b}+10Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-10Ba^2b^3c^4dg^4+2Bab^4c^5g^4}{2Ba^5d^5g^4+10Ba^4bcd^4g^4-20Ba^3b^2c^2d^3g^4+20Ba^2b^3c^3d^2g^4-10Bab^4c^4dg^4+2Bb^5c^5g^4}\right)}{5b}$$
$$+\frac{2Bcg^4\cdot(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4)\log\left(x+\frac{12Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-\cdots}{\cdots}\right.}{5\cdots}$$
$$+x^4\left(Aab^3g^4-\frac{Bab^3g^4}{10}+\frac{Bb^4cg^4}{10d}\right)+x^3\cdot\left(2Aa^2b^2g^4-\frac{8Ba^2b^2g^4}{15}+\frac{2Bab^3cg^4}{3d}-\frac{2Bb^4c^2g^4}{15d^2}\right)$$
$$+x^2\cdot\left(2Aa^3bg^4-\frac{6Ba^3bg^4}{5}+\frac{2Ba^2b^2cg^4}{d}-\frac{Bab^3c^2g^4}{d^2}+\frac{Bb^4c^3g^4}{5d^3}\right)$$
$$+x\left(Aa^4g^4-\frac{8Ba^4g^4}{5}+\frac{4Ba^3bcg^4}{d}-\frac{4Ba^2b^2c^2g^4}{d^2}+\frac{2Bab^3c^3g^4}{d^3}-\frac{2Bb^4c^4g^4}{5d^4}\right)$$
$$+\left(Ba^4g^4x+2Ba^3bg^4x^2+2Ba^2b^2g^4x^3+Bab^3g^4x^4+\frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
A*b**4*g**4*x**5/5 - 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a*
*5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**
4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4
 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**
3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) +
2*B*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*
b**3*c**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2
*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 +
2*B*a*b**4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a
**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**
4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**
4*c**4)/d)/(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*
c**2*d**3*g**4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 +
 2*B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 - B*a*b**3*g**4/10 +
B*b**4*c*g**4/(10*d)) + x**3*(2*A*a**2*b**2*g**4 - 8*B*a**2*b**2*g**4/15 +
 2*B*a*b**3*c*g**4/(3*d) - 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*
b*g**4 - 6*B*a**3*b*g**4/5 + 2*B*a**2*b**2*c*g**4/d - B*a*b**3*c**2*g**4/d
**2 + B*b**4*c**3*g**4/(5*d**3)) + x*(A*a**4*g**4 - 8*B*a**4*g**4/5 + 4*B*
a**3*b*c*g**4/d - 4*B*a**2*b**2*c**2*g**4/d**2 + 2*B*a*b**3*c**3*g**4/d**3
 - 2*B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2...
```

3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(170) = 340$.

Time = 0.24 (sec) , antiderivative size = 882, normalized size of antiderivative = 4.85

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx$$
$$= \frac{1}{5}Ab^4g^4x^5 + Aab^3g^4x^4 + 2Aa^2b^2g^4x^3 + 2Aa^3bg^4x^2$$
$$+ \left(x\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right) - \frac{2a\log(bx+a)}{b} + \frac{2c\log(dx-\right.$$
$$+2\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right) + \frac{2a^2\log(bx+a)}{b^2} - \frac{2c^2\log(}{d}\right.$$
$$+2\left(x^3\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right) - \frac{2a^3\log(bx+a)}{b^3} + \frac{2c^3\log(}{d}\right.$$
$$+\frac{1}{3}\left(3x^4\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right) + \frac{6a^4\log(bx+a)}{b^4} - \frac{6c^4\log}{}\right.$$
$$+\frac{1}{30}\left(6x^5\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right) - \frac{12a^5\log(bx+a)}{b^5} + \frac{12c}{}\right.$$
$$+ Aa^4g^4x$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*
x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*
a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c
*log(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2
) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))
 + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d)
)*B*a^3*b*g^4 + 2*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x
/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(
b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(d^2*e*x^2/(b^2
*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x
^2 + 2*a*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(d^2*e*x^2/(b^2*x^2
 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 +
 2*a*b*x + a^2)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*
(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c
^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A
*a^4*g^4*x
```

3.201.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(170) = 340$.

Time = 63.43 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.68

$$
\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx \\
&= \frac{1}{5}Ab^4g^4x^5 - \frac{2\,Ba^5g^4\log(bx+a)}{5\,b} + \frac{(Bb^4cg^4+10\,Aab^3dg^4-Bab^3dg^4)x^4}{10\,d} \\
&\quad - \frac{2\,(Bb^4c^2g^4-5\,Bab^3cdg^4-15\,Aa^2b^2d^2g^4+4\,Ba^2b^2d^2g^4)x^3}{15\,d^2} \\
&\quad + \frac{1}{5}\left(Bb^4g^4x^5+5\,Bab^3g^4x^4+10\,Ba^2b^2g^4x^3+10\,Ba^3bg^4x^2+5\,Ba^4g^4x\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right) \\
&\quad + \frac{(Bb^4c^3g^4-5\,Bab^3c^2dg^4+10\,Ba^2b^2cd^2g^4+10\,Aa^3bd^3g^4-6\,Ba^3bd^3g^4)x^2}{5\,d^3} \\
&\quad - \frac{(2\,Bb^4c^4g^4-10\,Bab^3c^3dg^4+20\,Ba^2b^2c^2d^2g^4-20\,Ba^3bcd^3g^4-5\,Aa^4d^4g^4+8\,Ba^4d^4g^4)x}{5\,d^4} \\
&\quad + \frac{2\,(Bb^4c^5g^4-5\,Bab^3c^4dg^4+10\,Ba^2b^2c^3d^2g^4-10\,Ba^3bc^2d^3g^4+5\,Ba^4cd^4g^4)\log(dx+c)}{5\,d^5}
\end{aligned}
$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
1/5*A*b^4*g^4*x^5 - 2/5*B*a^5*g^4*log(b*x + a)/b + 1/10*(B*b^4*c*g^4 + 10*
A*a*b^3*d*g^4 - B*a*b^3*d*g^4)*x^4/d - 2/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d
*g^4 - 15*A*a^2*b^2*d^2*g^4 + 4*B*a^2*b^2*d^2*g^4)*x^3/d^2 + 1/5*(B*b^4*g^
4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*
B*a^4*g^4*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)
) + 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 + 10
*A*a^3*b*d^3*g^4 - 6*B*a^3*b*d^3*g^4)*x^2/d^3 - 1/5*(2*B*b^4*c^4*g^4 - 10*
B*a*b^3*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 - 5*A*
a^4*d^4*g^4 + 8*B*a^4*d^4*g^4)*x/d^4 + 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c^4*
d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^4*
g^4)*log(d*x + c)/d^5
```

3.201. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

3.201.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 1024, normalized size of antiderivative = 5.63

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$

$$=x^2\left(\frac{(5\,a\,d+5\,b\,c)\,\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}\cdots\right.}{10\,b\,d}\right.$$

$$+\frac{a^2\,b\,g^4\,(5\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}$$

$$\left.-\frac{a\,c\,\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)}{2\,b\,d}\right)$$

$$-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{15\,b\,d}\right.$$

$$\left.-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{3\,d}+\frac{A\,a\,b^3\,c\,g^4}{3\,d}\right)$$

$$+x\left(\frac{a^3\,g^4\,(5\,A\,a\,d+10\,A\,b\,c-4\,B\,a\,d+4\,B\,b\,c)}{d}\right.$$

$$-\frac{(5\,a\,d+5\,b\,c)\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\cdots}{d}\right.}{5\,b\,d}\cdots\right.}{5\,b\,d\cdots}$$

$$\frac{a\,c\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right)}{b\,d}\cdots$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c)
)/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a
*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))
/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d - (a*c*(
(b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a
*d + 5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*
a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c)
)/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(3*d) +
(A*a*b^3*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c - 4*B*a*d + 4*B*b
*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*
c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d
+ 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d
 + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d
 + 2*B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/
(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3
*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c
- 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(c + d*x)^2)
/(a + b*x)^2)*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b
^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c -...
```

3.201. $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

3.202 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

3.202.1 Optimal result

Integrand size = 32, antiderivative size = 151

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{B(bc-ad)^3g^3x}{2d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2} + \frac{B(bc-ad)g^3(a+bx)^3}{6bd} - \frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4} + \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}$$

output
```
1/2*B*(-a*d+b*c)^3*g^3*x/d^3-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/6*B*
(-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g^
3*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b
```

3.202.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{g^3\left(\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{3d^4}+(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{4b}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*
x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4) + (a + b
*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(4*b)
```

3.202.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{g^4(a+bx)^3}{c+dx}dx}{2bg}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bg^3(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{2b}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{49}$$

$$\frac{Bg^3(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{2b}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{2b}$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(B*(b*c - a*d)*g^3*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2
*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(2*b) + (g^
3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(4*b)
```

3.202.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.202.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{-\frac{g^3A(bx+a)^4}{4}+g^3B\left(-\frac{(bx+a)^4\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{(ad-cb)^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^4}-\frac{(-a^2d^2+2abc\ldots}{\ldots}\right.\right.}{b}$
default	$-\frac{-\frac{g^3A(bx+a)^4}{4}+g^3B\left(-\frac{(bx+a)^4\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{(ad-cb)^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^4}-\frac{(-a^2d^2+2abc\ldots}{\ldots}\right.\right.}{b}$
parts	$\frac{Ag^3(bx+a)^4}{4b}-\frac{g^3B\left(-\frac{(bx+a)^4\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4}-\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(\frac{(ad-cb)^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^4}-\frac{(-a^2d^2+2abcd\ldots}{\ldots}\right.\right.}{b}$
risch	$\frac{g^3(bx+a)^4B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{4b}+\frac{g^3b^3Ax^4}{4}+g^3b^2Aax^3-\frac{g^3b^2Bax^3}{6}+\frac{g^3b^3Bcx^3}{6d}+\frac{3g^3bAa^2x^2}{2}-\frac{3g^3bBa^2\ldots}{4}$
parallelrisch	$\frac{24B\ln(bx+a)a^3bcd^3g^3+3Bx^4\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^4d^4g^3+12Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^3bd^4g^3+12B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^3bcd^3g^3-18\ldots}{\ldots}$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-1/4*g^3*A*(b*x+a)^4+g^3*B*(-1/4*(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(-1/2*a*d+1/2*c*b)*(1/d^4*(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b
*x+a)-d)-(-a^2*d^2+2*a*b*c*d-b^2*c^2)/d^3*(b*x+a)-1/2*(-a*d+b*c)/d^2*(b*x+
a)^2+1/3/d*(b*x+a)^3+1/d^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*
ln(1/(b*x+a)))))
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(141) = 282$.

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.27

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=\frac{3\,Ab^4d^4g^3x^4-6\,Ba^4d^4g^3\log(bx+a)+2\,(Bb^4cd^3+(6\,A-B)ab^3d^4)g^3x^3-3\,(Bb^4c^2d^2-4\,Bab^3cd^3-3\,(\ldots}{\ldots}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
1/12*(3*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3
+ (6*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(2
*A - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^
2*b^2*c*d^3 + (2*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*
d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4
*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4
*g^3*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b
*d^4)
```

3.202.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(131) = 262$.

Time = 2.19 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.68

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$

$$= \frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{2b}$$

$$+ \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)\log\left(x+\frac{5Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3-Bacg^3\cdot(2ad-bc)}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g}\right.}{2d^4}$$

$$+ x^3\left(Aab^2g^3-\frac{Bab^2g^3}{6}+\frac{Bb^3cg^3}{6d}\right)+x^2\cdot\left(\frac{3Aa^2bg^3}{2}-\frac{3Ba^2bg^3}{4}+\frac{Bab^2cg^3}{d}-\frac{Bb^3c^2g^3}{4d^2}\right)$$

$$+ x\left(Aa^3g^3-\frac{3Ba^3g^3}{2}+\frac{3Ba^2bcg^3}{d}-\frac{2Bab^2c^2g^3}{d^2}+\frac{Bb^3c^3g^3}{2d^3}\right)$$

$$+ \left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*
d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b*
*3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c
**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) + B*c*g*
*3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c
*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b
**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d
)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**
3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*
g**3 - B*a*b**2*g**3/6 + B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 -
3*B*a**2*b*g**3/4 + B*a*b**2*c*g**3/d - B*b**3*c**2*g**3/(4*d**2)) + x*(A*
a**3*g**3 - 3*B*a**3*g**3/2 + 3*B*a**2*b*c*g**3/d - 2*B*a*b**2*c**2*g**3/d
**2 + B*b**3*c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2
 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)**2/(a + b*x)**
2)
```

3.202.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(141) = 282$.

Time = 0.23 (sec) , antiderivative size = 645, normalized size of antiderivative = 4.27

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{1}{4}Ab^3g^3x^4+Aab^2g^3x^3+\frac{3}{2}Aa^2bg^3x^2$$
$$+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)-\frac{2a\log(bx+a)}{b}+\frac{2c\log(dx-$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)+\frac{2a^2\log(bx+a)}{b^2}-\frac{2c^2\log$$
$$+\left(x^3\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)-\frac{2a^3\log(bx+a)}{b^3}+\frac{2c^3\log(d$$
$$+\frac{1}{12}\left(3x^4\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)+\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log$$
$$+Aa^3g^3x$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d^2*e*x
^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e
/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a
^3*g^3 + 3/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2
*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x +
 a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x
^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x
+ a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3
*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b
^2*d^2))*B*a*b^2*g^3 + 1/12*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)
 + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))
+ 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*
d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*B*b^3*g^3 + A*a^3*g^3*x
```

3.202.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(141) = 282$.

Time = 10.54 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.37

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \frac{1}{4}Ab^3g^3x^4 - \frac{Ba^4g^3\log(bx+a)}{2b} + \frac{(Bb^3cg^3+6\,Aab^2dg^3-Bab^2dg^3)x^3}{6\,d} + \frac{1}{4}\left(Bb^3g^3x^4+4\,Bab^2g^3x^3+6\,Ba^2bg^3x^2+4\,Ba^3g^3x\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right) - \frac{(Bb^3c^2g^3-4\,Bab^2cdg^3-6\,Aa^2bd^2g^3+3\,Ba^2bd^2g^3)x^2}{4\,d^2} + \frac{(Bb^3c^3g^3-4\,Bab^2c^2dg^3+6\,Ba^2bcd^2g^3+2\,Aa^3d^3g^3-3\,Ba^3d^3g^3)x}{2\,d^3} - \frac{(Bb^3c^4g^3-4\,Bab^2c^3dg^3+6\,Ba^2bc^2d^2g^3-4\,Ba^3cd^3g^3)\log(-dx-c)}{2\,d^4}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
1/4*A*b^3*g^3*x^4 - 1/2*B*a^4*g^3*log(b*x + a)/b + 1/6*(B*b^3*c*g^3 + 6*A*
a*b^2*d*g^3 - B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*B*a*b^2*g^3*x^
3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)
/(b^2*x^2 + 2*a*b*x + a^2)) - 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3 - 6*A
*a^2*b*d^2*g^3 + 3*B*a^2*b*d^2*g^3)*x^2/d^2 + 1/2*(B*b^3*c^3*g^3 - 4*B*a*b
^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 + 2*A*a^3*d^3*g^3 - 3*B*a^3*d^3*g^3)*x/
d^3 - 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4
*B*a^3*c*d^3*g^3)*log(-d*x - c)/d^4
```

3.202. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

3.202.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.75

$$
\begin{aligned}
&\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx \\
&= \ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\,\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right) \\
&\quad - x^2\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{4\,b\,d}\right. \\
&\qquad\left. -\frac{a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right) \\
&\quad + x\left(\frac{(2\,a\,d+2\,b\,c)\,\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)\,(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\right.}{2\,b\,d}\right. \\
&\qquad + \frac{a^2\,g^3\,(4\,A\,a\,d+6\,A\,b\,c-3\,B\,a\,d+3\,B\,b\,c)}{d} \\
&\qquad\left. -\frac{a\,c\,\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)}{b\,d}\right) \\
&\quad + x^3\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{6\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{6\,d}\right) \\
&\quad - \frac{\ln(c+d\,x)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{2\,d^4} \\
&\quad + \frac{A\,b^3\,g^3\,x^4}{4}-\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{2\,b}
\end{aligned}
$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
log((e*(c + d*x)^2)/(a + b*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a
^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c -
B*a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c)
)/(4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g
^3)/(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d +
B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d
) - (2*a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d)
)/(2*b*d) + (a^2*g^3*(4*A*a*d + 6*A*b*c - 3*B*a*d + 3*B*b*c))/d - (a*c*((b
^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*
b*c))/(2*d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/
(6*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) - (log(c + d*x)*(B*b^3*c^4*g^3
- 4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4
) + (A*b^3*g^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(2*b)
```

3.202. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

3.203 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

3.203.1 Optimal result

Integrand size = 32, antiderivative size = 120

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = -\frac{2B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{3bd}+\frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}$$

output
```
-2/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+2/3*B*(-a
*d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a
)^2))/b
```

3.203.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{g^2\left(\frac{B(bc-ad)(d(a^2d+4abdx+b^2x(-2c+dx))+2(bc-ad)^2\log(c+dx))}{d^3}+(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{3b}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c
- a*d)^2*Log[c + d*x]))/d^3 + (a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2])))/(3*b)
```

3.203.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{2B(bc-ad)\int \frac{g^3(a+bx)^2}{c+dx}dx}{3bg}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2Bg^2(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{2Bg^2(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}+\frac{2Bg^2(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(2*B*(b*c - a*d)*g^2*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c
 - a*d)^2*Log[c + d*x])/d^3))/(3*b) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]))/(3*b)
```

3.203.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.203.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{-\frac{g^2A(bx+a)^3}{3}+g^2B\left(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2ad}{3}+\frac{2cb}{3}\right)\left(\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{1}{bx+a}\right)}{d^3}-\frac{(-ad+cb)\cdots}{d^2}\cdots\right)\right)}{b}$
default	$-\frac{-\frac{g^2A(bx+a)^3}{3}+g^2B\left(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2ad}{3}+\frac{2cb}{3}\right)\left(\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{1}{bx+a}\right)}{d^3}-\frac{(-ad+cb)\cdots}{d^2}\cdots\right)\right)}{b}$
parts	$\frac{Ag^2(bx+a)^3}{3b}-\frac{g^2B\left(-\frac{(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}-\left(-\frac{2ad}{3}+\frac{2cb}{3}\right)\left(\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{1}{bx+a}\right)}{d^3}-\frac{(-ad+cb)\cdots}{d^2}\cdots\right)\right)}{b}$
risch	$\frac{(bx+a)^3g^2B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{3b}+\frac{g^2b^2Ax^3}{3}+g^2bAax^2-\frac{g^2bBax^2}{3}+\frac{g^2b^2Bcx^2}{3d}+g^2Aa^2x-\frac{2g^2B\ln(dx+\cdots)}{3b}\cdots$
parallelrisch	$\frac{2Bx^3\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3d^3g^2+6Ax^2ab^2d^3g^2-2Bx^2ab^2d^3g^2+2Bx^2b^3cd^2g^2-12B\ln(bx+a)ab^2c^2dg^2-8Bxa^2bd^3g^2+\cdots}{\cdots}$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-1/3*g^2*A*(b*x+a)^3+g^2*B*(-1/3*(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(-2/3*a*d+2/3*c*b)*(1/d^3*(-a^2*d^2+2*a*b*c*d-b^2*c^2)*ln(1
/(b*x+a))-(-a*d+b*c)/d^2*(b*x+a)+1/2/d*(b*x+a)^2+1/d^3*(a*d-b*c)^2*ln(a*d/
(b*x+a)-b*c/(b*x+a)-d))))
```

3.203.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(112) = 224$.

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.04

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{Ab^3d^3g^2x^3-2\,Ba^3d^3g^2\log(bx+a)+(Bb^3cd^2+(3\,A-B)ab^2d^3)g^2x^2-(2\,Bb^3c^2d-6\,Bab^2cd^2-(3\,A-\cdots}{\cdots}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
1/3*(A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (3*
A - B)*a*b^2*d^3)*g^2*x^2 - (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 - (3*A - 4*B)
*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*
log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*
d^3)
```

3.203.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(107) = 214$.

Time = 1.47 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.31

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= \frac{Ab^2g^2x^3}{3} - \frac{2Ba^3g^2\log\left(x+\frac{\frac{2Ba^4d^3g^2}{b}+6Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right)}{3b}$$
$$+ \frac{2Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x+\frac{8Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2-2Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{2Bbc^2g^2\cdot(3a^2}{}}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right.}{3d^3}$$
$$+ x^2\left(Aabg^2-\frac{Babg^2}{3}+\frac{Bb^2cg^2}{3d}\right)+x\left(Aa^2g^2-\frac{4Ba^2g^2}{3}+\frac{2Babcg^2}{d}-\frac{2Bb^2c^2g^2}{3d^2}\right)$$
$$+ \left(Ba^2g^2x+Babg^2x^2+\frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
A*b**2*g**2*x**3/3 - 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**
3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d
**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3
*g**2))/(3*b) + 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (
8*B*a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B
*a*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*
d**2 - 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g
**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b
*g**2 - B*a*b*g**2/3 + B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 - 4*B*a**2*g*
*2/3 + 2*B*a*b*c*g**2/d - 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x +
B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3)*log(e*(c + d*x)**2/(a + b*x)**2)
```

3.203.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(112) = 224$.

Time = 0.21 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.63

$$\begin{aligned}\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx &= \frac{1}{3}Ab^2g^2x^3 + Aabg^2x^2\\ &+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)-\frac{2a\log(bx+a)}{b}+\frac{2c\log(dx-}{d}\right.\\ &+\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)+\frac{2a^2\log(bx+a)}{b^2}-\frac{2c^2\log(d}{d^2}\right.\\ &+\frac{1}{3}\left(x^3\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)-\frac{2a^3\log(bx+a)}{b^3}+\frac{2c^3\log}{c}\right.\\ &+Aa^2g^2x\end{aligned}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x +
a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^
2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d^2*e*
x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*
e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)
/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(d^2*e*x^2/(b^2*x^2
+ 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 +
2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2
*g^2*x
```

3.203.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.

Time = 1.80 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.02

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx$$
$$=\frac{1}{3}Ab^2g^2x^3-\frac{2Ba^3g^2\log(bx+a)}{3b}+\frac{(Bb^2cg^2+3Aabdg^2-Babdg^2)x^2}{3d}$$
$$+\frac{1}{3}\left(Bb^2g^2x^3+3Babg^2x^2+3Ba^2g^2x\right)\log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)$$
$$-\frac{(2Bb^2c^2g^2-6Babcdg^2-3Aa^2d^2g^2+4Ba^2d^2g^2)x}{3d^2}$$
$$+\frac{2(Bb^2c^3g^2-3Babc^2dg^2+3Ba^2cd^2g^2)\log(dx+c)}{3d^3}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
1/3*A*b^2*g^2*x^3 - 2/3*B*a^3*g^2*log(b*x + a)/b + 1/3*(B*b^2*c*g^2 + 3*A*
a*b*d*g^2 - B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*b*g^2*x^2 + 3*
B*a^2*g^2*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)
) - 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 - 3*A*a^2*d^2*g^2 + 4*B*a^2*d^2
*g^2)*x/d^2 + 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g^2)*
log(d*x + c)/d^3
```

3.203.9 Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.47

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$= x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)$$
$$+\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)$$
$$+\frac{\ln(c+d\,x)\,(6\,B\,a^2\,c\,d^2\,g^2-6\,B\,a\,b\,c^2\,d\,g^2+2\,B\,b^2\,c^3\,g^2)}{3\,d^3}$$
$$+\frac{A\,b^2\,g^2\,x^3}{3}-\frac{2\,B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
x^2*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/(6*d) - (A*b*g^2*(3*a
*d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B
*a*d + 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^
2*(3*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(
c + d*x)^2)/(a + b*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2)
 + (log(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2)
)/(3*d^3) + (A*b^2*g^2*x^3)/3 - (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

3.204 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

3.204.1 Optimal result

Integrand size = 30, antiderivative size = 78

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx=\frac{B(bc-ad)gx}{d}-\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}$$

output
```
B*(-a*d+b*c)*g*x/d-B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^2*(A+B*l
n(e*(d*x+c)^2/(b*x+a)^2))/b
```

3.204.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx=\frac{g\left(-\frac{2B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}+(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(g*((-2*B*(-(b*c) + a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a +
 b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(2*b)
```

3.204.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{g^2(a+bx)}{c+dx}dx}{bg}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bg(bc-ad)\int \frac{a+bx}{c+dx}dx}{b}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{Bg(bc-ad)\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{b}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}+\frac{Bg(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{b}$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
(B*(b*c - a*d)*g*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/b + (g*(a + b
*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b)
```

3.204.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.204.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.45

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2}+\frac{gbA\,x^2}{2}+gAax-\frac{B\,a^2g\ln(bx+a)}{b}+\frac{2gB\ln(-dx-c)ac}{d}-\frac{gbB\ln(-dx-c)c^2}{d^2}-$
derivativedivides	$-\frac{-\frac{gA(bx+a)^2}{2}+gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^2}+\frac{(-ad+cb)\ln\left(\frac{1}{bx+a}\right)}{d^2}\right.\right.}{b}$
default	$-\frac{-\frac{gA(bx+a)^2}{2}+gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^2}+\frac{(-ad+cb)\ln\left(\frac{1}{bx+a}\right)}{d^2}\right.\right.}{b}$
parts	$Ag\left(\frac{1}{2}b\,x^2+ax\right)-\frac{gB\left(-\frac{(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-(-ad+cb)\left(\frac{(ad-cb)\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{d^2}+\frac{(-ad+cb)}{d}\right.\right.}{b}$
parallelrisch	$B\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)ab\,d^2g+2Axab\,d^2g-2B\ln(bx+a)a^2d^2g+4B\ln(bx+a)abcdg-$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*B*x*(b*x+2*a)*ln(e*(d*x+c)^2/(b*x+a)^2)+1/2*g*b*A*x^2+g*A*a*x-B*a^2*
g/b*ln(b*x+a)+2*g/d*B*ln(-d*x-c)*a*c-g*b/d^2*B*ln(-d*x-c)*c^2-g*B*a*x+g*b/
d*B*c*x
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.91

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$

$$=\frac{Ab^2d^2gx^2-2\,Ba^2d^2g\log(bx+a)+2\,(Bb^2cd+(A-B)abd^2)gx-2\,(Bb^2c^2-2\,Babcd)g\log(dx+c)+(}{2\,bd^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*g*x^2 - 2*B*a^2*d^2*g*log(b*x + a) + 2*(B*b^2*c*d + (A - B)
*a*b*d^2)*g*x - 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*
x^2 + 2*B*a*b*d^2*g*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*
b*x + a^2)))/(b*d^2)
```

3.204.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(68) = 136$.

Time = 0.92 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$$
$$=\frac{Abgx^2}{2}-\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{b}$$
$$+\frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{d^2}$$
$$+x\left(Aag-Bag+\frac{Bbcg}{d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*
c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b + B*c*g*(2*a*d
- b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*
b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/d**2 + x*(A*a*g - B*a*g + B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(c
+ d*x)**2/(a + b*x)**2)
```

3.204.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.21 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.21

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{1}{2}Abgx^2$$
$$+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)-\frac{2a\log(bx+a)}{b}+\frac{2c\log(dx-}{d}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)+\frac{2a^2\log(bx+a)}{b^2}-\frac{2c^2\log}{c}\right.$$
$$+Aagx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxim
a")
```

output
```
1/2*A*b*g*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x +
a)/b + 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b
*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
 + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)
*x/(b*d))*B*b*g + A*a*g*x
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.69

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx$$
$$=\frac{1}{2}Abgx^2-\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}\left(Bbgx^2+2Bagx\right)\log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)$$
$$+\frac{(Bbcg+Aadg-Badg)x}{d}-\frac{(Bbc^2g-2Bacdg)\log(-dx-c)}{d^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac"
)
```

output
```
1/2*A*b*g*x^2 - B*a^2*g*log(b*x + a)/b + 1/2*(B*b*g*x^2 + 2*B*a*g*x)*log((
d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c*g + A*a
*d*g - B*a*d*g)*x/d - (B*b*c^2*g - 2*B*a*c*d*g)*log(-d*x - c)/d^2
```

3.204.9 Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.54

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx = x\left(\frac{g\,(2\,A\,a\,d+A\,b\,c-B\,a\,d+B\,b\,c)}{d} - \frac{A\,g\,(a\,d+b\,c)}{d}\right) + \ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right) + \frac{A\,b\,g\,x^2}{2} - \frac{B\,a^2\,g\,\ln(a+b\,x)}{b} + \frac{B\,c\,g\,\ln(c+d\,x)\,(2\,a\,d-b\,c)}{d^2}$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
x*((g*(2*A*a*d + A*b*c - B*a*d + B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e
*(c + d*x)^2)/(a + b*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 - (B*
a^2*g*log(a + b*x))/b + (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

3.205 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx$

3.205.1 Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg} - \frac{2B\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g-2*B*polylog(2
,1+(-a*d+b*c)/d/(b*x+a))/b/g
```

3.205.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx = \frac{\log(a+bx)\left(A+B\log(a+bx)-2B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-2B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bg}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

output
```
(Log[a + b*x]*(A + B*Log[a + b*x] - 2*B*Log[(b*(c + d*x))/(b*c - a*d)] + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)])/(b*g)
```

3.205.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{ag+bgx}dx$$

$$\Big\downarrow \text{2942}$$

$$-\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$-\frac{2B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2g}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2778}$$

$$\frac{2B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2005}$$

$$\frac{2B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}-\frac{2B\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

output
```
-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^
2]))/(b*g)) - (2*B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

3.205.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

3.205.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

method	result
derivativedivides	$-\frac{\frac{A\ln\left(\frac{1}{bx+a}\right)}{g}+\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}\right)\right)}{g}}{b}$
default	$-\frac{\frac{A\ln\left(\frac{1}{bx+a}\right)}{g}+\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}\right)\right)}{g}}{b}$
parts	$\frac{A\ln(bx+a)}{gb}-\frac{B\left(\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)}{ad-cb}\right.\right.}{gb}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{bg}+\frac{2B\operatorname{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)ad}{bg(ad-cb)}-\frac{2B\operatorname{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)c}{g(ad-cb)}+$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/g*A*ln(1/(b*x+a))+1/g*B*(ln(1/(b*x+a))*ln(e*(a*d/(b*x+a)-b*c/(b*x+
a)-d)^2/b^2)-(2*a*d-2*b*c)*(dilog(-(1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)+ln
(1/(b*x+a))*ln(-(1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c))))
```

3.205.5 Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="frica
s")
```

output
```
integral((B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))
 + A)/(b*g*x + a*g), x)
```

3.205.6 Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}{a+bx}\,dx}{g}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g),x)
```

output
```
(Integral(A/(a + b*x), x) + Integral(B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x
**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2))/(a + b*x), x))/g
```

3.205.7 Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx=\int \frac{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="maxim
a")
```

output
```
B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x*log(e) + b*c*log(
e) - 2*(2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c
*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

3.205.8 Giac [F]

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag + bgx}\,dx = \int \frac{B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A}{bgx + ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="giac"
)
```

output
```
integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)/(b*g*x + a*g), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag + bgx}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{a\,g + b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x), x)
```

3.206 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx$

3.206.1 Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A(c+dx)}{(bc-ad)g^2(a+bx)}+\frac{2B(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)}$$

output
```
-A*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+2*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-B*(d*
x+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)
```

3.206.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = \frac{2Bd(a+bx)\log(a+bx)-2Bd(a+bx)\log(c+dx)-(bc-ad)\left(A-2B+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2(a+bx)}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

output
```
(2*B*d*(a + b*x)*Log[a + b*x] - 2*B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)
*(A - 2*B + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2*(a + b
*x))
```

3.206.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2952, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{(ag+bgx)^2} \, dx$$

$$\Big\downarrow \; 2952$$

$$-\frac{\int \left(A+B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{A(c+dx)}{a+bx}+\frac{B(c+dx)\log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx}-\frac{2B(c+dx)}{a+bx}}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

output
```
-(((A*(c + d*x))/(a + b*x) - (2*B*(c + d*x))/(a + b*x) + (B*(c + d*x)*Log[
(e*(c + d*x)^2)/(a + b*x)^2])/(a + b*x))/((b*c - a*d)*g^2))
```

3.206.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.206.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

method	result
norman	$\frac{\frac{(A-2B)x}{ga}+\frac{Bc\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}+\frac{Bdx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{2Aa\,b^2d^2-2A\,b^3cd-4Ba\,b^2d^2+4B\,b^3cd-2Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3d^2-2B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3cd}{2g^2(bx+a)b^3d(ad-cb)}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{b\,g^2(bx+a)}-\frac{-2B\ln(-dx-c)bdx+2B\ln(bx+a)bdx-2B\ln(-dx-c)ad+2B\ln(bx+a)ad+Aad-Abc-2Bad+}{g^2(bx+a)b(ad-cb)}$
derivativedivides	$-\frac{\frac{A}{g^2(bx+a)}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2ad-2cb)\left(\frac{1}{(bx+a)(ad-cb)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^2}\right)\right)}{g^2}}{b}$
default	$-\frac{\frac{A}{g^2(bx+a)}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2ad-2cb)\left(\frac{1}{(bx+a)(ad-cb)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^2}\right)\right)}{g^2}}{b}$
parts	$-\frac{A}{g^2(bx+a)b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{bx+a}-(2ad-2cb)\left(\frac{1}{(bx+a)(ad-cb)}+\frac{d\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^2}\right)\right)}{g^2b}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

output
```
((A-2*B)/g/a*x+B*c/g/(a*d-b*c)*ln(e*(d*x+c)^2/(b*x+a)^2)+1/g*B*d/(a*d-b*c)
*x*ln(e*(d*x+c)^2/(b*x+a)^2))/g/(b*x+a)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{(A-2\,B)bc-(A-2\,B)ad+(Bbdx+Bbc)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="fri
cas")
```

output
```
-((A - 2*B)*b*c - (A - 2*B)*a*d + (B*b*d*x + B*b*c)*log((d^2*e*x^2 + 2*c*d
*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^
2*c - a^2*b*d)*g^2)
```

3.206.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(83) = 166$.

Time = 0.70 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.48

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x} + \frac{2Bd\log\left(x+\frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\,(ad-bc)} - \frac{2Bd\log\left(x+\frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\,(ad-bc)} + \frac{-A+2B}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**2,x)
```

output
```
-B*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) + 2*B*d*log(x
 + (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 -
 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c
)) - 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c)
 + 2*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*
g**2*(a*d - b*c)) + (-A + 2*B)/(a*b*g**2 + b**2*g**2*x)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -B\left(\frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^2g^2x+abg^2}-\frac{2}{b^2g^2x+abg^2}-\frac{2\,d\log\left(bx+a\right)}{(b^2c-abd)g^2}+\frac{2\,d\log\left(dx+\right.}{(b^2c-abd)}\right. - \frac{A}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="max
ima")
```

output
```
-B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x
 + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) - 2/(b^2*
g^2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(d*x +
c)/((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\left(2\left(b^2cg^2-abdg^2\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right|\right)}{b^4c^2g^4-2\,ab^3cdg^4+a^2b^2d^2g^4}-\frac{1}{(b^2cg^2-abdg^2)(bgx+ag)bg}\right)+\frac{\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)}{(bgx+ag)bg}\right)B - \frac{A}{(bgx+ag)bg}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="gia
c")
```

output
```
-(2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x
+ a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c
*g^2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) + log((d*x + c)^2*e/(b*x + a)^2)/((b
*g*x + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A-2\,B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)} + \frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^2,x)
```

output
```
(B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(b*g^2*(a*d - b*c)) -
(B*log((e*(c + d*x)^2)/(a + b*x)^2))/(b^2*g^2*(x + a/b)) - (A - 2*B)/(b^2*
g^2*x + a*b*g^2)
```

3.207 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx$

3.207.1 Optimal result

Integrand size = 32, antiderivative size = 139

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx=\frac{B}{2bg^3(a+bx)^2}-\frac{Bd}{b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}$$

output
```
1/2*B/b/g^3/(b*x+a)^2-B*d/b/(-a*d+b*c)/g^3/(b*x+a)-B*d^2*ln(b*x+a)/b/(-a*d
+b*c)^2/g^3+B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(d*x+c)^2/(b
*x+a)^2))/b/g^3/(b*x+a)^2
```

3.207.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx=-\frac{2Bd^2(a+bx)^2\log(a+bx)-2Bd^2(a+bx)^2\log(c+dx)+(bc-ad)\left(Abc-bBc-aAd+3aBd+2bB\right.}{2b(bc-ad)^2g^3(a+bx)^2}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

output
```
-1/2*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x]
+ (b*c - a*d)*(A*b*c - b*B*c - a*A*d + 3*a*B*d + 2*b*B*d*x + B*(b*c - a*d)
*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

3.207.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{2948}$$

$$-\frac{B(bc-ad)\int \frac{1}{g^2(a+bx)^3(c+dx)}dx}{bg}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^3(c+dx)}dx}{bg^3}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{54}$$

$$-\frac{B(bc-ad)\int \left(-\frac{d^3}{(bc-ad)^3(c+dx)}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)(a+bx)^3}\right)dx}{bg^3}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}-\frac{B(bc-ad)\left(\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}\right)}{bg^3}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

output
```
-((B*(b*c - a*d)*(-1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a +
 b*x)) + (d^2*Log[a + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)
^3))/(b*g^3)) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*b*g^3*(a + b*x
)^2)
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.207.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{\frac{A}{2g^3(bx+a)^2}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(ad-cb)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(ad-cb)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^3}\right)\right)}{g^3}}{b}$
default	$-\frac{\frac{A}{2g^3(bx+a)^2}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(ad-cb)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(ad-cb)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^3}\right)\right)}{g^3}}{b}$
parts	$-\frac{A}{2g^3(bx+a)^2b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2(bx+a)^2}-(ad-cb)\left(\frac{\frac{ad}{2(bx+a)^2}-\frac{bc}{2(bx+a)^2}+\frac{d}{bx+a}}{(ad-cb)^2}+\frac{d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-cb)^3}\right)\right)}{g^3b}$
norman	$\frac{\frac{Bdx}{g(ad-cb)}+\frac{Ba\,d^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{(a^2d^2-2abcd+b^2c^2)g}-\frac{Aabd-A\,b^2c-3Babd+B\,b^2c}{2g\,b^2(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2g(a^2d^2-2abcd+b^2c^2)}+\frac{B\,d^2b\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2(a^2d^2-2abcd+b^2c^2)g}}{g^2(bx+a)^2}$
parallelrisch	$-\frac{-2Bxa\,b^4d^3+2Bx\,b^5c\,d^2+A\,a^2b^3d^3+A\,b^5c^2d-3B\,a^2b^3d^3-B\,b^5c^2d-2Aa\,b^4c\,d^2+4Ba\,b^4c\,d^2-2B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a\,b^4c\cdots}{2g^3(bx+a)^2(a^2d^2-2abcd+b^2c^2)b^4d\cdots}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2b\,g^3(bx+a)^2}-\frac{2B\ln(bx+a)b^2d^2x^2-2B\ln(-dx-c)b^2d^2x^2+4B\ln(bx+a)ab\,d^2x-4B\ln(-dx-c)ab\,d^2x+2B\,a^2\cdots}{2(a^2d^2-2\cdots}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/2/g^3*A/(b*x+a)^2+1/g^3*B*(1/2/(b*x+a)^2*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(a*d-b*c)*(1/(a*d-b*c)^2*(1/2*a*d/(b*x+a)^2-1/2*b*c/(b*x+a)
^2+d/(b*x+a))+d^2/(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b*x+a)-d))))
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.73

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{(A-B)b^2c^2-2\,(A-2\,B)abcd+(A-3\,B)a^2d^2+2\,(Bb^2cd-Babd^2)x-(Bb^2d^2x^2+2\,Babd^2x-Bb^2\cdots}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3\cdots}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="fri
cas")
```

output
```
-1/2*((A - B)*b^2*c^2 - 2*(A - 2*B)*a*b*c*d + (A - 3*B)*a^2*d^2 + 2*(B*b^2
*c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b
*c*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.207.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(122) = 244$.

Time = 1.14 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.01

$$\begin{aligned}
&\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2} \\
&\quad + \frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\left(ad-bc\right)^2} \\
&\quad - \frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\left(ad-bc\right)^2} \\
&\quad + \frac{-Aad+Abc+3Bad-Bbc+2Bbdx}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot\left(2ab^3dg^3-2b^4cg^3\right)+x\left(4a^2b^2dg^3-4ab^3cg^3\right)}
\end{aligned}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**3,x)
```

output
```
-B*log(e*(c + d*x)**2/(a + b*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b
**3*g**3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*
c*d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B
*b**3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d -
b*c)**2) - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/
(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c
**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2
) + (-A*a*d + A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a*
*2*b**2*c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d
*g**3 - 4*a*b**3*c*g**3))
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(135) = 270$.

Time = 0.21 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.20

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag + bgx)^3}\,dx =$$

$$-\frac{1}{2}B\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \right.}{b^3g^3x^2 + 2\,ab^2g^3x + a^2b}\right.$$

$$-\frac{A}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="max
ima")
```

output
```
-1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + log(d^2*e*x^2/(b^2*x^2 + 2
*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a
*b*x + a^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a
)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a
^2*b*g^3)
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}+\frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3} - \frac{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)} - \frac{2\,Bbdx+Abc-Bbc-Aad+3\,Bad}{2\,(b^4cg^3x^2-ab^3dg^3x^2+2\,ab^3cg^3x-2\,a^2b^2dg^3x+a^2b^2cg^3-a^3bdg^3)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="gia
c")
```

output
```
-B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) + B*d^
2*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log
((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))/(b^3*g^3*x^2 +
 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(2*B*b*d*x + A*b*c - B*b*c - A*a*d + 3*B
*a*d)/(b^4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3
*x + a^2*b^2*c*g^3 - a^3*b*d*g^3)
```

3.207.9 Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.48

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = \frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2} - \frac{B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)} - \frac{\frac{A\,a\,d-A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,\left(a\,d-b\,c\right)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^3,x)
```

output
```
(2*B*d^2*atanh((b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*
d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x)^2)/(a + b*x
)^2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((A*a*d - A*b*c - 3*B*a*d + B*
b*c)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2
*a*b^2*g^3*x)
```

3.207. $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx$

3.208 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$

3.208.1 Optimal result

Integrand size = 32, antiderivative size = 177

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2B}{9bg^4(a+bx)^3} - \frac{Bd}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)} + \frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4} - \frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}$$

output
```
2/9*B/b/g^4/(b*x+a)^3-1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+2/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)+2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-2/3*B*d^3*ln(d*
x+c)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g^4/(b*x+a)
^3
```

3.208.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{\frac{B\left(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}-3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
```

output
```
((B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a
+ b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]
))/(b*c - a*d)^3 - 3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(9*b*g^4*(a
 + b*x)^3)
```

3.208.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{ 2948}$$

$$-\frac{2B(bc-ad)\int \frac{1}{g^3(a+bx)^4(c+dx)}dx}{3bg}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2B(bc-ad)\int \frac{1}{(a+bx)^4(c+dx)}dx}{3bg^4}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{2B(bc-ad)\int \left(\frac{d^4}{(bc-ad)^4(c+dx)}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)(a+bx)^4}\right)dx}{3bg^4}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}-\frac{2B(bc-ad)\left(-\frac{d^3\log(a+bx)}{(bc-ad)^4}+\frac{d^3\log(c+dx)}{(bc-ad)^4}-\frac{d^2}{(a+bx)(bc-ad)^3}+\frac{d}{2(a+bx)^2(bc-ad)^2}-\frac{1}{3(a+bx)^3(bc-ad)}\right)}{3bg^4}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
```

output
```
(-2*B*(b*c - a*d)*(-1/3*1/((b*c - a*d)*(a + b*x)^3) + d/(2*(b*c - a*d)^2*(
a + b*x)^2) - d^2/((b*c - a*d)^3*(a + b*x)) - (d^3*Log[a + b*x])/(b*c - a*
d)^4 + (d^3*Log[c + d*x])/(b*c - a*d)^4))/(3*b*g^4) - (A + B*Log[(e*(c + d
*x)^2)/(a + b*x)^2])/(3*b*g^4*(a + b*x)^3)
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.208. $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$

3.208.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{\frac{A}{3g^4(bx+a)^3}+\frac{B\Big(\frac{\ln\big(\frac{e(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d)^2}{b^2}\big)}{3(bx+a)^3}-(\frac{2ad}{3}-\frac{2cb}{3})\Big(\frac{\frac{a^2d^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{b^2c^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)^2}+\frac{d^2}{bx+a}}{(ad-cb)^3}}{g^4}}{b}$
default	$-\frac{\frac{A}{3g^4(bx+a)^3}+\frac{B\Big(\frac{\ln\big(\frac{e(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d)^2}{b^2}\big)}{3(bx+a)^3}-(\frac{2ad}{3}-\frac{2cb}{3})\Big(\frac{\frac{a^2d^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{b^2c^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)^2}+\frac{d^2}{bx+a}}{(ad-cb)^3}}{g^4}}{b}$
parts	$-\frac{A}{3g^4(bx+a)^3b}-\frac{B\Big(\frac{\ln\big(\frac{e(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d)^2}{b^2}\big)}{3(bx+a)^3}-(\frac{2ad}{3}-\frac{2cb}{3})\Big(\frac{\frac{a^2d^2}{3(bx+a)^3}-\frac{2abcd}{3(bx+a)^3}+\frac{b^2c^2}{3(bx+a)^3}+\frac{a\,d^2}{2(bx+a)^2}-\frac{bcd}{2(bx+a)^2}}{(ad-cb)^3}}{g^4b}$
risch	$-\frac{B\ln\big(\frac{e(dx+c)^2}{(bx+a)^2}\big)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-dx-c)b^3d^3x^3+6B\ln(bx+a)b^3d^3x^3-18B\ln(-dx-c)a\,b^2d^3x^2+18B\ln(bx+a)a\,b^2d^3x^2}{}$
parallelrisch	$-\frac{-18A\,a^2b^5c\,d^3+18Aa\,b^6c^2d^2-18B\,x^2\ln\big(\frac{e(dx+c)^2}{(bx+a)^2}\big)a\,b^6d^4-18Bx\ln\big(\frac{e(dx+c)^2}{(bx+a)^2}\big)a^2b^5d^4-18B\ln\big(\frac{e(dx+c)^2}{(bx+a)^2}\big)a^2b^5c\,d}{}$
norman	$\frac{B\,a^2d^3x\ln\big(\frac{e(dx+c)^2}{(bx+a)^2}\big)}{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)g}+\frac{Bab\,d^3x^2\ln\big(\frac{e(dx+c)^2}{(bx+a)^2}\big)}{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)g}+\frac{(3A\,a^2d^2-6Aabcd+3A\,b^2c^2-6B\,a^2d^2+6Babcd-2B\,b^2}{3ga(a^2d^2-2abcd+b^2c^2)}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/3/g^4*A/(b*x+a)^3+1/g^4*B*(1/3/(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(2/3*a*d-2/3*c*b)*(1/(a*d-b*c)^3*(1/3*a^2*d^2/(b*x+a)^3-2/3
*a*b*c*d/(b*x+a)^3+1/3*b^2*c^2/(b*x+a)^3+1/2*a*d^2/(b*x+a)^2-1/2*b*c*d/(b*
x+a)^2+d^2/(b*x+a))+d^3/(a*d-b*c)^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d))))
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(165) = 330$.

Time = 0.28 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.44

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{(3A-2B)b^3c^3 - 9(A-B)ab^2c^2d + 9(A-2B)a^2bcd^2 - (3A-11B)a^3d^3 - 6(Bb^3cd^2 - Bab^2d^3)x^2}{9((b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="fri
cas")
```

output
```
-1/9*((3*A - 2*B)*b^3*c^3 - 9*(A - B)*a*b^2*c^2*d + 9*(A - 2*B)*a^2*b*c*d^
2 - (3*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^
3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*l
og((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 -
 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3
*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 -
 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3
*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

3.208.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(162) = 324$.

Time = 1.78 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.82

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3}$$
$$+\frac{2Bd^3\log\left(x + \frac{-\frac{2Ba^4d^7}{(ad-bc)^3} + \frac{8Ba^3bcd^6}{(ad-bc)^3} - \frac{12Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{8Bab^3c^3d^4}{(ad-bc)^3} + 2Bad^4 - \frac{2Bb^4c^4d^3}{(ad-bc)^3} + 2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}$$
$$-\frac{2Bd^3\log\left(x + \frac{\frac{2Ba^4d^7}{(ad-bc)^3} - \frac{8Ba^3bcd^6}{(ad-bc)^3} + \frac{12Ba^2b^2c^2d^5}{(ad-bc)^3} - \frac{8Bab^3c^3d^4}{(ad-bc)^3} + 2Bad^4 + \frac{2Bb^4c^4d^3}{(ad-bc)^3} + 2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}$$
$$+\frac{-3Aa^2d^2 + 6Aabcd - 3Ab^2c^2 + 11Ba^2d^2 - 7Babcd + 2Bb^2c^2 + 6Bb}{9a^5bd^2g^4 - 18a^4b^2cdg^4 + 9a^3b^3c^2g^4 + x^3\cdot(9a^2b^4d^2g^4 - 18ab^5cdg^4 + 9b^6c^2g^4) + x^2\cdot(27a^3b^3d^2g^4 - 54a^2}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**4,x)
```

output
```
-B*log(e*(c + d*x)**2/(a + b*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x +
9*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) + 2*B*d**3*log(x + (-2*B*a**4*d**7/
(a*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d*
*5/(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B
*b**4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*
d - b*c)**3) - 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b
*c*d**6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b
**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)
**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d
**2 + 6*A*a*b*c*d - 3*A*b**2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**
2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(9*a**5*b*
d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b
**4*d**2*g**4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**
3*d**2*g**4 - 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b*
*2*d**2*g**4 - 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

3.208.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(165) = 330$.

Time = 0.22 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.71

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{1}{9}B\left(\frac{6\,b^2d^2x^2 + 2\,b^2c^2 - 7\,abcd + 11\,a^2d^2 - 3\,(b^2cd - 5\,abd^2)x}{(b^6c^2 - 2\,ab^5cd + a^2b^4d^2)g^4x^3 + 3\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)g^4x^2 + 3\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2)g\right.$$
$$- \frac{A}{3\,(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="max
ima")
```

output
```
1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 3*
log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a
^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*
a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d
+ 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b
^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b
^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

3.208.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(165) = 330$.

Time = 0.38 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.69

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2\,Bd^3\log\left(bx+a\right)}{3\left(b^4c^3g^4 - 3\,ab^3c^2dg^4 + 3\,a^2b^2cd^2g^4 - a^3bd^3g^4\right)} - \frac{2\,Bd^3\log\left(dx+c\right)}{3\left(b^4c^3g^4 - 3\,ab^3c^2dg^4 + 3\,a^2b^2cd^2g^4 - a^3bd^3g^4\right)} - \frac{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{3\left(b^4g^4x^3 + 3\,ab^3g^4x^2 + 3\,a^2b^2g^4x + a^3bg^4\right)} + \frac{6\,Bb^2d^2x^2 - 3\,Bb^2cdx + 15\,Babd^2x - 3\,Ab^2c^2 + 2\,Bb^2c^2 + 6\,Aabcd - 7 \ldots}{9\left(b^6c^2g^4x^3 - 2\,ab^5cdg^4x^3 + a^2b^4d^2g^4x^3 + 3\,ab^5c^2g^4x^2 - 6\,a^2b^4cdg^4x^2 + 3\,a^3b^3d^2g^4x^2 + 3\,a^2b^4c^2g^4x - 6 \ldots\right.}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="gia
c")
```

output
```
2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*
g^4 - a^3*b*d^3*g^4) - 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d
*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((d^2*e*x^2 + 2*c*d
*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 +
3*a^2*b^2*g^4*x + a^3*b*g^4) + 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B
*a*b*d^2*x - 3*A*b^2*c^2 + 2*B*b^2*c^2 + 6*A*a*b*c*d - 7*B*a*b*c*d - 3*A*a
^2*d^2 + 11*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^
2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^
4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x +
a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = \frac{2\,B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{3\,b\,g^4\,(a+b\,x)^3} - \frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{11\,B\,a^2\,d^2}{9\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{5\,B\,a\,d^2\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{2\,B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{7\,B\,a\,c\,d}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} - \frac{B\,b\,c\,d\,x}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3} + \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^4,x)
```

output
```
(B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d -
b*c)^3) - (B*log((e*(c + d*x)^2)/(a + b*x)^2))/(3*b*g^4*(a + b*x)^3) - (A*
b*c^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*B*b*c^2)/(9*g^4*(a*d - b*c)^
2*(a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a
^2*d^2)/(9*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(3*g^4*(a*d -
b*c)^2*(a + b*x)^3) + (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) +
(2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(9*g^4*(a*d -
b*c)^2*(a + b*x)^3) - (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.209 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$

3.209.1 Optimal result

Integrand size = 32, antiderivative size = 208

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \frac{B}{8bg^5(a+bx)^4} - \frac{Bd}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)} - \frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5} + \frac{Bd^4\log(c+dx)}{2b(bc-ad)^4g^5} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}$$

output
```
1/8*B/b/g^5/(b*x+a)^4-1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/4*B*d^2/b/(-a*d
+b*c)^2/g^5/(b*x+a)^2-1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/2*B*d^4*ln(b*
x+a)/b/(-a*d+b*c)^4/g^5+1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*l
n(e*(d*x+c)^2/(b*x+a)^2))/b/g^5/(b*x+a)^4
```

3.209.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \frac{\frac{B(3(bc-ad)^4+4d(-bc+ad)^3(a+bx)+6d^2(bc-ad)^2(a+bx)^2+12d^3(-bc+ad)(a+bx)^3-12d^4(a+bx)^4\log(a+bx)+12d^4(a+bx)^4\log(c+dx))}{(bc-ad)^4}-6\Big(}{24bg^5(a+bx)^4}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

output
```
((B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log
[a + b*x] + 12*d^4*(a + b*x)^4*Log[c + d*x]))/(b*c - a*d)^4 - 6*(A + B*Log
[(e*(c + d*x)^2)/(a + b*x)^2]))/(24*b*g^5*(a + b*x)^4)
```

3.209.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{(ag+bgx)^5}\,dx$$

$$\Big\downarrow \ 2948$$

$$-\frac{B(bc-ad)\int \frac{1}{g^4(a+bx)^5(c+dx)}dx}{2bg}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{B(bc-ad)\int \frac{1}{(a+bx)^5(c+dx)}dx}{2bg^5}-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \ 54$$

$$-\frac{B(bc-ad)\int \left(-\frac{d^5}{(bc-ad)^5(c+dx)}+\frac{bd^4}{(bc-ad)^5(a+bx)}-\frac{bd^3}{(bc-ad)^4(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{b}{(bc-ad)(a+bx}}{2bg^5}$$

$$\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}-$$

$$\frac{B(bc-ad)\left(\frac{d^4\log(a+bx)}{(bc-ad)^5}-\frac{d^4\log(c+dx)}{(bc-ad)^5}+\frac{d^3}{(a+bx)(bc-ad)^4}-\frac{d^2}{2(a+bx)^2(bc-ad)^3}+\frac{d}{3(a+bx)^3(bc-ad)^2}-\frac{1}{4(a+bx)^4(bc-ad)}\right)}{2bg^5}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

output
```
-1/2*(B*(b*c - a*d)*(-1/4*1/((b*c - a*d)*(a + b*x)^4) + d/(3*(b*c - a*d)^2
*(a + b*x)^3) - d^2/(2*(b*c - a*d)^3*(a + b*x)^2) + d^3/((b*c - a*d)^4*(a
+ b*x)) + (d^4*Log[a + b*x])/(b*c - a*d)^5 - (d^4*Log[c + d*x])/(b*c - a*d
)^5))/(b*g^5) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(4*b*g^5*(a + b*x
)^4)
```

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.209.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{\frac{A}{4g^5(bx+a)^4}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{ad}{2}-\frac{cb}{2}\right)\left(\frac{\frac{(ad-cb)\left(a^2d^2-2abcd+b^2c^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{3(bx+a)^3}+\frac{(ad-cb)d^2}{2(bx+a)^2}}{(ad-cb)^4}\right.}{g^5}}{b}$
default	$-\frac{\frac{A}{4g^5(bx+a)^4}+\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{ad}{2}-\frac{cb}{2}\right)\left(\frac{\frac{(ad-cb)\left(a^2d^2-2abcd+b^2c^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{3(bx+a)^3}+\frac{(ad-cb)d^2}{2(bx+a)^2}}{(ad-cb)^4}\right.}{g^5}}{b}$
parts	$-\frac{A}{4g^5(bx+a)^4b}-\frac{B\left(\frac{\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4(bx+a)^4}-\left(\frac{ad}{2}-\frac{cb}{2}\right)\left(\frac{\frac{(ad-cb)\left(a^2d^2-2abcd+b^2c^2\right)}{4(bx+a)^4}+\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{3(bx+a)^3}+\frac{(ad}{2(b}\right.}{(ad-cb)^4}\right.}{g^5b}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{4b\,g^5(bx+a)^4}-\frac{48Ba\,b^3c\,d^3x^2+72B\,a^2b^2c\,d^3x-24Ba\,b^3c^2d^2x-24A\,a^3bc\,d^3+36A\,a^2b^2c^2d^2-24Aa\,b^3c^3d-12Ba}{}$
parallelrisch	$\frac{24Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^9c\,d^4+6B\,x^4\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^6b^3c\,d^4+24B\,x^3\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^7b^2c\,d^4+36B\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)a^8bc}{}$
norman	$\frac{B\,a^3d^4x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{a\,d^4B\,b^2x^3\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)g}+\frac{\left(2A\,a^3d^3-6A\,a^2bc\,d^2+6Aa\,b^2\right.}{2ga(}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/4/g^5*A/(b*x+a)^4+1/g^5*B*(1/4/(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*
x+a)-d)^2/b^2)-(1/2*a*d-1/2*c*b)*(1/(a*d-b*c)^4*(1/4*(a*d-b*c)*(a^2*d^2-2*
a*b*c*d+b^2*c^2)/(b*x+a)^4+1/3*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^3+1/2
*(a*d-b*c)*d^2/(b*x+a)^2+d^3/(b*x+a))+d^4/(a*d-b*c)^5*ln(a*d/(b*x+a)-b*c/(
b*x+a)-d))))
```

3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(194) = 388$.

Time = 0.28 (sec) , antiderivative size = 658, normalized size of antiderivative = 3.16

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{3\,(2\,A-B)b^4c^4 - 8\,(3\,A-2\,B)ab^3c^3d + 36\,(A-B)a^2b^2c^2d^2 - 24\,(A-2\,B)a^3bcd^3 + (6\,A-25\,B)a^4d^4}{24\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4\,(ab^8c^4 - 4\,a^2b}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="fri
cas")
```

output
```
-1/24*(3*(2*A - B)*b^4*c^4 - 8*(3*A - 2*B)*a*b^3*c^3*d + 36*(A - B)*a^2*b^
2*c^2*d^2 - 24*(A - 2*B)*a^3*b*c*d^3 + (6*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*
d^3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*
d^4)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*
a^3*b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2
+ 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*
B*a^3*b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^
2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4
*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4
*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c
^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*
g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d
^3 + a^8*b*d^4)*g^5)
```

3.209.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(182) = 364$.

Time = 2.59 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.55

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4a^4bg^5 + 16a^3b^2g^5x + 24a^2b^3g^5x^2 + 16ab^4g^5x^3 + 4b^5g^5x^4} + \frac{Bd^4\log\left(x + \frac{-\frac{Ba^5d^9}{(ad-bc)^4} + \frac{5Ba^4bcd^8}{(ad-bc)^4} - \frac{10Ba^3b^2c^2d^7}{(ad-bc)^4} + \frac{10Ba^2b^3c^3d^6}{(ad-bc)^4} - \frac{5Bab^4c^4d^5}{(ad-bc)^4} + Bad^5 + \frac{Bb^5c^5d^4}{(ad-bc)^4} + Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4} - \frac{Bd^4\log\left(x + \frac{\frac{Ba^5d^9}{(ad-bc)^4} - \frac{5Ba^4bcd^8}{(ad-bc)^4} + \frac{10Ba^3b^2c^2d^7}{(ad-bc)^4} - \frac{10Ba^2b^3c^3d^6}{(ad-bc)^4} + \frac{5Bab^4c^4d^5}{(ad-bc)^4} + Bad^5 - \frac{Bb^5c^5d^4}{(ad-bc)^4} + Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4} + \frac{-6Aa^3d^3 + 18Aa^2bcd^2 - 18Aab^2c^2d +}{24a^7bd^3g^5 - 72a^6b^2cd^2g^5 + 72a^5b^3c^2dg^5 - 24a^4b^4c^3g^5 + x^4\cdot(24a^3b^5d^3g^5 - 72a^2b^6cd^2g^5 + 72ab^7c^2dg^5 -}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**5,x)
```

output
```
-B*log(e*(c + d*x)**2/(a + b*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
 24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4
*log(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 -
 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d -
 b*c)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d*
*4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) -
B*d**4*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)
**4 + 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(
a*d - b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c*
*5*d**4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**
4) + (-6*A*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*
c**3 + 25*B*a**3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3
*c**3 + 12*B*b**3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) +
x*(52*B*a**2*b*d**3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(24*a**7*b*d*
*3*g**5 - 72*a**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b*
*4*c**3*g**5 + x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 7
2*a*b**7*c**2*d*g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 -
 288*a**3*b**5*c*d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g*
*5) + x**2*(144*a**5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3
*b**5*c**2*d*g**5 - 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**...
```

3.209.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(194) = 388$.

Time = 0.22 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.36

$$\int \frac{A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{24}B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - \cdots}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\cdots}\right.$$

$$-\frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="max
ima")
```

output
```
-1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*
d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^
3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*
b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*
d^3)*g^5) + 6*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2
 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^
4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(
b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
 a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b
^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a
*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

3.209.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(194) = 388$.

Time = 0.41 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.04

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \frac{Bd^4\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{2\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)} - \frac{Bd^3}{2\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg} + \frac{Bd^2}{4\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2bg^2} - \frac{B\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)^2}+\frac{a^2d^2eg^2}{(bgx+ag)^2}+\frac{2\,bcdeg}{bgx+ag}-\frac{2\,ad^2eg}{bgx+ag}+d^2e}{b^2}\right)}{4\,(bgx+ag)^4bg} - \frac{Bd}{6\,(bgx+ag)^3(bc-ad)bg^2} - \frac{2\,Ab^3g^3-Bb^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="gia
c")
```

output
```
1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5
 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b
*d^4*g^5) - 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^
3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g
+ a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log((b^2*c^2*e*g^2/(b*g*x + a*
g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g*x + a*g)^2 + 2
*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*e)/b^2)/((b*g*x
 + a*g)^4*b*g) - 1/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^
3*g^3 - B*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

3.209. $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$

3.209.9 Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = \frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^4}\right)}{b\,g^5\,(a\,d-b\,c)^4} - \frac{B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)} - \frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3-25\,B\,a^3\,d^3+3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2-13\,B\,a\,b^2\,c^2\,d+23\,B\,a^2\,b\,c\,d^2}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{d^2\,x^2\,(B\,b^3\,c-7\,B\,a\,b^2\,d)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3+}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^5,x)
```

output
```
(B*d^4*atanh((2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*
b^2*c*d^3*g^5)/(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4) - (B*lo
g((e*(c + d*x)^2)/(a + b*x)^2))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*
a^2*b*x^2 + 4*a*b^2*x^3)) - ((6*A*a^3*d^3 - 6*A*b^3*c^3 - 25*B*a^3*d^3 + 3
*B*b^3*c^3 + 18*A*a*b^2*c^2*d - 18*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B
*a^2*b*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (
d^2*x^2*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^
3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^
4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2)
```

3.210 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

3.210.1 Optimal result

Integrand size = 34, antiderivative size = 515

$$\begin{aligned}
&\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \\
&= \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{15bd^3} + \frac{2B^2(bc-ad)^2g^4(a+bx)^3}{15bd^2} \\
&- \frac{10B^2(bc-ad)^5g^4\log(a+bx)}{3bd^5} - \frac{26B^2(bc-ad)^5g^4\log\left(\frac{c+dx}{a+bx}\right)}{15bd^5} \\
&+ \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd^3} \\
&- \frac{4B(bc-ad)^2g^4(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{15bd^2} \\
&+ \frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5bd} \\
&- \frac{4B(bc-ad)^4g^4(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5d^5} \\
&+ \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} \\
&- \frac{4B(bc-ad)^5g^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} \\
&+ \frac{8B^2(bc-ad)^5g^4\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}
\end{aligned}$$

output
```
26/15*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/15*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3
+2/15*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-10/3*B^2*(-a*d+b*c)^5*g^4*ln(b*
x+a)/b/d^5-26/15*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+2/5*B*(-a*
d+b*c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^3-4/15*B*(-a*d+
b*c)^2*g^4*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^2+1/5*B*(-a*d+b*c
)*g^4*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/5*B*(-a*d+b*c)^4*g^4
*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(
d*x+c)^2/(b*x+a)^2))^2/b-4/5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)^2/(b*x+a
)^2))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d
*(b*x+a)/b/(d*x+c))/b/d^5
```

3.210.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.02

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 - \frac{B(bc-ad)\left(12Abd(bc-ad)^3x+24B(bc-ad)^4\log(c+dx)-4B(bc-ad)^2(2bd(bc-ad)x-d^2(\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (B*(b*c - a
*d)*(12*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[c + d*x] - 4*B*(b*c
 - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c +
 d*x]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a +
b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a
*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*d*(b*c - a*d)^3*(a + b*
x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A +
B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(c +
 d*x)^2)/(a + b*x)^2]) - 12*(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) - 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])))/(3*d^5)))/(5*b)
```

3.210.3 Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.38, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.559, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^4 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \; 2952$$

$$g^4\left(-(bc-ad)^5\right)\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^6}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \; 2756$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}}{5b}\right)$$

$$\Big\downarrow \; 2789$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}\right)}{5b}\right)$$

$$\Big\downarrow \; 2756$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{2b}\right)}{d}+\int\frac{(a+bx)\left(A+B\cdots\right.}{(c+dx)\left(\cdots\right.}\cdots\right)}{5b}\cdots\right)$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int\left(\frac{b}{d^4\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d^2\left(d-\frac{b(c+\cdots}{a+\cdots}\right.}\cdots\right.}{2b}\cdots\right)}{d}\cdots\right)}{\cdots}\cdots\right)$$

$\downarrow$ 2009

3.210. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\log\left(\frac{c+\cdots}{a+\cdots}\right)}{d^4}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{5\cdots}\right.$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2756

3.210. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\,\mathrm{lo}\cdots\right.}{(c+dx)\left(d-\cdots\right.}}{\cdots}}{d}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 54

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\left(\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d\left(d-\frac{b(c+\cdots}{a+b\cdots}\right.}\cdots\right.}{3b}\cdots\right.}{d}\cdots\right.}{d}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2009

3.210. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^3}\right.}{}\right.}{d}\right.}{}\right.$$

$\downarrow$ 2789

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B}{3}\right.}{}\right.}{}\right.$$

$\downarrow$ 2756

3.210. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)}{d}+\int\frac{(a+bx)\left(A+B\log\cdots\right.}{(c+dx)\left(d-\cdots\right.}\cdots}{d}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 54$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d\cdots}{b}\cdots\right.}{d}\cdots}{d}\cdots\right.}{\cdots}\right.$$

3.210. $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$\Big\downarrow \ 2009$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}\cdots\right.}{\cdots}\right.}{\cdots}}{d}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \ 2789$$

3.210. $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\cdots}{\cdots}\cdots\right.}{\cdots}}{d}\cdots\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 2751

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{2B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\cdots\right.}{(c+dx)\left(d-\cdots\right.}\cdots}{\cdots}}{d}\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

3.210. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$\Big\downarrow \ 16$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{2B}{\cdots}\right.}{d}}{d}}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \ 2779$$

3.210. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{\frac{\frac{2B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}\cdots}{d}\cdots}{\cdots}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 2838$$

$$g^4\left(-(bc-ad)^5\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b\left(d-\frac{b(c+dx)}{a+bx}\right)^5}-\frac{4B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{b}\right)}{d}\cdots}{\cdots}\cdots\right.}{\cdots}\right.$$

3.210. $\int(ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx$

input
```
Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
-((b*c - a*d)^5*g^4*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(5*b*(d -
(b*(c + d*x))/(a + b*x))^5) - (4*B*((b*((A + B*Log[(e*(c + d*x)^2)/(a + b*
x)^2])/(4*b*(d - (b*(c + d*x))/(a + b*x))^4) - (B*(1/(3*d*(d - (b*(c + d*x
))/(a + b*x))^3) + 1/(2*d^2*(d - (b*(c + d*x))/(a + b*x))^2) + 1/(d^3*(d -
 (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^4 - Log[d - (b*(c
+ d*x))/(a + b*x)]/d^4))/(2*b)))/d + ((b*((A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2])/(3*b*(d - (b*(c + d*x))/(a + b*x))^3) - (2*B*(1/(2*d*(d - (b*(c +
 d*x))/(a + b*x))^2) + 1/(d^2*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*
x)/(a + b*x)]/d^3 - Log[d - (b*(c + d*x))/(a + b*x)]/d^3))/(3*b)))/d + ((b
*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*b*(d - (b*(c + d*x))/(a + b*
x))^2) - (B*(1/(d*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)
]/d^2 - Log[d - (b*(c + d*x))/(a + b*x)]/d^2))/b))/d + ((b*(((c + d*x)*(A
+ B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a
+ b*x))) + (2*B*Log[d - (b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*L
og[(e*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d)
+ (2*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d)/d)/d))/(5*b)))
```

3.210.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.210.4 Maple [F]

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.210.5 Fricas [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4
*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 +
6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d^2*e*x^2 +
2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A
*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4
)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

3.210.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. $2(490) = 980$.

Time = 0.38 (sec) , antiderivative size = 2660, normalized size of antiderivative = 5.17

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^
3*b*g^4*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x +
a)/b + 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2
*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a
*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c -
a*d)*x/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^
2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*lo
g(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2
) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(
d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*
x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(
d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
+ c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(
d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d
^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d
^4))*A*B*b^4*g^4 + A^2*a^4*g^4*x + 2/15*((6*g^4*log(e) - 25*g^4)*b^4*c^5 -
 (30*g^4*log(e) - 113*g^4)*a*b^3*c^4*d + 4*(15*g^4*log(e) - 49*g^4)*a^2...
```

3.210.8 Giac [F]

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^4 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^4*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^4 \left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.211 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

3.211.1 Optimal result

Integrand size = 34, antiderivative size = 422

$$\begin{aligned}
&\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \\
&= -\frac{5B^2(bc-ad)^3g^3x}{3d^3}+\frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2}+\frac{11B^2(bc-ad)^4g^3\log(a+bx)}{3bd^4} \\
&\quad+\frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^4}-\frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2bd^2} \\
&\quad+\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} \\
&\quad+\frac{B(bc-ad)^3g^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^4}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} \\
&\quad+\frac{B(bc-ad)^4g^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4} \\
&\quad-\frac{2B^2(bc-ad)^4g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}
\end{aligned}$$

output
```
-5/3*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1
1/3*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/3*B^2*(-a*d+b*c)^4*g^3*ln((d*x+
c)/(b*x+a))/b/d^4-1/2*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*
x+a)^2))/b/d^2+1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^
2))/b/d+B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^4+1/4
*g^3*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b+B*(-a*d+b*c)^4*g^3*(A+B
*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-2*B^2*(-a*d+b*
c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

3.211.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.95

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$=\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{2B(bc-ad)\left(6Abd(bc-ad)^2x+12B(bc-ad)^3\log(c+dx)-2B(bc-ad)\left(2bd(bc-ad)x-d^2(a\right.\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(b*c -
 a*d)*(6*A*b*d*(b*c - a*d)^2*x + 12*B*(b*c - a*d)^3*Log[c + d*x] - 2*B*(b*
c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c +
d*x]) - 6*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*d*(b
*c - a*d)^2*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 3*d^2*(-(b*c) + a
*d)*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 2*d^3*(a + b*x)
^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*(b*c - a*d)^3*Log[c + d*x]
*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*B*(b*c - a*d)^3*((2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

3.211.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.24, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.441, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2952}$$

$$g^3(bc-ad)^4\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^5}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{b}\right)$$

$$\Big\downarrow \text{2789}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}\right)}{b}\right)$$

$$\Big\downarrow \text{2756}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right.}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}}{d}\right.}{b}\right.$$

$\Big\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\int\left(\frac{b}{d^3\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^3}+\frac{a+bx}{d^3(c+dx)}\right.}{3b}\right.}{d}\right.}{b}\right.$$

$\Big\downarrow$ 2009

3.211. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{2B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^3}-\frac{\log\left(d-\frac{b(c}{a}\right.}{d^3}\right.}{}\right.}{d}\right.}{b}\right.$$

$\downarrow$ 2789

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)}\right.}{}\right.}{b}\right.$$

$\downarrow$ 2756

3.211. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c}{a}}{d}}{d}}{}\right)$$

$\Big\downarrow$ 54

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d\frac{c+dx}{a+bx}}{b}\right)}{d}+\frac{\int\frac{(a+bx)}{(c}}{}}{d}}{}\right)$$

$\Big\downarrow$ 2009

3.211. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx}{a+bx}\right)}{d^2}\right.}{b}\right.}{d}}{d}\right.}{}\right.$$

$\downarrow$ 2789

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}\right.}{d}}{}\right.}{}\right.$$

$\downarrow$ 2751

3.211. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{2B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c}{a}}{d}}{d}}{d}\right)}{}\right)$$

$\downarrow$ 16

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{2B\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{bd}\right)}{d}}{d}}{d}\right)}{}\right)$$

$\downarrow$ 2779

3.211. $\int(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{\frac{\frac{2B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\right.\right.}{d(a+bx)\left(\right.}\right.}{}}{d}\right.}{}\right.$$

$$\Big\downarrow \text{ 2838}$$

$$g^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b\left(d-\frac{b(c+dx)}{a+bx}\right)^4}-\frac{B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{b}\right)}{d}+\frac{\frac{2B\,\text{PolyLog}\left(2,\right.}{d}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

3.211. $\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

output
```
(b*c - a*d)^4*g^3*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(4*b*(d - (b
*(c + d*x))/(a + b*x))^4) - (B*((b*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])/(3*b*(d - (b*(c + d*x))/(a + b*x))^3) - (2*B*(1/(2*d*(d - (b*(c + d*x))
/(a + b*x))^2) + 1/(d^2*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a
+ b*x)]/d^3 - Log[d - (b*(c + d*x))/(a + b*x)]/d^3))/(3*b)))/d + ((b*((A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b*(d - (b*(c + d*x))/(a + b*x))^2)
 - (B*(1/(d*(d - (b*(c + d*x))/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^2
- Log[d - (b*(c + d*x))/(a + b*x)]/d^2))/b))/d + ((b*(((c + d*x)*(A + B*Lo
g[(e*(c + d*x)^2)/(a + b*x)^2]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x)
)) + (2*B*Log[d - (b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*
(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (2*B
*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/d)/d))/b)
```

3.211.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.211. $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.211.4 Maple [F]

$$\int (bgx+ag)^3 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.211.5 Fricas [F]

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*
a^3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2
+ 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g
^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

3.211.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1950 vs. $2(407) = 814$.

Time = 0.35 (sec) , antiderivative size = 1950, normalized size of antiderivative = 4.62

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log
(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
 + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c
)/d)*A*B*a^3*g^3 + 3*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*
e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*l
og(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a^2*
b*g^3 + 2*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^
2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)
/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^
2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2
*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a
*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d
^3)*x)/(b^3*d^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x - 1/3*((3*g^3*log(e) - 11*g^
3)*b^3*c^4 - 2*(6*g^3*log(e) - 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) - 5*g
^3)*a^2*b*c^2*d^2 - 6*(2*g^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d
^4 + 2*(b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*
c*d^3*g^3 + a^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3
*x^4*log(e)^2 + 4*(b^4*c*d^3*g^3*log(e) + (3*g^3*log(e)^2 - g^3*log(e))...
```

3.211.8 Giac [F]

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.212 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

3.212.1 Optimal result

Integrand size = 34, antiderivative size = 343

$$\begin{aligned}
&\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \\
&= \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^3g^2\log(a+bx)}{bd^3} - \frac{4B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3} \\
&\quad + \frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3bd} \\
&\quad - \frac{4B(bc-ad)^2g^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3d^3} \\
&\quad + \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} \\
&\quad - \frac{4B(bc-ad)^3g^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} \\
&\quad + \frac{8B^2(bc-ad)^3g^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}
\end{aligned}$$

output
```
4/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-4*B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-4/3*
B^2*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+2/3*B*(-a*d+b*c)*g^2*(b*x+a
)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+
B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*
x+a)^2))^2/b-4/3*B*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d
*(b*x+a)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(
d*x+c))/b/d^3
```

3.212.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.87

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$

$$= \frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 - \frac{2B(bc-ad)\left(2Abd(bc-ad)x+4B(bc-ad)^2\log(c+dx)-2B(bc-ad)(bdx+(-bc+ad)\log(c\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(b*c -
 a*d)*(2*A*b*d*(b*c - a*d)*x + 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*B*(b*c -
 a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*
Log[(e*(c + d*x)^2)/(a + b*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)
^2)/(a + b*x)^2]) - 2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2
)/(a + b*x)^2]) - 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
- Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/
d^3))/(3*b)
```

3.212.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2dx$$

$$\Big\downarrow \; 2952$$

$$g^2\left(-(bc-ad)^3\right)\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^4}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \; 2756$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{3b}\right)$$

$$\Big\downarrow \; 2789$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{b\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}}{d}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}\right)}{3b}\right)$$

$$\Big\downarrow \; 2756$$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int \frac{a+bx}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)}{d}+\frac{\int \frac{(a+bx)\left(A+B\ldots\right.}{(c+dx)\left(\ldots\right.}}{\ldots}\right.}{3b}\ldots\right.$$

3.212. $\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx$

$\downarrow$ 54

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\int\left(\frac{b}{d^2\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{b}{d\left(d-\frac{b(c+dx)}{a+bx}\right)^2}+\frac{a+bx}{d^2(c+dx)}\right)d}{b}\right.}{d}\right.}{3b}\right.$$

$\downarrow$ 2009

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+}{a+}\right.}{d^2}\right.}{}\right.}{}\right.}{3b}\right.$$

$\downarrow$ 2789

3.212. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}}{d}+\frac{b\left(\right.}{}\right.}{3b}\right.$$

$\Big\downarrow$ 2751

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{2B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\right.}{(c+dx)\left(d-\right.}}{}}{d}\right.}{}\right.$$

$\Big\downarrow$ 16

3.212. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{2B\ldots}{}\right.}{d}}{d}\right.}{}\right.$$

$\downarrow$ 2779

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{\frac{\frac{2B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}+\ldots}{d}\right.}{}\right.$$

$\downarrow$ 2838

3.212. $\int(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g^2\left(-(bc-ad)^3\right)\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{4B\left(\frac{b\left(\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{c+dx}{a+bx}\right)}{d^2}-\frac{\log\left(d-\frac{b(c+dx)}{a+bx}\right)}{d^2}+\frac{1}{d\left(d-\frac{b(c+dx)}{a+bx}\right)}\right)}{b}\right)}{d}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
-((b*c - a*d)^3*g^2*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(3*b*(d -
(b*(c + d*x))/(a + b*x))^3) - (4*B*((b*((A + B*Log[(e*(c + d*x)^2)/(a + b*
x)^2])/(2*b*(d - (b*(c + d*x))/(a + b*x))^2) - (B*(1/(d*(d - (b*(c + d*x))
/(a + b*x))) + Log[(c + d*x)/(a + b*x)]/d^2 - Log[d - (b*(c + d*x))/(a + b
*x)]/d^2))/b))/d + ((b*(((c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (2*B*Log[d - (b*(c + d*x)
)/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log
[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (2*B*PolyLog[2, (d*(a + b*x))/(b*(
c + d*x))])/d)/d)/d))/(3*b)))
```

3.212.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.212. $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

3.212.4 Maple [F]

$$\int (bgx + ag)^2 \left(A + B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.212.5 Fricas [F]

$$\int (ag + bgx)^2 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \int (bgx + ag)^2 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^
2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2
*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

3.212.6 Sympy [F(-1)]

Timed out.

$$\int (ag + bgx)^2 \left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

3.212.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. $2(328) = 656$.

Time = 0.34 (sec) , antiderivative size = 1333, normalized size of antiderivative = 3.89

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*
b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*
x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*
log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a
^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*lo
g(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(d^2*e*x
^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e
/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/
d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b
^2*g^2 + A^2*a^2*g^2*x + 4/3*((g^2*log(e) - 3*g^2)*b^2*c^3 - (3*g^2*log(e)
 - 7*g^2)*a*b*c^2*d + (3*g^2*log(e) - 4*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d
^3 - 8/3*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)
/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (2*b^3*c*
d^2*g^2*log(e) + (3*g^2*log(e)^2 - 2*g^2*log(e))*a*b^2*d^3)*B^2*x^2 - (4*(
g^2*log(e) - g^2)*b^3*c^2*d - 4*(3*g^2*log(e) - 2*g^2)*a*b^2*c*d^2 - (3*g^
2*log(e)^2 - 8*g^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x
^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*lo
g(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^
2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B...
```

3.212.8 Giac [F]

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \int (bgx+ag)^2 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.213 $\int (ag+bgx) \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

3.213.1 Optimal result

Integrand size = 32, antiderivative size = 211

$$\int (ag+bgx) \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{4B^2(bc-ad)^2g\log(a+bx)}{bd^2} + \frac{2B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} + \frac{2B(bc-ad)^2g\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{4B^2(bc-ad)^2g\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

output
```
4*B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+2*B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d
*x+c)^2/(b*x+a)^2))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/
b+2*B*(-a*d+b*c)^2*g*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x
+c))/b/d^2-4*B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

3.213.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.92

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$= \frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{4B(bc-ad)\left(Abdx+B(bc-ad)\log^2(c+dx)+Bd(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)-(bc-ad)\log(c+d\right.}{}\right.}{2b}$$

input
```
Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
(g*((a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(b*c - a
*d)*(A*b*d*x + B*(b*c - a*d)*Log[c + d*x]^2 + B*d*(a + b*x)*Log[(e*(c + d*
x)^2)/(a + b*x)^2] - (b*c - a*d)*Log[c + d*x]*(A - 2*B + 2*B*Log[(d*(a + b
*x))/(-(b*c) + a*d)] + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + (-2*b*B*c + 2
*a*B*d)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^2))/(2*b)
```

3.213.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2952, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2952}$$

$$g(bc-ad)^2\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{\left(d-\frac{b(c+dx)}{a+bx}\right)^3}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{ 2756}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{b}\right)$$

$\big\downarrow$ 2789

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{b\int\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{\left(d-\frac{b(c+dx)}{a+bx}\right)^2}d\frac{c+dx}{a+bx}}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}\right)}{b}\right)$$

$\big\downarrow$ 2751

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}-\frac{2B\int\frac{1}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{d}\right)}{d}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.\right.}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}\cdots}{d}\cdots}{b}\cdots\right.$$

$\big\downarrow$ 16

3.213. $\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)}+\frac{2B\log\left(d-\frac{b(c+dx)}{a+bx}\right.}{bd}\right.}{d}\right.}{b}\right.$$

$$\Big\downarrow \text{2779}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{\frac{2B\int\frac{(a+bx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\right.}{d(a+b}\right.}{}\right.}{b}\right.$$

$$\Big\downarrow \text{2838}$$

$$g(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b\left(d-\frac{b(c+dx)}{a+bx}\right)^2}-\frac{2B\left(\frac{\frac{2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d}}{d}+\frac{b\left(\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{(a+bx)}\right.\right.}{d(a+bx)\left(d-\frac{b(c+d}{a+b}\right.}\right.}{}\right.}{b}\right.$$

input
```
Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

3.213. $\int(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

output
```
(b*c - a*d)^2*g*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(2*b*(d - (b*(
c + d*x))/(a + b*x))^2) - (2*B*((b*(((c + d*x)*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2]))/(d*(a + b*x)*(d - (b*(c + d*x))/(a + b*x))) + (2*B*Log[d -
(b*(c + d*x))/(a + b*x)])/(b*d)))/d + (-(((A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) + (2*B*PolyLog[2, (d*(a
+ b*x))/(b*(c + d*x))])/d)/d))/b)
```

3.213.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.213. $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.213.4 Maple [F]

$$\int (bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)^{2}}{(bx+a)^{2}}\right)\right)^{2}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.213.5 Fricas [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx=\int (bgx+ag)\left(B\log\left(\frac{(dx+c)^{2}e}{(bx+a)^{2}}\right)+A\right)^{2}dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*e*x^2 + 2*c*
d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log(
(d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

3.213.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

3.213.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(208) = 416$.

Time = 0.32 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.46

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx=\frac{1}{2}A^2bgx^2$$
$$+2\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a\log(bx+a)}{b}+\frac{2\,c\log(dx\right.$$
$$+\left(x^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)+\frac{2\,a^2\log(bx+a)}{b^2}-\frac{2\,c^2\log(d}{d^2}\right.$$
$$+A^2agx-\frac{2\left((g\log(e)-2\,g)bc^2-2\,(g\log(e)-g)acd\right)B^2\log(dx+c)}{d^2}$$
$$+\frac{4\,(b^2c^2g-2\,abcdg+a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2}$$
$$+\frac{B^2b^2d^2gx^2\log(e)^2+2\left(2\,b^2cdg\log(e)+\left(g\log(e)^2-2\,g\log(e)\right)abd^2\right)B^2x+4\left(B^2b^2d^2gx^2+2\,B^2abd^2g\right.}{}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="max
ima")
```

output
```
1/2*A^2*b*g*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x
/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*
x + a)/b + 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a
*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b
*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*
d)*x/(b*d))*A*B*b*g + A^2*a*g*x - 2*((g*log(e) - 2*g)*b*c^2 - 2*(g*log(e)
- g)*a*c*d)*B^2*log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)
*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(2*b^2*c*d*
g*log(e) + (g*log(e)^2 - 2*g*log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2
 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^
2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 4*
(B^2*b^2*d^2*g*x^2*log(e) + 2*((g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x +
 ((g*log(e) - 2*g)*a^2*d^2 + 2*a*b*c*d*g)*B^2)*log(b*x + a) + 4*(B^2*b^2*d
^2*g*x^2*log(e) + 2*((g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x - 2*(B^2*b^
2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c
))/(b*d^2)
```

3.213.8 Giac [F]

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx=\int (bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.213. $\int (ag + bgx) \left(A + B \log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

3.214 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$

3.214.1 Optimal result

Integrand size = 34, antiderivative size = 132

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg} - \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bg} + \frac{8B^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

output
```
-ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b/g-4*B*(A+B*ln
(e*(d*x+c)^2/(b*x+a)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylog(
3,b*(d*x+c)/d/(b*x+a))/b/g
```

3.214.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{2AB\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+A^2\log(a+bx)+4AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-2AB\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

output
```
(2*A*B*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + A^2*Log[a + b*x] + 4*A*B*Log[
(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*Log[(
-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x)^2)/(a + b*x)^2] - B^2*Log[(-
(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x)^2)/(a + b*x)^2]^2 - 4*A*B*Pol
yLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 4*B^2*Log[(e*(c + d*x)^2)/(a + b*x
)^2]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d*
x))/(d*(a + b*x))])/(b*g)
```

3.214.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2952, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{ag+bgx}\,dx$$
$$\Big\downarrow \; 2952$$
$$\frac{\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{d-\frac{b(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{g}$$
$$\Big\downarrow \; 2754$$

$$\frac{\frac{4B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\frac{4B\left(2B\int \frac{(a+bx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{c+dx}d\frac{c+dx}{a+bx}-\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{b}}{g}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{4B\left(2B\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)-\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{b}}{g}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

output
```
(-(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2*Log[1 - (b*(c + d*x))/(d*(a
 + b*x))])/b) + (4*B*(-((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))]) + 2*B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x
))]))/b)/g
```

3.214.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

3.214. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.214.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}{bgx+ag}dx$$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

output
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

3.214.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="fri
cas")
```

output
```
integral((B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2
))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)
) + A^2)/(b*g*x + a*g), x)
```

3.214.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2}\right.}{a+bx}}{g}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g),x)
```

output
```
(Integral(A**2/(a + b*x), x) + Integral(B**2*log(c**2*e/(a**2 + 2*a*b*x +
b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
2*a*b*x + b**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(c**2*e/(a**2 +
 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x*
*2/(a**2 + 2*a*b*x + b**2*x**2))/(a + b*x), x))/g
```

3.214.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="max
ima")
```

output
```
4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - int
egrate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + 4*(B^2*b*d*x + B^2*b*c)*log
(b*x + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x - 4*(B^2*b*c*log(e)
+ A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) + 4*(B^2*b*c*log(e)
 + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x - 2*(2*B^2*b*d*x + (b*c + a*d)*B
^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*
g)*x), x)
```

3.214.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="gia
c")
```

output
```
integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x),x)
```

output
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x), x)
```

3.215 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

3.215.1 Optimal result

Integrand size = 34, antiderivative size = 157

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{4AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)} + \frac{4B^2(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

output
```
4*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+
4*B^2*(d*x+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+
B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(-a*d+b*c)/g^2/(b*x+a)
```

3.215.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{4B\left(2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-(bc-ad)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-d(a+bx)\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

output
```
-(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(2*B*(b*c - a*d + d*(
a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - (b*c - a*d)*(A + B*Log
[(e*(c + d*x)^2)/(a + b*x)^2]) - d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]) + d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x
)^2)/(a + b*x)^2]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B
*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a
+ b*x)))
```

3.215.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2952, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2733$$

$$-\frac{\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{a+bx}-4B\int\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{a+bx}-4B\left(\frac{A(c+dx)}{a+bx}+\frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx}-\frac{2B(c+dx)}{a+bx}\right)}{g^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

output
```
-((((c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(a + b*x) - 4*B*
((A*(c + d*x))/(a + b*x) - (2*B*(c + d*x))/(a + b*x) + (B*(c + d*x)*Log[(e
*(c + d*x)^2)/(a + b*x)^2])/(a + b*x)))/((b*c - a*d)*g^2))
```

3.215.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.215. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

3.215.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\frac{(A^2-4BA+8B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{2(A-2B)cB\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}+\frac{2d(A-2B)Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
parallelrisch	$-\frac{2A^2ab^2d^2-2A^2b^3cd+16B^2ab^2d^2-16B^2b^3cd-2B^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2b^3d^2+8B^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^3d^2-2B^2\ln\left(\frac{e(dx+}{(bx+}\right.}{2g^2(bx+a)b^3}$
parts	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}-\frac{4B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}-\frac{4B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}-\frac{2A}{}$
derivativedivides	$-\frac{-\frac{A^2}{g^2(bx+a)}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{g^2(bx+a)}+\frac{8B^2}{g^2(bx+a)}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}+\frac{4B^2d\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(ad-cb)}}{}$
default	$-\frac{-\frac{A^2}{g^2(bx+a)}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{g^2(bx+a)}+\frac{8B^2}{g^2(bx+a)}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}+\frac{4B^2d\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(ad-cb)}}{}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}-\frac{4B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}-\frac{4B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}-\frac{2A}{}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOS
E)
```

output
```
((A^2-4*A*B+8*B^2)/g/a*x+B^2*c/g/(a*d-b*c)*ln(e*(d*x+c)^2/(b*x+a)^2)^2+B^2
*d/g/(a*d-b*c)*x*ln(e*(d*x+c)^2/(b*x+a)^2)^2+2*(A-2*B)*c*B/g/(a*d-b*c)*ln(
e*(d*x+c)^2/(b*x+a)^2)+2*d*(A-2*B)*B/g/(a*d-b*c)*x*ln(e*(d*x+c)^2/(b*x+a)^
2))/g/(b*x+a)
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$

$$-\frac{(A^2-4\,AB+8\,B^2)bc-(A^2-4\,AB+8\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2+2\,((AB-2}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="f
ricas")
```

output
```
-((A^2 - 4*A*B + 8*B^2)*b*c - (A^2 - 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2
*b*c)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2
*((A*B - 2*B^2)*b*d*x + (A*B - 2*B^2)*b*c)*log((d^2*e*x^2 + 2*c*d*e*x + c^
2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2
*b*d)*g^2)
```

3.215.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(134) = 268$.

Time = 1.20 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$$

$$= \frac{4Bd(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd-\frac{4Ba^2d^3(A-2B)}{ad-bc}+\frac{8Babcd^2(A-2B)}{ad-bc}-\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\,(ad-bc)}$$

$$-\frac{4Bd(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd+\frac{4Ba^2d^3(A-2B)}{ad-bc}-\frac{8Babcd^2(A-2B)}{ad-bc}+\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\,(ad-bc)}$$

$$+\frac{(-2AB+4B^2)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x}+\frac{(B^2c+B^2dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2+4AB-8B^2}{abg^2+b^2g^2x}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**2,x)
```

output
```
4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d - 8*B**2*a*d**2 - 8*B*
*2*b*c*d - 4*B*a**2*d**3*(A - 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A - 2*B)/
(a*d - b*c) - 4*B*b**2*c**2*d*(A - 2*B)/(a*d - b*c))/(8*A*B*b*d**2 - 16*B*
*2*b*d**2))/(b*g**2*(a*d - b*c)) - 4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 +
 4*A*B*b*c*d - 8*B**2*a*d**2 - 8*B**2*b*c*d + 4*B*a**2*d**3*(A - 2*B)/(a*d
 - b*c) - 8*B*a*b*c*d**2*(A - 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A - 2*B)
/(a*d - b*c))/(8*A*B*b*d**2 - 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*
A*B + 4*B**2)*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) +
(B**2*c + B**2*d*x)*log(e*(c + d*x)**2/(a + b*x)**2)**2/(a**2*d*g**2 - a*b
*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 + 4*A*B - 8*B**2)/(a*b*g*
*2 + b**2*g**2*x)
```

3.215.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(157) = 314$.

Time = 0.24 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.65

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$=4\left(\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{}{b^2x^2}\right.\right.$$
$$-2\,AB\left(\frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^2g^2x+abg^2}-\frac{2}{b^2g^2x+abg^2}-\frac{2\,d\log(bx+a)}{(b^2c-abd)g^2}+\frac{2\,d\log(dx}{(b^2c-a}\right.$$
$$-\frac{B^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="m
axima")
```

output
```
4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) +
 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) +
((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a
*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x
 + a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2
)*x))*B^2 - 2*A*B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b
*g^2) - 2/(b^2*g^2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) +
 2*d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d^2*e*x^2/(b^2*x^2 + 2*
a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*
b*x + a^2))^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(157) = 314$.

Time = 0.65 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.47

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx =$$
$$-\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{(bgx+ag)bg}\right)\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)^2}+\frac{a^2d^2eg^2}{(bgx+ag)^2}+\frac{2\,bcdeg}{bgx+ag}-\frac{2\,ad^2eg}{bgx+ag}+d^2e}{b^2}\right)^2$$
$$-\frac{4\,(ABd-2\,B^2d)\log\left(\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right)}{b^2cg^2-abdg^2}$$
$$-\frac{2\,(AB-2\,B^2)\log\left(\frac{\frac{b^2c^2eg^2}{(bgx+ag)^2}-\frac{2\,abcdeg^2}{(bgx+ag)^2}+\frac{a^2d^2eg^2}{(bgx+ag)^2}+\frac{2\,bcdeg}{bgx+ag}-\frac{2\,ad^2eg}{bgx+ag}+d^2e}{b^2}\right)}{(bgx+ag)bg}-\frac{A^2-4\,AB+8\,B^2}{(bgx+ag)bg}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="g
iac")
```

output
```
-(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log((b^2*c^2*e*
g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g
*x + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*
e)/b^2)^2 - 4*(A*B*d - 2*B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a
*g) + d)/(b^2*c*g^2 - a*b*d*g^2) - 2*(A*B - 2*B^2)*log((b^2*c^2*e*g^2/(b*g
*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2/(b*g*x + a*g
)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) + d^2*e)/b^2)/
((b*g*x + a*g)*b*g) - (A^2 - 4*A*B + 8*B^2)/((b*g*x + a*g)*b*g)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 3.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = \frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\,\left(\frac{4\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}} - \frac{A^2-4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right) + \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-2\,B)\,8\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^2,x)
```

output
```
(log((e*(c + d*x)^2)/(a + b*x)^2)*((4*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^
2)))/(x/d + a/(b*d)) - (A^2 + 8*B^2 - 4*A*B)/(b^2*g^2*x + a*b*g^2) - log((
e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a
*d - b*c))) + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(
a*d - b*c))*(A - 2*B)*8i)/(b*g^2*(a*d - b*c))
```

3.216 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

3.216.1 Optimal result

Integrand size = 34, antiderivative size = 299

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{4ABd(c+dx)}{(bc-ad)^2g^3(a+bx)}+\frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2}-\frac{4B^2d(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)^2g^3(a+bx)} + \frac{bB(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)^2} + \frac{d(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

output
```
-4*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3
/(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-4*B^2*d*(d*x+c)*ln(e*(
d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)^2/g^3/(b*x+a)+b*B*(d*x+c)^2*(A+B*ln(e*(d*x+
c)^2/(b*x+a)^2))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)^2/
(b*x+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2
/(b*x+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

3.216.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.51

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2-\frac{2B\left(4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

output
```
-1/2*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(4*B*d*(a + b*x)*(
b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c
 - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x]
+ 2*d^2*(a + b*x)^2*Log[c + d*x]) + (b*c - a*d)^2*(A + B*Log[(e*(c + d*x)^
2)/(a + b*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) + 2*d^2*(a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) - 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/
(b*g^3*(a + b*x)^2)
```

3.216.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2767$$

$$\frac{\int \left(d\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2-\frac{b(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{a+bx}\right)d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{(a+bx)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2(a+bx)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{a+bx}-\frac{4ABd(c+dx)}{a+bx}-\frac{4B^2d(c+}{}}{g^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

output
```
((-4*A*B*d*(c + d*x))/(a + b*x) + (8*B^2*d*(c + d*x))/(a + b*x) - (b*B^2*(
c + d*x)^2)/(a + b*x)^2 - (4*B^2*d*(c + d*x)*Log[(e*(c + d*x)^2)/(a + b*x)
^2])/(a + b*x) + (b*B*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])
)/(a + b*x)^2 + (d*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(
a + b*x) - (b*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*(
a + b*x)^2))/((b*c - a*d)^2*g^3)
```

3.216.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

3.216. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.216.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.64

method	result
norman	$\frac{\frac{(A^2ad-A^2bc-4ABad+2ABbc+8B^2ad-2B^2bc)x}{ag(ad-cb)}+\frac{Bc(2Aad-Abc-4Bad+Bbc)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(a^2d^2-2abcd+b^2c^2)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(a^2d^2-2abcd+b^2c^2)}+\frac{bBd\cdots}{\cdots}}{\cdots}$
derivativedivides	$-\frac{\frac{A^2}{2g^3(bx+a)^2}+\frac{B^2}{g^3(bx+a)^2}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^3(bx+a)^2}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{2g^3(bx+a)^2}+\frac{6B^2d}{g^3(ad-cb)(bx+a)}+\frac{3B^2d^2\ln\left(\cdots\right.}{g^3\left(a\cdots\right.}}{\cdots}$
default	$-\frac{\frac{A^2}{2g^3(bx+a)^2}+\frac{B^2}{g^3(bx+a)^2}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^3(bx+a)^2}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{2g^3(bx+a)^2}+\frac{6B^2d}{g^3(ad-cb)(bx+a)}+\frac{3B^2d^2\ln\left(\cdots\right.}{g^3\left(a\cdots\right.}}{\cdots}$
parts	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b(7B^2ad-B^2bc)x^2}{a^2g(ad-cb)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(a^2d^2-2abcd+b^2c^2)}+\frac{2(4B^2ad-B^2bc)x}{ag(ad-cb)}+\frac{B^2c(2ad-cb)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{2g(a^2d^2-2abcd+b^2c^2)}-\frac{(4ad\cdots}{\cdots}}{g^2\cdots}$
parallelrisch	$-\frac{-2A^2ab^4cd^2-6ABa^2b^3d^3-2ABb^5c^2d-16B^2ab^4cd^2-2B^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)b^5c^2d-4ABx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)ab^4d^3-4AB\cdots}{\cdots}$
risch	$-\frac{A^2}{2g^3(bx+a)^2b}+\frac{\frac{b(7B^2ad-B^2bc)x^2}{a^2g(ad-cb)}+\frac{B^2ad^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(a^2d^2-2abcd+b^2c^2)}+\frac{2(4B^2ad-B^2bc)x}{ag(ad-cb)}+\frac{B^2c(2ad-cb)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{2g(a^2d^2-2abcd+b^2c^2)}-\frac{(4ad\cdots}{\cdots}}{g^2\cdots}$

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOS
E)
```

output
```
((A^2*a*d-A^2*b*c-4*A*B*a*d+2*A*B*b*c+8*B^2*a*d-2*B^2*b*c)/a/g/(a*d-b*c)*x
+B*c*(2*A*a*d-A*b*c-4*B*a*d+B*b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(d*x
+c)^2/(b*x+a)^2)+B^2*a*d^2/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)^2/
(b*x+a)^2)^2+b*B/g*d^2*(A-3*B)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^2*ln(e*(d*x+c
)^2/(b*x+a)^2)+1/2*B^2*c*(2*a*d-b*c)/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(d
*x+c)^2/(b*x+a)^2)^2+1/2*(A^2*a*d-A^2*b*c-6*A*B*a*d+2*A*B*b*c+14*B^2*a*d-2
*B^2*b*c)/a^2/g*b/(a*d-b*c)*x^2+2*B/g*d*(A*a*d-2*B*a*d-B*b*c)/(a^2*d^2-2*a
*b*c*d+b^2*c^2)*x*ln(e*(d*x+c)^2/(b*x+a)^2)+1/2*b*B^2*d^2/(a^2*d^2-2*a*b*c
*d+b^2*c^2)/g*x^2*ln(e*(d*x+c)^2/(b*x+a)^2)^2)/g^2/(b*x+a)^2
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.38

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{(A^2-2\,AB+2\,B^2)b^2c^2-2\,(A^2-4\,AB+8\,B^2)abcd+(A^2-6\,AB+14\,B^2)a^2d^2-(B^2b^2d^2x^2+2\,B^2\cdots}{\cdots}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="f
ricas")
```

output
```
-1/2*((A^2 - 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 - 4*A*B + 8*B^2)*a*b*c*d + (A
^2 - 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^
2*c^2 + 2*B^2*a*b*c*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*
b*x + a^2))^2 + 4*((A*B - 3*B^2)*b^2*c*d - (A*B - 3*B^2)*a*b*d^2)*x - 2*((
A*B - 3*B^2)*b^2*d^2*x^2 - (A*B - B^2)*b^2*c^2 + 2*(A*B - 2*B^2)*a*b*c*d -
 2*(B^2*b^2*c*d - (A*B - 2*B^2)*a*b*d^2)*x)*log((d^2*e*x^2 + 2*c*d*e*x + c
^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g
^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2
- 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

3.216.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(279) = 558$.

Time = 2.19 (sec) , antiderivative size = 877, normalized size of antiderivative = 2.93

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$$

$$= \frac{2Bd^2(A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-6B^2ad^3-6B^2bcd^2-\frac{2Ba^3d^5(A-3B)}{(ad-bc)^2}+\frac{6Ba^2bcd^4(A-3B)}{(ad-bc)^2}-\frac{6Bab^2c^2d^3(A-3B)}{(ad-bc)^2}+\frac{2Bb^3c^3d^2(A-}{(ad-bc)^2}}{4ABbd^3-12B^2bd^3}\right.}{bg^3(ad-bc)^2}$$

$$-\frac{2Bd^2(A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-6B^2ad^3-6B^2bcd^2+\frac{2Ba^3d^5(A-3B)}{(ad-bc)^2}-\frac{6Ba^2bcd^4(A-3B)}{(ad-bc)^2}+\frac{6Bab^2c^2d^3(A-3B)}{(ad-bc)^2}-\frac{2Bb^3c^3d^2}{(ad-b}}{4ABbd^3-12B^2bd^3}\right.}{bg^3(ad-bc)^2}$$

$$+\frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+}$$

$$+\frac{(-ABad+ABbc+3B^2ad-B^2bc+2B^2bdx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a^3bdg^3-a^2b^2cg^3+2a^2b^2dg^3x-2ab^3cg^3x+ab^3dg^3x^2-b^4cg^3x^2}$$

$$+\frac{-A^2ad+A^2bc+6ABad-2ABbc-14B^2ad+2B^2bc+x(4ABbd-12B^2bd)}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x(4a^2b^2dg^3-4ab^3cg^3)}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**3,x)
```

output
```
2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3
- 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A - 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c*
d**4*(A - 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)
**2 + 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2
*b*d**3))/(b*g**3*(a*d - b*c)**2) - 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d*
*3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3 - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A -
 3*B)/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 + 6*B*a*
b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A - 3*B)/(a*
d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2
*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c +
 d*x)**2/(a + b*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b
*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**
2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*
c**2*g**3*x**2) + (-A*B*a*d + A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d
*x)*log(e*(c + d*x)**2/(a + b*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2
*a**2*b**2*d*g**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3
*x**2) + (-A**2*a*d + A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 14*B**2*a*d + 2*B
**2*b*c + x*(4*A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g*
*3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*
b**3*c*g**3))
```

3.216.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(297) = 594$.

Time = 0.28 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.35

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\left(\left(\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,d^2\log\left(bx + a\right)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{2\,c}{(b^3c^2 - \cdots}\right.\right.\right.$$

$$-AB\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \cdots\right.}{b^3g^3x^2 + 2\,ab^2g^3x + a^2b\cdots}\right.$$

$$-\frac{B^2\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right)^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)} - \frac{A^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="m
axima")
```

output
```
-(((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b
^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 -
2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*g^3))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b
^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (b^2*c^2 - 8*
a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)
^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d -
 a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*
b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2
*d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a
^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(
a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - A*B*((2*b*d
*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3
*x + (a^2*b^2*c - a^3*b*d)*g^3) + log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)
+ 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(
b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 -
2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*g^3)) - 1/2*B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*
c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^
3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2...
```

3.216.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="g
iac")
```

output
```
integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^3, x)
```

3.216.9 Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.69

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$=\frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(\frac{2\,B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,(a\,d-b\,c)}{b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$-\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)$$
$$-\frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c-6\,A\,B\,a\,d+2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{2\,x\,(3\,B^2\,b\,d-A\,B\,b\,d)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}$$
$$-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)(A-3\,B)\,2\mathrm{i}}{(a\,d-b\,c)\,(6\,B^2\,d^2-2\,A\,B\,d^2)}\right)(A-3\,B)\,4\mathrm{i}}{b\,g^3\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^3,x)
```

output
```
(log((e*(c + d*x)^2)/(a + b*x)^2)*((2*B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x)^
2)/(a + b*x)^2)^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*
b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((A^2*a*d - A^2*b*c + 14*B^2*a*d
 - 2*B^2*b*c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (2*x*(3*B^2*b*d -
A*B*b*d))/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*d^2*
atan((B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*
(A - 3*B)*2i)/((a*d - b*c)*(6*B^2*d^2 - 2*A*B*d^2)))*(A - 3*B)*4i)/(b*g^3*
(a*d - b*c)^2)
```

3.217 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.217.1 Optimal result

Integrand size = 34, antiderivative size = 407

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)} + \frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} + \frac{4B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4} + \frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)} - \frac{2bBd(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^3g^4(a+bx)^2} + \frac{4b^2B(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^3g^4} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3}$$

output
```
-8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)
^3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+4/3*B^2
*d^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+4*B*d^2*(d*x+c)*(A+B*ln(e*(d
*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)-2*b*B*d*(d*x+c)^2*(A+B*ln(e*(
d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2+4/9*b^2*B*(d*x+c)^3*(A+B*l
n(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-4/3*B*d^3*ln((d*x+c)/
(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*
(d*x+c)^2/(b*x+a)^2))^2/b/g^4/(b*x+a)^3
```

3.217.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{-9\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 + \frac{2B\left(6A(bc-ad)^3-4B(bc-ad)^3-9Ad(bc-ad)^2(a+bx)+15Bd(bc-ad)^2(a+bx)+18Ad^2(bc-ad)(a+bx)^2\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

output
```
(-9*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(6*A*(b*c - a*d)^3 -
 4*B*(b*c - a*d)^3 - 9*A*d*(b*c - a*d)^2*(a + b*x) + 15*B*d*(b*c - a*d)^2*
(a + b*x) + 18*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(-(b*c) + a*d)*(a
+ b*x)^2 + 18*A*d^3*(a + b*x)^3*Log[a + b*x] - 66*B*d^3*(a + b*x)^3*Log[a
+ b*x] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 - 18*A*d^3*(a + b*x)^3*Log[c
+ d*x] + 66*B*d^3*(a + b*x)^3*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(d*(
a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 18*B*d^3*(a + b*x)^3*Log[c + d*x]
^2 - 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 6*
B*(b*c - a*d)^3*Log[(e*(c + d*x)^2)/(a + b*x)^2] - 9*B*d*(b*c - a*d)^2*(a
+ b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 18*B*d^2*(b*c - a*d)*(a + b*x)^2
*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 18*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[
(e*(c + d*x)^2)/(a + b*x)^2] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]*Log[(e*(c
 + d*x)^2)/(a + b*x)^2] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)] - 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
))/(b*c - a*d)^3)/(27*b*g^4*(a + b*x)^3)
```

3.217.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2952, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2d\frac{c+dx}{a+bx}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2756}$$

$$-\frac{\frac{4B\int \frac{(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{c+dx}d\frac{c+dx}{a+bx}}{3b}-\frac{\left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b}}{g^4(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$-\frac{\frac{4B\left(-2B\int \left(\frac{d^3(a+bx)\log\left(\frac{c+dx}{a+bx}\right)}{c+dx}-\frac{1}{6}b\left(18d^2-\frac{9b(c+dx)d}{a+bx}+\frac{2b^2(c+dx)^2}{(a+bx)^2}\right)\right)d\frac{c+dx}{a+bx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{(a+bx)}\right.\right.}{2(a+bx)^2}}{3b}}{g^4(bc-ad}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{4B\left(-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2(a+bx)^2}+d^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-\frac{3bd^2(c+dx)\left(B\log\left(\frac{e(c+d}{(a+b}\right.\right.}{a+bx}}{3b}}{g^4(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

output
```
-((-1/3*((d - (b*(c + d*x))/(a + b*x))^3*(A + B*Log[(e*(c + d*x)^2)/(a + b
*x)^2])^2)/b + (4*B*(-2*B*((-3*b*d^2*(c + d*x))/(a + b*x) + (3*b^2*d*(c +
d*x)^2)/(4*(a + b*x)^2) - (b^3*(c + d*x)^3)/(9*(a + b*x)^3) + (d^3*Log[(c
+ d*x)/(a + b*x)]^2)/2) - (3*b*d^2*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a
 + b*x)^2]))/(a + b*x) + (3*b^2*d*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(
a + b*x)^2]))/(2*(a + b*x)^2) - (b^3*(c + d*x)^3*(A + B*Log[(e*(c + d*x)^2
)/(a + b*x)^2]))/(3*(a + b*x)^3) + d^3*Log[(c + d*x)/(a + b*x)]*(A + B*Log
[(e*(c + d*x)^2)/(a + b*x)^2])))/(3*b))/((b*c - a*d)^3*g^4))
```

3.217.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.217. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.217.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.71

method	result
derivativedivides	$-\frac{\frac{A^2}{3g^4(bx+a)^3}+\frac{8B^2}{27g^4(bx+a)^3}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{9g^4(bx+a)^3}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{3g^4(bx+a)^3}+\frac{10B^2d}{9g^4(ad-cb)(bx+a)^2}+\frac{}{9g^4(}$
default	$-\frac{\frac{A^2}{3g^4(bx+a)^3}+\frac{8B^2}{27g^4(bx+a)^3}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{9g^4(bx+a)^3}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{3g^4(bx+a)^3}+\frac{10B^2d}{9g^4(ad-cb)(bx+a)^2}+\frac{}{9g^4(}$
parts	Expression too large to display
parallelrisch	Expression too large to display
norman	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/b*(1/3/g^4*A^2/(b*x+a)^3+8/27/g^4*B^2/(b*x+a)^3-4/9/g^4*B^2/(b*x+a)^3*l
n(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/3/g^4*B^2/(b*x+a)^3*ln(e*(a*d/(b*
x+a)-b*c/(b*x+a)-d)^2/b^2)^2+10/9/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2+44/9/g^4*B
^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)+22/9/g^4*d^3*B^2/(a^3*d^3-3*a^2
*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/
3/g^4*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(b*x
+a)-b*c/(b*x+a)-d)^2/b^2)^2-2/3/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2*ln(e*(a*d/(b
*x+a)-b*c/(b*x+a)-d)^2/b^2)-4/3/g^4*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b
*x+a)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+2/g^4*A*B*(1/3/(b*x+a)^3*ln(
e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-(2/3*a*d-2/3*c*b)*(1/(a*d-b*c)^3*(1/3
*a^2*d^2/(b*x+a)^3-2/3*a*b*c*d/(b*x+a)^3+1/3*b^2*c^2/(b*x+a)^3+1/2*a*d^2/(
b*x+a)^2-1/2*b*c*d/(b*x+a)^2+d^2/(b*x+a))+d^3/(a*d-b*c)^4*ln(a*d/(b*x+a)-b
*c/(b*x+a)-d))))
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.77

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{(9A^2-12AB+8B^2)b^3c^3-27(A^2-2AB+2B^2)ab^2c^2d+27(A^2-4AB+8B^2)a^2bcd^2-(9A^2-}{}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="f
ricas")
```

output
```
-1/27*((9*A^2 - 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 - 2*A*B + 2*B^2)*a*b^2*c
^2*d + 27*(A^2 - 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 - 66*A*B + 170*B^2)*a
^3*d^3 - 12*((3*A*B - 11*B^2)*b^3*c*d^2 - (3*A*B - 11*B^2)*a*b^2*d^3)*x^2
+ 9*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c
^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d^2*e*x^2 + 2*c*d*e*x + c
^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 6*((3*A*B - 5*B^2)*b^3*c^2*d - 18*(A*
B - 3*B^2)*a*b^2*c*d^2 + (15*A*B - 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B - 11*B
^2)*b^3*d^3*x^3 + (3*A*B - 2*B^2)*b^3*c^3 - 9*(A*B - B^2)*a*b^2*c^2*d + 9*
(A*B - 2*B^2)*a^2*b*c*d^2 - 3*(2*B^2*b^3*c*d^2 - 3*(A*B - 3*B^2)*a*b^2*d^3
)*x^2 + 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 3*(A*B - 2*B^2)*a^2*b*d^3)*
x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c
^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3
 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c
^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3
 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

3.217.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. 2(382) = 764.

Time = 13.02 (sec) , antiderivative size = 1561, normalized size of antiderivative = 3.84

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**4,x)
```

output
```
4*B*d**3*(3*A - 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a
*d**4 - 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A - 11*B)/(a*d - b*c)**3 + 16*
B*a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*
A - 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)*
*3 - 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B
**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) - 4*B*d**3*(3*A - 11*B)*log(x + (12
*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*d**4 - 44*B**2*b*c*d**3 + 4*B*a*
*4*d**7*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A - 11*B)/(a*d
 - b*c)**3 + 24*B*a**2*b**2*c**2*d**5*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a
*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A - 11
*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c
)**3) + (3*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B
**2*a*b*d**3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)*
*2/(a + b*x)**2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d*
*3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*
b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x -
 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*
c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3
 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*
x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 22*B**2*...
```

3.217.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. $2(397) = 794$.

Time = 0.32 (sec) , antiderivative size = 1576, normalized size of antiderivative = 3.87

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="m
axima")
```

output
```
2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) +
6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3
)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3)*g^4))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*
x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - (4*b^3*c^3 - 27*
a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^
2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x +
 a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d
*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^
3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^
3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^
3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))
/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*
g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3
*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 -
 a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b
^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 + 2/9*A*B*((6*b^2*d^2*x^2 + 2*b...
```

3.217.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="g
iac")
```

output
```
integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^4, x)
```

3.217.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.63

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{\frac{9A^2a^2d^2-18A^2abcd+9A^2b^2c^2-66ABa^2d^2+42ABabcd-12ABb^2c^2+170B^2a^2d^2-46B^2abcd+8B^2b^2c^2}{3(ad-bc)} + \frac{2x(-5cB^2b^2d+49a}{}}{x\,(27a^2b^3cg^4-27a^3b^2dg^4)-x^2\,(27a^2b^3dg^4-27ab^4cg^4)+x^3\,(9b^5cg^4-9ab^4}$$
$$-\ln\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2\left(\frac{B^2}{3b^2g^4\left(3a^2x+\frac{a^3}{b}+b^2x^3+3abx^2\right)} - \frac{B^2d^3}{3bg^4\,(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}\right)$$
$$-\frac{\ln\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\left(\frac{2AB}{3b^2dg^4}-\frac{2B^2d^3\left(a\left(\frac{3a^2d^2-4abcd+b^2c^2}{3bd^3}+\frac{2a(ad-bc)}{3bd^2}\right)+\frac{2(3a^3d^3-6a^2bcd^2+4ab^2c^2d-b^3c^3)}{3bd^4}\right)}{3bg^4(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}+\frac{2B^2d^3x^2}{3bg^4(a^3d^3}\right.}{\frac{3a^2x}{d}+\frac{a^3}{bd}+\frac{b^2x^3}{d}}$$
$$-\frac{Bd^3\,\mathrm{atan}\left(\frac{Bd^3\left(\frac{a^3bd^3g^4-a^2b^2cd^2g^4-ab^3c^2dg^4+b^4c^3g^4}{a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4}+2bdx\right)(3A-11B)\,(a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4)\,4i}{bg^4\,(ad-bc)^3\,(44B^2d^3-12ABd^3)}\right)(3A-11}{9bg^4\,(ad-bc)^3}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^4,x)
```

output
```
((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 - 66*A*B
*a^2*d^2 - 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d + 42*A*B*a*b*c
*d)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 15*A*B*a*b*d^
2 + 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d - 3*A*B*b^2*d))/(
a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g
^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4
 - 9*a^4*b*d*g^4) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(3*b^2*g^4*(3*
a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x)^2)/(a + b*x)^2)
*((2*A*B)/(3*b^2*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)
/(3*b*d^3) + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*
b^2*c^2*d - 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) + (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2)
- (4*b*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) - (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*
b*d^3) + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*
d))/(3*d^3) + (4*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (
3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c
^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*...
```

3.217. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

3.218 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.218.1 Optimal result

Integrand size = 34, antiderivative size = 501

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)} - \frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2} + \frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3} - \frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4} - \frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{b(bc-ad)^4g^5} - \frac{4Bd^3(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)} + \frac{3bBd^2(c+dx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^4g^5(a+bx)^2} - \frac{4b^2Bd(c+dx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3(bc-ad)^4g^5(a+bx)^3} + \frac{b^3B(c+dx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4(bc-ad)^4g^5(a+bx)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^4g^5} - \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4}$$

output
```
8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c
)^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b
^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-B^2*d^4*ln((d*x+c)/(b*x+a))^2/
b/(-a*d+b*c)^4/g^5-4*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b
*c)^4/g^5/(b*x+a)+3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*
d+b*c)^4/g^5/(b*x+a)^2-4/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2
))/(-a*d+b*c)^4/g^5/(b*x+a)^3+1/4*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)^2/(b*x
+a)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^4+B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*
x+c)^2/(b*x+a)^2))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^
2/b/g^5/(b*x+a)^4
```

3.218.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.36

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$$
$$= \frac{-18\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 + \frac{B\left(18A(bc-ad)^4-9B(bc-ad)^4+28Bd(bc-ad)^3(a+bx)+24Ad(-bc+ad)^3(a+bx)+36Ad^2(bc-ad)^2(a\cdots\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

output
```
(-18*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (B*(18*A*(b*c - a*d)^4 -
 9*B*(b*c - a*d)^4 + 28*B*d*(b*c - a*d)^3*(a + b*x) + 24*A*d*(-(b*c) + a*d
)^3*(a + b*x) + 36*A*d^2*(b*c - a*d)^2*(a + b*x)^2 - 78*B*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 300*B*d^3*(b*c - a*d)*(a + b*x)^3 + 72*A*d^3*(-(b*c) + a*d
)*(a + b*x)^3 - 72*A*d^4*(a + b*x)^4*Log[a + b*x] + 300*B*d^4*(a + b*x)^4*
Log[a + b*x] - 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 72*A*d^4*(a + b*x)^4*
Log[c + d*x] - 300*B*d^4*(a + b*x)^4*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*
Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 72*B*d^4*(a + b*x)^4*Log[
c + d*x]^2 + 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a
*d)] + 18*B*(b*c - a*d)^4*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 24*B*d*(-(b*c
) + a*d)^3*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 36*B*d^2*(b*c - a*
d)^2*(a + b*x)^2*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 72*B*d^3*(-(b*c) + a*d
)*(a + b*x)^3*Log[(e*(c + d*x)^2)/(a + b*x)^2] - 72*B*d^4*(a + b*x)^4*Log[
a + b*x]*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 72*B*d^4*(a + b*x)^4*Log[c + d
*x]*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)] + 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)]))/(b*c - a*d)^4)/(72*b*g^5*(a + b*x)^4)
```

3.218.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2952, 2756, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \left(d-\frac{b(c+dx)}{a+bx}\right)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2d\frac{c+dx}{a+bx}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2756}$$

$$\frac{B\int \frac{(a+bx)\left(d-\frac{b(c+dx)}{a+bx}\right)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{c+dx}d\frac{c+dx}{a+bx}-\frac{\left(d-\frac{b(c+dx)}{a+bx}\right)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b}}{g^5(bc-ad)^4}$$

$$\Big\downarrow \text{2772}$$

$$\frac{B\left(-2B\int\left(\frac{(c+dx)^3b^4}{4(a+bx)^3}-\frac{4d(c+dx)^2b^3}{3(a+bx)^2}+\frac{3d^2(c+dx)b^2}{a+bx}-4d^3b+\frac{d^4(a+bx)\log\left(\frac{c+dx}{a+bx}\right)}{c+dx}\right)d\frac{c+dx}{a+bx}+\frac{b^4(c+dx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4(a+bx)^4}-\frac{4b^3d(c+dx)^3\left(B\log\left(\frac{e(}{(}\right.\right.}{3(a+bx)^3}}{b}$$

$\downarrow$ 2009

$$\frac{B\left(\frac{b^4(c+dx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4(a+bx)^4}-\frac{4b^3d(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d^2(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{(a+bx)^2}+d^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.\right.}{b}$$

input
```
Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

output
```
(-1/4*((d - (b*(c + d*x))/(a + b*x))^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x
)^2])^2)/b + (B*(-2*B*((-4*b*d^3*(c + d*x))/(a + b*x) + (3*b^2*d^2*(c + d*
x)^2)/(2*(a + b*x)^2) - (4*b^3*d*(c + d*x)^3)/(9*(a + b*x)^3) + (b^4*(c +
d*x)^4)/(16*(a + b*x)^4) + (d^4*Log[(c + d*x)/(a + b*x)]^2)/2) - (4*b*d^3*
(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(a + b*x) + (3*b^2*d^2
*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(a + b*x)^2 - (4*b^
3*d*(c + d*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*(a + b*x)^3)
+ (b^4*(c + d*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(4*(a + b*x)^
4) + d^4*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])
))/b)/((b*c - a*d)^4*g^5)
```

3.218.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

3.218. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.218.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.77

method	result
derivativedivides	$-\frac{\frac{A^2}{4g^5(bx+a)^4}+\frac{B^2}{8g^5(bx+a)^4}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4g^5(bx+a)^4}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{4g^5(bx+a)^4}+\frac{25B^2d^3}{6g^5\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2}}{}$
default	$-\frac{\frac{A^2}{4g^5(bx+a)^4}+\frac{B^2}{8g^5(bx+a)^4}-\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4g^5(bx+a)^4}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{4g^5(bx+a)^4}+\frac{25B^2d^3}{6g^5\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2}}{}$
parts	Expression too large to display
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOS
E)
```

output

```
-1/b*(1/4/g^5*A^2/(b*x+a)^4+1/8/g^5*B^2/(b*x+a)^4-1/4/g^5*B^2/(b*x+a)^4*ln
(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/4/g^5*B^2/(b*x+a)^4*ln(e*(a*d/(b*x
+a)-b*c/(b*x+a)-d)^2/b^2)^2+25/6/g^5*B^2*d^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^
2*c^2*d-b^3*c^3)/(b*x+a)+7/18/g^5*d*B^2/(a*d-b*c)/(b*x+a)^3+13/12/g^5*d^2*
B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2+25/12/g^5*d^4*B^2/(a^4*d^4-4*a^3
*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(e*(a*d/(b*x+a)-b*c/(b
*x+a)-d)^2/b^2)-1/4/g^5*d^4*B^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4
*a*b^3*c^3*d+b^4*c^4)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)^2-1/g^5*B^2*
d^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)*ln(e*(a*d/(b*x+a
)-b*c/(b*x+a)-d)^2/b^2)-1/3/g^5*d*B^2/(a*d-b*c)/(b*x+a)^3*ln(e*(a*d/(b*x+a
)-b*c/(b*x+a)-d)^2/b^2)-1/2/g^5*d^2*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a
)^2*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+2/g^5*A*B*(1/4/(b*x+a)^4*ln(e*
(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-(1/2*a*d-1/2*c*b)*(1/(a*d-b*c)^4*(1/4*(
a*d-b*c)*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4+1/3*d*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/(b*x+a)^3+1/2*(a*d-b*c)*d^2/(b*x+a)^2+d^3/(b*x+a))+d^4/(a*d-b*c)^5*
ln(a*d/(b*x+a)-b*c/(b*x+a)-d))))
```

3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. 2(491) = 982.

Time = 0.29 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.17

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{9\,(2\,A^2-2\,AB+B^2)b^4c^4-8\,(9\,A^2-12\,AB+8\,B^2)ab^3c^3d+108\,(A^2-2\,AB+2\,B^2)a^2b^2c^2d^2-72\,(A}{}$$

input

```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="f
ricas")
```

output
```
-1/72*(9*(2*A^2 - 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 - 12*A*B + 8*B^2)*a*b^3*
c^3*d + 108*(A^2 - 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 - 4*A*B + 8*B^
2)*a^3*b*c*d^3 + (18*A^2 - 150*A*B + 415*B^2)*a^4*d^4 + 12*((6*A*B - 25*B^
2)*b^4*c*d^3 - (6*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((6*A*B - 13*B^2)*b^4*c
^2*d^2 - 16*(3*A*B - 11*B^2)*a*b^3*c*d^3 + (42*A*B - 163*B^2)*a^2*b^2*d^4)
*x^2 - 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d
^2 + 4*B^2*a^3*b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a
*b*x + a^2))^2 + 4*((6*A*B - 7*B^2)*b^4*c^3*d - 12*(3*A*B - 5*B^2)*a*b^3*c
^2*d^2 + 108*(A*B - 3*B^2)*a^2*b^2*c*d^3 - (78*A*B - 271*B^2)*a^3*b*d^4)*x
 - 6*((6*A*B - 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B - B^2)*b^4*c^4 + 8*(3*A*B -
2*B^2)*a*b^3*c^3*d - 36*(A*B - B^2)*a^2*b^2*c^2*d^2 + 24*(A*B - 2*B^2)*a^3
*b*c*d^3 - 4*(3*B^2*b^4*c*d^3 - 2*(3*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2
*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 6*(A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B
^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 6*(A*B - 2*B^2
)*a^3*b*d^4)*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a
^2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^
4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 -
4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d
+ 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b...
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

3.218.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2278 vs. 2(491) = 982.

Time = 0.40 (sec) , antiderivative size = 2278, normalized size of antiderivative = 4.55

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="m
axima")
```

output
```
-1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5
*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4
*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b
^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6
*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b
*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2
*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 -
 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*lo
g(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2
) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a
^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^
4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(
b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^
4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2
+ 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*
d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^
4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25
*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*...
```

3.218.8 Giac [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.76

$$\int \frac{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag + bgx)^5}\,dx$$

$$= \frac{1}{4}\left(\frac{B^2d^4}{b^5c^4g^5 - 4\,ab^4c^3dg^5 + 6\,a^2b^3c^2d^2g^5 - 4\,a^3b^2cd^3g^5 + a^4bd^4g^5} - \frac{B^2}{(bgx+ag)^4bg}\right)\log\left(\frac{b^2c^2eg^2}{(bgx+ag)^2} - \frac{2\,abcde}{(bgx+a}\right.$$

$$-\frac{1}{12}\left(\frac{12\,B^2d^3}{(b^3c^3g^3 - 3\,ab^2c^2dg^3 + 3\,a^2bcd^2g^3 - a^3d^3g^3)(bgx+ag)bg} - \frac{6\,B^2d^2}{(b^2c^2g - 2\,abcdg + a^2d^2g)(bgx+ag)^2}\right.$$

$$+\frac{(6\,ABd^4 - 25\,B^2d^4)\log\left(-\frac{bcg}{bgx+ag} + \frac{adg}{bgx+ag} - d\right)}{6\,(b^5c^4g^5 - 4\,ab^4c^3dg^5 + 6\,a^2b^3c^2d^2g^5 - 4\,a^3b^2cd^3g^5 + a^4bd^4g^5)}$$

$$-\frac{6\,ABd^3 - 25\,B^2d^3}{6\,(b^3c^3g^3 - 3\,ab^2c^2dg^3 + 3\,a^2bcd^2g^3 - a^3d^3g^3)(bgx+ag)bg}$$

$$+\frac{6\,ABbd^2 - 13\,B^2bd^2}{12\,(b^2c^2g - 2\,abcdg + a^2d^2g)(bgx+ag)^2b^2g^2}$$

$$-\frac{6\,ABb^2dg - 7\,B^2b^2dg}{18\,(bgx+ag)^3(bc-ad)b^3g^3} - \frac{2\,A^2b^3g^3 - 2\,ABb^3g^3 + B^2b^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

input

```
integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="g
iac")
```

output
```
1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*
a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log((b^2*c
^2*e*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^2 + a^2*d^2*e*g^2
/(b*g*x + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*g/(b*g*x + a*g) +
 d^2*e)/b^2)^2 - 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^
2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g -
2*a*b*c*d*g + a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3
*(b*c - a*d)*b*g^2) + 3*(2*A*B*b^3*g^3 - B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4
*g^4))*log((b^2*c^2*e*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*e*g^2/(b*g*x + a*g)^
2 + a^2*d^2*e*g^2/(b*g*x + a*g)^2 + 2*b*c*d*e*g/(b*g*x + a*g) - 2*a*d^2*e*
g/(b*g*x + a*g) + d^2*e)/b^2) + 1/6*(6*A*B*d^4 - 25*B^2*d^4)*log(-b*c*g/(b
*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 +
6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - 1/6*(6*A*B*
d^3 - 25*B^2*d^3)/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 -
a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/12*(6*A*B*b*d^2 - 13*B^2*b*d^2)/((b^2*
c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(b*g*x + a*g)^2*b^2*g^2) - 1/18*(6*A*B*b^
2*d*g - 7*B^2*b^2*d*g)/((b*g*x + a*g)^3*(b*c - a*d)*b^3*g^3) - 1/8*(2*A^2*
b^3*g^3 - 2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

3.218.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 1882, normalized size of antiderivative = 3.76

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^5,x)
```

output
```
(log((e*(c + d*x)^2)/(a + b*x)^2)*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 -
5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 +
 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^
2*b^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B
)/(2*b^2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6
*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(
3*d^3) + (a*(a*d - b*c))/d^2) - a*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c
))/d^2) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(2*d^3)))/(2*b*g^5*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d
^4*x^3*(b*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c - a*b^2
*d)/(2*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^
3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*
c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*
d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c
*d)/(3*d^3) + (a*(a*d - b*c))/d^2) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d
- 10*a^2*b*c*d^2)/(2*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)
/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - log((e*(c + d*x)^2)/(a + b*x)...
```

3.218. $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

3.219 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

3.219.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \operatorname{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

3.219.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

3.219.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
$Aborted
```

3.219.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.219.4 Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\left(bgx+ag\right)^{2}}{A+B\ln\left(\frac{e\left(dx+c\right)^{2}}{\left(bx+a\right)^{2}}\right)}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

3.219.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d^2*e*x^2 + 2*c*d*e
*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A), x)
```

3.219.6 Sympy [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 258, normalized size of antiderivative = 7.59

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = g^2\left(\int \frac{a^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \frac{b^2x^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \frac{2abx}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right)$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
g**2*(Integral(a**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d
*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**
2))), x) + Integral(b**2*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**
2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x
+ b**2*x**2))), x) + Integral(2*a*b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x +
b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
2*a*b*x + b**2*x**2))), x))
```

3.219.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

3.219.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{(bgx+ag)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

3.219.9 Mupad [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

3.220 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

3.220.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

3.220.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

3.220.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{ag+bgx}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

output
```
$Aborted
```

3.220.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.220.4 Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

3.220.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="frica
s")
```

output
```
integral((b*g*x + a*g)/(B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2
*a*b*x + a^2)) + A), x)
```

3.220.6 Sympy [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.16

$$\begin{aligned}&\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx\\&=g\left(\int \frac{a}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right.\\&\qquad\left.+\int \frac{bx}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right)\end{aligned}$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
g*(Integral(a/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(
a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))),
x) + Integral(b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e
*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)
)), x))
```

3.220.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxim
a")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

3.220.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac"
)
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

3.220.9 Mupad [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx=\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

3.221 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

3.221.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

3.221.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

3.221.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
$Aborted
```

3.221.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.221.4 Maple [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

3.221.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fri
cas")
```

output
```
integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d^2*e*x^2 + 2*c*d*e*x
 + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

3.221.6 Sympy [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$=\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bbx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{g}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
Integral(1/(A*a + A*b*x + B*a*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*
c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2)) + B*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 +
 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)/g
```

3.221.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="max
ima")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

3.221.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gia
c")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

3.221.9 Mupad [N/A]

Not integrable

Time = 4.97 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.222 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

3.222.1 Optimal result

Integrand size = 34, antiderivative size = 91

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = -\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

output
```
-1/2*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)/exp(1/
2*A/B)/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

3.222.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

3.222.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2952, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{2952}$$

$$-\frac{\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{2737}$$

$$-\frac{(c+dx)\int \frac{\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2g^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
-1/2*((c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*
B)])/(B*(b*c - a*d)*E^(A/(2*B))*g^2*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*
x)^2])
```

3.222.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.222.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

3.222.5 Fricas [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="f
ricas")
```

output
```
integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2
*B*a*b*g^2*x + B*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2
*a*b*x + a^2))), x)
```

3.222.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}}}{g^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2
*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2
/(a**2 + 2*a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b*
*2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*
a*b*x + b**2*x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2)
+ 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b
**2*x**2))), x)/g**2
```

3.222.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

3.222.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.223 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

3.223.1 Optimal result

Integrand size = 34, antiderivative size = 151

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B(bc-ad)^2eg^3}$$

output
```
-1/2*b*Ei((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3
+1/2*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/ex
p(1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

3.223.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

3.223.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(\frac{d}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}-\frac{b(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\right)d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2Be}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

output
```
((d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)
])/(2*B*E^(A/(2*B))*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)^2]) - (b*ExpI
ntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/B])/(2*B*e*E^(A/B)))/((
b*c - a*d)^2*g^3)
```

3.223.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.223.4 Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{3} \left(A+B\ln\left(\frac{e\left(dx+c\right)^{2}}{\left(bx+a\right)^{2}}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

3.223.5 Fricas [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="f
ricas")
```

output
```
integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^
3 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(
(d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

3.223.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$
$$= \frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+3Ba^2bx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\cdots\right.}}{\cdots}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

output
```
Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3
*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**
2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a**2*b*x*log(c**
2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2)
+ d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a*b**2*x**2*log(c**2*e/(
a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**
2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + B*b**3*x**3*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/
(a**2 + 2*a*b*x + b**2*x**2))), x)/g**3
```

3.223.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="m
axima")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

3.223.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.224 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

3.224.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.224.2 Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

3.224.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
$Aborted
```

3.224.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.224.4 Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.224.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.68

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d^2*e*x^2 + 2*c*d
*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*
e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A^2), x)
```

3.224.6 Sympy [N/A]

Not integrable

Time = 16.08 (sec) , antiderivative size = 792, normalized size of antiderivative = 23.29

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$
$$=\frac{-a^3cg^2-a^3dg^2x-3a^2bcg^2x-3a^2bdg^2x^2-3ab^2cg^2x^2-3ab^2dg^2x^3-b^3cg^2x^3-b^3dg^2x^4}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}$$
$$+\frac{g^2\left(\int \frac{a^3d}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{3a^2bc}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,d\right.}{}$$

input
```
integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-a**3*c*g**2 - a**3*d*g**2*x - 3*a**2*b*c*g**2*x - 3*a**2*b*d*g**2*x**2 -
 3*a*b**2*c*g**2*x**2 - 3*a*b**2*d*g**2*x**3 - b**3*c*g**2*x**3 - b**3*d*g
**2*x**4)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c + d*
x)**2/(a + b*x)**2)) + g**2*(Integral(a**3*d/(A + B*log(c**2*e/(a**2 + 2*a
*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*a**2*b*c/(A + B*log(c**2*e/
(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d*
*2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**3*c*x**2/(A +
 B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b
**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(4*b*
*3*d*x**3/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2
 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) +
 Integral(6*a*b**2*c*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*
c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2))), x) + Integral(9*a*b**2*d*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x +
 b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
 2*a*b*x + b**2*x**2))), x) + Integral(6*a**2*b*d*x/(A + B*log(c**2*e/(a**
2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e
*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x))/(2*B*(a*d - b*c))
```

3.224.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 312, normalized size of antiderivative = 9.18

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
-1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b
^2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d
)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b
*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*
g^2 + a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2
*b*d*g^2)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x +
 c) - (b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2), x)
```

3.224.8 Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(bgx+ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

3.224.9 Mupad [N/A]

Not integrable

Time = 9.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.225 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

3.225.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.225.2 Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

3.225.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

input
```
Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

output
```
$Aborted
```

3.225.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.225.4 Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.225.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.34

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fri
cas")
```

output
```
integral((b*g*x + a*g)/(B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 +
 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 +
2*a*b*x + a^2)) + A^2), x)
```

3.225.6 Sympy [N/A]

Not integrable

Time = 10.73 (sec) , antiderivative size = 559, normalized size of antiderivative = 17.47

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\frac{-a^2cg-a^2dgx-2abcgx-2abdgx^2-b^2cgx^2-b^2dgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}$$
$$+\frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{2abc}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right.}{}$$

input
```
integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b
**2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c
+ d*x)**2/(a + b*x)**2)) + g*(Integral(a**2*d/(A + B*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/
(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(c**2*e/(a
**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2
*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*lo
g(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x
**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**2*d*
x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*
a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Inte
gral(4*a*b*d*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/
(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))),
 x))/(2*B*(a*d - b*c))
```

3.225.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 234, normalized size of antiderivative = 7.31

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="max
ima")
```

output
```
-1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2
*d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c)
- (b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b^2*
d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c
*log(e) - a*d*log(e))*B^2), x)
```

3.225.8 Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag + bgx}{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{bgx + ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gia
c")
```

output
```
integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

3.225.9 Mupad [N/A]

Not integrable

Time = 11.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{ag + bgx}{\left(A + B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{a\,g + b\,g\,x}{\left(A + B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

output
```
int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.226 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

3.226.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.226.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

output
```
Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x]
```

3.226.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

output
```
$Aborted
```

3.226.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.226.4 Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.226.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.79

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="f
ricas")
```

output
```
integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*e*x^2 + 2
*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*l
og((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

3.226.6 Sympy [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.65

$$\begin{aligned}&\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx\\&= \frac{-c-dx}{2ABadg-2ABbcg+(2B^2adg-2B^2bcg)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\\&\quad+\frac{d\int \frac{1}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{2Bg\,(ad-bc)}\end{aligned}$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-c - d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(
e*(c + d*x)**2/(a + b*x)**2)) + d*Integral(1/(A + B*log(c**2*e/(a**2 + 2*a
*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

3.226.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="m
axima")
```

output
```
d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^
2*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2),
 x) - 1/2*(d*x + c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g
)*B^2*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B
^2)
```

3.226.8 Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="g
iac")
```

output
```
integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

3.226.9 Mupad [N/A]

Not integrable

Time = 13.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.227 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

3.227.1 Optimal result

Integrand size = 34, antiderivative size = 147

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = -\frac{e^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} + \frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

output
```
1/2*(d*x+c)/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))-1/4*(
d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)/exp(1/2*A/
B)/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

3.227.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x
]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),
x]
```

3.227.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2952, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2952}$$

$$-\frac{\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}d\frac{c+dx}{a+bx}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2734}$$

$$-\frac{\frac{\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{2B}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \text{ 2737}$$

$$-\frac{\frac{(c+dx)\int \frac{\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4B(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

$$\Big\downarrow \ 2609$$

$$-\frac{\frac{e^{-\frac{A}{2B}}(c+dx)\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{c+dx}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^2(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

output
```
-((((c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)
])/(4*B^2*E^(A/(2*B))*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)^2]) - (c +
d*x)/(2*B*(a + b*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/((b*c - a*d
)*g^2))
```

3.227.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

3.227. $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.227.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.227.5 Fricas [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2
*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/
(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*
a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))),
x)
```

3.227.6 Sympy [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$
$$= \frac{-c-dx}{2ABa^2dg^2-2ABabcg^2+2ABabdg^2x-2ABb^2cg^2x+(2B^2a^2dg^2-2B^2abcg^2+2B^2abdg^2x-2B^2b^2cg^2x)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}$$
$$+\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+\cdots}}{2Bg^2}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
(-c - d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*
A*B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d
*g**2*x - 2*B**2*b**2*c*g**2*x)*log(e*(c + d*x)**2/(a + b*x)**2)) + Integr
al(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2*a*b*x
 + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2
 + 2*a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**
2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x
+ b**2*x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*
d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x*
*2))), x)/(2*B*g**2)
```

3.227.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"maxima")
```

output
```
1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2
*log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^
2*log(e))*B^2)*x - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g
^2)*B^2)*log(b*x + a) + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^
2*d*g^2)*B^2)*log(d*x + c)) + integrate(1/2/(B^2*a^2*g^2*log(e) + A*B*a^2*
g^2 + (B^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B
*a*b*g^2)*x - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x
+ a) + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)),
x)
```

3.227.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.228 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

3.228.1 Optimal result

Integrand size = 34, antiderivative size = 206

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \frac{de^{-\frac{A}{2B}}(c+dx)\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2eg^3} + \frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

output
```
-1/2*b*Ei((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g
^3+1/2*(d*x+c)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))+
1/4*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/e
xp(1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

3.228.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} \, dx = \int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} \, dx$$

input
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x
]
```

output
```
Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),
x]
```

3.228.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2952, 2757, 2737, 2609, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^3 \left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2} \, dx$$

$$\Big\downarrow \text{ 2952}$$

$$\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}d\frac{c+dx}{a+bx}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2757}$$

$$\frac{-\frac{d\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{2B}+\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^3(bc-ad)^2}$$

$$\Big\downarrow \text{ 2737}$$

$$\frac{-\frac{d(c+dx)\int \frac{\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4B(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}+\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^3(bc-ad)^2}$$

$\Big\downarrow$ 2609

$$\frac{\frac{\int \frac{d-\frac{b(c+dx)}{a+bx}}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}d\frac{c+dx}{a+bx}}{B}-\frac{de^{-\frac{A}{2B}}(c+dx)\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{(c+dx)\left(d-\frac{b(c+dx)}{a+bx}\right)}{2B(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}}{g^3(bc-ad)^2}$$

$\Big\downarrow$ 2767

$$\frac{\frac{\int \left(\frac{d}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}-\frac{b(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\right)d\frac{c+dx}{a+bx}}{B}-\frac{de^{-\frac{A}{2B}}(c+dx)\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{(c+dx)\left(d-b\right.}{2B(a+bx)\left(B\log\left(e\right.\right.}}{g^3(bc-ad)^2}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{de^{-\frac{A}{2B}}(c+dx)\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}+\frac{\frac{de^{-\frac{A}{2B}}(c+dx)\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\text{ExpIntegralEi}\left(\frac{A+B\log\left(\frac{e(c\right.\right.}{B}}{2Be}}{B}}{g^3(bc-ad)^2}$$

input
```
Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

output
```
(-1/4*(d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/
(2*B)])/(B^2*E^(A/(2*B))*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)^2]) + ((
d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)])
/(2*B*E^(A/(2*B))*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)^2]) - (b*ExpInt
egralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/B])/(2*B*e*E^(A/B)))/B -
((c + d*x)*(d - (b*(c + d*x))/(a + b*x)))/(2*B*(a + b*x)*(A + B*Log[(e*(c
+ d*x)^2)/(a + b*x)^2])))/((b*c - a*d)^2*g^3)
```

3.228. $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

3.228.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2757
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
 1), x], x] + Simp[d*(q/(b*n*(p + 1)))   Int[(d + e*x)^(q - 1)*(a + b*Log[c
*x^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && Gt
Q[q, 0]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

3.228. $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

3.228.4 Maple [F]

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}dx$$

input
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

output
```
int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

3.228.5 Fricas [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"fricas")
```

output
```
integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^
2*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B
^2*a^3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))
^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^
3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

output
```
Timed out
```

3.228.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"maxima")
```

output
```
1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d
*g^3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*
b^2*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c
*g^3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^
2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^
2)*log(b*x + a) + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 -
a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integr
ate(-1/2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^
3*log(e) - a*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B +
(a^3*b*c*g^3*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*
g^3)*A*B + (a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*
b^2*c*g^3 - a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))
*B^2)*x - 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*
d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^
4*d*g^3)*B^2)*log(b*x + a) + 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b
^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x
+ (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

3.228.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2}\,dx$$

input
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

output
```
int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.229 $\int \frac{1}{(ag+bgx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

3.229.1 Optimal result

Integrand size = 36, antiderivative size = 96

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

output
```
exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei((-A-B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

3.229.2 Mathematica [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

input
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]
```

3.229.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2973, 2949, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ag+bgx)^2 (B\log(e(a+bx)^n(c+dx)^{-n})+A)}dx$$
$$\Big\downarrow \text{2973}$$
$$\int \frac{1}{(ag+bgx)^2 (B\log(e(a+bx)^n(c+dx)^{-n})+A)}dx$$
$$\Big\downarrow \text{2949}$$
$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{g^2(bc-ad)}$$
$$\Big\downarrow \text{2747}$$
$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2n(a+bx)(bc-ad)}$$
$$\Big\downarrow \text{2609}$$
$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

input
```
Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
(E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a +
b*x))
```

3.229.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

3.229.4 Maple [F]

$$\int \frac{1}{\left(bgx+ag\right)^{2} \left(A+B\ln\left(e\left(bx+a\right)^{n} \left(dx+c\right)^{-n}\right)\right)}dx$$

input
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ag+bgx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"fricas")
```

output
```
e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/
(b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

3.229.7 Maxima [F]

$$\int \frac{1}{(ag+bgx)^2 (A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"maxima")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

3.229.8 Giac [F]

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm=
"giac")
```

output
```
integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}\,dx$$

input
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
```

output
```
int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

3.230 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.230.1 Optimal result

Integrand size = 27, antiderivative size = 355

$$\begin{aligned}
&\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx \\
&= \frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c^2}{5b^4d^4} \\
&\quad -\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,x^2}{10b^3d^3} \\
&\quad -\frac{B(bc-ad)g^3(5bdf-bcg-adg)x^3}{15b^2d^2}-\frac{B(bc-ad)g^4x^4}{20bd}-\frac{B(bf-ag)^5\log(a+bx)}{5b^5g} \\
&\quad +\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}+\frac{B(df-cg)^5\log(c+dx)}{5d^5g}
\end{aligned}$$

output
```
1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*
g^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^
3*f^3))*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+
b^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*(-a*
d*g-b*c*g+5*b*d*f)*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*x^4/b/d-1/5*B*(-a*g+b
*f)^5*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/5*B*(-
c*g+d*f)^5*ln(d*x+c)/d^5/g
```

3.230.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{\frac{B(-bc+ad)g^2x(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg(6c^2g^2-3cdg(10f+gx)+d^2(60f^2+15fgx+2g^2x^2))+b^3(-12c^3g^3+6c^2dg^2(10f-}{12b^4d^4}}{}$$

input
```
Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*
g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 +
 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*
c*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f
*g^2*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*Log[a + b*x])/b^5
 + (f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(d*f - c*g)^5*Log
[c + d*x])/d^5)/(5*g)
```

3.230.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g}-\frac{B(bc-ad)\int\frac{(f+gx)^5}{(a+bx)(c+dx)}dx}{5g}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g}-$$
$$\frac{B(bc-ad)\int\left(\frac{x^3g^5}{bd}+\frac{(5bdf-bcg-adg)x^2g^4}{b^2d^2}+\frac{((10d^2f^2-5cdgf+c^2g^2)b^2-adg(5df-cg)b+a^2d^2g^2)xg^3}{b^3d^3}+\frac{((10d^3f^3-10cd^2gf^2+5c^2dg^2f}{}\right.}{5g}$$

$\downarrow$ 2009

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g}-$$
$$\frac{B(bc-ad)\left(\frac{g^3x^2(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2))}{2b^3d^3}-\frac{g^2x(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b^3(}{b^4d^4}\right.}{5g}$$

input
```
Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*g) - (B*(b*c - a*d)*
(-((g^2*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10*d^2*f^2
 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2
 - c^3*g^3))*x)/(b^4*d^4)) + (g^3*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b
^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(2*b^3*d^3) + (g^4*(5*b*d*f -
b*c*g - a*d*g)*x^3)/(3*b^2*d^2) + (g^5*x^4)/(4*b*d) + ((b*f - a*g)^5*Log[a
 + b*x])/(b^5*(b*c - a*d)) - ((d*f - c*g)^5*Log[c + d*x])/(d^5*(b*c - a*d)
)))/(5*g)
```

3.230.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] && !(EqQ[m, -2] && IntegerQ[n])
```

3.230.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.67

method	result
risch	$\frac{g^3 B a^3 f x}{b^3} - \frac{2g^2 B a^2 f^2 x}{b^2} + \frac{2gBa f^3 x}{b} - \frac{g^3 B c^3 f x}{d^3} + \frac{2g^2 B c^2 f^2 x}{d^2} - \frac{2gBc f^3 x}{d} + \frac{g^4 A x^5}{5} - \frac{B \ln(dx+c) c f^4}{d}$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*g^3*B*a^3*f*x-2/b^2*g^2*B*a^2*f^2*x+2/b*g*B*a*f^3*x-1/d^3*g^3*B*c^3*
f*x+2/d^2*g^2*B*c^2*f^2*x-2/d*g*B*c*f^3*x+1/5*g^4*A*x^5-1/d*B*ln(d*x+c)*c*
f^4+1/b*B*ln(-b*x-a)*a*f^4+1/5/b^5*g^4*B*ln(-b*x-a)*a^5-1/5/d^5*g^4*B*ln(d
*x+c)*c^5+1/3/b*g^3*B*a*f*x^3-1/3/d*g^3*B*c*f*x^3-1/2/b^2*g^3*B*a^2*f*x^2+
1/b*g^2*B*a*f^2*x^2+1/2/d^2*g^3*B*c^2*f*x^2-1/d*g^2*B*c*f^2*x^2-1/15/b^2*g
^4*B*a^2*x^3+1/15/d^2*g^4*B*c^2*x^3+2*g*A*f^3*x^2+1/10/b^3*g^4*B*a^3*x^2-1
/10/d^3*g^4*B*c^3*x^2+A*f^4*x+1/5*(g*x+f)^5*B/g*ln(e*(b*x+a)/(d*x+c))-1/5/
g*B*ln(-b*x-a)*f^5+1/5/g*B*ln(d*x+c)*f^5-1/5/b^4*g^4*B*a^4*x+1/5/d^4*g^4*B
*c^4*x-1/b^4*g^3*B*ln(-b*x-a)*a^4*f+2/b^3*g^2*B*ln(-b*x-a)*a^3*f^2-2/b^2*g
*B*ln(-b*x-a)*a^2*f^3+1/d^4*g^3*B*ln(d*x+c)*c^4*f-2/d^3*g^2*B*ln(d*x+c)*c^
3*f^2+2/d^2*g*B*ln(d*x+c)*c^2*f^3+g^3*A*f*x^4+1/20/b*g^4*B*a*x^4-1/20/d*g^
4*B*c*x^4+2*g^2*A*f^2*x^3
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.79

$$\int (f+gx)^4 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{12\,Ab^5d^5g^4x^5 + 3\,(20\,Ab^5d^5fg^3 - (Bb^5cd^4 - Bab^4d^5)g^4)x^4 + 4\,(30\,Ab^5d^5f^2g^2 - 5\,(Bb^5cd^4 - Bab^4d^5)fg^3}{}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^
4*d^5)*g^4)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*
f*g^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(20*A*b^5*d^5*f^3*g -
 10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5
)*f*g^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 12*(5*A*b^5*d^5*f^4 -
 10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)
*f^2*g^2 - 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + (B*b^5*c^4*d - B*a^4*
b*d^5)*g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*
b^2*d^5*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(
5*B*b^5*c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*
B*b^5*c^4*d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 12*(B*b^5*d^5*g^4*x^5 +
5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2
+ 5*B*b^5*d^5*f^4*x)*log((b*e*x + a*e)/(d*x + c)))/(b^5*d^5)
```

3.230.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. $2(337) = 674$.

Time = 55.68 (sec) , antiderivative size = 1436, normalized size of antiderivative = 4.05

$$\int (f+gx)^4 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{Ag^4x^5}{5}$$
$$+ \frac{Ba(a^4g^4 - 5a^3bfg^3 + 10a^2b^2f^2g^2 - 10ab^3f^3g + 5b^4f^4)\log\left(x + \frac{Ba^5cd^4g^4 - 5Ba^4bcd^4fg^3 + 10Ba^3b^2cd^4f^2g^2 - 10Ba\cdots}{\cdots}\right.}{\cdots}$$
$$- \frac{Bc(c^4g^4 - 5c^3dfg^3 + 10c^2d^2f^2g^2 - 10cd^3f^3g + 5d^4f^4)\log\left(x + \frac{Ba^5cd^4g^4 - 5Ba^4bcd^4fg^3 + 10Ba^3b^2cd^4f^2g^2 - 10Ba^2\cdots}{\cdots}\right.}{\cdots}$$
$$+ x^4\left(Afg^3 + \frac{Bag^4}{20b} - \frac{Bcg^4}{20d}\right) + x^3 \cdot \left(2Af^2g^2 - \frac{Ba^2g^4}{15b^2} + \frac{Bafg^3}{3b} + \frac{Bc^2g^4}{15d^2} - \frac{Bcfg^3}{3d}\right)$$
$$+ x^2 \cdot \left(2Af^3g + \frac{Ba^3g^4}{10b^3} - \frac{Ba^2fg^3}{2b^2} + \frac{Baf^2g^2}{b} - \frac{Bc^3g^4}{10d^3} + \frac{Bc^2fg^3}{2d^2} - \frac{Bcf^2g^2}{d}\right)$$
$$+ x\left(Af^4 - \frac{Ba^4g^4}{5b^4} + \frac{Ba^3fg^3}{b^3} - \frac{2Ba^2f^2g^2}{b^2} + \frac{2Baf^3g}{b} + \frac{Bc^4g^4}{5d^4} - \frac{Bc^3fg^3}{d^3} + \frac{2Bc^2f^2g^2}{d^2}\right.$$
$$\left. - \frac{2Bcf^3g}{d}\right) + \left(Bf^4x + 2Bf^3gx^2 + 2Bf^2g^2x^3 + Bfg^3x^4 + \frac{Bg^4x^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*g**4*x**5/5 + B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2
- 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b
*c*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f
**3*g + B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2
- 10*a*b**3*f**3*g + 5*b**4*f**4)/b + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4
*d*f*g**3 + 10*B*a*b**4*c**3*d**2*f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g
 + 10*B*a*b**4*c*d**4*f**4 - B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*
a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(B*a**5*d**5*g**4 -
 5*B*a**4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d
**5*f**3*g + 5*B*a*b**4*d**5*f**4 + B*b**5*c**5*g**4 - 5*B*b**5*c**4*d*f*g
**3 + 10*B*b**5*c**3*d**2*f**2*g**2 - 10*B*b**5*c**2*d**3*f**3*g + 5*B*b**
5*c*d**4*f**4))/(5*b**5) - B*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2
*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x + (B*a**5*c*d**4*g**4 -
 5*B*a**4*b*c*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b*
*3*c*d**4*f**3*g + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*g**3 + 10*B*a*
b**4*c**3*d**2*f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*B*a*b**4*c*d*
*4*f**4 - B*a*b**4*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2
 - 10*c*d**3*f**3*g + 5*d**4*f**4) + B*b**5*c**2*(c**4*g**4 - 5*c**3*d*f*g
**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)/d)/(B*a**5*
d**5*g**4 - 5*B*a**4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10...
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.67

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{5}Ag^4x^5 + Afg^3x^4 + 2Af^2g^2x^3$$
$$+ 2Af^3gx^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^4$$
$$+ 2\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bf^3g$$
$$+ \left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d}{b^2d^2}\right.$$
$$+ \frac{1}{6}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2c}{b^3}\right.$$
$$+ \frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4(}{}\right.$$
$$+ Af^4x$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^4
 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^
2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*f^3*g + (2*x^3*log(b*e*x/(d*x
+ c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 -
((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^2*g^2 +
 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4
+ 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d -
 a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/60*(12*x
^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5
*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*
b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(
b^4*d^4))*B*g^4 + A*f^4*x
```

3.230.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10664 vs. $2(341) = 682$.

Time = 1.23 (sec) , antiderivative size = 10664, normalized size of antiderivative = 30.04

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/60*(12*(5*B*b^6*c^2*d^4*e^6*f^4 - 10*B*a*b^5*c*d^5*e^6*f^4 + 5*B*a^2*b^4
*d^6*e^6*f^4 - 10*B*b^6*c^3*d^3*e^6*f^3*g + 10*B*a*b^5*c^2*d^4*e^6*f^3*g +
 10*B*a^2*b^4*c*d^5*e^6*f^3*g - 10*B*a^3*b^3*d^6*e^6*f^3*g + 10*B*b^6*c^4*
d^2*e^6*f^2*g^2 - 10*B*a*b^5*c^3*d^3*e^6*f^2*g^2 - 10*B*a^3*b^3*c*d^5*e^6*
f^2*g^2 + 10*B*a^4*b^2*d^6*e^6*f^2*g^2 - 5*B*b^6*c^5*d*e^6*f*g^3 + 5*B*a*b
^5*c^4*d^2*e^6*f*g^3 + 5*B*a^4*b^2*c*d^5*e^6*f*g^3 - 5*B*a^5*b*d^6*e^6*f*g
^3 + B*b^6*c^6*e^6*g^4 - B*a*b^5*c^5*d*e^6*g^4 - B*a^5*b*c*d^5*e^6*g^4 + B
*a^6*d^6*e^6*g^4 - 20*(b*e*x + a*e)*B*b^5*c^2*d^5*e^5*f^4/(d*x + c) + 40*(
b*e*x + a*e)*B*a*b^4*c*d^6*e^5*f^4/(d*x + c) - 20*(b*e*x + a*e)*B*a^2*b^3*
d^7*e^5*f^4/(d*x + c) + 50*(b*e*x + a*e)*B*b^5*c^3*d^4*e^5*f^3*g/(d*x + c)
 - 70*(b*e*x + a*e)*B*a*b^4*c^2*d^5*e^5*f^3*g/(d*x + c) - 10*(b*e*x + a*e)
*B*a^2*b^3*c*d^6*e^5*f^3*g/(d*x + c) + 30*(b*e*x + a*e)*B*a^3*b^2*d^7*e^5*
f^3*g/(d*x + c) - 50*(b*e*x + a*e)*B*b^5*c^4*d^3*e^5*f^2*g^2/(d*x + c) + 5
0*(b*e*x + a*e)*B*a*b^4*c^3*d^4*e^5*f^2*g^2/(d*x + c) + 30*(b*e*x + a*e)*B
*a^2*b^3*c^2*d^5*e^5*f^2*g^2/(d*x + c) - 10*(b*e*x + a*e)*B*a^3*b^2*c*d^6*
e^5*f^2*g^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^4*b*d^7*e^5*f^2*g^2/(d*x + c)
 + 25*(b*e*x + a*e)*B*b^5*c^5*d^2*e^5*f*g^3/(d*x + c) - 25*(b*e*x + a*e)*B
*a*b^4*c^4*d^3*e^5*f*g^3/(d*x + c) - 20*(b*e*x + a*e)*B*a^3*b^2*c^2*d^5*e^
5*f*g^3/(d*x + c) + 15*(b*e*x + a*e)*B*a^4*b*c*d^6*e^5*f*g^3/(d*x + c) + 5
*(b*e*x + a*e)*B*a^5*d^7*e^5*f*g^3/(d*x + c) - 5*(b*e*x + a*e)*B*b^5*c^...
```

3.230.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 1392, normalized size of antiderivative = 3.92

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
int((f + g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^
2 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(10*b*d) + ((5*a*d + 5*b*c)*((((5
*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d)
 - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^
4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A
*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4
+ 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(
5*a*d + 5*b*c))/(5*b*d)))/(2*b*d)) + x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a
*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(2
0*b*d)) + log((e*(a + b*x))/(c + d*x))*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*
g^2*x^3 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g
+ 20*A*b*c*f^3*g + 10*B*a*d*f^3*g - 10*B*b*c*f^3*g + 30*A*a*c*f^2*g^2)/(5*
b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g
^2 + 30*A*b*c*f^2*g^2 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(5*b*d) + ((5
*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A
*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5
*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5
*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*d) - (
a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/
(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((...
```

3.230. $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.231 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

3.231.1 Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx &= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{4b^3d^3} \\ &\quad -\frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{8b^2d^2}-\frac{B(bc-ad)g^3x^3}{12bd}-\frac{B(bf-ag)^4\log(a+bx)}{4b^4g} \\ &\quad +\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g}+\frac{B(df-cg)^4\log(c+dx)}{4d^4g}\end{aligned}$$

output
```
-1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f
*g+6*d^2*f^2))*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b
^2/d^2-1/12*B*(-a*d+b*c)*g^3*x^3/b/d-1/4*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/
4*(g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/4*B*(-c*g+d*f)^4*ln(d*x+c)/d^4
/g
```

3.231.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.95

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(6bd(bc-ad)g^2(a^2d^2g^2+abdg(-4df+cg)+b^2(6d^2f^2-4cdfg+c^2g^2))x+3b^2d^2(bc-ad)g^3(4bdf-}{6b^4d^4}}{4g}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(6*b*d*(b*c - a*d)*
g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c
^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*b
^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f
 - c*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

3.231.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\Big\downarrow \; 2948$$
$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-\frac{B(bc-ad)\int\frac{(f+gx)^4}{(a+bx)(c+dx)}dx}{4g}$$
$$\Big\downarrow \; 93$$
$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-$$
$$\frac{B(bc-ad)\int\left(\frac{x^2g^4}{bd}+\frac{(4bdf-bcg-adg)xg^3}{b^2d^2}+\frac{((6d^2f^2-4cdgf+c^2g^2)b^2-adg(4df-cg)b+a^2d^2g^2)g^2}{b^3d^3}+\frac{(bf-ag)^4}{b^3(bc-ad)(a+bx)}+\frac{(df-cg)}{d^3(ad-bc)(c}\right.}{4g}$$
$$\Big\downarrow \; 2009$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-\frac{B(bc-ad)\left(\frac{g^2x(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2))}{b^3d^3}+\frac{(bf-ag)^4\log(a+bx)}{b^4(bc-ad)}+\frac{g^3x^2(-adg-bcg+4bdf)}{2b^2d^2}-\frac{(df-cg)^4\log(c+d}{d^4(bc-ad)}\right.}{4g}$$

input
```
Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*g) - (B*(b*c - a*d)*
((g^2*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
c^2*g^2))*x)/(b^3*d^3) + (g^3*(4*b*d*f - b*c*g - a*d*g)*x^2)/(2*b^2*d^2) +
 (g^4*x^3)/(3*b*d) + ((b*f - a*g)^4*Log[a + b*x])/(b^4*(b*c - a*d)) - ((d*
f - c*g)^4*Log[c + d*x])/(d^4*(b*c - a*d))))/(4*g)
```

3.231.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.231. $\int(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.231.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.81

method	result
risch	$\frac{(gx+f)^4 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{4g}+\frac{g^3 A x^4}{4}-\frac{B \ln(-dx-c) c f^3}{d}+\frac{g^2 B a f x^2}{2b}-\frac{g^2 B c f x^2}{2d}-\frac{g^2 B a^2 f x}{b^2}+\frac{3gBa f^2 x}{2b}+$
parallelrisch	$\frac{-24Bx a^2 b^2 d^4 f g^2+36Bxa b^3 d^4 f^2 g-36B \ln\left(\frac{e(bx+a)}{dx+c}\right) b^4 c^2 d^2 f^2 g+36B \ln(bx+a) b^4 c^2 d^2 f^2 g+24B \ln(bx+a) a^3 b d^4 f g^2-}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/4*(g*x+f)^4*B/g*ln(e*(b*x+a)/(d*x+c))+1/4*g^3*A*x^4-1/d*B*ln(-d*x-c)*c*f
^3+1/2/b*g^2*B*a*f*x^2-1/2/d*g^2*B*c*f*x^2-1/b^2*g^2*B*a^2*f*x+3/2/b*g*B*a
*f^2*x+1/d^2*g^2*B*c^2*f*x-3/2/d*g*B*c*f^2*x+1/4/g*B*ln(-d*x-c)*f^4-1/4/g*
B*ln(b*x+a)*f^4+1/b*B*ln(b*x+a)*a*f^3+1/4/d^4*g^3*B*ln(-d*x-c)*c^4-1/4/b^4
*g^3*B*ln(b*x+a)*a^4+1/4/b^3*g^3*B*a^3*x-1/4/d^3*g^3*B*c^3*x-1/d^3*g^2*B*l
n(-d*x-c)*c^3*f+3/2/d^2*g*B*ln(-d*x-c)*c^2*f^2+1/b^3*g^2*B*ln(b*x+a)*a^3*f
-3/2/b^2*g*B*ln(b*x+a)*a^2*f^2+g^2*A*f*x^3+1/12/b*g^3*B*a*x^3-1/12/d*g^3*B
*c*x^3+3/2*g*A*f^2*x^2-1/8/b^2*g^3*B*a^2*x^2+1/8/d^2*g^3*B*c^2*x^2+A*f^3*x
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(215) = 430.

Time = 0.38 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.96

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$=\frac{6\,Ab^4d^4g^3x^4+2\,(12\,Ab^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3)x^3+3\,(12\,Ab^4d^4f^2g-4\,(Bb^4cd^3-Bab^3d^4)fg^2+}{}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3
*d^4)*g^3)*x^3 + 3*(12*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g
^2 + (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(4*A*b^4*d^4*f^3 - 6*(B*
b^4*c*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 -
 (B*b^4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d
^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*
c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*l
og(d*x + c) + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f
^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^4)
```

3.231.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(207) = 414$.

Time = 8.13 (sec) , antiderivative size = 998, normalized size of antiderivative = 4.40

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ag^3x^4}{4}$$
$$-\frac{Ba(ag-2bf)\left(a^2g^2-2abfg+2b^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+\frac{Ba^2d^4(ag-2bf)\left(a^2g^2-2abfg+2b\cdots\right.}{b}\cdots}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g\cdots}\right.\cdots}{4b^4}$$
$$+\frac{Bc(cg-2df)\left(c^2g^2-2cdfg+2d^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+Bab^3c^4g^3-4Bab^3c^3dfg^2+6Bab\cdots}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g-\cdots}\right.\cdots}{4d^4}$$
$$+x^3\left(Afg^2+\frac{Bag^3}{12b}-\frac{Bcg^3}{12d}\right)+x^2\cdot\left(\frac{3Af^2g}{2}-\frac{Ba^2g^3}{8b^2}+\frac{Bafg^2}{2b}+\frac{Bc^2g^3}{8d^2}-\frac{Bcfg^2}{2d}\right)$$
$$+x\left(Af^3+\frac{Ba^3g^3}{4b^3}-\frac{Ba^2fg^2}{b^2}+\frac{3Baf^2g}{2b}-\frac{Bc^3g^3}{4d^3}+\frac{Bc^2fg^2}{d^2}-\frac{3Bcf^2g}{2d}\right)$$
$$+\left(Bf^3x+\frac{3Bf^2gx^2}{2}+Bfg^2x^3+\frac{Bg^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*lo
g(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**
3*f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)
/b + B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*
f**2*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*
a*b*f*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a
**2*b**2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*
c**3*d*f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*b**4
) + B*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**
4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B
*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g
 - 8*B*a*b**3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*
g + 2*d**2*f**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d*
*2*f**2)/d)/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**
4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**
2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*d**4) + x**3*(A*
f*g**2 + B*a*g**3/(12*b) - B*c*g**3/(12*d)) + x**2*(3*A*f**2*g/2 - B*a**2*
g**3/(8*b**2) + B*a*f*g**2/(2*b) + B*c**2*g**3/(8*d**2) - B*c*f*g**2/(2*d)
) + x*(A*f**3 + B*a**3*g**3/(4*b**3) - B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/(
2*b) - B*c**3*g**3/(4*d**3) + B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/(2*d)) ...
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.83

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{1}{4}\,Ag^3x^4 + Afg^2x^3 + \frac{3}{2}\,Af^2gx^2$$
$$+ \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)Bf^3$$
$$+ \frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log\left(bx+a\right)}{b^2}+\frac{c^2\log\left(dx+c\right)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bf^2g$$
$$+\frac{1}{2}\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6\,x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6\,a^4\log\left(bx+a\right)}{b^4}+\frac{6\,c^4\log\left(dx+c\right)}{d^4}-\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2}{}\right.$$
$$+Af^3x$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^3 + 3/2*(x^2*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)
/d^2 - (b*c - a*d)*x/(b*d))*B*f^2*g + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/24*(6*x^4
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log
(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

3.231.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6073 vs. $2(215) = 430$.

Time = 0.79 (sec) , antiderivative size = 6073, normalized size of antiderivative = 26.75

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/24*(6*(4*B*b^5*c^2*d^3*e^5*f^3 - 8*B*a*b^4*c*d^4*e^5*f^3 + 4*B*a^2*b^3*d
^5*e^5*f^3 - 6*B*b^5*c^3*d^2*e^5*f^2*g + 6*B*a*b^4*c^2*d^3*e^5*f^2*g + 6*B
*a^2*b^3*c*d^4*e^5*f^2*g - 6*B*a^3*b^2*d^5*e^5*f^2*g + 4*B*b^5*c^4*d*e^5*f
*g^2 - 4*B*a*b^4*c^3*d^2*e^5*f*g^2 - 4*B*a^3*b^2*c*d^4*e^5*f*g^2 + 4*B*a^4
*b*d^5*e^5*f*g^2 - B*b^5*c^5*e^5*g^3 + B*a*b^4*c^4*d*e^5*g^3 + B*a^4*b*c*d
^4*e^5*g^3 - B*a^5*d^5*e^5*g^3 - 12*(b*e*x + a*e)*B*b^4*c^2*d^4*e^4*f^3/(d
*x + c) + 24*(b*e*x + a*e)*B*a*b^3*c*d^5*e^4*f^3/(d*x + c) - 12*(b*e*x + a
*e)*B*a^2*b^2*d^6*e^4*f^3/(d*x + c) + 24*(b*e*x + a*e)*B*b^4*c^3*d^3*e^4*f
^2*g/(d*x + c) - 36*(b*e*x + a*e)*B*a*b^3*c^2*d^4*e^4*f^2*g/(d*x + c) + 12
*(b*e*x + a*e)*B*a^3*b*d^6*e^4*f^2*g/(d*x + c) - 16*(b*e*x + a*e)*B*b^4*c^
4*d^2*e^4*f*g^2/(d*x + c) + 16*(b*e*x + a*e)*B*a*b^3*c^3*d^3*e^4*f*g^2/(d*
x + c) + 12*(b*e*x + a*e)*B*a^2*b^2*c^2*d^4*e^4*f*g^2/(d*x + c) - 8*(b*e*x
 + a*e)*B*a^3*b*c*d^5*e^4*f*g^2/(d*x + c) - 4*(b*e*x + a*e)*B*a^4*d^6*e^4*
f*g^2/(d*x + c) + 4*(b*e*x + a*e)*B*b^4*c^5*d*e^4*g^3/(d*x + c) - 4*(b*e*x
 + a*e)*B*a*b^3*c^4*d^2*e^4*g^3/(d*x + c) - 4*(b*e*x + a*e)*B*a^3*b*c^2*d^
4*e^4*g^3/(d*x + c) + 4*(b*e*x + a*e)*B*a^4*c*d^5*e^4*g^3/(d*x + c) + 12*(
b*e*x + a*e)^2*B*b^3*c^2*d^5*e^3*f^3/(d*x + c)^2 - 24*(b*e*x + a*e)^2*B*a*
b^2*c*d^6*e^3*f^3/(d*x + c)^2 + 12*(b*e*x + a*e)^2*B*a^2*b*d^7*e^3*f^3/(d*
x + c)^2 - 30*(b*e*x + a*e)^2*B*b^3*c^3*d^4*e^3*f^2*g/(d*x + c)^2 + 54*(b*
e*x + a*e)^2*B*a*b^2*c^2*d^5*e^3*f^2*g/(d*x + c)^2 - 18*(b*e*x + a*e)^2...
```

3.231.9 Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 741, normalized size of antiderivative = 3.26

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g-6\,B\,b\,c\,f^2\,g}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c}{}\right.}{4\,b\,d}$$

$$\left.-\frac{a\,c\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\,g^2+12\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2-4\,B\,b\,c\,f\,g^2}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)$$

$$+x^3\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{12\,b\,d}-\frac{A\,g^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\frac{A\,g^3\,x^4}{4}-\frac{\ln(a+b\,x)\,\left(B\,a^4\,g^3-4\,B\,a^3\,b\,f\,g^2+6\,B\,a^2\,b^2\,f^2\,g-4\,B\,a\,b^3\,f^3\right)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,c^4\,g^3-4\,B\,c^3\,d\,f\,g^2+6\,B\,c^2\,d^2\,f^2\,g-4\,B\,c\,d^3\,f^3\right)}{4\,d^4}$$

input
```
int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a
*d*f^2*g - 6*B*b*c*f^2*g)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 + 4*A
*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d
 + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g
^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b
*d) + (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a
*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*
b*d)))/(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3
+ 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*
c))/(8*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^
2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(8*b*d) + (A*a*c*g^3)/(2*b*d)) + log(
(e*(a + b*x))/(c + d*x))*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 + B*
f*g^2*x^3) + x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*
A*b*d*f*g^2)/(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + (A*g^3*x^4)/4
- (log(a + b*x)*(B*a^4*g^3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b
*f*g^2))/(4*b^4) + (log(c + d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*
f^2*g - 4*B*c^3*d*f*g^2))/(4*d^4)
```

3.231. $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.232 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.232.1 Optimal result

Integrand size = 27, antiderivative size = 150

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = -\frac{B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{6bd} - \frac{B(bf-ag)^3\log(a+bx)}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} + \frac{B(df-cg)^3\log(c+dx)}{3d^3g}$$

output
```
-1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^2*
x^2/b/d-1/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)
/(d*x+c)))/g+1/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

3.232.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B\left(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3\log\right.}{2b^3d^3}}{3g}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(2*b*d*(b*c - a*d)*
g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b*f
 - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3))/(
3*g)
```

3.232.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \; 2948$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{B(bc-ad)\int\frac{(f+gx)^3}{(a+bx)(c+dx)}dx}{3g}$$

$$\Big\downarrow \; 93$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{B(bc-ad)\int\left(\frac{xg^3}{bd}+\frac{(3bdf-bcg-adg)g^2}{b^2d^2}+\frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)}+\frac{(df-cg)^3}{d^2(ad-bc)(c+dx)}\right)dx}{3g}$$

$$\Big\downarrow \; 2009$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{B(bc-ad)\left(\frac{(bf-ag)^3\log(a+bx)}{b^3(bc-ad)}+\frac{g^2x(-adg-bcg+3bdf)}{b^2d^2}-\frac{(df-cg)^3\log(c+dx)}{d^3(bc-ad)}+\frac{g^3x^2}{2bd}\right)}{3g}$$

input
```
Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*g) - (B*(b*c - a*d)*
((g^2*(3*b*d*f - b*c*g - a*d*g)*x)/(b^2*d^2) + (g^3*x^2)/(2*b*d) + ((b*f -
 a*g)^3*Log[a + b*x])/(b^3*(b*c - a*d)) - ((d*f - c*g)^3*Log[c + d*x])/(d^
3*(b*c - a*d))))/(3*g)
```

3.232.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.232.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.77

method	result
risch	$\frac{(gx+f)^3 B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{3g}+\frac{g^2 A x^3}{3}+gAf x^2+\frac{g^2 Ba x^2}{6b}-\frac{g^2 Bc x^2}{6d}+A f^2 x-\frac{g^2 B \ln(dx+c) c^3}{3d^3}+\frac{gB \ln(}{}$
parallelrisch	$\frac{B x^2 a b^2 d^3 g^2-B x^2 b^3 c d^2 g^2+6Bxa b^2 d^3 fg-2Bx a^2 b d^3 g^2-2B c^3 g^2 b^3+2B a^3 d^3 g^2-6Bx b^3 c d^2 fg-Ba b^2 c^2 d g^2+6B b^3}{}$
parts	$\frac{A(gx+f)^3}{3g}-\frac{B(ad-cb)e\left(-2eg\left(acdg-a d^2 f-b c^2 g+bcdf\right)\left(-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}\right.\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(A d^2\left(\frac{e^2 g^2\left(a^2 d^2-2abcd+b^2 c^2\right)}{3d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}+\frac{g^2 c^2-2gfdc+f^2 d^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{eg\left(acdg-a d^2 f-b c^2 g+bcdf\right)}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}\right)+B d^2\left(\frac{e^2}{}\right.\right.}{}$
default	$-\frac{e(ad-cb)\left(A d^2\left(\frac{e^2 g^2\left(a^2 d^2-2abcd+b^2 c^2\right)}{3d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}+\frac{g^2 c^2-2gfdc+f^2 d^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{eg\left(acdg-a d^2 f-b c^2 g+bcdf\right)}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}\right)+B d^2\left(\frac{e^2}{}\right.\right.}{}$

input
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/3*(g*x+f)^3*B/g*ln(e*(b*x+a)/(d*x+c))+1/3*g^2*A*x^3+g*A*f*x^2+1/6/b*g^2*
B*a*x^2-1/6/d*g^2*B*c*x^2+A*f^2*x-1/3/d^3*g^2*B*ln(d*x+c)*c^3+1/d^2*g*B*ln
(d*x+c)*c^2*f-1/d*B*ln(d*x+c)*c*f^2+1/3/g*B*ln(d*x+c)*f^3+1/3/b^3*g^2*B*ln
(-b*x-a)*a^3-1/b^2*g*B*ln(-b*x-a)*a^2*f+1/b*B*ln(-b*x-a)*a*f^2-1/3/g*B*ln(
-b*x-a)*f^3-1/3/b^2*g^2*B*a^2*x+1/b*g*B*a*f*x+1/3/d^2*g^2*B*c^2*x-1/d*g*B*
c*f*x
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{2\,Ab^3d^3g^2x^3+(6\,Ab^3d^3fg-(Bb^3cd^2-Bab^2d^3)g^2)x^2+2\,(3\,Ab^3d^3f^2-3\,(Bb^3cd^2-Bab^2d^3)fg+(Bb^3c^2\cdots}{\cdots}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*
g^2)*x^2 + 2*(3*A*b^3*d^3*f^2 - 3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + (B*b^3
*c^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g +
B*a^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B
*b^3*c^3*g^2)*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 +
3*B*b^3*d^3*f^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^3)
```

3.232.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(131) = 262$.

Time = 3.12 (sec) , antiderivative size = 658, normalized size of antiderivative = 4.39

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{Ag^2x^3}{3}$$
$$+\frac{Ba(a^2g^2-3abfg+3b^2f^2)\log\left(x+\frac{Ba^3cd^2g^2-3Ba^2bcd^2fg+\frac{Ba^2d^3\left(a^2g^2-3abfg+3b^2f^2\right)}{b}+Bab^2c^3g^2-3Bab^2c^2dfg+6Bab^2cd^2\cdots}{Ba^3d^3g^2-3Ba^2bd^3fg+3Bab^2d^3f^2+Bb^3c^3g^2-3Bb^3c^2dfg+3Bb\cdots}\cdots\right.}{3b^3}$$
$$-\frac{Bc(c^2g^2-3cdfg+3d^2f^2)\log\left(x+\frac{Ba^3cd^2g^2-3Ba^2bcd^2fg+Bab^2c^3g^2-3Bab^2c^2dfg+6Bab^2cd^2f^2-Bab^2c(c^2g^2-3cdfg+3d^2\cdots}{Ba^3d^3g^2-3Ba^2bd^3fg+3Bab^2d^3f^2+Bb^3c^3g^2-3Bb^3c^2dfg+3Bb^3\cdots}\cdots\right.}{3d^3}$$
$$+x^2\left(Afg+\frac{Bag^2}{6b}-\frac{Bcg^2}{6d}\right)+x\left(Af^2-\frac{Ba^2g^2}{3b^2}+\frac{Bafg}{b}+\frac{Bc^2g^2}{3d^2}-\frac{Bcfg}{d}\right)$$
$$+\left(Bf^2x+Bfgx^2+\frac{Bg^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*g**2*x**3/3 + B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (B*a**3*
c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a**2*d**3*(a**2*g**2 - 3*a*b*f*g +
 3*b**2*f**2)/b + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*
c*d**2*f**2 - B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(B*a**3*d*
*3*g**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d**3*f**2 + B*b**3*c**3*g**2 -
3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/(3*b**3) - B*c*(c**2*g**2 - 3
*c*d*f*g + 3*d**2*f**2)*log(x + (B*a**3*c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*
g + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d**2*f**2 -
B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + B*b**3*c**2*(c**2*g**2
- 3*c*d*f*g + 3*d**2*f**2)/d)/(B*a**3*d**3*g**2 - 3*B*a**2*b*d**3*f*g + 3*
B*a*b**2*d**3*f**2 + B*b**3*c**3*g**2 - 3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d
**2*f**2))/(3*d**3) + x**2*(A*f*g + B*a*g**2/(6*b) - B*c*g**2/(6*d)) + x*(
A*f**2 - B*a**2*g**2/(3*b**2) + B*a*f*g/b + B*c**2*g**2/(3*d**2) - B*c*f*g
/d) + (B*f**2*x + B*f*g*x**2 + B*g**2*x**3/3)*log(e*(a + b*x)/(c + d*x))
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.75

$$
\begin{aligned}
&\int (f+gx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= \frac{1}{3} A g^2 x^3 + A f g x^2 + \left(x\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right) B f^2 \\
&\quad + \left(x^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right) B f g \\
&\quad + \frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd - abd^2)x^2 - 2(b^2c^2 - a}{b^2d^2}\right. \\
&\quad + A f^2 x
\end{aligned}
$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*lo
g(b*x + a)/b - c*log(d*x + c)/d)*B*f^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*
d))*B*f*g + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*
x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

3.232.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3076 vs. $2(140) = 280$.

Time = 0.61 (sec) , antiderivative size = 3076, normalized size of antiderivative = 20.51

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/6*(2*(3*B*b^4*c^2*d^2*e^4*f^2 - 6*B*a*b^3*c*d^3*e^4*f^2 + 3*B*a^2*b^2*d^
4*e^4*f^2 - 3*B*b^4*c^3*d*e^4*f*g + 3*B*a*b^3*c^2*d^2*e^4*f*g + 3*B*a^2*b^
2*c*d^3*e^4*f*g - 3*B*a^3*b*d^4*e^4*f*g + B*b^4*c^4*e^4*g^2 - B*a*b^3*c^3*
d*e^4*g^2 - B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2 - 6*(b*e*x + a*e)*B*
b^3*c^2*d^3*e^3*f^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^2*c*d^4*e^3*f^2/(d*
x + c) - 6*(b*e*x + a*e)*B*a^2*b*d^5*e^3*f^2/(d*x + c) + 9*(b*e*x + a*e)*B
*b^3*c^3*d^2*e^3*f*g/(d*x + c) - 15*(b*e*x + a*e)*B*a*b^2*c^2*d^3*e^3*f*g/
(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^4*e^3*f*g/(d*x + c) + 3*(b*e*x + a
*e)*B*a^3*d^5*e^3*f*g/(d*x + c) - 3*(b*e*x + a*e)*B*b^3*c^4*d*e^3*g^2/(d*x
 + c) + 3*(b*e*x + a*e)*B*a*b^2*c^3*d^2*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e
)*B*a^2*b*c^2*d^3*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^3*c*d^4*e^3*g^2/
(d*x + c) + 3*(b*e*x + a*e)^2*B*b^2*c^2*d^4*e^2*f^2/(d*x + c)^2 - 6*(b*e*x
 + a*e)^2*B*a*b*c*d^5*e^2*f^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^2*d^6*e^
2*f^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*b^2*c^3*d^3*e^2*f*g/(d*x + c)^2 +
12*(b*e*x + a*e)^2*B*a*b*c^2*d^4*e^2*f*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B
*a^2*c*d^5*e^2*f*g/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*b^2*c^4*d^2*e^2*g^2/(
d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a*b*c^3*d^3*e^2*g^2/(d*x + c)^2 + 3*(b*e*
x + a*e)^2*B*a^2*c^2*d^4*e^2*g^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))
/(b^3*d^3*e^3 - 3*(b*e*x + a*e)*b^2*d^4*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*
b*d^5*e/(d*x + c)^2 - (b*e*x + a*e)^3*d^6/(d*x + c)^3) + (6*A*b^6*c^2*d...
```

3.232.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.37

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)$$
$$-x\left(\frac{\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right.$$
$$\left.-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+3\,B\,a\,d\,f\,g-3\,B\,b\,c\,f\,g}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right)$$
$$+\frac{\ln(a+b\,x)\,\left(B\,a^3\,g^2-3\,B\,a^2\,b\,f\,g+3\,B\,a\,b^2\,f^2\right)}{3\,b^3}$$
$$-\frac{\ln(c+d\,x)\,\left(B\,c^3\,g^2-3\,B\,c^2\,d\,f\,g+3\,B\,c\,d^2\,f^2\right)}{3\,d^3}+\frac{A\,g^2\,x^3}{3}$$

input
```
int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a*d*g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(6*
b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x))/(c + d*x))*((B
*g^2*x^3)/3 + B*f^2*x + B*f*g*x^2) - x*((((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a
*d*g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*b*c))/(3*b*d
))*(3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*d*f*g + 6
*A*b*c*f*g + 3*B*a*d*f*g - 3*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/(b*d)) + (lo
g(a + b*x)*(B*a^3*g^2 + 3*B*a*b^2*f^2 - 3*B*a^2*b*f*g))/(3*b^3) - (log(c +
 d*x)*(B*c^3*g^2 + 3*B*c*d^2*f^2 - 3*B*c^2*d*f*g))/(3*d^3) + (A*g^2*x^3)/3
```

3.233 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.233.1 Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=-\frac{B(bc-ad)gx}{2bd}-\frac{B(bf-ag)^2\log(a+bx)}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g}+\frac{B(df-cg)^2\log(c+dx)}{2d^2g}$$

output
```
-1/2*B*(-a*d+b*c)*g*x/b/d-1/2*B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2
*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/2*B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

3.233.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{-Bd^2(bf-ag)^2\log(a+bx)+b\Big(d(B(-bc+ad)g^2x+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)}{c+dx}\right)+b\cdots}{2b^2d^2g}$$

input
```
Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(-(B*d^2*(b*f - a*g)^2*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*x + A*b*
d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x))/(c + d*x)] + b*B*(d
*f - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

3.233.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bc-ad)\int\frac{(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bc-ad)\int\left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(ad-bc)(c+dx)}\right)dx}{2g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bc-ad)\left(\frac{(bf-ag)^2\log(a+bx)}{b^2(bc-ad)}-\frac{(df-cg)^2\log(c+dx)}{d^2(bc-ad)}+\frac{g^2x}{bd}\right)}{2g}$$

input
```
Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*g) - (B*(b*c - a*d)*
((g^2*x)/(b*d) + ((b*f - a*g)^2*Log[a + b*x])/(b^2*(b*c - a*d)) - ((d*f -
c*g)^2*Log[c + d*x])/(d^2*(b*c - a*d))))/(2*g)
```

3.233.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.233.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

method	result
risch	$\frac{Bx(gx+2f)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{A\,x^{2}g}{2}+Afx+\frac{B\ln(-dx-c)c^{2}g}{2d^{2}}-\frac{B\ln(-dx-c)cf}{d}-\frac{B\ln(bx+a)a^{2}g}{2b^{2}}+\frac{B\ln(b\cdots}{\cdots}$
parallelrisch	$\frac{B\,x^{2}\ln\left(\frac{e(bx+a)}{dx+c}\right)b^{2}d^{2}g+A\,x^{2}b^{2}d^{2}g+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^{2}d^{2}f+2A\,b^{2}d^{2}fx-B\ln(bx+a)a^{2}d^{2}g+2B\ln(bx+a)ab\,d^{2}f+B\ln\cdots}{\cdots}$
parts	$A\left(\frac{1}{2}g\,x^{2}+fx\right)-\frac{B(ad-cb)e\left(deg(ad-cb)\left(-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^{2}b^{2}d}+\frac{\ln\left(\frac{be}{d}+\cdots\right)}{\cdots}\cdots\right)\cdots\right)}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-A\,d^{2}\left(\frac{eg(ad-cb)}{2d^{2}\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^{2}}+\frac{cg-df}{d^{2}\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)-B\,d^{2}\left(-\frac{eg(ad-cb)\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(a\cdots}{d(\cdots}\right)\cdots\right)}{2b^{2}e^{2}\cdots}\cdots\right)}{\cdots}\cdots\right)\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(-A\,d^{2}\left(\frac{eg(ad-cb)}{2d^{2}\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^{2}}+\frac{cg-df}{d^{2}\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)-B\,d^{2}\left(-\frac{eg(ad-cb)\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(a\cdots}{d(\cdots}\right)\cdots\right)}{2b^{2}e^{2}\cdots}\cdots\right)}{\cdots}\cdots\right)\right)}{\cdots}$

input
```
int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x*(g*x+2*f)*ln(e*(b*x+a)/(d*x+c))+1/2*A*x^2*g+A*f*x+1/2/d^2*B*ln(-d*
x-c)*c^2*g-1/d*B*ln(-d*x-c)*c*f-1/2/b^2*B*ln(b*x+a)*a^2*g+1/b*B*ln(b*x+a)*
a*f+1/2/b*B*x*a*g-1/2/d*B*x*c*g
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+(2\,Ab^2d^2f-(Bb^2cd-Babd^2)g)x+(2\,Babd^2f-Ba^2d^2g)\log(bx+a)-(2\,Bb^2cdf-Bb^2c^2g}{2\,b^2d^2}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + (2*A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + (2*
B*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - (2*B*b^2*c*d*f - B*b^2*c^2*g)*lo
g(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b*e*x + a*e)/(d*x +
c)))/(b^2*d^2)
```

3.233.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(90) = 180.

Time = 1.42 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Agx^2}{2}-\frac{Ba(ag-2bf)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2(ag-2bf)}{b}+Babc^2g-4Babcdf-Bacd(ag-2bf)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2b^2}$$
$$+\frac{Bc(cg-2df)\log\left(x+\frac{Ba^2cdg+Babc^2g-4Babcdf-Babc(cg-2df)+\frac{Bb^2c^2(cg-2df)}{d}}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2d^2}$$
$$+x\left(Af+\frac{Bag}{2b}-\frac{Bcg}{2d}\right)+\left(Bfx+\frac{Bgx^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g -
2*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d
**2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*b**2) + B*c*(
c*g - 2*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*
c*(c*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d*
*2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*d**2) + x*(A*f + B*a*g/(2*b) -
B*c*g/(2*d)) + (B*f*x + B*g*x**2/2)*log(e*(a + b*x)/(c + d*x))
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}Agx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bg+Afx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/2*A*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a
^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*g + A*
f*x
```

3.233.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(101) = 202.

Time = 0.49 (sec) , antiderivative size = 1145, normalized size of antiderivative = 10.50

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}\left(\frac{\left(2\,Bb^3c^2de^3f-4\,Bab^2cd^2e^3f+2\,Ba^2bd^3e^3f-Bb^3c^3e^3g+Bab^2c^2de^3g+Ba^2bcd^2e^3g-Ba^3d^3e^3g-\right.}{b^2a}\right.$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/2*((2*B*b^3*c^2*d*e^3*f - 4*B*a*b^2*c*d^2*e^3*f + 2*B*a^2*b*d^3*e^3*f -
B*b^3*c^3*e^3*g + B*a*b^2*c^2*d*e^3*g + B*a^2*b*c*d^2*e^3*g - B*a^3*d^3*e^
3*g - 2*(b*e*x + a*e)*B*b^2*c^2*d^2*e^2*f/(d*x + c) + 4*(b*e*x + a*e)*B*a*
b*c*d^3*e^2*f/(d*x + c) - 2*(b*e*x + a*e)*B*a^2*d^4*e^2*f/(d*x + c) + 2*(b
*e*x + a*e)*B*b^2*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b*c^2*d^2*e^
2*g/(d*x + c) + 2*(b*e*x + a*e)*B*a^2*c*d^3*e^2*g/(d*x + c))*log((b*e*x +
a*e)/(d*x + c))/(b^2*d^2*e^2 - 2*(b*e*x + a*e)*b*d^3*e/(d*x + c) + (b*e*x
+ a*e)^2*d^4/(d*x + c)^2) + (2*A*b^4*c^2*d*e^3*f - 4*A*a*b^3*c*d^2*e^3*f +
 2*A*a^2*b^2*d^3*e^3*f - A*b^4*c^3*e^3*g - B*b^4*c^3*e^3*g + A*a*b^3*c^2*d
*e^3*g + 3*B*a*b^3*c^2*d*e^3*g + A*a^2*b^2*c*d^2*e^3*g - 3*B*a^2*b^2*c*d^2
*e^3*g - A*a^3*b*d^3*e^3*g + B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*A*b^3*c^2
*d^2*e^2*f/(d*x + c) + 4*(b*e*x + a*e)*A*a*b^2*c*d^3*e^2*f/(d*x + c) - 2*(
b*e*x + a*e)*A*a^2*b*d^4*e^2*f/(d*x + c) + 2*(b*e*x + a*e)*A*b^3*c^3*d*e^2
*g/(d*x + c) + (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)
*A*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 3*(b*e*x + a*e)*B*a*b^2*c^2*d^2*e^2*g/(
d*x + c) + 2*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d*x + c) + 3*(b*e*x + a*e)
*B*a^2*b*c*d^3*e^2*g/(d*x + c) - (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/
(b^3*d^2*e^2 - 2*(b*e*x + a*e)*b^2*d^3*e/(d*x + c) + (b*e*x + a*e)^2*b*d^4
/(d*x + c)^2) + (2*B*b^3*c^2*d*e*f - 4*B*a*b^2*c*d^2*e*f + 2*B*a^2*b*d^3*e
*f - B*b^3*c^3*e*g + B*a*b^2*c^2*d*e*g + B*a^2*b*c*d^2*e*g - B*a^3*d^3*...
```

3.233.9 Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)$$
$$+x\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)$$
$$-\frac{\ln(a+b\,x)\,\left(B\,a^2\,g-2\,B\,a\,b\,f\right)}{2\,b^2}+\frac{\ln(c+d\,x)\,\left(B\,c^2\,g-2\,B\,c\,d\,f\right)}{2\,d^2}+\frac{A\,g\,x^2}{2}$$

input
```
int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
log((e*(a + b*x))/(c + d*x))*(B*f*x + (B*g*x^2)/2) + x*((2*A*a*d*g + 2*A*b
*c*g + 2*A*b*d*f + B*a*d*g - B*b*c*g)/(2*b*d) - (A*g*(2*a*d + 2*b*c))/(2*b
*d)) - (log(a + b*x)*(B*a^2*g - 2*B*a*b*f))/(2*b^2) + (log(c + d*x)*(B*c^2
*g - 2*B*c*d*f))/(2*d^2) + (A*g*x^2)/2
```

3.233. $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.234 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

3.234.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}$$

output
```
A*x+B*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/b-B*(-a*d+b*c)*ln(d*x+c)/b/d
```

3.234.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}$$

input
```
Integrate[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

output
```
A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c
+ d*x])/(b*d)
```

3.234.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}+Ax$$

input
```
Int[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

output
```
A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c
+ d*x])/(b*d)
```

3.234.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.234.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result	size
risch	$Ax+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)-\frac{Bc\ln(dx+c)}{d}+\frac{Ba\ln(-bx-a)}{b}$	51
parallelrisch	$\frac{B\left(x\ln\left(\frac{e(bx+a)}{dx+c}\right)bd+\ln(bx+a)ad-\ln(bx+a)bc+\ln\left(\frac{e(bx+a)}{dx+c}\right)bc\right)}{bd}+Ax$	70
default	$Ax-B(ad-cb)\,e\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}\right)$	162
parts	$Ax-B(ad-cb)\,e\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}\right)$	162
derivativedivides	$-\frac{e(ad-cb)\left(\frac{dA}{be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d}+\frac{dB\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{d^2B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)}{d^2}$	201

input
```
int(A+B*ln(e*(b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
A*x+B*x*ln(e*(b*x+a)/(d*x+c))-B/d*c*ln(d*x+c)+B/b*a*ln(-b*x-a)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx =\frac{Bbdx\log\left(\frac{bex+ae}{dx+c}\right)+Abdx+Bad\log(bx+a)-Bbc\log(dx+c)}{bd}$$

input
```
integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="fricas")
```

output
```
(B*b*d*x*log((b*e*x + a*e)/(d*x + c)) + A*b*d*x + B*a*d*log(b*x + a) - B*b
*c*log(d*x + c))/(b*d)
```

3.234.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = Ax+\frac{Ba\log\left(x+\frac{\frac{Ba^2d}{b}+Bac}{Bad+Bbc}\right)}{b} - \frac{Bc\log\left(x+\frac{Bac+\frac{Bbc^2}{d}}{Bad+Bbc}\right)}{d} + Bx\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate(A+B*ln(e*(b*x+a)/(d*x+c)),x)
```

output
```
A*x + B*a*log(x + (B*a**2*d/b + B*a*c)/(B*a*d + B*b*c))/b - B*c*log(x + (B
*a*c + B*b*c**2/d)/(B*a*d + B*b*c))/d + B*x*log(e*(a + b*x)/(c + d*x))
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \left(x\log\left(\frac{(bx+a)e}{dx+c}\right)+\frac{\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}}{e}\right)B+Ax$$

input
```
integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

output
```
(x*log((b*x + a)*e/(d*x + c)) + (a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)/
e)*B + A*x
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(52) = 104.

Time = 0.44 (sec) , antiderivative size = 406, normalized size of antiderivative = 7.81

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = -\left((b^2c^2e^2-2\,abcde^2+a^2d^2e^2)\left(\frac{\log\left(\frac{|bex+ae|}{|dx+c|}\right)}{bde}-\frac{\log\left(\left|-be+\frac{(bex+ae)d}{dx+c}\right|\right)}{bde}\right)-\frac{(b^2c^2e^2-2\,abcde^2+a^2d\cdots}{(\cdots}\right. + Ax$$

input
```
integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="giac")
```

output
```
-((b^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*(log(abs(b*e*x + a*e)/abs(d*
x + c))/(b*d*e) - log(abs(-b*e + (b*e*x + a*e)*d/(d*x + c)))/(b*d*e)) - (b
^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*log((a - b*(a/(b*c - a*d) - (b*e
*x + a*e)*c/((b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*e*x + a*e)*d/
((b*c*e - a*d*e)*(d*x + c))))*e/(c - d*(a/(b*c - a*d) - (b*e*x + a*e)*c/((
b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*e*x + a*e)*d/((b*c*e - a*d*
e)*(d*x + c)))))/((b*e - (b*e*x + a*e)*d/(d*x + c))*d))*B*(b*c/((b*c*e - a
*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))) + A*x
```

3.234.9 Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = A\,x+B\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)+\frac{B\,a\,\ln(a+b\,x)}{b}-\frac{B\,c\,\ln(c+d\,x)}{d}$$

input
```
int(A + B*log((e*(a + b*x))/(c + d*x)),x)
```

output
```
A*x + B*x*log((e*(a + b*x))/(c + d*x)) + (B*a*log(a + b*x))/b - (B*c*log(c
 + d*x))/d
```

3.235 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx$

3.235.1 Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{B\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{B\,\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}$$

output
```
-B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(g*
x+f)/g+B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*polylog(2,b*(g*x+f)/(-a*g
+b*f))/g+B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

3.235.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = \frac{\left(A-B\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-B\,\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)+B\,\text{Po}}{g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

output
```
((A - B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x))/(c + d*x)]
 + B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*PolyLog[2, (b*(f
+ g*x))/(b*f - a*g)] + B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

3.235.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2946, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{f+gx}\,dx$$

$$\Big\downarrow \text{2946}$$

$$-\frac{bB\int \frac{\log(f+gx)}{a+bx}dx}{g}+\frac{Bd\int \frac{\log(f+gx)}{c+dx}dx}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}$$

$$\Big\downarrow \text{2841}$$

$$-\frac{bB\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{g\int\frac{\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{f+gx}dx}{b}\right)}{g}+\frac{Bd\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{g\int\frac{\log\left(-\frac{g(c+dx)}{df-cg}\right)}{f+gx}dx}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}$$

$$\Big\downarrow \text{ 2840}$$

$$-\frac{bB\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(f+gx)}{bf-ag}\right)}{f+gx}d(f+gx)}{b}\right)}{g}+\frac{Bd\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(f+gx)}{df-cg}\right)}{f+gx}d(f+gx)}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}-\frac{bB\left(\frac{\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{b}+\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}\right)}{g}+\frac{Bd\left(\frac{\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{d}+\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}\right)}{g}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

output
```
((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x])/g - (b*B*((Log[-((g*(a
 + b*x))/(b*f - a*g))]*Log[f + g*x])/b + PolyLog[2, (b*(f + g*x))/(b*f - a
*g)]/b))/g + (B*d*((Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/d + Po
lyLog[2, (d*(f + g*x))/(d*f - c*g)]/d))/g
```

3.235. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx$

3.235.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2946
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Simp[b*B*(n/g)   Int[Log[f + g*x]/
(a + b*x), x], x] + Simp[B*d*(n/g)   Int[Log[f + g*x]/(c + d*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*
d, 0]
```

3.235.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(140) = 280$.

Time = 3.98 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.66

method	result
parts	$\frac{A\ln(gx+f)}{g}-\frac{B(ad-cb)e\left(-\frac{d^2(cg-df)\left(\frac{\operatorname{dilog}\left(\frac{(cg-df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-aeg+bef}{-aeg+bef}\right)}{cg-df}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{(cg-df)\left(\frac{be}{d}\cdots\right.}{\cdots}\right)}{cg-df}\cdots\right)}{eg(ad-cb)}\cdots\right.}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-d^2A\left(-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}{eg(ad-cb)(-cg+df)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{eg(ad-cb)}\right)-d^2B\cdots\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(-d^2A\left(-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}{eg(ad-cb)(-cg+df)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{eg(ad-cb)}\right)-d^2B\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
A*ln(g*x+f)/g-B/d^2*(a*d-b*c)*e*(-d^2*(c*g-d*f)/e/g/(a*d-b*c)*(dilog((((c*g
-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)
+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f))+d^3/e/g/(a*d-b*c)*(dilog(-((b*e
/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln
(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

3.235. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx$

3.235.5 Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(g*x + f), x)
```

3.235.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x)
```

output
```
Timed out
```

3.235.7 Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="maxima")
```

output
```
-B*integrate(-(log(b*x + a) - log(d*x + c) + log(e))/(g*x + f), x) + A*log
(g*x + f)/g
```

3.235.8 Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/(g*x + f), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{f+g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x), x)
```

3.236 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx$

3.236.1 Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(f+gx)}+\frac{B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*ln((g*
x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

3.236.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx=\frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}+\frac{B(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)) + (B*(b*(d*f - c*g)*Log
[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]))
/((b*f - a*g)*(d*f - c*g)))/g
```

3.236.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2954, 2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^2} \, dx$$

$$\Big\downarrow \text{ 2954}$$

$$(bc-ad) \int \frac{A+B \log \left(\frac{e(a+bx)}{c+dx}\right)}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2751}$$

$$(bc-ad) \left(\frac{(a+bx) \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(c+dx)(bf-ag) \left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{B \int \frac{1}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \text{ 16}$$

$$(bc-ad) \left(\frac{(a+bx) \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(c+dx)(bf-ag) \left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{B \log \left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}{(bf-ag)(df-cg)}\right)$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*g)
*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (B*Log[b*f -
 a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)*(d*f - c*g)))
```

3.236.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.236.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(87) = 174$.

Time = 1.19 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.78

method	result
parts	$-\frac{A}{(gx+f)g}-B(ad-cb)\,e\left(\frac{\ln\left((cg-df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-aeg+bef\right)}{e(ag-bf)(cg-df)}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\cdots\right.}{e(ag-bf)\left(cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-df\left(\frac{be}{d}+\cdots\right.\right.}\right.$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(gx+f)}-\frac{-B\ln(-dx-c)ad\,g^2x+B\ln(-dx-c)bdf\,gx+B\ln(gx+f)ad\,g^2x-B\ln(gx+f)bc\,g^2x+B\ln(-bx-a\cdots}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A}{\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)(-cg+df)}+d^2B\left(-\frac{\ln\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)}{e(ag-bf)(-cg+df)}+\frac{\cdots}{e(ag-\cdots}\right.\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A}{\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)(-cg+df)}+d^2B\left(-\frac{\ln\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)}{e(ag-bf)(-cg+df)}+\frac{\cdots}{e(ag-\cdots}\right.\right.}{d^2}$
parallelrisch	$\frac{B\ln(bx+a)a^2cd\,f^2-B\ln(bx+a)ab\,c^2f^2-B\ln(gx+f)a^2cd\,f^2+B\ln(gx+f)ab\,c^2f^2-Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2cdfg+Bx\ln\left(\frac{e(b\cdots}{d\cdots}\right.}{\cdots}$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/(g*x+f)/g-B*(a*d-b*c)*e*(1/e/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-a*e*g+b*e*f)/(c*g-d*f)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(
a*d-b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(87) = 174$.

Time = 3.55 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.93

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx =$$
$$-\frac{Abdf^2 + Aacg^2 - (Abc + Aad)fg - (Bbdf^2 - Bbcfg + (Bbdfg - Bbcg^2)x)\log(bx+a) + (Bbdf^2 - B\cdots}{\cdots bdf^3g + acfg\cdots}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="fricas")
```

output
```
-(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - (B*b*d*f^2 - B*b*c*f*g + (
B*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + (B*b*d*f^2 - B*a*d*f*g + (B*b*d*f
*g - B*a*d*g^2)*x)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a*d)
*f*g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log((b*
e*x + a*e)/(d*x + c)))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d
*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**2,x)
```

output
```
Timed out
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.59

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = B\left(\frac{b\log(bx+a)}{bfg-ag^2}-\frac{d\log(dx+c)}{dfg-cg^2}+\frac{(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg}-\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{g^2x+fg}\right) - \frac{A}{g^2x+fg}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="maxima")
```

output
```
B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*c
- a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*e*x/(d*x
 + c) + a*e/(d*x + c))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(87) = 174.

Time = 0.50 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = \left(\frac{(Bb^2c^2e-2\,Babcde+Ba^2d^2e)\log\left(-bef+aeg+\frac{(bex+ae)df}{dx+c}-\frac{(bex+ae)cg}{dx+c}\right)}{bdf^2-bcfg-adfg+acg^2}+\frac{(Bb^2c^2e^2-2}{bdef^2-bcefg-adefg+}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="giac")
```

output
```
((B*b^2*c^2*e - 2*B*a*b*c*d*e + B*a^2*d^2*e)*log(-b*e*f + a*e*g + (b*e*x +
 a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b*d*f^2 - b*c*f*g - a*
d*f*g + a*c*g^2) + (B*b^2*c^2*e^2 - 2*B*a*b*c*d*e^2 + B*a^2*d^2*e^2)*log((
b*e*x + a*e)/(d*x + c))/(b*d*e*f^2 - b*c*e*f*g - a*d*e*f*g + a*c*e*g^2 - (
b*e*x + a*e)*d^2*f^2/(d*x + c) + 2*(b*e*x + a*e)*c*d*f*g/(d*x + c) - (b*e*
x + a*e)*c^2*g^2/(d*x + c)) - (B*b^2*c^2*e - 2*B*a*b*c*d*e + B*a^2*d^2*e)*
log((b*e*x + a*e)/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^2) + (A*
b^2*c^2*e^2 - 2*A*a*b*c*d*e^2 + A*a^2*d^2*e^2)/(b*d*e*f^2 - b*c*e*f*g - a*
d*e*f*g + a*c*e*g^2 - (b*e*x + a*e)*d^2*f^2/(d*x + c) + 2*(b*e*x + a*e)*c*
d*f*g/(d*x + c) - (b*e*x + a*e)*c^2*g^2/(d*x + c)))*(b*c/((b*c*e - a*d*e)*
(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

3.236.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.91

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = \frac{B\,d\,\ln(c+d\,x)}{c\,g^2-d\,f\,g} - \frac{B\,\ln\left(\frac{a\,e+b\,e\,x}{c+d\,x}\right)}{x\,g^2+f\,g} - \frac{B\,b\,\ln(a+b\,x)}{a\,g^2-b\,f\,g} - \frac{A}{x\,g^2+f\,g} - \frac{B\,a\,d\,\ln(f+g\,x)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g} + \frac{B\,b\,c\,\ln(f+g\,x)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^2,x)
```

output
```
(B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a*e + b*e*x)/(c + d*x)))/(f*g
 + g^2*x) - (B*b*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x) - (B*a*d*
log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) + (B*b*c*log(f + g*x
))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)
```

3.237 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$

3.237.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = -\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)}{2g(bf-ag)^2} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} - \frac{Bd^2\log(c+dx)}{2g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)\log(f+gx)}{2(bf-ag)^2(df-cg)^2}$$

output
```
-1/2*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*ln(b*x+a)/g/(-a*
g+b*f)^2+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^2-1/2*B*d^2*ln(d*x+c)/
g/(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f
)^2/(-c*g+d*f)^2
```

3.237.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2} + B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2} + \frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)} + \frac{d^2\log(c+dx)}{-bc+ad} - \frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2) + B*(b*c - a*d)*((b^2
*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f - a
*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g
+ a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

3.237.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^3}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^2}dx}{2g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \text{93}$$

$$\frac{B(bc-ad)\int \left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(cg-df)^2(c+dx)}-\frac{g^2(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^2}\right)dx}{2g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}-\frac{d^2\log(c+dx)}{(bc-ad)(df-cg)^2}-\frac{g}{(f+gx)(bf-ag)(df-cg)}+\frac{g\log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}\right)}{2g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(g*(f + g*x)^2) + (B*(b*c - a*d)
*(-(g/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*Log[a + b*x])/((b*c - a*
d)*(b*f - a*g)^2) - (d^2*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) + (g*(2
*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/(2*g
)
```

3.237.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.237.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(176) = 352$.

Time = 1.89 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.79

method	result
parts	$-\frac{A}{2(gx+f)^2 g}-\frac{B(ad-cb)e\left(-\frac{g\,d^2(ad-cb)e\left(-\frac{\frac{\ln\left(cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-aeg+bef\right)}{cg-df}+\frac{\cdots}{(cg-df)\left(cg\left(\frac{be}{d}+\cdots\right.\right.}}{2(ag-bf)^2e^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-A\,d^2\left(-\frac{d}{(cg-df)(-cg+df)\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\cdots}{2(cg-df)(-cg+df)\left(aeg-bef-\cdots\right.}\cdots\right.\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(-A\,d^2\left(-\frac{d}{(cg-df)(-cg+df)\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\cdots}{2(cg-df)(-cg+df)\left(aeg-bef-\cdots\right.}\cdots\right.\right.}{\cdots}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

3.237. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$

output
```
-1/2*A/(g*x+f)^2/g-B/d^2*(a*d-b*c)*e*(-g*d^2*(a*d-b*c)*e/(c*g-d*f)*(-1/2/(
a*g-b*f)^2/e^2*(1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))-2*a*e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(
c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+
b*e*f)^2/(a*g-b*f)^2/e^2)-d^3/(c*g-d*f)*(1/e/(a*g-b*f)*ln((c*g-d*f)*(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(c*g-d*f)-ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)))
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(173) = 346$.

Time = 45.30 (sec) , antiderivative size = 1017, normalized size of antiderivative = 5.56

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = -\frac{Ab^2d^2f^4 + Aa^2c^2g^4 - ((2A-B)b^2cd + (2A+B)abd^2)f^3g + ((A-B)b^2c^2 + 4Aabcd + (A+B)a^2d^2)}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="fricas")
```

output
```
-1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - ((2*A - B)*b^2*c*d + (2*A + B)*a*b*d
^2)*f^3*g + ((A - B)*b^2*c^2 + 4*A*a*b*c*d + (A + B)*a^2*d^2)*f^2*g^2 - ((
2*A - B)*a*b*c^2 + (2*A + B)*a^2*c*d)*f*g^3 + ((B*b^2*c*d - B*a*b*d^2)*f^2
*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x - (B
*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2
- 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*
d*f^2*g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + (B*b^2*d^2*f^4 - 2*B*a*b*d^
2*f^3*g + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a
^2*d^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g
^3)*x)*log(d*x + c) - (2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^
2*d^2)*f^2*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2
)*g^4)*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2
)*f*g^3)*x)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d +
 B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a
*b*c^2 + B*a^2*c*d)*f*g^3)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^2*f^6*g +
a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*c
^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f
^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f
*g^6 - 2*(b^2*c*d + a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*...
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**3,x)
```

output
```
Timed out
```

3.237.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(173) = 346$.

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.92

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$$
$$=\frac{1}{2}\left(\frac{b^2\log(bx+a)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{d^2\log(dx+c)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{(2\,(b^2cd-abd^2)f-(b^2c^2-}{b^2d^2f^4+a^2c^2g^4-2\,(b^2cd+abd^2)f^3g+(b^2c^2+}\right.$$
$$-\frac{A}{2\,(g^3x^2+2\,fg^2x+f^2g)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="maxima")
```

output
```
1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x +
c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c
^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*
b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*
c*d)*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*
f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*e*x/(d*x + c) + a*e/(d*x +
 c))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^2*g
)
```

3.237.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2969 vs. $2(173) = 346$.

Time = 0.52 (sec) , antiderivative size = 2969, normalized size of antiderivative = 16.22

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="giac")
```

output
```
1/2*((2*B*b^3*c^2*d*e*f - 4*B*a*b^2*c*d^2*e*f + 2*B*a^2*b*d^3*e*f - B*b^3*
c^3*e*g + B*a*b^2*c^2*d*e*g + B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log(-b*e*
f + a*e*g + (b*e*x + a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b^
2*d^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*
d*f^2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*
g^4) + (2*B*b^3*c^2*d*e^3*f - 4*B*a*b^2*c*d^2*e^3*f + 2*B*a^2*b*d^3*e^3*f
- B*b^3*c^3*e^3*g + B*a*b^2*c^2*d*e^3*g + B*a^2*b*c*d^2*e^3*g - B*a^3*d^3*
e^3*g - 2*(b*e*x + a*e)*B*b^2*c^2*d^2*e^2*f/(d*x + c) + 4*(b*e*x + a*e)*B*
a*b*c*d^3*e^2*f/(d*x + c) - 2*(b*e*x + a*e)*B*a^2*d^4*e^2*f/(d*x + c) + 2*
(b*e*x + a*e)*B*b^2*c^3*d*e^2*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b*c^2*d^2*
e^2*g/(d*x + c) + 2*(b*e*x + a*e)*B*a^2*c*d^3*e^2*g/(d*x + c))*log((b*e*x
+ a*e)/(d*x + c))/(b^2*d^2*e^2*f^4 - 2*b^2*c*d*e^2*f^3*g - 2*a*b*d^2*e^2*f
^3*g + b^2*c^2*e^2*f^2*g^2 + 4*a*b*c*d*e^2*f^2*g^2 + a^2*d^2*e^2*f^2*g^2 -
 2*a*b*c^2*e^2*f*g^3 - 2*a^2*c*d*e^2*f*g^3 + a^2*c^2*e^2*g^4 - 2*(b*e*x +
a*e)*b*d^3*e*f^4/(d*x + c) + 6*(b*e*x + a*e)*b*c*d^2*e*f^3*g/(d*x + c) + 2
*(b*e*x + a*e)*a*d^3*e*f^3*g/(d*x + c) - 6*(b*e*x + a*e)*b*c^2*d*e*f^2*g^2
/(d*x + c) - 6*(b*e*x + a*e)*a*c*d^2*e*f^2*g^2/(d*x + c) + 2*(b*e*x + a*e)
*b*c^3*e*f*g^3/(d*x + c) + 6*(b*e*x + a*e)*a*c^2*d*e*f*g^3/(d*x + c) - 2*(
b*e*x + a*e)*a*c^3*e*g^4/(d*x + c) + (b*e*x + a*e)^2*d^4*f^4/(d*x + c)^2 -
 4*(b*e*x + a*e)^2*c*d^3*f^3*g/(d*x + c)^2 + 6*(b*e*x + a*e)^2*c^2*d^2*...
```

3.237.9 Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.28

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$$
$$= \frac{\ln(f+g\,x)\,\left(g\,\left(B\,a^2\,d^2-B\,b^2\,c^2\right)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c\,d}$$
$$-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-B\,a\,d\,f\,g+B\,b\,c\,f\,g}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{x\,\left(B\,a\,d\,g^2-B\,b\,c\,g^2\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}$$
$$+\frac{B\,b^2\,\ln(a+b\,x)}{2\,a^2\,g^3-4\,a\,b\,f\,g^2+2\,b^2\,f^2\,g}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)}-\frac{B\,d^2\,\ln(c+d\,x)}{2\,c^2\,g^3-4\,c\,d\,f\,g^2+2\,d^2\,f^2\,g}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^3,x)
```

output
```
(log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))
/(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^2*g^2 -
4*a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^3*g + 8*
a*b*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - B*a*d
*f*g + B*b*c*f*g)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x*(B*a*d*g^2
- B*b*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*f^2*g + 2*g^3*x^
2 + 4*f*g^2*x) + (B*b^2*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*g - 4*a*b*f*g
^2) - (B*log((e*(a + b*x))/(c + d*x)))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x)) - (
B*d^2*log(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d*f*g^2)
```

3.237. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$

3.238 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$

3.238.1 Optimal result

Integrand size = 27, antiderivative size = 275

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx &= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\ &\quad + \frac{b^3B\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} - \frac{Bd^3\log(c+dx)}{3g(df-cg)^3} \\ &\quad + \frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log(f+gx)}{3(bf-ag)^3(df-cg)^3}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*ln(b*x+a)/g/(-
a*g+b*f)^3+1/3*(-A-B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^3-1/3*B*d^3*ln(d*x+c
)/g/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

3.238.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}+B(bc-ad)\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2}+\frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log(c+}{(bc-ad)(-d}\right.}{3g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3) + B*(b*c - a*d)*(-1/2
*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g))/
((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*d)*
(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a
^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2
))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

3.238.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^4}\,dx$$

$$\Big\downarrow \ 2948$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}dx}{3g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}$$

$$\Big\downarrow \ 93$$

$$\frac{B(bc-ad)\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(cg-df)^3(c+dx)}+\frac{g^2((3d^2f^2-3cdgf+c^2g^2)b^2-adg(3df-cg)b+a^2d^2g^2)}{(bf-ag)^3(df-cg)^3(f+gx)}-\frac{g^2(-2b}{(bf-ag)^2(}}{3g}$$

$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}$$

$\downarrow$ 2009

$$\frac{B(bc-ad)\left(\frac{g\log(f+gx)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}-\frac{d^3\log(c+dx)}{(bc-ad)(df-cg)^3}-\frac{g(-adg-bcg+}{(f+gx)(bf-ag)^2(}}{3g}$$

$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(g*(f + g*x)^3) + (B*(b*c - a*d)
*(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (g*(2*b*d*f - b*c*g - a*d
*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c -
a*d)*(b*f - a*g)^3) - (d^3*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^3) + (g*
(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^
2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

3.238.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.238. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$

3.238.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1503 vs. $2(266) = 532$.

Time = 3.05 (sec) , antiderivative size = 1504, normalized size of antiderivative = 5.47

method	result	size
parts	Expression too large to display	1504
derivativedivides	Expression too large to display	1821
default	Expression too large to display	1821
risch	Expression too large to display	2293
parallelrisch	Expression too large to display	2896

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/(g*x+f)^3/g-B/d^2*(a*d-b*c)*e*(2*d^3*e*g*(a*d-b*c)/(c*g-d*f)^2*(-1/
2/(a*g-b*f)^2/e^2*(1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b
*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1/
2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))-2*a*e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e
*g+b*e*f)^2/(a*g-b*f)^2/e^2)+d^2*e^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*g^2/(c*g-
d*f)^2*(-1/3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/e^3*(1/2*e^2*(a^2*g^2-2
*a*b*f*g+b^2*f^2)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^2-1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)-e*(a*g-b*f)/(c
*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))-a*e*g+b*e*f))-1/3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(3*a^2*e^2*g^2-6*a*b*
e^2*f*g-3*a*c*e*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+3*a*d*e*f*g*(b*e/d+(a*d-
b*c)*e/d/(d*x+c))+3*b^2*e^2*f^2+3*b*c*e*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-
3*b*d*e*f^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+c^2*g^2*(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2-2*c*d*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+d^2*f^2*(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^2)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^3/(a*g-b*f)/(a^2...
```

3.238.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="fricas")
```

output
```
Timed out
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**4,x)
```

output
```
Timed out
```

3.238.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(263) = 526$.

Time = 0.26 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.08

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$$
$$=\frac{1}{6}\left(\frac{2\,b^3\log(bx+a)}{b^3f^3g-3\,ab^2f^2g^2+3\,a^2bfg^3-a^3g^4}-\frac{2\,d^3\log(dx+c)}{d^3f^3g-3\,cd^2f^2g^2+3\,c^2dfg^3-c^3g^4}+\frac{}{b^3d^3f^6+a^3c^3g^6-3\,(b\right.$$
$$-\frac{A}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="maxima")
```

output
```
1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*e*x/(d*x
 + c) + a*e/(d*x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B -
1/3*A/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

3.238.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9339 vs. $2(263) = 526$.

Time = 0.78 (sec) , antiderivative size = 9339, normalized size of antiderivative = 33.96

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="giac")
```

output

```
1/6*(2*(3*B*b^4*c^2*d^2*e*f^2 - 6*B*a*b^3*c*d^3*e*f^2 + 3*B*a^2*b^2*d^4*e*
f^2 - 3*B*b^4*c^3*d*e*f*g + 3*B*a*b^3*c^2*d^2*e*f*g + 3*B*a^2*b^2*c*d^3*e*
f*g - 3*B*a^3*b*d^4*e*f*g + B*b^4*c^4*e*g^2 - B*a*b^3*c^3*d*e*g^2 - B*a^3*
b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)*log(-b*e*f + a*e*g + (b*e*x + a*e)*d*f/(d
*x + c) - (b*e*x + a*e)*c*g/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g -
3*a*b^2*d^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*
d^3*f^4*g^2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*
g^3 - a^3*d^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^
3*c*d^2*f^2*g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + 2
*(3*B*b^4*c^2*d^2*e^4*f^2 - 6*B*a*b^3*c*d^3*e^4*f^2 + 3*B*a^2*b^2*d^4*e^4*
f^2 - 3*B*b^4*c^3*d*e^4*f*g + 3*B*a*b^3*c^2*d^2*e^4*f*g + 3*B*a^2*b^2*c*d^
3*e^4*f*g - 3*B*a^3*b*d^4*e^4*f*g + B*b^4*c^4*e^4*g^2 - B*a*b^3*c^3*d*e^4*
g^2 - B*a^3*b*c*d^3*e^4*g^2 + B*a^4*d^4*e^4*g^2 - 6*(b*e*x + a*e)*B*b^3*c^
2*d^3*e^3*f^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^2*c*d^4*e^3*f^2/(d*x + c)
 - 6*(b*e*x + a*e)*B*a^2*b*d^5*e^3*f^2/(d*x + c) + 9*(b*e*x + a*e)*B*b^3*c
^3*d^2*e^3*f*g/(d*x + c) - 15*(b*e*x + a*e)*B*a*b^2*c^2*d^3*e^3*f*g/(d*x +
 c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^4*e^3*f*g/(d*x + c) + 3*(b*e*x + a*e)*B*
a^3*d^5*e^3*f*g/(d*x + c) - 3*(b*e*x + a*e)*B*b^3*c^4*d*e^3*g^2/(d*x + c)
+ 3*(b*e*x + a*e)*B*a*b^2*c^3*d^2*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)*B*a^
2*b*c^2*d^3*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^3*c*d^4*e^3*g^2/(d*...
```

3.238.9 Mupad [B] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 1154, normalized size of antiderivative = 4.20

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$$
$$= \frac{\ln(f+g\,x)\,\left(g\,(3\,B\,a^2\,b\,d^3\,f-\cdots\right.}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f\,g^5+27\,a^2\,b\,c^2\,d\,f^2\,g^4-27\,a^2\,b\,c\,d^2\,f^3\,\cdots}$$
$$-\frac{\frac{2\,A\,a^2\,c^2\,g^4+2\,A\,b^2\,d^2\,f^4+2\,A\,a^2\,d^2\,f^2\,g^2+2\,A\,b^2\,c^2\,f^2\,g^2+3\,B\,a^2\,d^2\,f^2\,g^2-3\,B\,b^2\,c^2\,f^2\,g^2-4\,A\,a\,b\,c^2\,f\,g^3-4\,A\,a\,b\,d^2\,f^3\,g+B\,a\,b\,c^2\,f\,g^3-4\,A\,a\cdots}{2\,(a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\cdots}}{\cdots}$$
$$-\frac{B\,b^3\,\ln(a+b\,x)}{3\,a^3\,g^4-9\,a^2\,b\,f\,g^3+9\,a\,b^2\,f^2\,g^2-3\,b^3\,f^3\,g}$$
$$+\frac{B\,d^3\,\ln(c+d\,x)}{3\,c^3\,g^4-9\,c^2\,d\,f\,g^3+9\,c\,d^2\,f^2\,g^2-3\,d^3\,f^3\,g}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{3\,g\,\left(f^3+3\,f^2\,g\,x+3\,f\,g^2\,x^2+g^3\,x^3\right)}$$

input

```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^4,x)
```

output
```
(log(f + g*x)*(g*(3*B*a^2*b*d^3*f - 3*B*b^3*c^2*d*f) - g^2*(B*a^3*d^3 - B*
b^3*c^3) - 3*B*a*b^2*d^3*f^2 + 3*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*b^3*
d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 - 9*a*
b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^2*g^
4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 + 27*a
*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 + 27*
a^2*b*c^2*d*f^2*g^4) - ((2*A*a^2*c^2*g^4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f
^2*g^2 + 2*A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 -
 4*A*a*b*c^2*f*g^3 - 4*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 4*A*a^2*c*d*f*g
^3 - 5*B*a*b*d^2*f^3*g - 4*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d
*f^3*g + 8*A*a*b*c*d*f^2*g^2)/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*
g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^
3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2)) + (x^2*(B*a^2*d^2*g^4 - B*b^2*c^
2*g^4 - 2*B*a*b*d^2*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4
 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g -
 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*
f*g^3 - 5*B*b^2*c^2*f*g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^
2*g^2 + 9*B*b^2*c*d*f^2*g^2))/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*
g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^
3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2)))/(3*f^3*g + 3*g^4*x^3 + 9*f^2...
```

3.238. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$

3.239 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$

3.239.1 Optimal result

Integrand size = 27, antiderivative size = 379

$$
\begin{aligned}
\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx
&= -\frac{B(bc-ad)}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{8(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&\quad -\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)}{4(bf-ag)^3(df-cg)^3(f+gx)} \\
&\quad +\frac{b^4B\log(a+bx)}{4g(bf-ag)^4}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}-\frac{Bd^4\log(c+dx)}{4g(df-cg)^4} \\
&\quad -\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log(f+gx)}{4(bf-ag)^4(df-cg)^4}
\end{aligned}
$$

output
```
-1/12*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*d*
g-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*(a^2
*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f
)^3/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*ln(e
*(b*x+a)/(d*x+c)))/g/(g*x+f)^4-1/4*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/4*B*(-
a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*
c*d*f*g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

3.239.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}+B(bc-ad)\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g(a^2d^2g^2+abdg(-3df+cg)+b^2(\cdots}{(bf-ag)^3(df-cg)^3\cdots}\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4) + B*(b*c - a*d)*(-1/3
*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g))/
(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(-
3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f
 - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d
^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*
g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*
Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

3.239.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^5}dx$$

$$\Big\downarrow \; 2948$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}dx}{4g}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4g(f+gx)^4}$$

$$\Big\downarrow \; 93$$

$$\frac{B(bc-ad)\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(cg-df)^4(c+dx)}+\frac{g^2(2bdf-bcg-adg)\left(2d^2f^2b^2+c^2g^2b^2-2cdfgb^2-2ad^2fgb+a^2d^2g^2\right)}{(bf-ag)^4(df-cg)^4(f+gx)} \cdots\right)}{4g}$$

$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4g(f+gx)^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{B(bc-ad)\left(-\frac{g\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(f+gx)(bf-ag)^3(df-cg)^3}-\frac{g\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-\left(b^2\left(c^2g^2-2cdfg+\cdots\right)\right)\right)}{(bf-ag)^4(df-cg)^4} \cdots\right)}{4g}$$

$$\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4g(f+gx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(g*(f + g*x)^4) + (B*(b*c - a*d)
*(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (g*(2*b*d*f - b*c*g - a*d
*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 - a*b*d
*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*
(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4)
- (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(2*b*d*f - b*c*g - a
*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2)
)*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

3.239.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.239. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.239.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2849 vs. $2(368) = 736$.

Time = 6.65 (sec) , antiderivative size = 2850, normalized size of antiderivative = 7.52

method	result	size
parts	Expression too large to display	2850
derivativedivides	Expression too large to display	3309
default	Expression too large to display	3309
risch	Expression too large to display	4450
parallelrisch	Expression too large to display	5539

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/(g*x+f)^4/g-B/d^2*(a*d-b*c)*e*(-3*d^4*e*(a*d-b*c)*g/(c*g-d*f)^3*(-1
/2/(a*g-b*f)^2/e^2*(1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)+e*(a*g-b*f)/(c*g-d*f)/(c*g*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f))+1
/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*
(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2*a*e*g+2*b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*
e*g+b*e*f)^2/(a*g-b*f)^2/e^2)-3*d^3*e^2*g^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c
*g-d*f)^3*(-1/3/(a*g-b*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/e^3*(1/2*e^2*(a^2*g^
2-2*a*b*f*g+b^2*f^2)/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^2-1/(c*g-d*f)*ln(c*g*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)-e*(a*g-b*f)
/(c*g-d*f)/(c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-a*e*g+b*e*f))-1/3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(3*a^2*e^2*g^2-6*a
*b*e^2*f*g-3*a*c*e*g^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+3*a*d*e*f*g*(b*e/d+(a
*d-b*c)*e/d/(d*x+c))+3*b^2*e^2*f^2+3*b*c*e*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))-3*b*d*e*f^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+c^2*g^2*(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2-2*c*d*f*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+d^2*f^2*(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(c*g*(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)^3/(a*g-b*f)/(...
```

3.239.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="fricas")
```

output
```
Timed out
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**5,x)
```

output
```
Timed out
```

3.239.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1757 vs. $2(365) = 730$.

Time = 0.33 (sec) , antiderivative size = 1757, normalized size of antiderivative = 4.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="maxima")
```

output
```
1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

3.239.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20791 vs. $2(365) = 730$.

Time = 0.99 (sec) , antiderivative size = 20791, normalized size of antiderivative = 54.86

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="giac")
```

output
```
1/24*(6*(4*B*b^5*c^2*d^3*e*f^3 - 8*B*a*b^4*c*d^4*e*f^3 + 4*B*a^2*b^3*d^5*e
*f^3 - 6*B*b^5*c^3*d^2*e*f^2*g + 6*B*a*b^4*c^2*d^3*e*f^2*g + 6*B*a^2*b^3*c
*d^4*e*f^2*g - 6*B*a^3*b^2*d^5*e*f^2*g + 4*B*b^5*c^4*d*e*f*g^2 - 4*B*a*b^4
*c^3*d^2*e*f*g^2 - 4*B*a^3*b^2*c*d^4*e*f*g^2 + 4*B*a^4*b*d^5*e*f*g^2 - B*b
^5*c^5*e*g^3 + B*a*b^4*c^4*d*e*g^3 + B*a^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3
)*log(-b*e*f + a*e*g + (b*e*x + a*e)*d*f/(d*x + c) - (b*e*x + a*e)*c*g/(d*
x + c))/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d
^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*
f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^
4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*
f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 -
 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5
 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6
- 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) + 6*(4*B*b^5*c^2*d^
3*e^5*f^3 - 8*B*a*b^4*c*d^4*e^5*f^3 + 4*B*a^2*b^3*d^5*e^5*f^3 - 6*B*b^5*c^
3*d^2*e^5*f^2*g + 6*B*a*b^4*c^2*d^3*e^5*f^2*g + 6*B*a^2*b^3*c*d^4*e^5*f^2*
g - 6*B*a^3*b^2*d^5*e^5*f^2*g + 4*B*b^5*c^4*d*e^5*f*g^2 - 4*B*a*b^4*c^3*d^
2*e^5*f*g^2 - 4*B*a^3*b^2*c*d^4*e^5*f*g^2 + 4*B*a^4*b*d^5*e^5*f*g^2 - B*b^
5*c^5*e^5*g^3 + B*a*b^4*c^4*d*e^5*g^3 + B*a^4*b*c*d^4*e^5*g^3 - B*a^5*d^5*
e^5*g^3 - 12*(b*e*x + a*e)*B*b^4*c^2*d^4*e^4*f^3/(d*x + c) + 24*(b*e*x ...
```

3.239.9 Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 2518, normalized size of antiderivative = 6.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^5,x)
```

output
```
(log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^
3*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4
*f^3 + 4*B*b^4*c*d^3*f^3))/(4*a^4*c^4*g^8 + 4*b^4*d^4*f^8 + 4*a^4*d^4*f^4*
g^4 + 4*b^4*c^4*f^4*g^4 + 24*a^2*b^2*c^4*f^2*g^6 + 24*a^2*b^2*d^4*f^6*g^2
+ 24*a^4*c^2*d^2*f^2*g^6 + 24*b^4*c^2*d^2*f^6*g^2 - 16*a^3*b*c^4*f*g^7 - 1
6*a*b^3*d^4*f^7*g - 16*a^4*c^3*d*f*g^7 - 16*b^4*c*d^3*f^7*g - 16*a*b^3*c^4
*f^3*g^5 - 16*a^3*b*d^4*f^5*g^3 - 16*a^4*c*d^3*f^3*g^5 - 16*b^4*c^3*d*f^5*
g^3 + 64*a*b^3*c*d^3*f^6*g^2 + 64*a*b^3*c^3*d*f^4*g^4 + 64*a^3*b*c*d^3*f^4
*g^4 + 64*a^3*b*c^3*d*f^2*g^6 - 96*a*b^3*c^2*d^2*f^5*g^3 - 96*a^2*b^2*c*d^
3*f^5*g^3 - 96*a^2*b^2*c^3*d*f^3*g^5 - 96*a^3*b*c^2*d^2*f^3*g^5 + 144*a^2*
b^2*c^2*d^2*f^4*g^4) - ((6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*d^3*f
^3*g^3 - 6*A*b^3*c^3*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3
 + 18*A*a*b^2*c^3*f^2*g^4 + 18*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4
 + 18*A*a^3*c*d^2*f^2*g^4 + 31*B*a^2*b*d^3*f^4*g^2 + 18*A*b^3*c^2*d*f^4*g^
2 + 7*B*a^3*c*d^2*f^2*g^4 - 31*B*b^3*c^2*d*f^4*g^2 - 18*A*a^2*b*c^3*f*g^5
- 18*A*a*b^2*d^3*f^5*g + 2*B*a^2*b*c^3*f*g^5 - 18*A*a^3*c^2*d*f*g^5 - 26*B
*a*b^2*d^3*f^5*g - 18*A*b^3*c*d^2*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c
*d^2*f^5*g + 54*A*a*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^
2*b*c*d^2*f^3*g^3 + 54*A*a^2*b*c^2*d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 -
15*B*a^2*b*c*d^2*f^3*g^3)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g...
```

3.239. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$

3.240 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.240.1 Optimal result

Integrand size = 29, antiderivative size = 874

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{B^2(bc-ad)^3g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{4b^3d^3} + \frac{B^2(bc-ad)^2g^3(c+dx)^2}{12b^2d^4}$$
$$+ \frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4}$$
$$- \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^4d^3}$$
$$- \frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d^4}$$
$$- \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd^4}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\right.}{2b^4d^4}$$
$$- \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g}$$
$$+ \frac{B^2(bc-ad)^4g^3\log(c+dx)}{6b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log(c+dx)}{4b^4d^4}$$
$$+ \frac{B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{2b^4d^4}$$
$$- \frac{B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2b^4d^4}$$

output
```
1/6*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*
g+4*b*d*f)*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4+1/6*B^2*(
-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d
*g-3*b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/b^4/d^4-1/2*B*(-a*d+b*c)*g*(a^2*d^
2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*(b*x+a)*
(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4
*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3
*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*
g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*l
n((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^4-1/4*(-a*g+b*f)
^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/g+1/6*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3
*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^
2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(d
*x+c)/b^4/d^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2
*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))
/b^4/d^4
```

3.240.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 733, normalized size of antiderivative = 0.84

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B\left(6Abd(bc-ad)g^2(a^2d^2g^2+abdg(-4df+cg)+b^2(6d^2f^2-4cdfg+c^2g^2))x+6Bd(bc-ad)g^2(a^2c\right.}{}}{}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(6*A*b*d*(b*c - a
*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g
 + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*
g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x))/(c
 + d*x)] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A + B*Log[(
e*(a + b*x))/(c + d*x)]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*
d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c + d*x] - 6*b^4*(
d*f - c*g)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c -
a*d)*g^4*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*Log[a + b*
x] - 2*b^3*c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g + a*d
*g)*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])
) - 3*B*d^4*(b*f - a*g)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x)
)/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b^4*B*(d
*f - c*g)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))/(4*g)
```

3.240.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 1069, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2g(bc-ad)}\right.$$

$$\Big\downarrow \text{ 2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\left(\frac{(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)g^4}{bd^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(bc-ad)^3(4bdf-3bc}{b^2c}\right.}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$(bc-ad)\left(\frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(-\frac{B(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)g^4}{3b^4d^4}+\frac{(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right.}{}\right.$$

input
```
Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^4*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(4*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d*x
))^4) - (B*(-1/6*(B*(b*c - a*d)^4*g^4)/(b^2*d^4*(b - (d*(a + b*x))/(c + d*
x))^2) - (B*(b*c - a*d)^4*g^4)/(3*b^3*d^4*(b - (d*(a + b*x))/(c + d*x))) -
 (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g))/(2*b^3*d^4*(b - (d*(a +
 b*x))/(c + d*x))) - (B*(b*c - a*d)^4*g^4*Log[(a + b*x)/(c + d*x)])/(3*b^4
*d^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a + b*x)/(c
+ d*x)])/(2*b^4*d^4) + ((b*c - a*d)^4*g^4*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(3*b*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + ((b*c - a*d)^3*g^3*(4*b*
d*f - 3*b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^2*d^4*(b
 - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(a^2*d^2*g^2 - 2*a*b*d
*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a + b*x)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*d^3*(c + d*x)*(b - (d*(a + b*x))/(c
 + d*x))) + ((b*f - a*g)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^4*
B) + (B*(b*c - a*d)^4*g^4*Log[b - (d*(a + b*x))/(c + d*x)])/(3*b^4*d^4) +
(B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*Log[b - (d*(a + b*x))/(c
+ d*x)])/(2*b^4*d^4) + (B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*
f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*Log[b - (d*(a + b*x))/
(c + d*x)])/(b^4*d^4) + ((b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^
2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(A + B*Log...
```

3.240. $\int(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

3.240.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.240.4 Maple [F]

$$\int (gx+f)^3 \left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.240.5 Fricas [F]

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^
3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*e*x + a*e)/(d*x
+ c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log(
(b*e*x + a*e)/(d*x + c)), x)
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.240.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2140 vs. $2(843) = 1686$.

Time = 0.32 (sec) , antiderivative size = 2140, normalized size of antiderivative = 2.45

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b*e*x/(d*x
+ c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^3 + 3*(
x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(
d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*f^2*g + (2*x^3*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^
2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*f*g^2 + 1/1
2*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6
*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^
2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^3 + A^2*f^3*x - 1
/12*(6*a^3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b + 2*(18*c*d^3
*f^2*g - 6*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (24*c*d^3*f^3*log(e) - (6*g^
3*log(e) + 11*g^3)*c^4 + 12*(2*f*g^2*log(e) + 3*f*g^2)*c^3*d - 36*(f^2*g*l
og(e) + f^2*g)*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/2*(4*a*b^3*d^4
*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - (4*c*d^3*f^
3 - 6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x + a)*log((b*d
*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d
^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 2*(a*b^3*d^4*g^3*log(e) + (6*
d^4*f*g^2*log(e)^2 - c*d^3*g^3*log(e))*b^4)*B^2*x^3 - ((3*g^3*log(e) - g^3
)*a^2*b^2*d^4 - 2*(6*d^4*f*g^2*log(e) - c*d^3*g^3)*a*b^3 - (18*d^4*f^2*g*l
og(e)^2 - 12*c*d^3*f*g^2*log(e) + (3*g^3*log(e) + g^3)*c^2*d^2)*b^4)*B^...
```

3.240.8 Giac [F]

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (f+g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.241 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.241.1 Optimal result

Integrand size = 29, antiderivative size = 532

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} + \frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}$$
$$- \frac{2B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3d^2}$$
$$- \frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd^3}$$
$$+ \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{3b^3d^3}$$
$$- \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g}$$
$$+ \frac{B^2(bc-ad)^3g^2\log(c+dx)}{3b^3d^3} + \frac{2B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3}$$
$$+ \frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*
x+c))/b^3/d^3-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(
e*(b*x+a)/(d*x+c)))/b^3/d^2-1/3*B*(-a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*
(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b^3/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g+1/3
*(g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+1/3*B^2*(-a*d+b*c)^3*g^2*ln(d*x
+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b*d*f)*ln(d*x+c)/b^3/
d^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*
d*f*g+3*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

3.241.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.91

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{}}{}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a
*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f - b*c
*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*d^2*(b*c - a*d)*g
^3*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*Log[a +
b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*B*(b*c - a*d)^2*g^2*(-3*b*d*
f + b*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b
*x))/(c + d*x)])*Log[c + d*x] - B*(b*c - a*d)*g^3*(a^2*d^2*Log[a + b*x] -
b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f - a*g)^3*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*((2*Log[(d*(a + b*x))/(
-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)])))/(b^3*d^3))/(3*g)
```

3.241.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d}{3g(bc-ad)}\right.$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int\left(\frac{(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)g^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad)^2(3bdf-2}{b}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(-\frac{g(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f}{b^3d^3}\right.}{}\right.$$

input
```
Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

3.241. $\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(3*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d*x
))^3) - (2*B*(-1/2*(B*(b*c - a*d)^3*g^3)/(b^2*d^3*(b - (d*(a + b*x))/(c +
d*x))) - (B*(b*c - a*d)^3*g^3*Log[(a + b*x)/(c + d*x)])/(2*b^3*d^3) + ((b*
c - a*d)^3*g^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b*d^3*(b - (d*(a +
 b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f - 2*b*c*g - a*d*g)*(a +
 b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*d^2*(c + d*x)*(b - (d*(a
+ b*x))/(c + d*x))) + ((b*f - a*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2)/(2*b^3*B) + (B*(b*c - a*d)^3*g^3*Log[b - (d*(a + b*x))/(c + d*x)])/(2*b
^3*d^3) + (B*(b*c - a*d)^2*g^2*(3*b*d*f - 2*b*c*g - a*d*g)*Log[b - (d*(a +
 b*x))/(c + d*x)])/(b^3*d^3) - ((b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*
f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))
/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) - (B*(b*c - a
*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3)))/(3*(b*c - a
*d)*g))
```

3.241.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.241. $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.241.4 Maple [F]

$$\int (gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.241.5 Fricas [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x
+ B^2*f^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x +
 A*B*f^2)*log((b*e*x + a*e)/(d*x + c)), x)
```

3.241.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.241.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. $2(511) = 1022$.

Time = 0.30 (sec) , antiderivative size = 1300, normalized size of antiderivative = 2.44

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c))
+ a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c -
a*d)*x/(b*d))*A*B*f*g + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 1/3*(2*a^2*c*
d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (6*c*d^2*f^2*log(e) + (2*g^2*log
(e) + 3*g^2)*c^3 - 6*(f*g*log(e) + f*g)*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*
d^3) + 2/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f^2
 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d)
 + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^
3*g^2*x^3*log(e)^2 + (a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*log(e)^2 - c*d^2*g
^2*log(e))*b^3)*B^2*x^2 - ((2*g^2*log(e) - g^2)*a^2*b*d^3 - 2*(3*d^3*f*g*l
og(e) - c*d^2*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 6*c*d^2*f*g*log(e) + (2*g
^2*log(e) + g^2)*c^2*d)*b^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*
f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d
^3*g^2)*B^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2
 + 3*B^2*b^3*d^3*f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^3)*lo
g(d*x + c)^2 + (2*B^2*b^3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (6*d^3*f*g
*log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g - a^2*b*d^3*g^2 ...
```

3.241.8 Giac [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.242 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.242.1 Optimal result

Integrand size = 27, antiderivative size = 270

$$\begin{aligned}
&\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= -\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d} \\
&\quad+\frac{B(bc-ad)(2bdf-bcg-adg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d^2} \\
&\quad-\frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} \\
&\quad+\frac{B^2(bc-ad)^2g\log(c+dx)}{b^2d^2}+\frac{B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

output
```
-B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d+B*(-a*d+b*c)*(-a
*d*g-b*c*g+2*b*d*f)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b
^2/d^2-1/2*(-a*g+b*f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g+1/2*(g*x+f)^2*
(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+B^2*(-a*d+b*c)^2*g*ln(d*x+c)/b^2/d^2+B^2*(
-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

3.242.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$=\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B\left(2Abd(bc-ad)g^2x+2Bd(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+2d^2(bf-ag)^2\log(a+bx)\left(A+B\log\right.\right.}{}}{}$$

input
```
Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a
*d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] +
 2*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2
*B*(b*c - a*d)^2*g^2*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b*x]*(Log[a
 + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)
] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
)/(b^2*d^2))/(2*g)
```

3.242.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.45, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a}{c}}{g(bc-ad)}\right.$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)g^2}{bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{(bc-ad)(2bdf-bcg-}{b^2d}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(-\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+}{c+d}\right.\right.}{b^2d^2}\right.}{}\right.$$

input
```
Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)*g*(b - (d*(a + b*x))/(c + d*x
))^2) - (B*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a*g)^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2*B) + (B*(b*c - a*d)^2*g^2*Log[b
 - (d*(a + b*x))/(c + d*x)])/(b^2*d^2) - ((b*c - a*d)*g*(2*b*d*f - b*c*g -
 a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c +
 d*x))])/(b^2*d^2) - (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/((b*c - a*d)*g))
```

3.242. $\int(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

3.242.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.242.4 Maple [F]

$$\int (gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.242.5 Fricas [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*e*x + a*e)/(d*x + c))^
2 + 2*(A*B*g*x + A*B*f)*log((b*e*x + a*e)/(d*x + c)), x)
```

3.242.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.242.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(265) = 530$.

Time = 0.27 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.49

$$\begin{aligned}
&\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{1}{2}A^2gx^2 + 2\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)ABf \\
&\quad + \left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)ABg \\
&\quad + A^2fx - \frac{(acdg+(2\,cdf\log(e)-(g\log(e)+g)c^2)b)B^2\log(dx+c)}{bd^2} \\
&\quad + \frac{(2\,abd^2f-a^2d^2g-(2\,cdf-c^2g)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2} \\
&\quad + \frac{B^2b^2d^2gx^2\log(e)^2+2\left(abd^2g\log(e)+\left(d^2f\log(e)^2-cdg\log(e)\right)b^2\right)B^2x+(B^2b^2d^2gx^2+2\,B^2b^2d^2fx}{}
\end{aligned}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
1/2*A^2*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)
/b - c*log(d*x + c)/d)*A*B*f + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
 a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*g
+ A^2*f*x - (a*c*d*g + (2*c*d*f*log(e) - (g*log(e) + g)*c^2)*b)*B^2*log(d*
x + c)/(b*d^2) + (2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*g)*b^2)*(log(b*
x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(a*b*d^2*g*log(e)
 + (d^2*f*log(e)^2 - c*d*g*log(e))*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2
*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a)^2 + (B^2*b^2*d^
2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)*log(d*x + c)^2 +
2*(B^2*b^2*d^2*g*x^2*log(e) + (a*b*d^2*g + (2*d^2*f*log(e) - c*d*g)*b^2)*B
^2*x - ((g*log(e) - g)*a^2*d^2 - (2*d^2*f*log(e) - c*d*g)*a*b)*B^2)*log(b*
x + a) - 2*(B^2*b^2*d^2*g*x^2*log(e) + (a*b*d^2*g + (2*d^2*f*log(e) - c*d*
g)*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^
2*d^2*g)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d^2)
```

3.242.8 Giac [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.243 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

3.243.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b} + \frac{2B^2(bc-ad)\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

output
```
2*B*(-a*d+b*c)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+(b
*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+2*B^2*(-a*d+b*c)*polylog(2,d*(b*x+a)
/b/(d*x+c))/b/d
```

3.243.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.71

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\Big(2ad\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) - 2bc\Big(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\Big)\log(c+dx) - aBd\Big(\log(a+bx)}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*a*d*Log[a + b*x]*(A + B*L
og[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*
c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*((2*Log[(
d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (
b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

3.243.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2936, 2944, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2936}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}dx}{b}$$

$$\Big\downarrow \text{2944}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2858}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int\frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(-\frac{B(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2778$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int \frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2005$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(\frac{B(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2752$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}-\frac{2B(bc-ad)\left(-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}-\frac{B\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{d}\right)}{b}$$

3.243. $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/b - (2*B*(b*c - a*d)*(-
((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/d)
- (B*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/d))/b
```

3.243.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2944
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
- a*d)/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x
] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && EqQ[d*f - c*g, 0]
```

3.243.4 Maple [F]

$$\int \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.243.5 Fricas [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx=\int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2, x)
```

3.243.6 Sympy [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.243.7 Maxima [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
2*(x*log((b*x + a)*e/(d*x + c)) + (a*e*log(b*x + a)/b - c*e*log(d*x + c)/d
)/e)*A*B + A^2*x + B^2*((b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)
^2 - 2*(b*d*x*log(e) + (b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(b*d) + i
ntegrate(((log(e)^2 + 2*log(e))*b^2*d*x^2 + a*b*c*log(e)^2 + (b^2*c*log(e)
^2 + (log(e)^2 + 2*log(e))*a*b*d)*x + 2*(b^2*d*x^2*log(e) + a*b*c*log(e) +
 a^2*d + (a*b*d*(log(e) + 2) + b^2*c*(log(e) - 1))*x)*log(b*x + a))/(b^2*d
*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

3.243.8 Giac [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.244 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$

3.244.1 Optimal result

Integrand size = 29, antiderivative size = 277

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g} \\ &+ \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} \\ &- \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g} \\ &+ \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} \\ &+ \frac{2B^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g} \\ &- \frac{2B^2\,\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}\end{aligned}$$

output
```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+(A+B*ln(e*(b*x+a
)/(d*x+c)))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*(A+B*ln(e*
(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*(A+B*ln(e*(b*x+a)/(
d*x+c)))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g+2*B^2*polylog(
3,d*(b*x+a)/b/(d*x+c))/g-2*B^2*polylog(3,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*
x+c))/g
```

3.244.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1348 vs. $2(277) = 554$.

Time = 0.37 (sec) , antiderivative size = 1348, normalized size of antiderivative = 4.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

output
```
(-(B^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((d*f
 - c*g)*(a + b*x))]^2) + A^2*Log[f + g*x] - 2*A*B*Log[a/b + x]*Log[f + g*x
] + B^2*Log[a/b + x]^2*Log[f + g*x] + 2*A*B*Log[c/d + x]*Log[f + g*x] - 2*
B^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + B^2*Log[c/d + x]^2*Log[f + g*
x] + 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[f + g*x] - 2*B^2*Log[a/b + x]*
Log[(e*(a + b*x))/(c + d*x)]*Log[f + g*x] + 2*B^2*Log[c/d + x]*Log[(e*(a +
 b*x))/(c + d*x)]*Log[f + g*x] + B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[f
+ g*x] + 2*A*B*Log[a/b + x]*Log[(b*(f + g*x))/(b*f - a*g)] - B^2*Log[a/b +
 x]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[a/b + x]*Log[(e*(a + b*x)
)/(c + d*x)]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[a/b + x]*Log[(g*(c
 + d*x))/(-(d*f) + c*g)]*Log[(b*(f + g*x))/(b*f - a*g)] - B^2*Log[(g*(c +
d*x))/(-(d*f) + c*g)]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*Log[(g*(c +
 d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))
]*Log[(b*(f + g*x))/(b*f - a*g)] - B^2*Log[((b*f - a*g)*(c + d*x))/((d*f -
 c*g)*(a + b*x))]^2*Log[(b*(f + g*x))/(b*f - a*g)] - 2*A*B*Log[c/d + x]*Lo
g[(d*(f + g*x))/(d*f - c*g)] + 2*B^2*Log[a/b + x]*Log[c/d + x]*Log[(d*(f +
 g*x))/(d*f - c*g)] - B^2*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] -
2*B^2*Log[c/d + x]*Log[(e*(a + b*x))/(c + d*x)]*Log[(d*(f + g*x))/(d*f - c
*g)] - 2*B^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*
x))/(d*f - c*g)] + B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(d*(f + ...
```

3.244.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2954, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.244. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{f+gx}\,dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{2B\,\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g(bc-ad)}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g(bc-ad)}-\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x
))/(b*(c + d*x))])/((b*c - a*d)*g)) + ((A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)
*g) - (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(
b*(c + d*x))])/((b*c - a*d)*g) + (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c - a*d)
*g) + (2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g) - (2
*B^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*c -
a*d)*g))
```

3.244.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.244.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(277) = 554$.

Time = 4.26 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.95

method	result
parts	$\frac{A^2\ln(gx+f)}{g}+B^2(ad-cb)\,e\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\operatorname{Li}_2\left(\frac{d\left(\ldots\right.}{\ldots}\right.}{eg(ad-cb)}\right.$
derivativedivides	$-\frac{e(ad-cb)\left(-d^2A^2\left(-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}{eg(ad-cb)(-cg+df)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{eg(ad-cb)}\right)-d^2B\ldots\right.}{\ldots}$
default	$-\frac{e(ad-cb)\left(-d^2A^2\left(-\frac{(cg-df)\ln\left(aeg-bef-cg\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+df\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}{eg(ad-cb)(-cg+df)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{eg(ad-cb)}\right)-d^2B\ldots\right.}{\ldots}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x,method=_RETURNVERBOSE)
```

3.244. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$

output
```
A^2*ln(g*x+f)/g+B^2*(a*d-b*c)*e*(-1/e/g/(a*d-b*c)*(ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))*polylog(2,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*polylog(
3,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))))+1/e/g/(a*d-b*c)*(ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2*ln(1+(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x
+c)))+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,-(c*g-d*f)/(-a*e*g+b*e*f
)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*polylog(3,-(c*g-d*f)/(-a*e*g+b*e*f)*(b*
e/d+(a*d-b*c)*e/d/(d*x+c)))))-2*B*A/d^2*(a*d-b*c)*e*(-d^2*(c*g-d*f)/e/g/(a
*d-b*c)*(dilog(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e
*g+b*e*f))/(c*g-d*f)+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f))+d^3/e/g/(a*
d-b*c)*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

3.244.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g*x + f), x)
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f),x)
```

output
```
Timed out
```

3.244.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="maxima")
```

output
```
A^2*log(g*x + f)/g - integrate(-(B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B
*log(e) + 2*(B^2*log(e) + A*B)*log(b*x + a) - 2*(B^2*log(b*x + a) + B^2*lo
g(e) + A*B)*log(d*x + c))/(g*x + f), x)
```

3.244.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{f+g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x), x)
```

3.245 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$

3.245.1 Optimal result

Integrand size = 29, antiderivative size = 196

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bf-ag)(f+gx)}+\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}+\frac{2B^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*(A
+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*
g+b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)
/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

3.245.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 402 vs. 2(196) = 392.

Time = 0.29 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.05

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$$
$$= \frac{-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}+\frac{B\left(2b(df-cg)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(bf-ag)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+2(bc-ad)g\left(A+B\log\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)) + (B*(2*b*(d*f - c*g)
*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(b*f - a*g)*(A +
B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) -
 2*B*(b*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))
/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - P
olyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

3.245.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2954, 2755, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)^2}\,dx$$
$$\Big\downarrow \ 2954$$

$$(bc-ad)\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$\downarrow$ 2755

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2B\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$\downarrow$ 2754

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2B\left(\frac{B\int\frac{(c+dx)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{df-cg}-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{df-}\right.}{bf-ag}$$

$\downarrow$ 2838

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2B\left(-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{df-cg}-\frac{B\,\text{PolyLog}\left(2,\right.}{df-}\right.}{bf-ag}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*f - a*
g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - (2*B*(-(((
A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f
- a*g)*(c + d*x))])/(d*f - c*g)) - (B*PolyLog[2, ((d*f - c*g)*(a + b*x))/(
(b*f - a*g)*(c + d*x))])/(d*f - c*g)))/(b*f - a*g))
```

3.245. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}dx$

3.245.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.245.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^2}\,dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

3.245.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**2,x)
```

output
```
Timed out
```

3.245.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="maxima")
```

output
```
2*A*B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (
b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*e*x/
(d*x + c) + a*e/(d*x + c))/(g^2*x + f*g)) - B^2*(log(d*x + c)^2/(g^2*x + f
*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g*x + c*g)*log(b*x +
a)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((g*log(e) - g)*d*x
+ c*g*log(e) - d*f + (d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^3*x^3
+ c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - A^2/
(g^2*x + f*g)
```

3.245.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^2, x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2, x)
```

3.246 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$

3.246.1 Optimal result

Integrand size = 29, antiderivative size = 369

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx = \frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)} + \frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(bf-ag)^2} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{B^2(bc-ad)^2g\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} + \frac{B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}$$

output
```
B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c*g+d*f)
/(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(-a*g+b*f)^2-1/2*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*ln((g*x+f)/(d*x+c))/(
-a*g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*
x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-
c*g+d*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,(-c*g+d*f)*(b*x
+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

3.246.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B(f+gx)\left(2(bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\log\left(\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

output
```
-1/2*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a*d)
*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^2*(d
*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
2*d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
 d*x] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*Log[(e
*(a + b*x))/(c + d*x)])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)*(b*(d*f
 - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log
[f + g*x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*x] - 2
*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]) - B*d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) -
 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*x)
)/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + Poly
Log[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])
))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

3.246.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)^3}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2}\right)$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{(bf-ag)^2(a+bx)}+\frac{(bc-ad)g(-2bdf+bcg+adg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^2(df-cg)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)\left(bf-ag-\frac{(df-cg)(a+bx}{c+dx}\right.}\right.}{g(bc-ad)}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{B\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2B(bf-ag)^2}+\frac{g^2(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(c+dx)(bf-ag)^2(df-cg)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)(df-}{(c+dx)(bf-}\right.}{(bf-ag)^2(df-cg)^2}\right.}{g(bc}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

3.246. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c +
d*x))^2) + (B*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/((b*f - a*g)^2*(d*f - c*g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a
+ b*x))/(c + d*x))) + (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*B*(b
*f - a*g)^2) + (B*(b*c - a*d)^2*g^2*Log[b*f - a*g - ((d*f - c*g)*(a + b*x)
)/(c + d*x)])/((b*f - a*g)^2*(d*f - c*g)^2) + ((b*c - a*d)*g*(2*b*d*f - b*
c*g - a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a
+ b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2) + (B*(b*c
- a*d)*g*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*
f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2)))/((b*c - a*d)*g))
```

3.246.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.246. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$

3.246.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^3}dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

3.246.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**3,x)
```

output
```
Timed out
```

3.246.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="maxima")
```

output
```
(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(
d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 -
 a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^
2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*
g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*e*x/(d*x + c) + a*e/(d*x + c))
/(g^3*x^2 + 2*f*g^2*x + f^2*g))*A*B - 1/2*B^2*(log(d*x + c)^2/(g^3*x^2 + 2
*f*g^2*x + f^2*g) + 2*integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g*x +
 c*g)*log(b*x + a)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - ((2*g*
log(e) - g)*d*x + 2*c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d
*x + c))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g^2 + c
*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 + 2*f*g^2
*x + f^2*g)
```

3.246.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^3, x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{{\left(f+g\,x\right)}^3} \,d x$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3, x)
```

3.247 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$

3.247.1 Optimal result

Integrand size = 29, antiderivative size = 714

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx = \frac{B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)}$$
$$+\frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+\frac{2B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(bf-ag)^3}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}$$
$$-\frac{B^2(bc-ad)^3g^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df}{(bf}\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)+1/3*B^2
*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3-1/3*B*(-a*
d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(-c*g+d*f)^3/(
g*x+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e*(b*
x+a)/(d*x+c)))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^3-
1/3*B^2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3
*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f
)^3/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(
b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a
^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog
(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

3.247.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.25

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2bdf+bcg}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

output
```
-1/3*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*((b*c - a*d)*g
*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c
 - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*
(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log
[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*(f +
g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*(b*c - a*d)*g
*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*
g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - 2*B*
(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d*f - c*g)*Log[a +
 b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + B*
(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f
- c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x]
 + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + b^3*
B*(d*f - c*g)^3*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d^3*(b
*f - a*g)^3*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*c - a
*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d
*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*...
```

3.247.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 885, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)^4}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{2B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3}\right)$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{2B\int\left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{(bf-ag)^3(a+bx)}+\frac{(bc-ad)g(-((3d^2f^2-3cdgf+c^2g^2)b^2)+adg(3df-cg)b-a^2d^2g^2)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^3(df-cg)^2\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}+\right.}{3g(bc-ad)}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{2B(bf-ag)^3}+\frac{B(bc-ad)^3g^3\log\left(\frac{a+bx}{c+dx}\right)}{2(bf-ag)^3(df-cg)^3}+\frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^3(df-cg)^2(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}-\frac{(bc}{2(bf-ag}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

3.247. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$

output
```
(b*c - a*d)*(-1/3*((b - (d*(a + b*x))/(c + d*x))^3*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c +
d*x))^3) + (2*B*((B*(b*c - a*d)^3*g^3)/(2*(b*f - a*g)^2*(d*f - c*g)^3*(b*f
 - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (B*(b*c - a*d)^3*g^3*Log[(a
 + b*x)/(c + d*x)])/(2*(b*f - a*g)^3*(d*f - c*g)^3) - ((b*c - a*d)^3*g^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)*(d*f - c*g)^3*(b*f - a
*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f -
 b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f -
a*g)^3*(d*f - c*g)^2*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d
*x))) + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*B*(b*f - a*g)^3) -
 (B*(b*c - a*d)^3*g^3*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/
(2*(b*f - a*g)^3*(d*f - c*g)^3) + (B*(b*c - a*d)^2*g^2*(3*b*d*f - b*c*g -
2*a*d*g)*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)^
3*(d*f - c*g)^3) + ((b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b
^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^3*
(d*f - c*g)^3) + (B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b
^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/(
(b*f - a*g)*(c + d*x))])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*(b*c - a*d)*g)
)
```

3.247.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.247. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.247.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}{(gx+f)^{4}}dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

3.247.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^{2}}{(f+gx)^{4}}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^{2}}{(gx+f)^{4}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4),
x)
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**4,x)
```

output
```
Timed out
```

3.247.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="maxima")
```

output
```
1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*e*x/(d*x
 + c) + a*e/(d*x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B
- 1/3*B^2*(log(d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) +
3*integrate(-1/3*(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 3*(d*g*x + c*g)*log(
b*x + a)^2 + 6*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((3*g*log(e) -
 g)*d*x + 3*c*g*log(e) - d*f + 3*(d*g*x + c*g)*log(b*x + a))*log(d*x + ...
```

3.247.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^4, x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^4}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4, x)
```

3.248 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx$

3.248.1 Optimal result

Integrand size = 29, antiderivative size = 1159

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = -\frac{B^2(bc-ad)^2g^3(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$-\frac{B^2(bc-ad)^3g^3(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c+dx)}{4(bf-ag)^3(df-cg)^4(f+gx)}$$
$$-\frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}+\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)^4(f+gx)^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+\frac{b^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}$$
$$+\frac{B^2(bc-ad)^4g^3\log\left(\frac{f+gx}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}-\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{f+gx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}$$
$$+\frac{B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\log\left(\frac{f+gx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,\mathrm{l}}{2(bf-ag)^4(df-cg)^4}$$
$$-\frac{B^2(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx}{(bf-ag)(c+dx}\right.}{2(bf-ag)^4(df-cg)^4}$$

output
```
-1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-1
/6*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+1/4*B^2*
(-a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^
4/(g*x+f)-1/6*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+
d*f)^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c
))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/6*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a
*d*g-b*c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c
*g+d*f)^4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*
f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*g+b*f)^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/
(-a*g+b*f)^4-1/4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^4+1/6*B^2*(-a*d+b
*c)^4*g^3*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)
^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f
)^4+1/2*B^2*(-a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*
g^2-4*c*d*f*g+6*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/
2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*
g^2-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x
+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d
*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d...
```

3.248.2 Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 1301, normalized size of antiderivative = 1.12

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx =$$

$$-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(f+gx)\left(2(bc-ad)g(bf-ag)^3(df-cg)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-3(bc-ad)g(bf-ag)^2(df-cg)^2(-2bdf+}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

output
```
-1/12*(3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a
*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 3
*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*g)*(f +
 g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)*g*(b*f - a*g)*(
d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*
f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*b^4*(
d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Lo
g[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g +
a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*x)^3*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - 6*B*(b*c - a*d)*g*(a^2*d^2*g^2
+ a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x
)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c -
 a*d)*g*Log[f + g*x]) + 3*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g
*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f + g*x)
*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*d)*g*(
-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + B*(b*c - a*d)*g*(f + g
*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(b*c - a*d)*g*(b*f - a*
g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x) - 2*b^3*(d*f - c*g)
^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b*f - a*g)^3*(f + g*x)^2*Log[c + d...
```

3.248.3 Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 1398, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)^5}dx$$

$$\Big\downarrow \text{ 2954}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2798}$$

$$(bc-ad)\left(\frac{B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4}\right)$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^4}{(bf-ag)^4(a+bx)}+\frac{(bc-ad)g(2bdf-bcg-adg)(-2d^2f^2b^2-c^2g^2b^2+2cdfgb^2+2ad^2fgb-a^2d^2g^2)\left(A+B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{(bf-ag)^4(df-cg)^3\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^4}{2B(bf-ag)^4}+\frac{B(bc-ad)^3g^3(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{2(bf-ag)^4(df-cg)^4}-\frac{B(bc-ad)^4g^4\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4}+\frac{(bc-ad)^2g^2((6d^2f^2-4cdg}{(bf-}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

3.248. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx$

output
```
(b*c - a*d)*(-1/4*((b - (d*(a + b*x))/(c + d*x))^4*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c +
d*x))^4) + (B*(-1/6*(B*(b*c - a*d)^4*g^4)/((b*f - a*g)^2*(d*f - c*g)^4*(b*
f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) - (B*(b*c - a*d)^4*g^4)/(3
*(b*f - a*g)^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x
))) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g))/(2*(b*f - a*g)^3*(
d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - (B*(b*c -
a*d)^4*g^4*Log[(a + b*x)/(c + d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B*
(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/(2
*(b*f - a*g)^4*(d*f - c*g)^4) + ((b*c - a*d)^4*g^4*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(3*(b*f - a*g)*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a +
 b*x))/(c + d*x))^3) - ((b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(b*f - a*
g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*a^2*d^2*
g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*g)^4*(d*f - c*g)^3*
(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (b^4*(A + B*L
og[(e*(a + b*x))/(c + d*x)])^2)/(2*B*(b*f - a*g)^4) + (B*(b*c - a*d)^4*g^4
*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/(3*(b*f - a*g)^4*(d*f
 - c*g)^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*Log[b*f -...
```

3.248.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.248. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx$

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.248.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}{\left(gx+f\right)^{5}}dx$$

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

output
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

3.248.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^{2}}{(f+gx)^{5}}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^{2}}{\left(gx+f\right)^{5}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="fricas")
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 +
5*f^4*g*x + f^5), x)
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**5,x)
```

output
```
Timed out
```

3.248.7 Maxima [F]

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="maxima")
```

output
```
1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

3.248.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^5, x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^5}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5,x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5, x)
```

3.249 $\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx$

3.249.1 Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\log\left(-\frac{x}{1-x}\right) - \frac{(1+x)\log\left(-\frac{1+x}{1-x}\right)}{x}$$

output
```
2*ln(-x/(1-x))-(1+x)*ln((-1-x)/(1-x))/x
```

3.249.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\log(x) - \frac{\log\left(\frac{1+x}{-1+x}\right)}{x} - \log\left(1-x^2\right)$$

input
```
Integrate[Log[(1 + x)/(-1 + x)]/x^2,x]
```

output
```
2*Log[x] - Log[(1 + x)/(-1 + x)]/x - Log[1 - x^2]
```

3.249.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2953, 2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{x+1}{x-1}\right)}{x^2}dx$$

$$\Big\downarrow \text{2953}$$

$$-2\int \frac{\log\left(-\frac{x+1}{1-x}\right)}{\left(1-\frac{x+1}{1-x}\right)^2}d\left(-\frac{x+1}{1-x}\right)$$

$$\Big\downarrow \text{2751}$$

$$-2\left(-\int \frac{1}{1-\frac{x+1}{1-x}}d\left(-\frac{x+1}{1-x}\right)-\frac{(x+1)\log\left(-\frac{x+1}{1-x}\right)}{(1-x)\left(1-\frac{x+1}{1-x}\right)}\right)$$

$$\Big\downarrow \text{16}$$

$$-2\left(-\frac{(x+1)\log\left(-\frac{x+1}{1-x}\right)}{(1-x)\left(1-\frac{x+1}{1-x}\right)}-\log\left(1-\frac{x+1}{1-x}\right)\right)$$

input
```
Int[Log[(1 + x)/(-1 + x)]/x^2,x]
```

output
```
-2*(-(((1 + x)*Log[-((1 + x)/(1 - x))])/((1 - x)*(1 - (1 + x)/(1 - x)))) -
 Log[1 - (1 + x)/(1 - x)])
```

3.249.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.249.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{\ln\left(\frac{x+1}{-1+x}\right)}{x}+2\ln\left(x\right)-\ln\left(x^{2}-1\right)$	29
parts	$-\frac{\ln\left(\frac{x+1}{-1+x}\right)}{x}-\ln\left(-1+x\right)+2\ln\left(x\right)-\ln\left(x+1\right)$	33
parallelrisch	$\frac{2\ln(x)x-2\ln(-1+x)x-x\ln\left(\frac{x+1}{-1+x}\right)-\ln\left(\frac{x+1}{-1+x}\right)}{x}$	43
derivativedivides	$2\ln\left(2+\frac{2}{-1+x}\right)-\frac{2\ln\left(1+\frac{2}{-1+x}\right)\left(1+\frac{2}{-1+x}\right)}{2+\frac{2}{-1+x}}$	46
default	$2\ln\left(2+\frac{2}{-1+x}\right)-\frac{2\ln\left(1+\frac{2}{-1+x}\right)\left(1+\frac{2}{-1+x}\right)}{2+\frac{2}{-1+x}}$	46

input
```
int(ln((x+1)/(-1+x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln((x+1)/(-1+x))+2*ln(x)-ln(x^2-1)
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = -\frac{x\log\left(x^2-1\right)-2\,x\log\left(x\right)+\log\left(\frac{x+1}{x-1}\right)}{x}$$

input
```
integrate(log((1+x)/(-1+x))/x^2,x, algorithm="fricas")
```

output
```
-(x*log(x^2 - 1) - 2*x*log(x) + log((x + 1)/(x - 1)))/x
```

3.249.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\log\left(x\right)-\log\left(x^2-1\right)-\frac{\log\left(\frac{x+1}{x-1}\right)}{x}$$

input
```
integrate(ln((1+x)/(-1+x))/x**2,x)
```

output
```
2*log(x) - log(x**2 - 1) - log((x + 1)/(x - 1))/x
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = -\frac{\log\left(\frac{x+1}{x-1}\right)}{x}-\log\left(x+1\right)-\log\left(x-1\right)+2\,\log\left(x\right)$$

input
```
integrate(log((1+x)/(-1+x))/x^2,x, algorithm="maxima")
```

output
```
-log((x + 1)/(x - 1))/x - log(x + 1) - log(x - 1) + 2*log(x)
```

3.249.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(29) = 58.

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.94

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = \frac{2\log\left(\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}+1}{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}-1}\right)}{\frac{x+1}{x-1}+1} - 2\log\left(\frac{|x+1|}{|x-1|}\right) + 2\log\left(\left|\frac{x+1}{x-1}+1\right|\right)$$

input
```
integrate(log((1+x)/(-1+x))/x^2,x, algorithm="giac")
```

output
```
2*log((((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) + 1)/(((x + 1)/(x - 1)
+ 1)/((x + 1)/(x - 1) - 1) - 1))/((x + 1)/(x - 1) + 1) - 2*log(abs(x + 1)/
abs(x - 1)) + 2*log(abs((x + 1)/(x - 1) + 1))
```

3.249.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx = 2\ln(x) - \ln\left(x^2-1\right) - \frac{\ln\left(\frac{x+1}{x-1}\right)}{x}$$

input
```
int(log((x + 1)/(x - 1))/x^2,x)
```

output
```
2*log(x) - log(x^2 - 1) - log((x + 1)/(x - 1))/x
```

3.250 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

3.250.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.250.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

3.250.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
$Aborted
```

3.250.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.250.4 Maple [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.250.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x
)
```

3.250.6 Sympy [N/A]

Not integrable

Time = 8.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral((f + g*x)**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

3.250.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.250.8 Giac [N/A]

Not integrable

Time = 13.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.250.9 Mupad [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{(f+g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.251 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

3.251.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.251.2 Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

3.251.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{f+gx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
$Aborted
```

3.251.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.251.4 Maple [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.251.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral((g*x + f)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

3.251.6 Sympy [N/A]

Not integrable

Time = 4.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

input
```
integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral((f + g*x)/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

3.251.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.251.8 Giac [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.251.9 Mupad [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.252 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

3.252.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\text{Int}\left(\frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

output
```
Unintegrable(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.252.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x]
```

3.252.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2938}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A} \, dx$$

$$\Big\downarrow \text{2938}$$

$$\int \frac{1}{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}dx$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
```

output
```
$Aborted
```

3.252.3.1 Defintions of rubi rules used

rule 2938
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x] && EqQ[n + mn, 0]
```

3.252.4 Maple [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B \ln \left(\frac{e(bx+a)}{dx+c}\right)}dx$$

input
```
int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.252.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral(1/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

3.252.6 Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

input
```
integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/(A + B*log(e*(a + b*x)/(c + d*x))), x)
```

3.252.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.252.8 Giac [N/A]

Not integrable

Time = 11.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

3.252.9 Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

input
```
int(1/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
int(1/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.253 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

3.253.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.253.2 Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

3.253.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
$Aborted
```

3.253.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.253.4 Maple [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.253.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b*e*x + a*e)/(d*x + c))), x)
```

3.253.6 Sympy [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{\left(A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)\right)(f+gx)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)), x)
```

3.253.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.253.8 Giac [N/A]

Not integrable

Time = 17.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.253.9 Mupad [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.254 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

3.254.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.254.2 Mathematica [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

3.254.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
$Aborted
```

3.254.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.254.4 Maple [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.254.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2
)*log((b*e*x + a*e)/(d*x + c))), x)
```

3.254.6 Sympy [N/A]

Not integrable

Time = 166.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)\right)(f+gx)^2}\,dx$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)**2), x)
```

3.254.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.254.8 Giac [N/A]

Not integrable

Time = 27.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.254.9 Mupad [N/A]

Not integrable

Time = 5.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.255 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

3.255.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.255.2 Mathematica [N/A]

Not integrable

Time = 21.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

3.255.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

output
```
$Aborted
```

3.255.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.255.4 Maple [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

3.255.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b*e*x + a*e)/(d*x + c))), x)
```

3.255.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
Timed out
```

3.255.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.255.8 Giac [N/A]

Not integrable

Time = 38.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

3.255.9 Mupad [N/A]

Not integrable

Time = 7.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.256 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.256.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.256.2 Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

3.256.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
$Aborted
```

3.256.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.256.4 Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.256.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2
*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.256.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 318, normalized size of antiderivative = 10.97

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
-(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g
+ c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)
/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d
)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((4*b*d*g^2*x^3 + b*c*f^
2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2*((2*d*
f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a) - (
b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*
B^2), x)
```

3.256.8 Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.256.9 Mupad [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{(f+g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.257 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.257.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.257.2 Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

3.257.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{f+gx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
$Aborted
```

3.257.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.257.4 Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.257.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral((g*x + f)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x
+ a*e)/(d*x + c)) + A^2), x)
```

3.257.6 Sympy [N/A]

Not integrable

Time = 22.02 (sec) , antiderivative size = 337, normalized size of antiderivative = 12.48

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$= \frac{acf+acgx+adfx+adgx^2+bcfx+bcgx^2+bdfx^2+bdgx^3}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{\int \frac{acg}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{adf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{bcf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{2adgx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{B(ad-bc)}$$

input
```
integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**
2 + b*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)
/(c + d*x))) - (Integral(a*c*g/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))
), x) + Integral(a*d*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) +
Integral(b*c*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral
(2*a*d*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b
*c*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b*d*f
*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(3*b*d*g*x**
2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d - b*c))
```

3.257.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 225, normalized size of antiderivative = 8.33

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
-(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a
)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c -
 a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b*d*g*x^2 + b*c*
f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c - a*d)*B^2*log(b*x
+ a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*
log(e))*B^2), x)
```

3.257.8 Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

3.257.9 Mupad [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{f+g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.258 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.258.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.258.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x]
```

3.258.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2938}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2938}$$

$$\int \frac{1}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
```

output
```
$Aborted
```

3.258.3.1 Defintions of rubi rules used

rule 2938
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x] && EqQ[n + mn, 0]
```

3.258.4 Maple [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.258.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(1/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(
d*x + c)) + A^2), x)
```

3.258.6 Sympy [N/A]

Not integrable

Time = 8.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 7.52

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$
$$=\frac{ac+adx+bcx+bdx^2}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)}$$
$$-\frac{\int \frac{ad}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{bc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{2bdx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{B\,(ad-bc)}$$

input
```
integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
(a*c + a*d*x + b*c*x + b*d*x**2)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c
)*log(e*(a + b*x)/(c + d*x))) - (Integral(a*d/(A + B*log(a*e/(c + d*x) + b
*e*x/(c + d*x))), x) + Integral(b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c +
d*x))), x) + Integral(2*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))
, x))/(B*(a*d - b*c))
```

3.258.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 8.14

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*
d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + i
ntegrate((2*b*d*x + b*c + a*d)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)
*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

3.258.8 Giac [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^(-2), x)
```

3.258.9 Mupad [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.258. $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.259 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$

3.259.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx = \text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}, x\right)$$

output
```
Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.259.2 Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

3.259.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
$Aborted
```

3.259.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.259.4 Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.259.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.66

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

output
```
integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*e*x + a*e)/(d*x + c
))^2 + 2*(A*B*g*x + A*B*f)*log((b*e*x + a*e)/(d*x + c))), x)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.259.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 452, normalized size of antiderivative = 15.59

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*log(e) - a*
d*f*log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2
)*x + ((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - ((b*c*g
 - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(d*x + c)) + integrate((b*d*g*x^
2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2
*log(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e)
 - a*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e)
 - a*d*f*g*log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*
d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2
)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x
 + c)), x)
```

3.259.8 Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

3.259.9 Mupad [N/A]

Not integrable

Time = 5.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.260 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.260.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.260.2 Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

3.260.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2} \, dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(f+gx)^2 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
$Aborted
```

3.260.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.260.4 Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{2} \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.260.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g
*x + B^2*f^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*
x + A*B*f^2)*log((b*e*x + a*e)/(d*x + c))), x)
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.260.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 688, normalized size of antiderivative = 23.72

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*log(e
) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) - a*d
*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) - a*d
*f*g*log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)
*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^2*
x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c))
 - integrate(-(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b
*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*
f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2
 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b
*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*log(e) - a*d*f^2*g*log(e))*B^2)*x +
 ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c
*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - ((b*c*
g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g
- a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)), x)
```

3.260.8 Giac [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

3.260.9 Mupad [N/A]

Not integrable

Time = 22.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.261 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

3.261.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.261.2 Mathematica [N/A]

Not integrable

Time = 48.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

3.261.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2956}$$

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

output
```
$Aborted
```

3.261.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.261.4 Maple [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

3.261.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.48

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2
*g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*l
og((b*e*x + a*e)/(d*x + c))), x)
```

3.261.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

3.261.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 921, normalized size of antiderivative = 31.76

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
-(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(
e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e)
- a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e)
 - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*
g*log(e) - a*d*f^2*g*log(e))*B^2)*x + ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*
c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3
- a*d*f^3)*B^2)*log(b*x + a) - ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2
 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f
^3)*B^2)*log(d*x + c)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a +
2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4*log(e) -
 a*d*g^4*log(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3*lo
g(e) - a*d*f*g^3*log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4*log
(e) - a*d*f^4*log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*
g^2*log(e) - a*d*f^2*g^2*log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B
 + (b*c*f^3*g*log(e) - a*d*f^3*g*log(e))*B^2)*x + ((b*c*g^4 - a*d*g^4)*B^2
*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B
^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(b*
x + a) - ((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3
+ 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x
+ (b*c*f^4 - a*d*f^4)*B^2)*log(d*x + c)), x)
```

3.261.8 Giac [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

3.261.9 Mupad [N/A]

Not integrable

Time = 31.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.262 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.262.1 Optimal result

Integrand size = 29, antiderivative size = 357

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{2B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c}{5b^4d^4}$$
$$-\frac{B(bc-ad)g^2(a^2d^2g^2-abdg(5df-cg)+b^2(10d^2f^2-5cdfg+c^2g^2))\,x^2}{5b^3d^3}$$
$$-\frac{2B(bc-ad)g^3(5bdf-bcg-adg)x^3}{15b^2d^2}-\frac{B(bc-ad)g^4x^4}{10bd}-\frac{2B(bf-ag)^5\log(a+bx)}{5b^5g}$$
$$+\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}+\frac{2B(df-cg)^5\log(c+dx)}{5d^5g}$$

output
```
2/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*
g^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^
3*f^3))*x/b^4/d^4-1/5*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b
^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-2/15*B*(-a*d+b*c)*g^3*(-a*d
*g-b*c*g+5*b*d*f)*x^3/b^2/d^2-1/10*B*(-a*d+b*c)*g^4*x^4/b/d-2/5*B*(-a*g+b*
f)^5*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+2/5*B
*(-c*g+d*f)^5*ln(d*x+c)/d^5/g
```

3.262.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.79

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{\frac{B(-bc+ad)g^2x\left(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10f-\right.\right.}{6b^4d^4}}{}$$

input
```
Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*
g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 +
 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*
c*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f
*g^2*x^2 + 3*g^3*x^3))))/(6*b^4*d^4) - (2*B*(b*f - a*g)^5*Log[a + b*x])/b^
5 + (f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(d*f - c*g
)^5*Log[c + d*x])/d^5)/(5*g)
```

3.262.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^4 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$
$$\Big\downarrow \text{2948}$$
$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5g}-\frac{2B(bc-ad)\int\frac{(f+gx)^5}{(a+bx)(c+dx)}dx}{5g}$$
$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5g}-$$
$$\frac{2B(bc-ad)\int\left(\frac{x^3g^5}{bd}+\frac{(5bdf-bcg-adg)x^2g^4}{b^2d^2}+\frac{((10d^2f^2-5cdgf+c^2g^2)b^2-adg(5df-cg)b+a^2d^2g^2)xg^3}{b^3d^3}+\frac{((10d^3f^3-10cd^2gf^2+5c^2dg^2}{}\right.}{5g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5g}-$$
$$\frac{2B(bc-ad)\left(\frac{g^3x^2(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2))}{2b^3d^3}-\frac{g^2x(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b}{b^4d^4}\right.}{5g}$$

input
```
Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*g) - (2*B*(b*c -
 a*d)*(-((g^2*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10*d
^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d
*f*g^2 - c^3*g^3))*x)/(b^4*d^4)) + (g^3*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*
g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(2*b^3*d^3) + (g^4*(5*b*
d*f - b*c*g - a*d*g)*x^3)/(3*b^2*d^2) + (g^5*x^4)/(4*b*d) + ((b*f - a*g)^5
*Log[a + b*x])/(b^5*(b*c - a*d)) - ((d*f - c*g)^5*Log[c + d*x])/(d^5*(b*c
- a*d))))/(5*g)
```

3.262.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.262.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.68

method	result
risch	$\frac{4g^2Bc^2f^2x}{d^2}+\frac{g^4Bax^4}{10b}-\frac{g^4Bcx^4}{10d}+2g^2Af^2x^3+\frac{2g^4Bc^2x^3}{15d^2}+2gAf^3x^2+\frac{g^4Ba^3x^2}{5b^3}-\frac{g^4Bc^3x^2}{5d^3}+$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((g*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
4/d^2*g^2*B*c^2*f^2*x+1/10/b*g^4*B*a*x^4-1/10/d*g^4*B*c*x^4+2*g^2*A*f^2*x^
3+2/15/d^2*g^4*B*c^2*x^3+2*g*A*f^3*x^2+1/5/b^3*g^4*B*a^3*x^2-1/5/d^3*g^4*B
*c^3*x^2+2/d^4*g^3*B*ln(d*x+c)*c^4*f-4/d^3*g^2*B*ln(d*x+c)*c^3*f^2+4/d^2*g
*B*ln(d*x+c)*c^2*f^3-2/b^4*g^3*B*ln(-b*x-a)*a^4*f+1/5*g^4*A*x^5-2/5/d^5*g^
4*B*ln(d*x+c)*c^5+1/5*(g*x+f)^5*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)-2/5/g*B*ln(-
b*x-a)*f^5+1/d^2*g^3*B*c^2*f*x^2-2/15/b^2*g^4*B*a^2*x^3+A*f^4*x+g^3*A*f*x^
4+2/5/g*B*ln(d*x+c)*f^5+4/b^3*g^2*B*ln(-b*x-a)*a^3*f^2+2/b^3*g^3*B*a^3*f*x
-4/b^2*g^2*B*a^2*f^2*x+4/b*g*B*a*f^3*x-2/d^3*g^3*B*c^3*f*x-4/d*g*B*c*f^3*x
-2/5/b^4*g^4*B*a^4*x+2/5/d^4*g^4*B*c^4*x-4/b^2*g*B*ln(-b*x-a)*a^2*f^3+2/3/
b*g^3*B*a*f*x^3-2/3/d*g^3*B*c*f*x^3-1/b^2*g^3*B*a^2*f*x^2+2/b*g^2*B*a*f^2*
x^2-2/d*g^2*B*c*f^2*x^2+2/5/b^5*g^4*B*ln(-b*x-a)*a^5+2/b*B*ln(-b*x-a)*a*f^
4-2/d*B*ln(d*x+c)*c*f^4
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.85

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{6\,Ab^5d^5g^4x^5+3\,(10\,Ab^5d^5fg^3-(Bb^5cd^4-Bab^4d^5)g^4)x^4+4\,(15\,Ab^5d^5f^2g^2-5\,(Bb^5cd^4-Bab^4d^5)fg^3+}{}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
1/30*(6*A*b^5*d^5*g^4*x^5 + 3*(10*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*g^4)*x^4 + 4*(15*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f
*g^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(10*A*b^5*d^5*f^3*g -
10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)
*f*g^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 6*(5*A*b^5*d^5*f^4 - 2
0*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 20*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f
^2*g^2 - 10*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + 2*(B*b^5*c^4*d - B*a^4
*b*d^5)*g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3
*b^2*d^5*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*
(5*B*b^5*c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5
*B*b^5*c^4*d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 6*(B*b^5*d^5*g^4*x^5 +
5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2
+ 5*B*b^5*d^5*f^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*
x + c^2)))/(b^5*d^5)
```

3.262.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(347) = 694$.

Time = 57.42 (sec) , antiderivative size = 1477, normalized size of antiderivative = 4.14

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*g**4*x**5/5 + 2*B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**
2 - 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (2*B*a**5*c*d**4*g**4 - 10*B*a
**4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4*f**2*g**2 - 20*B*a**2*b**3*c*d
**4*f**3*g + 2*B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**
2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4)/b + 2*B*a*b**4*c**5*g**4 - 10*B*a
*b**4*c**4*d*f*g**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d
**3*f**3*g + 20*B*a*b**4*c*d**4*f**4 - 2*B*a*c*d**4*(a**4*g**4 - 5*a**3*b*
f*g**3 + 10*a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(2*B*a*
*5*d**5*g**4 - 10*B*a**4*b*d**5*f*g**3 + 20*B*a**3*b**2*d**5*f**2*g**2 - 2
0*B*a**2*b**3*d**5*f**3*g + 10*B*a*b**4*d**5*f**4 + 2*B*b**5*c**5*g**4 - 1
0*B*b**5*c**4*d*f*g**3 + 20*B*b**5*c**3*d**2*f**2*g**2 - 20*B*b**5*c**2*d*
*3*f**3*g + 10*B*b**5*c*d**4*f**4))/(5*b**5) - 2*B*c*(c**4*g**4 - 5*c**3*d
*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x +
 (2*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4
*f**2*g**2 - 20*B*a**2*b**3*c*d**4*f**3*g + 2*B*a*b**4*c**5*g**4 - 10*B*a*
b**4*c**4*d*f*g**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d*
*3*f**3*g + 20*B*a*b**4*c*d**4*f**4 - 2*B*a*b**4*c*(c**4*g**4 - 5*c**3*d*f
*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4) + 2*B*b**
5*c**2*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f
**3*g + 5*d**4*f**4)/d)/(2*B*a**5*d**5*g**4 - 10*B*a**4*b*d**5*f*g**3 +...
```

3.262.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(343) = 686$.

Time = 0.23 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.39

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{1}{5}Ag^4x^5 + Afg^3x^4 + 2Af^2g^2x^3 + 2Af^3gx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2} + \frac{2abex}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{2a\log(bx+a)}{b} - \frac{2c\log(dx-}{d}\right.$$
$$+2\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2} + \frac{2abex}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) - \frac{2a^2\log(bx+a)}{b^2} + \frac{2c^2\log(}{d}\right.$$
$$+2\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2} + \frac{2abex}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(}{d}\right.$$
$$+\frac{1}{3}\left(3x^4\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2} + \frac{2abex}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log}{}\right.$$
$$+\frac{1}{30}\left(6x^5\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2} + \frac{2abex}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{12a^5\log(bx+a)}{b^5} - \frac{12c}{}\right.$$
$$+Af^4x$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b^2
*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a
^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)
*B*f^4 + 2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x
^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a
)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^3*g + 2*(x^3*l
og(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^
2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log
(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d
^2))*B*f^2*g^2 + 1/3*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*
b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4
*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^
3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*
f*g^3 + 1/30*(6*x^5*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d
^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 12*a^5*log(b*
x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*
(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*
c^4 - a^4*d^4)*x)/(b^4*d^4))*B*g^4 + A*f^4*x
```

3.262.8 Giac [F(-1)]

Timed out.

$$\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \text{Timed out}$$

input
```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
Timed out
```

3.262.9 Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 1403, normalized size of antiderivative = 3.93

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=\text{Too large to display}$$

input
```
int((f + g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*g^2*x^
3 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g +
 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*g^2
)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4
- 2*B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))
*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3
 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^
4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B
*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(2*
b*d)) + x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A
*b*d*f*g^3)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x*((5*A*b*d*f^4
 + 20*A*a*d*f^3*g + 20*A*b*c*f^3*g + 20*B*a*d*f^3*g - 20*B*b*c*f^3*g + 30*
A*a*c*f^2*g^2)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*
g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*
g^2)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^
4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d
))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g
^3 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*
g^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*
B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))...
```

3.263 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.263.1 Optimal result

Integrand size = 29, antiderivative size = 229

$$\begin{aligned}\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx &= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{2b^3d^3} \\ &\quad -\frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{4b^2d^2}-\frac{B(bc-ad)g^3x^3}{6bd}-\frac{B(bf-ag)^4\log(a+bx)}{2b^4g} \\ &\quad +\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g}+\frac{B(df-cg)^4\log(c+dx)}{2d^4g}\end{aligned}$$

output
```
-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f
*g+6*d^2*f^2))*x/b^3/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b
^2/d^2-1/6*B*(-a*d+b*c)*g^3*x^3/b/d-1/2*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/4
*(g*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+1/2*B*(-c*g+d*f)^4*ln(d*x+c)/
d^4/g
```

3.263.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$=\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B\left(6bd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+3b^2d^2(bc-ad)g^3(4bdf\cdots}{3b^4d^4}}{4g}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(6*b*d*(b*c - a
*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g
 + c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 +
 2*b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*
(d*f - c*g)^4*Log[c + d*x]))/(3*b^4*d^4))/(4*g)
```

3.263.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-\frac{B(bc-ad)\int\frac{(f+gx)^4}{(a+bx)(c+dx)}dx}{2g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-$$
$$\frac{B(bc-ad)\int\left(\frac{x^2g^4}{bd}+\frac{(4bdf-bcg-adg)xg^3}{b^2d^2}+\frac{\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-adg(4df-cg)b+a^2d^2g^2\right)g^2}{b^3d^3}+\frac{(bf-ag)^4}{b^3(bc-ad)(a+bx)}+\frac{(df-cg)\cdots}{d^3(ad-bc)(c\cdots}\right.}{2g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-$$
$$\frac{B(bc-ad)\left(\frac{g^2x(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2))}{b^3d^3}+\frac{(bf-ag)^4\log(a+bx)}{b^4(bc-ad)}+\frac{g^3x^2(-adg-bcg+4bdf)}{2b^2d^2}-\frac{(df-cg)^4\log(c+d}{d^4(bc-ad)}\right.}{2g}$$

input
```
Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*g) - (B*(b*c - a
*d)*((g^2*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*
g + c^2*g^2))*x)/(b^3*d^3) + (g^3*(4*b*d*f - b*c*g - a*d*g)*x^2)/(2*b^2*d^
2) + (g^4*x^3)/(3*b*d) + ((b*f - a*g)^4*Log[a + b*x])/(b^4*(b*c - a*d)) -
((d*f - c*g)^4*Log[c + d*x])/(d^4*(b*c - a*d))))/(2*g)
```

3.263.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.263. $\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.263.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{g^{2} B c f \,x^{2}}{d}-\frac{2 g^{2} B \,a^{2} f x}{b^{2}}+\frac{3 g B a \,f^{2} x}{b}+\frac{2 g^{2} B \,c^{2} f x}{d^{2}}-\frac{2 B \ln \left(-d x-c\right) c \,f^{3}}{d}+\frac{B \ln \left(-d x-c\right) f^{4}}{2 g}+\frac{g^{2} B a f \,x^{2}}{b}-$
parts	$\frac{A \left(g x+f\right)^{4}}{4 g}+\frac{B \left(-\left(-\frac{\left(d x+c\right)^{4} \ln \left(\frac{e \left(\frac{a d}{d x+c}-\frac{b c}{d x+c}+b\right)^{2}}{d^{2}}\right)}{4}-\left(-\frac{a d}{2}+\frac{c b}{2}\right) \left(\frac{\left(-a^{3} d^{3}+3 a^{2} b c \,d^{2}-3 a \,b^{2} c^{2} d+b^{3} c^{3}\right) \ln \left(\frac{1}{d x+c}\right)}{b^{4}}-\right.\right.\right.}{}$
derivativedivides	$-\frac{-\frac{A \left(\frac{g^{3} \left(d x+c\right)^{4}}{4}+\frac{3 g \left(g^{2} c^{2}-2 g f d c+f^{2} d^{2}\right) \left(d x+c\right)^{2}}{2}-g^{2} \left(c g-d f\right) \left(d x+c\right)^{3}-\left(c^{3} g^{3}-3 c^{2} d f \,g^{2}+3 c \,d^{2} f^{2} g-d^{3} f^{3}\right) \left(d x+c\right)\right)}{d^{3}}-\frac{B \left(-\left(-\right.\right.}{}}{}$
default	$-\frac{-\frac{A \left(\frac{g^{3} \left(d x+c\right)^{4}}{4}+\frac{3 g \left(g^{2} c^{2}-2 g f d c+f^{2} d^{2}\right) \left(d x+c\right)^{2}}{2}-g^{2} \left(c g-d f\right) \left(d x+c\right)^{3}-\left(c^{3} g^{3}-3 c^{2} d f \,g^{2}+3 c \,d^{2} f^{2} g-d^{3} f^{3}\right) \left(d x+c\right)\right)}{d^{3}}-\frac{B \left(-\left(-\right.\right.}{}}{}$
parallelrisch	$\frac{-24 B x \,a^{2} b^{2} d^{4} f \,g^{2}+36 B x a \,b^{3} d^{4} f^{2} g+36 B \ln \left(b x+a\right) b^{4} c^{2} d^{2} f^{2} g+24 B \ln \left(b x+a\right) a^{3} b \,d^{4} f \,g^{2}-36 B \ln \left(b x+a\right) a^{2} b^{2} d^{4} f^{2} g-24}{}$

input
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/d*g^2*B*c*f*x^2-2/b^2*g^2*B*a^2*f*x+3/b*g*B*a*f^2*x+2/d^2*g^2*B*c^2*f*x
-2/d*B*ln(-d*x-c)*c*f^3+1/2/g*B*ln(-d*x-c)*f^4+1/b*g^2*B*a*f*x^2-3/d*g*B*c
*f^2*x-1/2/g*B*ln(b*x+a)*f^4+1/4*(g*x+f)^4*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)+1
/2/d^4*g^3*B*ln(-d*x-c)*c^4+3/d^2*g*B*ln(-d*x-c)*c^2*f^2-1/6/d*g^3*B*c*x^3
-1/4/b^2*g^3*B*a^2*x^2+1/4/d^2*g^3*B*c^2*x^2+A*f^3*x+1/2/b^3*g^3*B*a^3*x-1
/2/d^3*g^3*B*c^3*x+2/b^3*g^2*B*ln(b*x+a)*a^3*f-3/b^2*g*B*ln(b*x+a)*a^2*f^2
-2/d^3*g^2*B*ln(-d*x-c)*c^3*f-1/2/b^4*g^3*B*ln(b*x+a)*a^4+g^2*A*f*x^3+1/6/
b*g^3*B*a*x^3+3/2*g*A*f^2*x^2+1/4*g^3*A*x^4+2/b*B*ln(b*x+a)*a*f^3
```

3.263.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(217) = 434.

Time = 0.39 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.04

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{3\,Ab^4d^4g^3x^4+2\,(6\,Ab^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3)x^3+3\,(6\,Ab^4d^4f^2g-4\,(Bb^4cd^3-Bab^3d^4)fg^2+(B}{}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
1/12*(3*A*b^4*d^4*g^3*x^4 + 2*(6*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*g^3)*x^3 + 3*(6*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2
 + (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(2*A*b^4*d^4*f^3 - 6*(B*b^
4*c*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (
B*b^4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4
*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*
d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log
(d*x + c) + 3*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2
*g*x^2 + 4*B*b^4*d^4*f^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 +
 2*c*d*x + c^2)))/(b^4*d^4)
```

3.263.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. 2(211) = 422.

Time = 8.17 (sec) , antiderivative size = 998, normalized size of antiderivative = 4.36

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx=\frac{Ag^3x^4}{4}$$

$$-\frac{Ba(ag-2bf)\left(a^2g^2-2abfg+2b^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+\frac{Ba^2d^4(ag-2bf)\left(a^2g^2-2abfg+2b\cdots\right)}{b}\cdots}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g\cdots}\cdots\right)}{2b^4}$$

$$+\frac{Bc(cg-2df)\left(c^2g^2-2cdfg+2d^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+Bab^3c^4g^3-4Bab^3c^3dfg^2+6Bab^3\cdots}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g-\cdots}\cdots\right)}{2d^4}$$

$$+x^3\left(Afg^2+\frac{Bag^3}{6b}-\frac{Bcg^3}{6d}\right)+x^2\cdot\left(\frac{3Af^2g}{2}-\frac{Ba^2g^3}{4b^2}+\frac{Bafg^2}{b}+\frac{Bc^2g^3}{4d^2}-\frac{Bcfg^2}{d}\right)$$

$$+x\left(Af^3+\frac{Ba^3g^3}{2b^3}-\frac{2Ba^2fg^2}{b^2}+\frac{3Baf^2g}{b}-\frac{Bc^3g^3}{2d^3}+\frac{2Bc^2fg^2}{d^2}-\frac{3Bcf^2g}{d}\right)$$

$$+\left(Bf^3x+\frac{3Bf^2gx^2}{2}+Bfg^2x^3+\frac{Bg^3x^4}{4}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*lo
g(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**
3*f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)
/b + B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*
f**2*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*
a*b*f*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a
**2*b**2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*
c**3*d*f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*b**4
) + B*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**
4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B
*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g
 - 8*B*a*b**3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*
g + 2*d**2*f**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d*
*2*f**2)/d)/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**
4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**
2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*d**4) + x**3*(A*
f*g**2 + B*a*g**3/(6*b) - B*c*g**3/(6*d)) + x**2*(3*A*f**2*g/2 - B*a**2*g*
*3/(4*b**2) + B*a*f*g**2/b + B*c**2*g**3/(4*d**2) - B*c*f*g**2/d) + x*(A*f
**3 + B*a**3*g**3/(2*b**3) - 2*B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/b - B*c**
3*g**3/(2*d**3) + 2*B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/d) + (B*f**3*x + ...
```

3.263. $\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$

3.263.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(217) = 434$.

Time = 0.23 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.72

$$\int (f+gx)^3 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{1}{4}\,Ag^3x^4 + Afg^2x^3 + \frac{3}{2}\,Af^2gx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx\ldots}{d}\right.$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\log\ldots}{\ldots}\right.$$
$$+\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log(d\ldots}{d^3}\right.$$
$$+\frac{1}{12}\left(3\,x^4\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{6\,a^4\log(bx+a)}{b^4} + \frac{6\,c^4\,\mathrm{l}\ldots}{}\right.$$
$$+ Af^3x$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2
+ 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 +
2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^3 + 3/2*(x^
2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x +
 c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*
log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^2*g + (x^3*log(b^2*e*x^2/(d^
2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*
x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 -
((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1
/12*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 +
2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4
 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
- a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

3.263.8 Giac [A] (verification not implemented)

Time = 75.81 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.79

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{4}Ag^3x^4+\frac{(6\,Abdfg^2-Bbcg^3+Badg^3)x^3}{6\,bd} + \frac{1}{4}\left(Bg^3x^4+4\,Bfg^2x^3+6\,Bf^2gx^2+4\,Bf^3x\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{(6\,Ab^2d^2f^2g-4\,Bb^2cdfg^2+4\,Babd^2fg^2+Bb^2c^2g^3-Ba^2d^2g^3)x^2}{4\,b^2d^2} + \frac{(4\,Bab^3f^3-6\,Ba^2b^2f^2g+4\,Ba^3bfg^2-Ba^4g^3)\log(bx+a)}{2\,b^4} - \frac{(4\,Bcd^3f^3-6\,Bc^2d^2f^2g+4\,Bc^3dfg^2-Bc^4g^3)\log(-dx-c)}{2\,d^4} + \frac{(2\,Ab^3d^3f^3-6\,Bb^3cd^2f^2g+6\,Bab^2d^3f^2g+4\,Bb^3c^2dfg^2-4\,Ba^2bd^3fg^2-Bb^3c^3g^3+Ba^3d^3g^3)x}{2\,b^3d^3}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
1/4*A*g^3*x^4 + 1/6*(6*A*b*d*f*g^2 - B*b*c*g^3 + B*a*d*g^3)*x^3/(b*d) + 1/
4*(B*g^3*x^4 + 4*B*f*g^2*x^3 + 6*B*f^2*g*x^2 + 4*B*f^3*x)*log((b^2*e*x^2 +
 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(6*A*b^2*d^2*f^2*g -
4*B*b^2*c*d*f*g^2 + 4*B*a*b*d^2*f*g^2 + B*b^2*c^2*g^3 - B*a^2*d^2*g^3)*x^2
/(b^2*d^2) + 1/2*(4*B*a*b^3*f^3 - 6*B*a^2*b^2*f^2*g + 4*B*a^3*b*f*g^2 - B*
a^4*g^3)*log(b*x + a)/b^4 - 1/2*(4*B*c*d^3*f^3 - 6*B*c^2*d^2*f^2*g + 4*B*c
^3*d*f*g^2 - B*c^4*g^3)*log(-d*x - c)/d^4 + 1/2*(2*A*b^3*d^3*f^3 - 6*B*b^3
*c*d^2*f^2*g + 6*B*a*b^2*d^3*f^2*g + 4*B*b^3*c^2*d*f*g^2 - 4*B*a^2*b*d^3*f
*g^2 - B*b^3*c^3*g^3 + B*a^3*d^3*g^3)*x/(b^3*d^3)
```

3.263.9 Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.24

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$

$$= \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)$$

$$+x\left(\frac{2\,A\,b\,d\,f^3+6\,A\,a\,c\,f\,g^2+6\,A\,a\,d\,f^2\,g+6\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g-6\,B\,b\,c\,f^2\,g}{2\,b\,d}\right.$$

$$+\frac{(2\,a\,d+2\,b\,c)\,\left(\frac{\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)\,(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,A\,a\,c\,g^3+6\,A\,a\,d\,f\,g^2+6\,A\,b\,c\,f\,g^2\cdots}{\cdots}\right.\cdots}{2\,b\,d}$$

$$\left.-\frac{a\,c\,\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{2\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)\,(2\,a\,d+2\,b\,c)}{4\,b\,d}\right.$$

$$-\frac{2\,A\,a\,c\,g^3+6\,A\,a\,d\,f\,g^2+6\,A\,b\,c\,f\,g^2+6\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2-4\,B\,b\,c\,f\,g^2}{4\,b\,d}$$

$$\left.+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)$$

$$+x^3\left(\frac{2\,A\,a\,d\,g^3+2\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+6\,A\,b\,d\,f\,g^2}{6\,b\,d}-\frac{A\,g^3\,(2\,a\,d+2\,b\,c)}{6\,b\,d}\right)$$

$$+\frac{A\,g^3\,x^4}{4}-\frac{\ln(a+b\,x)\,\left(B\,a^4\,g^3-4\,B\,a^3\,b\,f\,g^2+6\,B\,a^2\,b^2\,f^2\,g-4\,B\,a\,b^3\,f^3\right)}{2\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,c^4\,g^3-4\,B\,c^3\,d\,f\,g^2+6\,B\,c^2\,d^2\,f^2\,g-4\,B\,c\,d^3\,f^3\right)}{2\,d^4}$$

input
```
int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2
)/2 + B*f*g^2*x^3) + x*((2*A*b*d*f^3 + 6*A*a*c*f*g^2 + 6*A*a*d*f^2*g + 6*A
*b*c*f^2*g + 6*B*a*d*f^2*g - 6*B*b*c*f^2*g)/(2*b*d) + ((2*a*d + 2*b*c)*(((
(2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d
) - (A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(2*a*d + 2*b*c))/(2*b*d) - (2*A*a*c*g
^3 + 6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b
*c*f*g^2)/(2*b*d) + (A*a*c*g^3)/(b*d)))/(2*b*d) - (a*c*((2*A*a*d*g^3 + 2*A
*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d
+ 2*b*c))/(2*b*d)))/(b*d)) - x^2*((((2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3
 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(
2*a*d + 2*b*c))/(4*b*d) - (2*A*a*c*g^3 + 6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6
*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b*d) + (A*a*c*g^3)/(2*b*d
)) + x^3*((2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g
^2)/(6*b*d) - (A*g^3*(2*a*d + 2*b*c))/(6*b*d)) + (A*g^3*x^4)/4 - (log(a +
b*x)*(B*a^4*g^3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b*f*g^2))/(2
*b^4) + (log(c + d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*f^2*g - 4*B
*c^3*d*f*g^2))/(2*d^4)
```

3.263. $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.264 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

3.264.1 Optimal result

Integrand size = 29, antiderivative size = 152

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = -\frac{2B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{3bd} - \frac{2B(bf-ag)^3\log(a+bx)}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} + \frac{2B(df-cg)^3\log(c+dx)}{3d^3g}$$

output
```
-2/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/3*B*(-a*d+b*c)*g^2*
x^2/b/d-2/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)
^2/(d*x+c)^2))/g+2/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

3.264.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3}{b^3d^3}}{3g}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(2*b*d*(b*c - a
*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*
(b*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(b^3*d^3))
/(3*g)
```

3.264.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2B(bc-ad)\int\frac{(f+gx)^3}{(a+bx)(c+dx)}dx}{3g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2B(bc-ad)\int\left(\frac{xg^3}{bd}+\frac{(3bdf-bcg-adg)g^2}{b^2d^2}+\frac{(bf-ag)^3}{b^2(bc-ad)(a+bx)}+\frac{(df-cg)^3}{d^2(ad-bc)(c+dx)}\right)dx}{3g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2B(bc-ad)\left(\frac{(bf-ag)^3\log(a+bx)}{b^3(bc-ad)}+\frac{g^2x(-adg-bcg+3bdf)}{b^2d^2}-\frac{(df-cg)^3\log(c+dx)}{d^3(bc-ad)}+\frac{g^3x^2}{2bd}\right)}{3g}$$

input
```
Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g) - (2*B*(b*c -
 a*d)*((g^2*(3*b*d*f - b*c*g - a*d*g)*x)/(b^2*d^2) + (g^3*x^2)/(2*b*d) + (
(b*f - a*g)^3*Log[a + b*x])/(b^3*(b*c - a*d)) - ((d*f - c*g)^3*Log[c + d*x
])/(d^3*(b*c - a*d))))/(3*g)
```

3.264.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.264.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.78

method	result
risch	$\frac{(gx+f)^3 B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3g}+\frac{g^2Ax^3}{3}+gAf\,x^2+\frac{g^2Ba\,x^2}{3b}-\frac{g^2Bc\,x^2}{3d}+A\,f^2x-\frac{2g^2B\ln(dx+c)c^3}{3d^3}+ 2gB \ldots$
parts	$\frac{A(gx+f)^3}{3g}-\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right)\right)(g \ldots}{\ldots}$
derivativedivides	$-\frac{A\left(-\left(g^2c^2-2gfdc+f^2d^2\right)(dx+c)+g(cg-df)(dx+c)^2-\frac{g^2(dx+c)^3}{3}\right)}{d^2}+\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\ldots\right.\right.}{\ldots}$
default	$-\frac{A\left(-\left(g^2c^2-2gfdc+f^2d^2\right)(dx+c)+g(cg-df)(dx+c)^2-\frac{g^2(dx+c)^3}{3}\right)}{d^2}+\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\ldots\right.\right.}{\ldots}$
parallelrisch	$\frac{2Bx^2ab^2d^3g^2-2Bx^2b^3cd^2g^2+12Bxab^2d^3fg+6Bx^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^3d^3fg-4Bxa^2bd^3g^2-4Bc^3g^2b^3+4Ba^3d^3g^2-6\ldots}{\ldots}$

input
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(g*x+f)^3*B/g*ln(e*(b*x+a)^2/(d*x+c)^2)+1/3*g^2*A*x^3+g*A*f*x^2+1/3/b*
g^2*B*a*x^2-1/3/d*g^2*B*c*x^2+A*f^2*x-2/3/d^3*g^2*B*ln(d*x+c)*c^3+2/d^2*g*
B*ln(d*x+c)*c^2*f-2/d*B*ln(d*x+c)*c*f^2+2/3/g*B*ln(d*x+c)*f^3+2/3/b^3*g^2*
B*ln(-b*x-a)*a^3-2/b^2*g*B*ln(-b*x-a)*a^2*f+2/b*B*ln(-b*x-a)*a*f^2-2/3/g*B
*ln(-b*x-a)*f^3-2/3/b^2*g^2*B*a^2*x+2/b*g*B*a*f*x+2/3/d^2*g^2*B*c^2*x-2/d*
g*B*c*f*x
```

3.264.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(142) = 284$.

Time = 0.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.98

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{Ab^3d^3g^2x^3 + (3\,Ab^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2)x^2 + (3\,Ab^3d^3f^2 - 6\,(Bb^3cd^2 - Bab^2d^3)fg + 2\,(Bb^3c^2d}{}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
1/3*(A*b^3*d^3*g^2*x^3 + (3*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^
2)*x^2 + (3*A*b^3*d^3*f^2 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + 2*(B*b^3*c
^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*
a^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b
^3*c^3*g^2)*log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*
b^3*d^3*f^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^
2)))/(b^3*d^3)
```

3.264.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(139) = 278$.

Time = 3.27 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.55

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{Ag^2x^3}{3}$$
$$+ \frac{2Ba(a^2g^2-3abfg+3b^2f^2)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+\frac{2Ba^2d^3(a^2g^2-3abfg+3b^2f^2)}{b}+2Bab^2c^3g^2-6Bab^2c^2dfg+12Ba}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2dfg}\right.}{3b^3}$$
$$- \frac{2Bc(c^2g^2-3cdfg+3d^2f^2)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+2Bab^2c^3g^2-6Bab^2c^2dfg+12Bab^2cd^2f^2-2Bab^2c(c^2g^2-3cdfg}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2dfg+}\right.}{3d^3}$$
$$+ x^2\left(Afg+\frac{Bag^2}{3b}-\frac{Bcg^2}{3d}\right) + x\left(Af^2-\frac{2Ba^2g^2}{3b^2}+\frac{2Bafg}{b}+\frac{2Bc^2g^2}{3d^2}-\frac{2Bcfg}{d}\right)$$
$$+ \left(Bf^2x+Bfgx^2+\frac{Bg^2x^3}{3}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*g**2*x**3/3 + 2*B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (2*B*a
**3*c*d**2*g**2 - 6*B*a**2*b*c*d**2*f*g + 2*B*a**2*d**3*(a**2*g**2 - 3*a*b
*f*g + 3*b**2*f**2)/b + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*
B*a*b**2*c*d**2*f**2 - 2*B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))
/(2*B*a**3*d**3*g**2 - 6*B*a**2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b*
*3*c**3*g**2 - 6*B*b**3*c**2*d*f*g + 6*B*b**3*c*d**2*f**2))/(3*b**3) - 2*B
*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)*log(x + (2*B*a**3*c*d**2*g**2 - 6
*B*a**2*b*c*d**2*f*g + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B
*a*b**2*c*d**2*f**2 - 2*B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) +
 2*B*b**3*c**2*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)/d)/(2*B*a**3*d**3*g**
2 - 6*B*a**2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c**3*g**2 - 6*B*
b**3*c**2*d*f*g + 6*B*b**3*c*d**2*f**2))/(3*d**3) + x**2*(A*f*g + B*a*g**2
/(3*b) - B*c*g**2/(3*d)) + x*(A*f**2 - 2*B*a**2*g**2/(3*b**2) + 2*B*a*f*g/
b + 2*B*c**2*g**2/(3*d**2) - 2*B*c*f*g/d) + (B*f**2*x + B*f*g*x**2 + B*g**
2*x**3/3)*log(e*(a + b*x)**2/(c + d*x)**2)
```

3.264.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(142) = 284$.

Time = 0.21 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.76

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{1}{3} Ag^2x^3 + Afgx^2$$
$$+ \left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log(bx+a)}{b} - \frac{2\,c\log(dx-}{d}\right.$$
$$+ \left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log(bx+a)}{b^2} + \frac{2\,c^2\log(d}{d^2}\right.$$
$$+ \frac{1}{3}\left(x^3\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log}{c}\right.$$
$$+ Af^2x$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2
*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*
a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^2 + (x^2*log(b^2*e*x^2/(d^2*x^2
 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 +
 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*
c - a*d)*x/(b*d))*B*f*g + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2)
 + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x
^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

3.264.8 Giac [A] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \frac{1}{3}\,Ag^2x^3+\frac{1}{3}\,(Bg^2x^3+3\,Bfgx^2+3\,Bf^2x)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{(3\,Abdfg-Bbcg^2+Badg^2)x^2}{3\,bd}+\frac{2\,(3\,Bab^2f^2-3\,Ba^2bfg+Ba^3g^2)\log(bx+a)}{3\,b^3} - \frac{2\,(3\,Bcd^2f^2-3\,Bc^2dfg+Bc^3g^2)\log(-dx-c)}{3\,d^3} + \frac{(3\,Ab^2d^2f^2-6\,Bb^2cdfg+6\,Babd^2fg+2\,Bb^2c^2g^2-2\,Ba^2d^2g^2)x}{3\,b^2d^2}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
1/3*A*g^2*x^3 + 1/3*(B*g^2*x^3 + 3*B*f*g*x^2 + 3*B*f^2*x)*log((b^2*e*x^2 +
 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/3*(3*A*b*d*f*g - B*b*c*
g^2 + B*a*d*g^2)*x^2/(b*d) + 2/3*(3*B*a*b^2*f^2 - 3*B*a^2*b*f*g + B*a^3*g^
2)*log(b*x + a)/b^3 - 2/3*(3*B*c*d^2*f^2 - 3*B*c^2*d*f*g + B*c^3*g^2)*log(
-d*x - c)/d^3 + 1/3*(3*A*b^2*d^2*f^2 - 6*B*b^2*c*d*f*g + 6*B*a*b*d^2*f*g +
 2*B*b^2*c^2*g^2 - 2*B*a^2*d^2*g^2)*x/(b^2*d^2)
```

3.264.9 Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.38

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$= \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right) + x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right) - x\left(\frac{\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{3\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)(3\,a\,d+3\,b\,c)}{3\,b\,d} - \frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+6\,B\,a\,d\,f\,g-6\,B\,b\,c\,f\,g}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right) + \frac{\ln(a+b\,x)\,\left(2\,B\,a^3\,g^2-6\,B\,a^2\,b\,f\,g+6\,B\,a\,b^2\,f^2\right)}{3\,b^3} - \frac{\ln(c+d\,x)\,\left(2\,B\,c^3\,g^2-6\,B\,c^2\,d\,f\,g+6\,B\,c\,d^2\,f^2\right)}{3\,d^3}+\frac{A\,g^2\,x^3}{3}$$

input
```
int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^2*x^3)/3 + B*f^2*x + B*f*g*x^2) + x
^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/
(6*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*((((3*A*a*d*g^2 + 3*A*b*c*g
^2 + 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*
b*c))/(3*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A
*a*d*f*g + 6*A*b*c*f*g + 6*B*a*d*f*g - 6*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/
(b*d)) + (log(a + b*x)*(2*B*a^3*g^2 + 6*B*a*b^2*f^2 - 6*B*a^2*b*f*g))/(3*b
^3) - (log(c + d*x)*(2*B*c^3*g^2 + 6*B*c*d^2*f^2 - 6*B*c^2*d*f*g))/(3*d^3)
 + (A*g^2*x^3)/3
```

3.265 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.265.1 Optimal result

Integrand size = 27, antiderivative size = 104

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=-\frac{B(bc-ad)gx}{bd}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g}+\frac{B(df-cg)^2\log(c+dx)}{d^2g}$$

output
```
-B*(-a*d+b*c)*g*x/b/d-B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2*(A+B*ln
(e*(b*x+a)^2/(d*x+c)^2))/g+B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

3.265.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{-2Bd^2(bf-ag)^2\log(a+bx)+b\Big(d(2B(-bc+ad)g^2x+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\cdots}{2b^2d^2g}$$

input
```
Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
(-2*B*d^2*(b*f - a*g)^2*Log[a + b*x] + b*(d*(2*B*(-(b*c) + a*d)*g^2*x + A*
b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x)^2)/(c + d*x)^2] +
2*b*B*(d*f - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

3.265.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bc-ad)\int\frac{(f+gx)^2}{(a+bx)(c+dx)}dx}{g}$$

$$\Big\downarrow \text{93}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bc-ad)\int\left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\frac{(df-cg)^2}{d(ad-bc)(c+dx)}\right)dx}{g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bc-ad)\left(\frac{(bf-ag)^2\log(a+bx)}{b^2(bc-ad)}-\frac{(df-cg)^2\log(c+dx)}{d^2(bc-ad)}+\frac{g^2x}{bd}\right)}{g}$$

input
```
Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*g) - (B*(b*c - a
*d)*((g^2*x)/(b*d) + ((b*f - a*g)^2*Log[a + b*x])/(b^2*(b*c - a*d)) - ((d*
f - c*g)^2*Log[c + d*x])/(d^2*(b*c - a*d))))/g
```

3.265.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.265.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18

method	result
risch	$\frac{Bx(gx+2f)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2}+\frac{Ax^2g}{2}+Afx-\frac{B\ln(bx+a)a^2g}{b^2}+\frac{2B\ln(bx+a)af}{b}+\frac{B\ln(-dx-c)c^2g}{d^2}-\frac{2B\ln}{}$
parts	$A\left(\frac{1}{2}g\,x^2+fx\right)+\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+\right.}{b(ad-cb)}\right.\right.\right.}{}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2g+A\,x^2b^2d^2g+2Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)b^2d^2f+2A\,b^2d^2fx-2B\ln(bx+a)a^2d^2g+4B\ln(bx+a)ab\,d^2f}{}$
derivativedivides	$-\frac{-\frac{A\left(-(cg-df)(dx+c)+\frac{g(dx+c)^2}{2}\right)}{d}-\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}\right.}{b(ad-cb)}\right.\right.\right.}{}}{}$
default	$-\frac{-\frac{A\left(-(cg-df)(dx+c)+\frac{g(dx+c)^2}{2}\right)}{d}-\frac{B\left(\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}\right.}{b(ad-cb)}\right.\right.\right.}{}}{}$

input
```
int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x*(g*x+2*f)*ln(e*(b*x+a)^2/(d*x+c)^2)+1/2*A*x^2*g+A*f*x-1/b^2*B*ln(b
*x+a)*a^2*g+2/b*B*ln(b*x+a)*a*f+1/d^2*B*ln(-d*x-c)*c^2*g-2/d*B*ln(-d*x-c)*
c*f+1/b*B*x*a*g-1/d*B*x*c*g
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.67

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{Ab^2d^2gx^2+2\,(Ab^2d^2f-(Bb^2cd-Babd^2)g)x+2\,(2\,Babd^2f-Ba^2d^2g)\log{(bx+a)}-2\,(2\,Bb^2cdf-Bb^2}{2\,b^2d^2}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*g*x^2 + 2*(A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + 2*(
2*B*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - 2*(2*B*b^2*c*d*f - B*b^2*c^2*g
)*log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b^2*e*x^2 + 2*a*
b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*d^2)
```

3.265.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(88) = 176$.

Time = 1.41 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.02

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{Agx^2}{2}-\frac{Ba(ag-2bf)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2(ag-2bf)}{b}+Babc^2g-4Babcdf-Bacd(ag-2bf)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{b^2}$$
$$+\frac{Bc(cg-2df)\log\left(x+\frac{Ba^2cdg+Babc^2g-4Babcdf-Babc(cg-2df)+\frac{Bb^2c^2(cg-2df)}{d}}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{d^2}$$
$$+x\left(Af+\frac{Bag}{b}-\frac{Bcg}{d}\right)+\left(Bfx+\frac{Bgx^2}{2}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g -
2*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d
**2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/b**2 + B*c*(c*g
- 2*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c
*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f
 + B*b**2*c**2*g - 2*B*b**2*c*d*f))/d**2 + x*(A*f + B*a*g/b - B*c*g/d) + (
B*f*x + B*g*x**2/2)*log(e*(a + b*x)**2/(c + d*x)**2)
```

3.265.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(102) = 204$.

Time = 0.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.37

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{1}{2}Agx^2$$
$$+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log(bx+a)}{b}-\frac{2\,c\log(dx\ldots}{d}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log(bx+a)}{b^2}+\frac{2\,c^2\log\ldots}{\ldots}\right.$$
$$+Afx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

output
```
1/2*A*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*
x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)
/b - 2*c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
 c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(
b*d))*B*g + A*f*x
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.34

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\frac{1}{2}\,Agx^2+\frac{1}{2}\,\left(Bgx^2+2\,Bfx\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{(Abdf-Bbcg+Badg)x}{bd}+\frac{(2\,Babf-Ba^2g)\log\left(bx+a\right)}{b^2}-\frac{(2\,Bcdf-Bc^2g)\log\left(-dx-c\right)}{d^2}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
1/2*A*g*x^2 + 1/2*(B*g*x^2 + 2*B*f*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2)) + (A*b*d*f - B*b*c*g + B*a*d*g)*x/(b*d) + (2*B*
a*b*f - B*a^2*g)*log(b*x + a)/b^2 - (2*B*c*d*f - B*c^2*g)*log(-d*x - c)/d^
2
```

3.265.9 Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\,\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)+x\,\left(\frac{A\,a\,d\,g+A\,b\,c\,g+A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{b\,d}-\frac{A\,g\,\left(a\,d+b\,c\right)}{b\,d}\right)+\frac{A\,g\,x^2}{2}-\frac{B\,a\,\ln\left(a+b\,x\right)\,\left(a\,g-2\,b\,f\right)}{b^2}+\frac{B\,c\,\ln\left(c+d\,x\right)\,\left(c\,g-2\,d\,f\right)}{d^2}$$

input
```
int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
log((e*(a + b*x)^2)/(c + d*x)^2)*(B*f*x + (B*g*x^2)/2) + x*((A*a*d*g + A*b
*c*g + A*b*d*f + B*a*d*g - B*b*c*g)/(b*d) - (A*g*(a*d + b*c))/(b*d)) + (A*
g*x^2)/2 - (B*a*log(a + b*x)*(a*g - 2*b*f))/b^2 + (B*c*log(c + d*x)*(c*g -
 2*d*f))/d^2
```

3.266 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

3.266.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

output
```
A*x+B*(b*x+a)*ln(e*(b*x+a)^2/(d*x+c)^2)/b-2*B*(-a*d+b*c)*ln(d*x+c)/b/d
```

3.266.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx=Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

input
```
Integrate[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

output
```
A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*
Log[c + d*x])/(b*d)
```

3.266.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}+Ax$$

input
```
Int[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

output
```
A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*
Log[c + d*x])/(b*d)
```

3.266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.266.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

method	result	s
risch	$Ax+Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)-\frac{2Bc\ln(dx+c)}{d}+\frac{2Ba\ln(-bx-a)}{b}$	5
parallelrisch	$\frac{B\left(2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)bd+4\ln(bx+a)ad-4\ln(bx+a)bc+2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)bc\right)}{2bd}+Ax$	7
default	$Ax-\frac{B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right)\right)}{d}$	1
parts	$Ax-\frac{B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right)\right)}{d}$	1
derivativedivides	$-\frac{-A(dx+c)+B\left(-(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(-2ad+2cb)\left(\frac{\ln\left(\frac{1}{dx+c}\right)}{b}+\frac{(-ad+cb)\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{b(ad-cb)}\right)\right)}{d}$	1

input
```
int(A+B*ln(e*(b*x+a)^2/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
A*x+B*x*ln(e*(b*x+a)^2/(d*x+c)^2)-2*B/d*c*ln(d*x+c)+2*B/b*a*ln(-b*x-a)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$$
$$=\frac{Bbdx\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+Abdx+2\,Bad\log\left(bx+a\right)-2\,Bbc\log\left(dx+c\right)}{bd}$$

input
```
integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="fricas")
```

output
```
(B*b*d*x*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) +
A*b*d*x + 2*B*a*d*log(b*x + a) - 2*B*b*c*log(d*x + c))/(b*d)
```

3.266.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(48) = 96.

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.93

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = Ax+\frac{2Ba\log\left(x+\frac{\frac{2Ba^2d}{b}+2Bac}{2Bad+2Bbc}\right)}{b} - \frac{2Bc\log\left(x+\frac{2Bac+\frac{2Bbc^2}{d}}{2Bad+2Bbc}\right)}{d} + Bx\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

input
```
integrate(A+B*ln(e*(b*x+a)**2/(d*x+c)**2),x)
```

output
```
A*x + 2*B*a*log(x + (2*B*a**2*d/b + 2*B*a*c)/(2*B*a*d + 2*B*b*c))/b - 2*B*
c*log(x + (2*B*a*c + 2*B*b*c**2/d)/(2*B*a*d + 2*B*b*c))/d + B*x*log(e*(a +
 b*x)**2/(c + d*x)**2)
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \left(x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+\frac{2\left(\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}\right)}{e}\right)B + Ax$$

input
```
integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="maxima")
```

output
```
(x*log((b*x + a)^2*e/(d*x + c)^2) + 2*(a*e*log(b*x + a)/b - c*e*log(d*x +
c)/d)/e)*B + A*x
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \left(2\,(bc-ad)\left(\frac{a\log\left(|bx+a|\right)}{b^2c-abd}-\frac{c\log\left(|dx+c|\right)}{bcd-ad^2}\right)+x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)\right)B+Ax$$

input
```
integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="giac")
```

output
```
(2*(b*c - a*d)*(a*log(abs(b*x + a))/(b^2*c - a*b*d) - c*log(abs(d*x + c))/
(b*c*d - a*d^2)) + x*log((b*x + a)^2*e/(d*x + c)^2))*B + A*x
```

3.266.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = A\,x+B\,x\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)+\frac{2\,B\,a\,\ln\left(a+b\,x\right)}{b}-\frac{2\,B\,c\,\ln\left(c+d\,x\right)}{d}$$

input
```
int(A + B*log((e*(a + b*x)^2)/(c + d*x)^2),x)
```

output
```
A*x + B*x*log((e*(a + b*x)^2)/(c + d*x)^2) + (2*B*a*log(a + b*x))/b - (2*B
*c*log(c + d*x))/d
```

3.267 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx$

3.267.1 Optimal result

Integrand size = 29, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g} - \frac{2B\,\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{2B\,\mathrm{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}$$

output
```
-2*B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))
*ln(g*x+f)/g+2*B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-2*B*polylog(2,b*(g*
x+f)/(-a*g+b*f))/g+2*B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

3.267.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx$$
$$=\frac{\left(A-2B\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-2B\,\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)+2}{g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

output
```
((A - 2*B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2] + 2*B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - 2*B*PolyLog
[2, (b*(f + g*x))/(b*f - a*g)] + 2*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)]
)/g
```

3.267.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2946, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{f+gx}\,dx$$

$$\Big\downarrow \text{ 2946}$$

$$-\frac{2bB\int \frac{\log(f+gx)}{a+bx}dx}{g}+\frac{2Bd\int \frac{\log(f+gx)}{c+dx}dx}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}$$

$$\Big\downarrow \text{ 2841}$$

$$-\frac{2bB\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{g\int\frac{\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{f+gx}dx}{b}\right)}{g}+\frac{2Bd\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{g\int\frac{\log\left(-\frac{g(c+dx)}{df-cg}\right)}{f+gx}dx}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}$$

$$\Big\downarrow \; 2840$$

$$-\frac{2bB\left(\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(f+gx)}{bf-ag}\right)}{f+gx}d(f+gx)}{b}\right)}{g}+\frac{2Bd\left(\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(f+gx)}{df-cg}\right)}{f+gx}d(f+gx)}{d}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}$$

$$\Big\downarrow \; 2838$$

$$\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}-\frac{2bB\left(\frac{\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{b}+\frac{\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{b}\right)}{g}+\frac{2Bd\left(\frac{\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{d}+\frac{\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{d}\right)}{g}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

output
```
((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/g - (2*b*B*((Log[-
((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/b + PolyLog[2, (b*(f + g*x))/(b
*f - a*g)]/b))/g + (2*B*d*((Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x]
)/d + PolyLog[2, (d*(f + g*x))/(d*f - c*g)]/d))/g
```

3.267. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx$

3.267.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2946
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Simp[b*B*(n/g)   Int[Log[f + g*x]/
(a + b*x), x], x] + Simp[B*d*(n/g)   Int[Log[f + g*x]/(c + d*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*
d, 0]
```

3.267.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(144) = 288$.

Time = 3.37 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.75

method	result
parts	$\frac{A\ln(gx+f)}{g}+B\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}+\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\right.}{g}\right.$
derivativedivides	$-\frac{-dA\left(-\frac{\ln\left(\frac{1}{dx+c}\right)}{g}+\frac{\ln\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}{g}\right)-dB\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\right.}{g}\right.}{}$
default	$-\frac{-dA\left(-\frac{\ln\left(\frac{1}{dx+c}\right)}{g}+\frac{\ln\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}{g}\right)-dB\left(-\frac{\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-(2ad-2cb)\left(\frac{\operatorname{dilog}\left(\frac{\frac{ad-cb}{dx+c}+b}{b}\right)}{ad-cb}\right.}{g}\right.}{}$
risch	Expression too large to display

3.267. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
A*ln(g*x+f)/g+B*(-(ln(1/(d*x+c))*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-(
2*a*d-2*b*c)*(dilog(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)+ln(1/(d*x+c))*ln(((
a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)))/g+(ln((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)*ln
(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-2/(c*g-d*f)*(a*d-b*c)*(dilog((((c*g-
d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)+ln((c*g-d*
f)/(d*x+c)-g)*ln((((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*
f))/(a*d-b*c)))/g*(c*g-d*f))
```

3.267.5 Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="fricas")
```

output
```
integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
 + A)/(g*x + f), x)
```

3.267.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f),x)
```

output
```
Timed out
```

3.267.7 Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="maxima")
```

output
```
-B*integrate(-(2*log(b*x + a) - 2*log(d*x + c) + log(e))/(g*x + f), x) + A
*log(g*x + f)/g
```

3.267.8 Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = \int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(g*x + f), x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = \int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{f+g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x), x)
```

3.268 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$

3.268.1 Optimal result

Integrand size = 29, antiderivative size = 90

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx=\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(f+gx)}+\frac{2B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*
ln((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

3.268.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx=\frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}+\frac{2B(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)) + (2*B*(b*(d*f - c*
g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f +
g*x]))/((b*f - a*g)*(d*f - c*g)))/g
```

3.268.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2954, 2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2751}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{2B\int \frac{1}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \text{16}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{2B\log\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}{(bf-ag)(df-cg)}\right)$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f -
a*g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + (2*B*Log
[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)*(d*f - c*g))
)
```

3.268. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}dx$

3.268.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.268.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(90) = 180$.

Time = 0.67 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.72

method	result
derivativedivides	$-\frac{-\frac{d^{2} A}{\left(\frac{cg-df}{dx+c}-g\right)(cg-df)}+\frac{-\frac{bBd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)}{ag-bf}-\frac{Bd(ad-cb)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)}{(ag-bf)(dx+c)}}{\frac{cg}{dx+c}-\frac{fd}{dx+c}-g}+\frac{2Bd(ad-cb)\ln\left(\frac{cg}{dx+c}-\cdots\right.}{ac\,g^{2}-adfg-bcfg+b\cdots}}{d}$
default	$-\frac{-\frac{d^{2} A}{\left(\frac{cg-df}{dx+c}-g\right)(cg-df)}+\frac{-\frac{bBd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)}{ag-bf}-\frac{Bd(ad-cb)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)}{(ag-bf)(dx+c)}}{\frac{cg}{dx+c}-\frac{fd}{dx+c}-g}+\frac{2Bd(ad-cb)\ln\left(\frac{cg}{dx+c}-\cdots\right.}{ac\,g^{2}-adfg-bcfg+b\cdots}}{d}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)}{g(gx+f)}-\frac{-2B\ln(-dx-c)ad\,g^{2}x+2B\ln(-dx-c)bdfgx+2B\ln(gx+f)ad\,g^{2}x-2B\ln(gx+f)bc\,g^{2}x+2B\ln\cdots}{\cdots}$
parallelrisch	$\frac{4B\ln(gx+f)xab\,c^{2}fg+4B\ln(bx+a)x\,a^{2}cdfg-2Bx\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)a^{2}cdfg+2Bx\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)abcd\,f^{2}-4B\ln(bx+a)xab\cdots}{\cdots}$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

3.268. $\int\frac{A+B\log\left(\frac{e(a+bx)^{2}}{(c+dx)^{2}}\right)}{(f+gx)^{2}}\,dx$

output
```
-1/d*(-d^2*A/((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)+(-b*B*d/(a*g-b*f)*ln(e*(a*d/(
d*x+c)-b*c/(d*x+c)+b)^2/d^2)-B*d*(a*d-b*c)/(a*g-b*f)/(d*x+c)*ln(e*(a*d/(d*
x+c)-b*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)+2*B*d*(a*d-b*c)/(a
*c*g^2-a*d*f*g-b*c*f*g+b*d*f^2)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g))
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(90) = 180$.

Time = 3.50 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.10

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = -\frac{Abdf^2+Aacg^2-(Abc+Aad)fg-2\left(Bbdf^2-Bbcfg+(Bbdfg-Bbcg^2)x\right)\log(bx+a)+2\left(Bbdf^2-\right.}{bdf^3g}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="fricas"
)
```

output
```
-(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - 2*(B*b*d*f^2 - B*b*c*f*g +
 (B*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + 2*(B*b*d*f^2 - B*a*d*f*g + (B*b
*d*f*g - B*a*d*g^2)*x)*log(d*x + c) - 2*((B*b*c - B*a*d)*g^2*x + (B*b*c -
B*a*d)*f*g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*l
og((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d*f^3*g
+ a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f
*g^3)*x)
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**2,x)
```

output
```
Timed out
```

3.268.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(90) = 180$.

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.13

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$$
$$= B\left(\frac{2\,b\log(bx+a)}{bfg-ag^2} - \frac{2\,d\log(dx+c)}{dfg-cg^2} + \frac{2\,(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg} - \frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \right.}{g^2x+fg}\right.$$
$$- \frac{A}{g^2x+fg}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="maxima"
)
```

output
```
B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2) + 2
*(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^2*
e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^
2*e/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

3.268.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(90) = 180$.

Time = 0.52 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.04

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$$
$$= \left((bcg^2-adg^2)\left(\frac{(2\,bdf-bcg-adg)\log\left(\frac{\left|2\,bdfg-\frac{2\,bdf^2g}{gx+f}-bcg^2-adg^2+\frac{2\,bcfg^2}{gx+f}+\frac{2\,adfg^2}{gx+f}-\frac{2\,acg^3}{gx+f}-|-bcg^2+adg^2|\right|}{\left|2\,bdfg-\frac{2\,bdf^2g}{gx+f}-bcg^2-adg^2+\frac{2\,bcfg^2}{gx+f}+\frac{2\,adfg^2}{gx+f}-\frac{2\,acg^3}{gx+f}+|-bcg^2+adg^2|\right|}\right)}{(bdf^2g-bcfg^2-adfg^2+acg^3)|-bcg^2+adg^2|} - \frac{\log}{}\right.\right.$$
$$- \frac{A}{(gx+f)g}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="giac")
```

output
```
((b*c*g^2 - a*d*g^2)*((2*b*d*f - b*c*g - a*d*g)*log(abs(2*b*d*f*g - 2*b*d*
f^2*g/(g*x + f) - b*c*g^2 - a*d*g^2 + 2*b*c*f*g^2/(g*x + f) + 2*a*d*f*g^2/
(g*x + f) - 2*a*c*g^3/(g*x + f) - abs(-b*c*g^2 + a*d*g^2))/abs(2*b*d*f*g -
 2*b*d*f^2*g/(g*x + f) - b*c*g^2 - a*d*g^2 + 2*b*c*f*g^2/(g*x + f) + 2*a*d
*f*g^2/(g*x + f) - 2*a*c*g^3/(g*x + f) + abs(-b*c*g^2 + a*d*g^2)))/((b*d*f
^2*g - b*c*f*g^2 - a*d*f*g^2 + a*c*g^3)*abs(-b*c*g^2 + a*d*g^2)) - log(abs
(b*d - 2*b*d*f/(g*x + f) + b*d*f^2/(g*x + f)^2 + b*c*g/(g*x + f) + a*d*g/(
g*x + f) - b*c*f*g/(g*x + f)^2 - a*d*f*g/(g*x + f)^2 + a*c*g^2/(g*x + f)^2
))/(b*d*f^2*g^2 - b*c*f*g^3 - a*d*f*g^3 + a*c*g^4)) - log((b*x + a)^2*e/(d
*x + c)^2)/((g*x + f)*g))*B - A/((g*x + f)*g)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = \frac{2\,B\,d\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g} - \frac{B\,\ln\left(\frac{e\,a^2+2\,e\,a\,b\,x+e\,b^2\,x^2}{c^2+2\,c\,d\,x+d^2\,x^2}\right)}{x\,g^2+f\,g} - \frac{2\,B\,b\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g} - \frac{A}{x\,g^2+f\,g} - \frac{2\,B\,a\,d\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g} + \frac{2\,B\,b\,c\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^2,x)
```

output
```
(2*B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a^2*e + b^2*e*x^2 + 2*a*b*e
*x)/(c^2 + d^2*x^2 + 2*c*d*x)))/(f*g + g^2*x) - (2*B*b*log(a + b*x))/(a*g^
2 - b*f*g) - A/(f*g + g^2*x) - (2*B*a*d*log(f + g*x))/(a*c*g^2 + b*d*f^2 -
 a*d*f*g - b*c*f*g) + (2*B*b*c*log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g
- b*c*f*g)
```

3.269 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$

3.269.1 Optimal result

Integrand size = 29, antiderivative size = 175

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = -\frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(c+dx)}{g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg)\log(f+gx)}{(bf-ag)^2(df-cg)^2}$$

output
```
-B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+b^2*B*ln(b*x+a)/g/(-a*g+b*f)^2
+1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^2-B*d^2*ln(d*x+c)/g/(-c*g+
d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)^2/(-c*g+d*
f)^2
```

3.269.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}+2B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)}+\frac{d^2\log(c+dx)}{-bc+ad}-\frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2) + 2*B*(b*c - a*d)
*((b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b
*f - a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f +
b*c*g + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

3.269.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^3}\,dx$$

$$\Big\downarrow \ 2948$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^2}dx}{g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \ 93$$

$$\frac{B(bc-ad)\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(cg-df)^2(c+dx)}-\frac{g^2(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{g^2}{(bf-ag)(df-cg)(f+gx)^2}\right)dx}{g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}-\frac{d^2\log(c+dx)}{(bc-ad)(df-cg)^2}-\frac{g}{(f+gx)(bf-ag)(df-cg)}+\frac{g\log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}\right)}{g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(g*(f + g*x)^2) + (B*(b*c -
a*d)*(-(g/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*Log[a + b*x])/((b*c
- a*d)*(b*f - a*g)^2) - (d^2*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) + (
g*(2*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/
g
```

3.269.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.269. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$

3.269.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(176) = 352$.

Time = 1.22 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.59

method	result
derivativedivides	$-\frac{-d^3 A\left(-\frac{1}{(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}-\frac{g}{2(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)^2}\right)+\frac{\frac{Ba\,d^3g^2-Bbc\,d^2g^2}{g^2(ag-bf)(dx+c)^2}+\frac{b^2(cg-df)Bd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\cdots\right.}{d^2}\cdots\right.}{\left(g^2a^2-2abfg+f^2b^2\right)(\cdots}}{\cdots}}{\cdots}$
default	$-\frac{-d^3 A\left(-\frac{1}{(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}-\frac{g}{2(cg-df)^2\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)^2}\right)+\frac{\frac{Ba\,d^3g^2-Bbc\,d^2g^2}{g^2(ag-bf)(dx+c)^2}+\frac{b^2(cg-df)Bd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\cdots\right.}{d^2}\cdots\right.}{\left(g^2a^2-2abfg+f^2b^2\right)(\cdots}}{\cdots}}{\cdots}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-d^3*A*(-1/(c*g-d*f)^2/(c*g/(d*x+c)-f/(d*x+c)*d-g)-1/2*g/(c*g-d*f)^2
/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2)+((B*a*d^3*g^2-B*b*c*d^2*g^2)/g^2/(a*g-b*f)
/(d*x+c)^2+b^2*(c*g-d*f)*B*d/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)*ln(e*(a*d
/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-(B*a*d^3*g^2-B*b*c*d^2*g^2)/g/(a*c*g^2-a*d*
f*g-b*c*f*g+b*d*f^2)/(d*x+c)+1/2*B*d*(a^2*d^2*g-2*a*b*d^2*f-b^2*c^2*g+2*b^
2*c*d*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln(e*(a*d/(d*x+c)-b*c/(d*x+
c)+b)^2/d^2)-1/2*b^2*g*B*d/(a^2*g^2-2*a*b*f*g+b^2*f^2)*ln(e*(a*d/(d*x+c)-b
*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2-B*d*(a^2*d^2*g-2*a*b*d
^2*f-b^2*c^2*g+2*b^2*c*d*f)/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2
*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d
*f^3*g+b^2*d^2*f^4)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g))
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(173) = 346$.

Time = 47.45 (sec) , antiderivative size = 1036, normalized size of antiderivative = 5.92

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx =$$
$$-\frac{Ab^2d^2f^4+Aa^2c^2g^4-2\left((A-B)b^2cd+(A+B)abd^2\right)f^3g+\left((A-2\,B)b^2c^2+4\,Aabcd+(A+2\,B)a^2d\right.}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="fricas"
)
```

output
```
-1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*((A - B)*b^2*c*d + (A + B)*a*b*d^2
)*f^3*g + ((A - 2*B)*b^2*c^2 + 4*A*a*b*c*d + (A + 2*B)*a^2*d^2)*f^2*g^2 -
2*((A - B)*a*b*c^2 + (A + B)*a^2*c*d)*f*g^3 + 2*((B*b^2*c*d - B*a*b*d^2)*f
^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x -
2*(B*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*
g^2 - 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^
2*c*d*f^2*g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + 2*(B*b^2*d^2*f^4 - 2*B*
a*b*d^2*f^3*g + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3
 + B*a^2*d^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d
^2*f*g^3)*x)*log(d*x + c) - 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^
2 - B*a^2*d^2)*f^2*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B
*a^2*d^2)*g^4)*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B
*a^2*d^2)*f*g^3)*x)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b
^2*c*d + B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2
- 2*(B*a*b*c^2 + B*a^2*c*d)*f*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^
2*x^2 + 2*c*d*x + c^2)))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a
*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a
^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f
^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f
*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d + a*b*d^2)*...
```

3.269.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**3,x)
```

output
```
Timed out
```

3.269.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(173) = 346$.

Time = 0.23 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.31

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$$
$$=\frac{1}{2}\left(\frac{2\,b^2\log(bx+a)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{2\,d^2\log(dx+c)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{2\left(2\left(b^2cd-abd^2\right)f-\left(b^2c^2-\right.\right.}{b^2d^2f^4+a^2c^2g^4-2\left(b^2cd+abd^2\right)f^3g+\left(b^2c^2+\right.}\right.$$
$$-\frac{A}{2\left(g^3x^2+2\,fg^2x+f^2g\right)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="maxima"
)
```

output
```
1/2*(2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*
x + c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f -
(b^2*c^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*
d + a*b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2
+ a^2*c*d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g
 + (b*d*f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 +
 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2
*c*d*x + c^2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2
*x + f^2*g)
```

3.269.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(173) = 346$.

Time = 0.50 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = \frac{Bb^3\log\left(|bx+a|\right)}{b^3f^2g - 2\,ab^2fg^2 + a^2bg^3} - \frac{Bd^3\log\left(|dx+c|\right)}{d^3f^2g - 2\,cd^2fg^2 + c^2dg^3}$$
$$+ \frac{(2\,Bb^2cdf - 2\,Babd^2f - Bb^2c^2g + Ba^2d^2g)\log\left(gx+f\right)}{b^2d^2f^4 - 2\,b^2cdf^3g - 2\,abd^2f^3g + b^2c^2f^2g^2 + 4\,abcdf^2g^2 + a^2d^2f^2g^2 - 2\,abc^2fg^3 - 2\,a^2cdfg^3 + a^2c^2g^4}$$
$$- \frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{2\,(g^3x^2 + 2\,fg^2x + f^2g)}$$
$$- \frac{2\,Bbcg^2x - 2\,Badg^2x + Abdf^2 - Abcfg + 2\,Bbcfg - Aadfg - 2\,Badfg + Aac}{2\,(bdf^2g^3x^2 - bcfg^4x^2 - adfg^4x^2 + acg^5x^2 + 2\,bdf^3g^2x - 2\,bcf^2g^3x - 2\,adf^2g^3x + 2\,acfg^4x + bdf^4g -}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="giac")
```

output
```
B*b^3*log(abs(b*x + a))/(b^3*f^2*g - 2*a*b^2*f*g^2 + a^2*b*g^3) - B*d^3*lo
g(abs(d*x + c))/(d^3*f^2*g - 2*c*d^2*f*g^2 + c^2*d*g^3) + (2*B*b^2*c*d*f -
 2*B*a*b*d^2*f - B*b^2*c^2*g + B*a^2*d^2*g)*log(g*x + f)/(b^2*d^2*f^4 - 2*
b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^
2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) - 1/2*B*l
og((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(g^3*x^2 + 2
*f*g^2*x + f^2*g) - 1/2*(2*B*b*c*g^2*x - 2*B*a*d*g^2*x + A*b*d*f^2 - A*b*c
*f*g + 2*B*b*c*f*g - A*a*d*f*g - 2*B*a*d*f*g + A*a*c*g^2)/(b*d*f^2*g^3*x^2
 - b*c*f*g^4*x^2 - a*d*f*g^4*x^2 + a*c*g^5*x^2 + 2*b*d*f^3*g^2*x - 2*b*c*f
^2*g^3*x - 2*a*d*f^2*g^3*x + 2*a*c*f*g^4*x + b*d*f^4*g - b*c*f^3*g^2 - a*d
*f^3*g^2 + a*c*f^2*g^3)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.35

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$$
$$= \frac{\ln(f+g\,x)\,\left(g\,(B\,a^2\,d^2-B\,b^2\,c^2)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\,f^2\,g^2-2\,b^2\,c\,d\,f^3\,g}$$
$$-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-2\,B\,a\,d\,f\,g+2\,B\,b\,c\,f\,g}{2\,(a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g)}-\frac{x\,(B\,a\,d\,g^2-B\,b\,c\,g^2)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{f^2\,g+2\,f\,g^2\,x+g^3\,x^2}$$
$$+\frac{B\,b^2\,\ln(a+b\,x)}{a^2\,g^3-2\,a\,b\,f\,g^2+b^2\,f^2\,g}-\frac{B\,d^2\,\ln(c+d\,x)}{c^2\,g^3-2\,c\,d\,f\,g^2+d^2\,f^2\,g}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{2\,g\,(f^2+2\,f\,g\,x+g^2\,x^2)}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^3,x)
```

output
```
(log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))
/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^
2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*
f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - 2*B*a*d*f*g +
 2*B*b*c*f*g)/(2*(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)) - (x*(B*a*d*g^2
- B*b*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(f^2*g + g^3*x^2 +
2*f*g^2*x) + (B*b^2*log(a + b*x))/(a^2*g^3 + b^2*f^2*g - 2*a*b*f*g^2) - (B
*d^2*log(c + d*x))/(c^2*g^3 + d^2*f^2*g - 2*c*d*f*g^2) - (B*log((e*(a + b*
x)^2)/(c + d*x)^2))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x))
```

3.270 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$

3.270.1 Optimal result

Integrand size = 29, antiderivative size = 277

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx &= -\frac{B(bc-ad)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\ &\quad + \frac{2b^3B\log(a+bx)}{3g(bf-ag)^3} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3} - \frac{2Bd^3\log(c+dx)}{3g(df-cg)^3} \\ &\quad + \frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\log(f+gx)}{3(bf-ag)^3(df-cg)^3}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-2/3*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+2/3*b^3*B*ln(b*x+a)/g/(-
a*g+b*f)^3+1/3*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^3-2/3*B*d^3*ln(d
*x+c)/g/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^
2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

3.270.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}+2B(bc-ad)\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2}+\frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log}{(bc-ad)(}\right.}{3g}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3) + 2*B*(b*c - a*d)
*(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*
d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c -
 a*d)*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) +
 (g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c
^2*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

3.270.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^4}\,dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{2B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}dx}{3g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{2B(bc-ad)\int \left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(cg-df)^3(c+dx)}+\frac{g^2\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2-adg(3df-cg)b+a^2d^2g^2\right)}{(bf-ag)^3(df-cg)^3(f+gx)}-\frac{g^2(-}{(bf-ag)}\right.}{3g}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2B(bc-ad)\left(\frac{g\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}-\frac{d^3\log(c+dx)}{(bc-ad)(df-cg)^3}-\frac{g(-adg-bcg}{(f+gx)(bf-ag)}\right.}{3g}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(g*(f + g*x)^3) + (2*B*(b*c
- a*d)*(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (g*(2*b*d*f - b*c*g
 - a*d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((
b*c - a*d)*(b*f - a*g)^3) - (d^3*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^3)
 + (g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
c^2*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3))/(3*g)
```

3.270.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.270. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$

3.270.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. 2(268) = 536.

Time = 2.04 (sec) , antiderivative size = 1293, normalized size of antiderivative = 4.67

method	result	size
derivativedivides	Expression too large to display	1293
default	Expression too large to display	1293
risch	Expression too large to display	2444
parallelrisch	Expression too large to display	2946

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(d^4*A*(-1/(c*g-d*f)^3/(c*g/(d*x+c)-f/(d*x+c)*d-g)-1/3*g^2/(c*g-d*f)^
3/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3-g/(c*g-d*f)^3/(c*g/(d*x+c)-f/(d*x+c)*d-g)^
2)+((c*g-d*f)*b^3*g*B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x
+c)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/3*(2*B*a^2*d^4*g^4-4*B*a*b*d
^4*f*g^3-2*B*b^2*c^2*d^2*g^4+4*B*b^2*c*d^3*f*g^3)/g/(a^2*c^2*g^4-2*a^2*c*d
*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b
^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)/(d*x+c)-1/3*(3*B*a^2*d^4*g^4-B
*a*b*c*d^3*g^4-5*B*a*b*d^4*f*g^3-2*B*b^2*c^2*d^2*g^4+5*B*b^2*c*d^3*f*g^3)/
(a^2*g^2-2*a*b*f*g+b^2*f^2)/g^3/(d*x+c)^3+1/3*(5*B*a^2*d^4*g^4-B*a*b*c*d^3
*g^4-9*B*a*b*d^4*f*g^3-4*B*b^2*c^2*d^2*g^4+9*B*b^2*c*d^3*f*g^3)/(c*g-d*f)/
g^2/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2-1/3*B*d*(a^3*d^3*g^2-3*a^2*b*d^3
*f*g+3*a*b^2*d^3*f^2-b^3*c^3*g^2+3*b^3*c^2*d*f*g-3*b^3*c*d^2*f^2)/(a^3*g^3
-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x
+c)+b)^2/d^2)-1/3*b^3*g^2*B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3
)*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-(c^2*g^2-2*c*d*f*g+d^2*f^2)*b^3*
B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^2*ln(e*(a*d/(d*x
+c)-b*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3+2/3*B*d*(a^3*d^3*
g^2-3*a^2*b*d^3*f*g+3*a*b^2*d^3*f^2-b^3*c^3*g^2+3*b^3*c^2*d*f*g-3*b^3*c*d^
2*f^2)/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-
3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d...
```

3.270.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="fricas"
)
```

output
```
Timed out
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**4,x)
```

output
```
Timed out
```

3.270.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(265) = 530$.

Time = 0.25 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.25

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$$
$$= \frac{1}{3}\left(\frac{2\,b^3\log(bx+a)}{b^3f^3g-3\,ab^2f^2g^2+3\,a^2bfg^3-a^3g^4} - \frac{2\,d^3\log(dx+c)}{d^3f^3g-3\,cd^2f^2g^2+3\,c^2dfg^3-c^3g^4} + \frac{}{b^3d^3f^6+a^3c^3g^6-3\,(}\right.$$
$$-\frac{A}{3\,(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g)}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="maxima"
)
```

output
```
1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*e*x^2/(d
^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2
*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B -
1/3*A/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

3.270.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(265) = 530$.

Time = 0.84 (sec) , antiderivative size = 1359, normalized size of antiderivative = 4.91

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="giac")
```

output
```
2/3*B*b^4*log(abs(b*x + a))/(b^4*f^3*g - 3*a*b^3*f^2*g^2 + 3*a^2*b^2*f*g^3
 - a^3*b*g^4) - 2/3*B*d^4*log(abs(d*x + c))/(d^4*f^3*g - 3*c*d^3*f^2*g^2 +
 3*c^2*d^2*f*g^3 - c^3*d*g^4) + 2/3*(3*B*b^3*c*d^2*f^2 - 3*B*a*b^2*d^3*f^2
 - 3*B*b^3*c^2*d*f*g + 3*B*a^2*b*d^3*f*g + B*b^3*c^3*g^2 - B*a^3*d^3*g^2)*
log(g*x + f)/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3*b^3*
c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3*f^3*
g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + 3*
a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*a^2*b*
c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) - 1/3*B*log((b^2*e*x^2 + 2*a*
b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g
^2*x + f^3*g) - 1/3*(4*B*b^2*c*d*f*g^3*x^2 - 4*B*a*b*d^2*f*g^3*x^2 - 2*B*b
^2*c^2*g^4*x^2 + 2*B*a^2*d^2*g^4*x^2 + 9*B*b^2*c*d*f^2*g^2*x - 9*B*a*b*d^2
*f^2*g^2*x - 5*B*b^2*c^2*f*g^3*x + 5*B*a^2*d^2*f*g^3*x + B*a*b*c^2*g^4*x -
 B*a^2*c*d*g^4*x + A*b^2*d^2*f^4 - 2*A*b^2*c*d*f^3*g + 5*B*b^2*c*d*f^3*g -
 2*A*a*b*d^2*f^3*g - 5*B*a*b*d^2*f^3*g + A*b^2*c^2*f^2*g^2 - 3*B*b^2*c^2*f
^2*g^2 + 4*A*a*b*c*d*f^2*g^2 + A*a^2*d^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 2
*A*a*b*c^2*f*g^3 + B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 - B*a^2*c*d*f*g^3 +
 A*a^2*c^2*g^4)/(b^2*d^2*f^4*g^4*x^3 - 2*b^2*c*d*f^3*g^5*x^3 - 2*a*b*d^2*f
^3*g^5*x^3 + b^2*c^2*f^2*g^6*x^3 + 4*a*b*c*d*f^2*g^6*x^3 + a^2*d^2*f^2*g^6
*x^3 - 2*a*b*c^2*f*g^7*x^3 - 2*a^2*c*d*f*g^7*x^3 + a^2*c^2*g^8*x^3 + 3*...
```

3.270.9 Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$$

$$= \frac{\ln(f+g\,x)\,\left(g\,\left(6\,B\,a^2\,b\,d^3\,f-6\right.\right.}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f\,g^5+27\,a^2\,b\,c^2\,d\,f^2\,g^4-27\,a^2\,b\,c\,d^2\,f^3\,g}$$

$$-\frac{\frac{A\,a^2\,c^2\,g^4+A\,b^2\,d^2\,f^4+A\,a^2\,d^2\,f^2\,g^2+A\,b^2\,c^2\,f^2\,g^2+3\,B\,a^2\,d^2\,f^2\,g^2-3\,B\,b^2\,c^2\,f^2\,g^2-2\,A\,a\,b\,c^2\,f\,g^3-2\,A\,a\,b\,d^2\,f^3\,g+B\,a\,b\,c^2\,f\,g^3-2\,A\,a^2\,c\,d\,f}{a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\,f^2\,g}}{}$$

$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{3\,g\,\left(f^3+3\,f^2\,g\,x+3\,f\,g^2\,x^2+g^3\,x^3\right)}-\frac{2\,B\,b^3\,\ln(a+b\,x)}{3\,a^3\,g^4-9\,a^2\,b\,f\,g^3+9\,a\,b^2\,f^2\,g^2-3\,b^3\,f^3\,g}$$

$$+\frac{2\,B\,d^3\,\ln(c+d\,x)}{3\,c^3\,g^4-9\,c^2\,d\,f\,g^3+9\,c\,d^2\,f^2\,g^2-3\,d^3\,f^3\,g}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^4,x)
```

output
```
(log(f + g*x)*(g*(6*B*a^2*b*d^3*f - 6*B*b^3*c^2*d*f) - g^2*(2*B*a^3*d^3 -
2*B*b^3*c^3) - 6*B*a*b^2*d^3*f^2 + 6*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*
b^3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 -
9*a*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^
2*g^4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 +
27*a*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 +
 27*a^2*b*c^2*d*f^2*g^4) - ((A*a^2*c^2*g^4 + A*b^2*d^2*f^4 + A*a^2*d^2*f^2
*g^2 + A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 - 2*A
*a*b*c^2*f*g^3 - 2*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 -
 5*B*a*b*d^2*f^3*g - 2*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d*f^3
*g + 4*A*a*b*c*d*f^2*g^2)/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b
^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b
^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (2*x^2*(B*a^2*d^2*g^4 - B*b^2*c^2*g^4
- 2*B*a*b*d^2*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2
*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2
*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*f*g^3
- 5*B*b^2*c^2*f*g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^2*g^2
+ 9*B*b^2*c*d*f^2*g^2))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2
*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2
*c*d*f^3*g + 4*a*b*c*d*f^2*g^2))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9...
```

3.270. $\displaystyle \int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$

3.271 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$

3.271.1 Optimal result

Integrand size = 29, antiderivative size = 381

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx &= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{4(bf-ag)^2(df-cg)^2(f+gx)^2}\\ &\quad-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)}{2(bf-ag)^3(df-cg)^3(f+gx)}\\ &\quad+\frac{b^4B\log(a+bx)}{2g(bf-ag)^4}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4}-\frac{Bd^4\log(c+dx)}{2g(df-cg)^4}\\ &\quad-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\log(f+gx)}{2(bf-ag)^4(df-cg)^4}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/4*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/2*B*(-a*d+b*c)*(a^2*
d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f)
^3/(-c*g+d*f)^3/(g*x+f)+1/2*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*ln(e*
(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^4-1/2*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/2*B
*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2
-2*c*d*f*g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

3.271.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$$
$$= \frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}+2B(bc-ad)\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g(a^2d^2g^2+abdg(-3df+cg)+}{(bf-ag)^3(df-c}\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4) + 2*B*(b*c - a*d)
*(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*
d*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*
d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^
3*(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4
) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g
+ a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*
g^2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

3.271.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^5}\,dx$$
$$\Big\downarrow \text{ 2948}$$
$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}dx}{2g}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4g(f+gx)^4}$$
$$\Big\downarrow \text{ 93}$$

$$\frac{B(bc-ad)\int\left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(cg-df)^4(c+dx)}+\frac{g^2(2bdf-bcg-adg)(2d^2f^2b^2+c^2g^2b^2-2cdfgb^2-2ad^2fgb+a^2d^2g^2)}{(bf-ag)^4(df-cg)^4(f+gx)}\right.}{2g}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4g(f+gx)^4}$$

$\big\downarrow$ 2009

$$\frac{B(bc-ad)\left(-\frac{g(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{(f+gx)(bf-ag)^3(df-cg)^3}-\frac{g\log(f+gx)(-adg-bcg+2bdf)(-a^2d^2g^2+2abd^2fg-(b^2(c^2g^2-2cdfg+\right.}{2g}$$

$$\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4g(f+gx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(g*(f + g*x)^4) + (B*(b*c -
a*d)*(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (g*(2*b*d*f - b*c*g -
 a*d*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 - a
*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g
)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)
^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(2*b*d*f - b*c*g
 - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*
g^2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(2*g)
```

3.271.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.271. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.271.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2298 vs. $2(370) = 740$.

Time = 5.50 (sec) , antiderivative size = 2299, normalized size of antiderivative = 6.03

method	result	size
derivativedivides	Expression too large to display	2299
default	Expression too large to display	2299
risch	Expression too large to display	4452
parallelrisch	Expression too large to display	5619

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-d^5*A*(-1/4*g^3/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g)^4-g^2/(c*g-
d*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3-3/2*g/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x
+c)*d-g)^2-1/(c*g-d*f)^4/(c*g/(d*x+c)-f/(d*x+c)*d-g))+((c*g-d*f)*b^4*g^2*B
*d/(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^2*g^2-4*a*b^3*f^3*g+b^4*f^4)/(d*x+c)
*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)+(c*g-d*f)*(c^2*g^2-2*c*d*f*g+d^2*
f^2)*b^4*B*d/(a^4*g^4-4*a^3*b*f*g^3+6*a^2*b^2*f^2*g^2-4*a*b^3*f^3*g+b^4*f^
4)/(d*x+c)^3*ln(e*(a*d/(d*x+c)-b*c/(d*x+c)+b)^2/d^2)-1/2*(B*a^3*d^5*g^6-3*
B*a^2*b*d^5*f*g^5+3*B*a*b^2*d^5*f^2*g^4-B*b^3*c^3*d^2*g^6+3*B*b^3*c^2*d^3*
f*g^5-3*B*b^3*c*d^4*f^2*g^4)/g/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*
f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*
d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+
9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^
2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)/(d*x+c)+1/12*(11*B*a^3*d^5*g^6-2*B*a^2*b*
c*d^4*g^6-31*B*a^2*b*d^5*f*g^5-3*B*a*b^2*c^2*d^3*g^6+10*B*a*b^2*c*d^4*f*g^
5+26*B*a*b^2*d^5*f^2*g^4-6*B*b^3*c^3*d^2*g^6+21*B*b^3*c^2*d^3*f*g^5-26*B*b
^3*c*d^4*f^2*g^4)/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/g^4/(d*x+c
)^4-1/6*(13*B*a^3*d^5*g^6-B*a^2*b*c*d^4*g^6-38*B*a^2*b*d^5*f*g^5-3*B*a*b^2
*c^2*d^3*g^6+8*B*a*b^2*c*d^4*f*g^5+34*B*a*b^2*d^5*f^2*g^4-9*B*b^3*c^3*d^2*
g^6+30*B*b^3*c^2*d^3*f*g^5-34*B*b^3*c*d^4*f^2*g^4)/g^3/(a^3*c*g^4-a^3*d*f*
g^3-3*a^2*b*c*f*g^3+3*a^2*b*d*f^2*g^2+3*a*b^2*c*f^2*g^2-3*a*b^2*d*f^3*g...
```

3.271.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="fricas"
)
```

output
```
Timed out
```

3.271.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**5,x)
```

output
```
Timed out
```

3.271.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1809 vs. $2(367) = 734$.

Time = 0.32 (sec) , antiderivative size = 1809, normalized size of antiderivative = 4.75

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="maxima"
)
```

output
```
1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3
- 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g
^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^
3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^
4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^
8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3
*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
 a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*
a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^
2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2
*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*
d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 -
3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^
3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c
*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f
^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c...
```

3.271.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2159 vs. $2(367) = 734$.

Time = 3.18 (sec) , antiderivative size = 2159, normalized size of antiderivative = 5.67

$$\int \frac{A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="giac")
```

output
```
-1/4*(4*B*b^4*c*d^3*f^3 - 4*B*a*b^3*d^4*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 6*B*
a^2*b^2*d^4*f^2*g + 4*B*b^4*c^3*d*f*g^2 - 4*B*a^3*b*d^4*f*g^2 - B*b^4*c^4*
g^3 + B*a^4*d^4*g^3)*log(abs(b*d - 2*b*d*f/(g*x + f) + b*d*f^2/(g*x + f)^2
 + b*c*g/(g*x + f) + a*d*g/(g*x + f) - b*c*f*g/(g*x + f)^2 - a*d*f*g/(g*x
+ f)^2 + a*c*g^2/(g*x + f)^2))/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*
d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4
*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d
^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^
4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4
- 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^
5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 +
 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g
^8) + 1/4*(2*B*b^5*c*d^4*f^4*g - 2*B*a*b^4*d^5*f^4*g - 4*B*b^5*c^2*d^3*f^3
*g^2 + 4*B*a^2*b^3*d^5*f^3*g^2 + 6*B*b^5*c^3*d^2*f^2*g^3 - 6*B*a*b^4*c^2*d
^3*f^2*g^3 + 6*B*a^2*b^3*c*d^4*f^2*g^3 - 6*B*a^3*b^2*d^5*f^2*g^3 - 4*B*b^5
*c^4*d*f*g^4 + 4*B*a*b^4*c^3*d^2*f*g^4 - 4*B*a^3*b^2*c*d^4*f*g^4 + 4*B*a^4
*b*d^5*f*g^4 + B*b^5*c^5*g^5 - B*a*b^4*c^4*d*g^5 + B*a^4*b*c*d^4*g^5 - B*a
^5*d^5*g^5)*log(abs(2*b*d*f*g - 2*b*d*f^2*g/(g*x + f) - b*c*g^2 - a*d*g^2
+ 2*b*c*f*g^2/(g*x + f) + 2*a*d*f*g^2/(g*x + f) - 2*a*c*g^3/(g*x + f) - ab
s(-b*c*g^2 + a*d*g^2))/abs(2*b*d*f*g - 2*b*d*f^2*g/(g*x + f) - b*c*g^2 ...
```

3.271.9 Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 2520, normalized size of antiderivative = 6.61

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^5,x)
```

output
```
(log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^
3*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4
*f^3 + 4*B*b^4*c*d^3*f^3))/(2*a^4*c^4*g^8 + 2*b^4*d^4*f^8 + 2*a^4*d^4*f^4*
g^4 + 2*b^4*c^4*f^4*g^4 + 12*a^2*b^2*c^4*f^2*g^6 + 12*a^2*b^2*d^4*f^6*g^2
+ 12*a^4*c^2*d^2*f^2*g^6 + 12*b^4*c^2*d^2*f^6*g^2 - 8*a^3*b*c^4*f*g^7 - 8*
a*b^3*d^4*f^7*g - 8*a^4*c^3*d*f*g^7 - 8*b^4*c*d^3*f^7*g - 8*a*b^3*c^4*f^3*
g^5 - 8*a^3*b*d^4*f^5*g^3 - 8*a^4*c*d^3*f^3*g^5 - 8*b^4*c^3*d*f^5*g^3 + 32
*a*b^3*c*d^3*f^6*g^2 + 32*a*b^3*c^3*d*f^4*g^4 + 32*a^3*b*c*d^3*f^4*g^4 + 3
2*a^3*b*c^3*d*f^2*g^6 - 48*a*b^3*c^2*d^2*f^5*g^3 - 48*a^2*b^2*c*d^3*f^5*g^
3 - 48*a^2*b^2*c^3*d*f^3*g^5 - 48*a^3*b*c^2*d^2*f^3*g^5 + 72*a^2*b^2*c^2*d
^2*f^4*g^4) - ((3*A*a^3*c^3*g^6 + 3*A*b^3*d^3*f^6 - 3*A*a^3*d^3*f^3*g^3 -
3*A*b^3*c^3*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3 + 9*A*a*
b^2*c^3*f^2*g^4 + 9*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4 + 9*A*a^3*
c*d^2*f^2*g^4 + 31*B*a^2*b*d^3*f^4*g^2 + 9*A*b^3*c^2*d*f^4*g^2 + 7*B*a^3*c
*d^2*f^2*g^4 - 31*B*b^3*c^2*d*f^4*g^2 - 9*A*a^2*b*c^3*f*g^5 - 9*A*a*b^2*d^
3*f^5*g + 2*B*a^2*b*c^3*f*g^5 - 9*A*a^3*c^2*d*f*g^5 - 26*B*a*b^2*d^3*f^5*g
 - 9*A*b^3*c*d^2*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c*d^2*f^5*g + 27*A
*a*b^2*c*d^2*f^4*g^2 - 27*A*a*b^2*c^2*d*f^3*g^3 - 27*A*a^2*b*c*d^2*f^3*g^3
 + 27*A*a^2*b*c^2*d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 - 15*B*a^2*b*c*d^2*
f^3*g^3)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*...
```

3.271. $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$

3.272 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

3.272.1 Optimal result

Integrand size = 31, antiderivative size = 869

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \frac{2B^2(bc-ad)^3g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{b^3d^3} + \frac{B^2(bc-ad)^2g^3(c+dx)^2}{3b^2d^4}$$
$$- \frac{B(bc-ad)g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\,(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^4d^3}$$
$$- \frac{B(bc-ad)g^2(4bdf-3bcg-adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b^2d^4}$$
$$- \frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^4}$$
$$- \frac{(bf-ag)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b^4g} + \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g}$$
$$- \frac{B(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^4d^4}$$
$$+ \frac{2B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{b^4d^4}$$
$$+ \frac{2B^2(bc-ad)^4g^3\log(c+dx)}{3b^4d^4} + \frac{B^2(bc-ad)^3g^2(4bdf-3bcg-adg)\log(c+dx)}{b^4d^4}$$
$$+ \frac{2B^2(bc-ad)^2g(a^2d^2g^2-2abdg(2df-cg)+b^2(6d^2f^2-8cdfg+3c^2g^2))\log(c+dx)}{b^4d^4}$$
$$- \frac{2B^2(bc-ad)(2bdf-bcg-adg)\,(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4d^4}$$

output
```
2/3*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*g+4*
b*d*f)*x/b^3/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4-B*(-a*d+b*c)*g
*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*
(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^4/d^3-1/2*B*(-a*d+b*c)*g^2*(-a*d
*g-3*b*c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d^4-1/3*
B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4-1/4*(-a*g
+b*f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b
*x+a)^2/(d*x+c)^2))^2/g-B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g
-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c
)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)
/(d*x+c))/b^4/d^4+B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln((b*x+a)
/(d*x+c))/b^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+B^2*(-a*d+b*c
)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+2*B^2*(-a*d+b*c)^2*g*(a
^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(
d*x+c)/b^4/d^4-2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*
d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/
b^4/d^4
```

3.272.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 746, normalized size of antiderivative = 0.86

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \frac{(f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B\left(6Abd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+6Bd(bc-ad)g^2(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(6*A*b*d*(b
*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c
*d*f*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*
f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*
x)^2)/(c + d*x)^2] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x
^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^
3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*Log[a + b
*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 12*B*(b*c - a*d)^2*g^2*(a^2
*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))
*Log[c + d*x] - 6*b^4*(d*f - c*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
])*Log[c + d*x] + 2*B*(b*c - a*d)*g^4*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d -
b*d*x) + 2*a^3*d^3*Log[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 6*B*(b*c - a*d
)*g^3*(-4*b*d*f + b*c*g + a*d*g)*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) +
 a*d)*x + b*c^2*Log[c + d*x])) - 6*B*d^4*(b*f - a*g)^4*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + 6*b^4*B*(d*f - c*g)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
])))/(3*b^4*d^4))/(4*g)
```

3.272.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 1074, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}}{g(bc-ad)}$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\left(\frac{(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)g^4}{bd^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(bc-ad)^3(4bdf-}{}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4(bc-ad)g\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)g^4}{3bd^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B(bc-ad)^4\log\left(\frac{a+b}{c+d}\right.}{3b^4d^4}\right.}{}\right.$$

input
```
Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

3.272. $\int(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^4*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*(b*c - a*d)*g*(b - (d*(a + b*x))/(c +
 d*x))^4) - (B*(-1/3*(B*(b*c - a*d)^4*g^4)/(b^2*d^4*(b - (d*(a + b*x))/(c
+ d*x))^2) - (2*B*(b*c - a*d)^4*g^4)/(3*b^3*d^4*(b - (d*(a + b*x))/(c + d*
x))) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g))/(b^3*d^4*(b - (d*
(a + b*x))/(c + d*x))) + ((b*c - a*d)^4*g^4*(A + B*Log[(e*(a + b*x)^2)/(c
+ d*x)^2]))/(3*b*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + ((b*c - a*d)^3*g^3
*(4*b*d*f - 3*b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*
b^2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(a^2*d^2*g^2
 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a +
 b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^4*d^3*(c + d*x)*(b - (d
*(a + b*x))/(c + d*x))) + ((b*f - a*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2])^2)/(4*b^4*B) - (2*B*(b*c - a*d)^4*g^4*Log[(a + b*x)/(c + d*x)])/(3
*b^4*d^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a + b*x)
/(c + d*x)])/(b^4*d^4) + (2*B*(b*c - a*d)^4*g^4*Log[b - (d*(a + b*x))/(c +
 d*x)])/(3*b^4*d^4) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - 3*b*c*g - a*d*g)*Log
[b - (d*(a + b*x))/(c + d*x)])/(b^4*d^4) + (2*B*(b*c - a*d)^2*g^2*(a^2*d^2
*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*
Log[b - (d*(a + b*x))/(c + d*x)])/(b^4*d^4) + ((b*c - a*d)*g*(2*b*d*f - b*
c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g ...
```

3.272.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.272.4 Maple [F]

$$\int (gx+f)^3 \left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.272.5 Fricas [F]

$$\int (f+gx)^3 \left(A + B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^3 \left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2 dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^
3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*e*x^2 + 2*a*b*
e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x
^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2
 + 2*c*d*x + c^2)), x)
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.272.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. 2(852) = 1704.

Time = 0.35 (sec) , antiderivative size = 2351, normalized size of antiderivative = 2.71

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxim
a")
```

output
```
1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b^2*e*x^2/(
d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^
2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^3
 + 3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2
*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2
+ 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*f^2*g + 2*(x^3*log(b
^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) +
 a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
 + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))
*A*B*f*g^2 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*
x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 -
3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^
3 + A^2*f^3*x - 1/3*(6*a^3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2
*b + 2*(18*c*d^3*f^2*g - 6*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (12*c*d^3*f^
3*log(e) - (3*g^3*log(e) + 11*g^3)*c^4 + 12*(f*g^2*log(e) + 3*f*g^2)*c^3*d
 - 18*(f^2*g*log(e) + 2*f^2*g)*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) +
2*(4*a*b^3*d^4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3
 - (4*c*d^3*f^3 - 6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x
 + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - ...
```

3.272.8 Giac [F]

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int (gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (f+g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.273 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

3.273.1 Optimal result

Integrand size = 31, antiderivative size = 542

$$\begin{aligned}
&\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx \\
&= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b^3d^2} \\
&\quad - \frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd^3} \\
&\quad - \frac{(bf-ag)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b^3g} + \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} \\
&\quad + \frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3b^3d^3} \\
&\quad + \frac{4B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{4B^2(bc-ad)^3g^2\log(c+dx)}{3b^3d^3} \\
&\quad + \frac{8B^2(bc-ad)^2g(3bdf-2bcg-adg)\log(c+dx)}{3b^3d^3} \\
&\quad + \frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}
\end{aligned}$$

output
```
4/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2-4/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*
d*f)*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^3/d^2-2/3*B*(-a*d+b*c)*g^2*
(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2
))^2/g+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c
*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c)
)/b^3/d^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/b^3/d^3+4/3*B^2*(-a
*d+b*c)^3*g^2*ln(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b
*d*f)*ln(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*
f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d
^3
```

3.273.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.92

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$
$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.}{}}{}$$

input
```
Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(2*A*b*d*(b
*c - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f
 - b*c*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] + b^2*d^2*(b*
c - a*d)*g^3*x^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(b*f - a
*g)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 4*B*(b*c - a
*d)^2*g^2*(-3*b*d*f + b*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A
 + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 2*B*(b*c - a*d)*g^3*
(a^2*d^2*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 2*B
*d^3*(b*f - a*g)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^3*B*(d*f - c*
g)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

3.273.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\int \frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3g(bc-ad)}\right.$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\int\left(\frac{(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)g^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad)^2(3bd\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{4B\left(-\frac{g(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2\right.}{b^3d^3}\right.}{}\right.$$

input
```
Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*(b*c - a*d)*g*(b - (d*(a + b*x))/(c +
 d*x))^3) - (4*B*(-((B*(b*c - a*d)^3*g^3)/(b^2*d^3*(b - (d*(a + b*x))/(c +
 d*x)))) + ((b*c - a*d)^3*g^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2
*b*d^3*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f - 2*
b*c*g - a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^3*d^
2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a*g)^3*(A + B*Log[(e*
(a + b*x)^2)/(c + d*x)^2])^2)/(4*b^3*B) - (B*(b*c - a*d)^3*g^3*Log[(a + b*
x)/(c + d*x)])/(b^3*d^3) + (B*(b*c - a*d)^3*g^3*Log[b - (d*(a + b*x))/(c +
 d*x)])/(b^3*d^3) + (2*B*(b*c - a*d)^2*g^2*(3*b*d*f - 2*b*c*g - a*d*g)*Log
[b - (d*(a + b*x))/(c + d*x)])/(b^3*d^3) - ((b*c - a*d)*g*(a^2*d^2*g^2 - a
*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^
3) - (2*B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*
f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*
d^3)))/(3*(b*c - a*d)*g))
```

3.273.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.273. $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.273.4 Maple [F]

$$\int (gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

input
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.273.5 Fricas [F]

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)^2 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x
+ B^2*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^
2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

3.273.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.273.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. 2(521) = 1042.

Time = 0.33 (sec) , antiderivative size = 1458, normalized size of antiderivative = 2.69

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxim
a")
```

output
```
1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^
2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b^2*e*x^
2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/
(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d
^2 - 2*(b*c - a*d)*x/(b*d))*A*B*f*g + 2/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*
c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*
d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x +
 4/3*(2*a^2*c*d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (3*c*d^2*f^2*log(e
) + (g^2*log(e) + 3*g^2)*c^3 - 3*(f*g*log(e) + 2*f*g)*c^2*d)*b^2)*B^2*log(
d*x + c)/(b^2*d^3) + 8/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2
- (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1
/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (2*a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*lo
g(e)^2 - 2*c*d^2*g^2*log(e))*b^3)*B^2*x^2 - (4*(g^2*log(e) - g^2)*a^2*b*d^
3 - 4*(3*d^3*f*g*log(e) - 2*c*d^2*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 12*c*
d^2*f*g*log(e) + 4*(g^2*log(e) + g^2)*c^2*d)*b^3)*B^2*x + 4*(B^2*b^3*d^3*g
^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 -
3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*log(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2...
```

3.273.8 Giac [F]

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int (gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

input
```
integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\int (f+g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2\,dx$$

input
```
int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.274 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

3.274.1 Optimal result

Integrand size = 29, antiderivative size = 281

$$\begin{aligned}\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= -\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^2d} \\ &\quad - \frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} \\ &\quad + \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2d^2} \\ &\quad + \frac{4B^2(bc-ad)^2g\log(c+dx)}{b^2d^2} + \frac{4B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}\end{aligned}$$

output
```
-2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d-1/2*(-a*g+
b*f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*(b*
x+a)^2/(d*x+c)^2))^2/g+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*
x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+4*B^2*(-a*d+b*c)^2*g*l
n(d*x+c)/b^2/d^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+
a)/b/(d*x+c))/b^2/d^2
```

3.274.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.25

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{4B\left(Abd(bc-ad)g^2x+Bd(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+d^2(bf-ag)^2\log(a+bx)\left(A+B\log\right.\right.}{}}{}$$

input
```
Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(A*b*d*(b*c
 - a*d)*g^2*x + B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x
)^2] + d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]) - 2*B*(b*c - a*d)^2*g^2*Log[c + d*x] - b^2*(d*f - c*g)^2*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b
*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)])))/(b^2*d^2))/(2*g)
```

3.274.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.43, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \; 2954$$

$$(bc-ad)\int \frac{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \; 2798$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\int\frac{(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{g(bc-ad)}\right.$$

$\big\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\int\left(\frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)g^2}{bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{(bc-ad)(2bdf}{}\right.}{}\right.$$

$\big\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\left(-\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e}{(}\right.\right.}{b^2d^2}\right.}{}\right.$$

input
```
Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
(b*c - a*d)*(((b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*(b*c - a*d)*g*(b - (d*(a + b*x))/(c +
 d*x))^2) - (2*B*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/
(c + d*x)^2]))/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*f - a
*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*b^2*B) + (2*B*(b*c -
a*d)^2*g^2*Log[b - (d*(a + b*x))/(c + d*x)])/(b^2*d^2) - ((b*c - a*d)*g*(2
*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - (
d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2) - (2*B*(b*c - a*d)*g*(2*b*d*f - b*c
*g - a*d*g)*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/((b*c - a
*d)*g))
```

3.274. $\int(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$

3.274.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.274.4 Maple [F]

$$\int (gx+f)\left(A+B\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)\right)^{2}dx$$

input
```
int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.274.5 Fricas [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*e*x^2 + 2*a*b*e*x +
a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*e*x^2 +
 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.274.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(276) = 552$.

Time = 0.31 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.80

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{1}{2}A^2gx^2$$
$$+2\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log(bx+a)}{b}-\frac{2\,c\log(dx}{d}\right.$$
$$+\left(x^2\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log(bx+a)}{b^2}+\frac{2\,c^2\log(d}{d^2}\right.$$
$$+A^2fx-\frac{2\,(2\,acdg+(2\,cdf\log(e)-(g\log(e)+2\,g)c^2)b)B^2\log(dx+c)}{bd^2}$$
$$+\frac{4\,(2\,abd^2f-a^2d^2g-(2\,cdf-c^2g)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2}$$
$$+\frac{B^2b^2d^2gx^2\log(e)^2+2\left(2\,abd^2g\log(e)+\left(d^2f\log(e)^2-2\,cdg\log(e)\right)b^2\right)B^2x+4\,(B^2b^2d^2gx^2+2\,B^2b^2d}{}$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima"
)
```

output
```
1/2*A^2*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(
d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x
+ a)/b - 2*c*log(d*x + c)/d)*A*B*f + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x
 + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x
/(b*d))*A*B*g + A^2*f*x - 2*(2*a*c*d*g + (2*c*d*f*log(e) - (g*log(e) + 2*g
)*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + 4*(2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f
 - c^2*g)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(
b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2
 + 2*(2*a*b*d^2*g*log(e) + (d^2*f*log(e)^2 - 2*c*d*g*log(e))*b^2)*B^2*x +
4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*
log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2
*g)*B^2*b^2)*log(d*x + c)^2 + 4*(B^2*b^2*d^2*g*x^2*log(e) + 2*(a*b*d^2*g +
 (d^2*f*log(e) - c*d*g)*b^2)*B^2*x - ((g*log(e) - 2*g)*a^2*d^2 - 2*(d^2*f*
log(e) - c*d*g)*a*b)*B^2)*log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2*log(e) + 2*(
a*b*d^2*g + (d^2*f*log(e) - c*d*g)*b^2)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2*B
^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a))*log(d*x + c)
)/(b^2*d^2)
```

3.274.8 Giac [F]

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int (f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.275 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$

3.275.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b}+\frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd}+\frac{8B^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4*B*(-a*d+b*c)*(A+B*ln(e*(b*x+
a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d+8*B^2*(-a*d+b*c)*polylog(2,d
*(b*x+a)/b/(d*x+c))/b/d
```

3.275.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.71

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B\left(ad\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-bc\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)-aBd\left(\log(a+bx\right.\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
x*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(a*d*Log[a + b*x]*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b*c*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

3.275.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2936, 2942, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2936}$$

$$\frac{(a+bx) \left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad) \int \frac{A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{c+dx}dx}{b}$$

$$\Big\downarrow \text{2942}$$

$$\frac{(a+bx) \left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad) \left(\frac{2B(bc-ad) \int \frac{\log \left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{d}-\frac{\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2858}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int\frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(-\frac{2B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2778}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int\frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(\frac{2B(bc-ad)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2752}$$

3.275. $\int\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2dx$

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}-\frac{4B(bc-ad)\left(-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{d}-\frac{2B\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{d}\right)}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/b - (4*B*(b*c - a*d
)*(-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x
))])/d) - (2*B*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/d))/b
```

3.275.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

3.275. $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

3.275.4 Maple [F]

$$\int \left(A+B\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)\right)^{2}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.275.5 Fricas [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx=\int \left(B\log\left(\frac{(bx+a)^{2}e}{(dx+c)^{2}}\right)+A\right)^{2}dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

output
```
integral(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
 + A^2, x)
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int \left(A+B\log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.275.7 Maxima [F]

$$\int \left(A+B\log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \int \left(B\log \left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

output
```
2*(x*log((b*x + a)^2*e/(d*x + c)^2) + 2*(a*e*log(b*x + a)/b - c*e*log(d*x
+ c)/d)/e)*A*B + A^2*x + B^2*(4*(b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(
d*x + c)^2 - (b*d*x*log(e) + 2*(b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(
b*d) + integrate(((log(e)^2 + 4*log(e))*b^2*d*x^2 + a*b*c*log(e)^2 + (b^2*
c*log(e)^2 + (log(e)^2 + 4*log(e))*a*b*d)*x + 4*(b^2*d*x^2*log(e) + a*b*c*
log(e) + 2*a^2*d + (a*b*d*(log(e) + 4) + b^2*c*(log(e) - 2))*x)*log(b*x +
a))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

3.275.8 Giac [F]

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx=\int \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.276 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$

3.276.1 Optimal result

Integrand size = 31, antiderivative size = 285

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} - \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{g} + \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g} + \frac{8B^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g} - \frac{8B^2\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

output
```
-(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*(b
*x+a)^2/(d*x+c)^2))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-4*B*(A
+B*ln(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+4*B*(A+B*ln
(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
g+8*B^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-8*B^2*polylog(3,(-c*g+d*f)*(b*x+a
)/(-a*g+b*f)/(d*x+c))/g
```

3.276.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1370 vs. $2(285) = 570$.

Time = 0.42 (sec) , antiderivative size = 1370, normalized size of antiderivative = 4.81

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

output
```
(-4*B^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((d*
f - c*g)*(a + b*x))]^2 + A^2*Log[f + g*x] - 4*A*B*Log[a/b + x]*Log[f + g*x
] + 4*B^2*Log[a/b + x]^2*Log[f + g*x] + 4*A*B*Log[c/d + x]*Log[f + g*x] -
8*B^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + 4*B^2*Log[c/d + x]^2*Log[f
+ g*x] + 2*A*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] - 4*B^2*Log[a
/b + x]*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] + 4*B^2*Log[c/d + x]
*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[f + g*x] + B^2*Log[(e*(a + b*x)^2)/(
c + d*x)^2]^2*Log[f + g*x] + 4*A*B*Log[a/b + x]*Log[(b*(f + g*x))/(b*f - a
*g)] - 4*B^2*Log[a/b + x]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 4*B^2*Log[a/b
 + x]*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[(b*(f + g*x))/(b*f - a*g)] + 8*
B^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(b*(f + g*x))/(b*f
- a*g)] - 4*B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(b*(f + g*x))/(b*f
 - a*g)] + 8*B^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d
*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f + g*x))/(b*f - a*g)] - 4*B^2*Log[(
(b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[(b*(f + g*x))/(b*f -
 a*g)] - 4*A*B*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] + 8*B^2*Log[a/b
 + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] - 4*B^2*Log[c/d + x]^2*L
og[(d*(f + g*x))/(d*f - c*g)] - 4*B^2*Log[c/d + x]*Log[(e*(a + b*x)^2)/(c
+ d*x)^2]*Log[(d*(f + g*x))/(d*f - c*g)] - 8*B^2*Log[a/b + x]*Log[(g*(c +
d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*x))/(d*f - c*g)] + 4*B^2*Log[(g*(c ...
```

3.276.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2954, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{f+gx} \, dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad) \int \frac{\left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right) \left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad) \int \left(\frac{d\left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g \left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(cg-df) \left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g \left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad) \left(\frac{4B \text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right) \left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g(bc-ad)}+\frac{\log \left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) \left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g(bc-ad)} -\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[1 - (d*(a +
 b*x))/(b*(c + d*x))])/((b*c - a*d)*g)) + ((A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])^2*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*
c - a*d)*g) - (4*B*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*PolyLog[2, (d*
(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g) + (4*B*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x)
)])/((b*c - a*d)*g) + (8*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*
c - a*d)*g) - (8*B^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c +
d*x))])/((b*c - a*d)*g))
```

3.276.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.276.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{gx+f}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
```

3.276.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="fricas"
)
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g*x + f), x)
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f),x)
```

output
```
Timed out
```

3.276.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="maxima"
)
```

output
```
A^2*log(g*x + f)/g - integrate(-(4*B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A
*B*log(e) + 4*(B^2*log(e) + A*B)*log(b*x + a) - 4*(2*B^2*log(b*x + a) + B^
2*log(e) + A*B)*log(d*x + c))/(g*x + f), x)
```

3.276.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f), x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{f+g\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x), x)
```

3.277 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$

3.277.1 Optimal result

Integrand size = 31, antiderivative size = 200

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bf-ag)(f+gx)} + \frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)} + \frac{8B^2(bc-ad)\,\mathrm{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*g+b*f)/(g*x+f)+4*B*(-a*d+b*c
)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/(-a*g+b*f)/(-c*g+d*f)+8*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-
a*g+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

3.277.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(200) = 400$.

Time = 0.29 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.04

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$$

$$= \frac{-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx} + \frac{4B\left(b(df-cg)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-d(bf-ag)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)+(bc-ad)g\left(A\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

output
```
(-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)) + (4*B*(b*(d*f -
c*g)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - d*(b*f - a*g)
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + (b*c - a*d)*g*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(
b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)]) - 2*B*(b*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(
g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f
 - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*
g)))/g
```

3.277.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2954, 2755, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)^2}\,dx$$

$$\Big\downarrow \; 2954$$

$$(bc-ad)\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2755}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{4B\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bf-ag-\frac{(df-cg)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bf-ag}\right)$$

$$\Big\downarrow \text{2754}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{4B\left(\frac{2B\int \frac{(c+dx)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{df-cg}-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\ldots}{\ldots}\right.}{bf-ag}\right.$$

$$\Big\downarrow \text{2838}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(c+dx)(bf-ag)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}-\frac{4B\left(-\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{df-cg}-\frac{2B\,\text{PolyLog}(\ldots}{\ldots}\right.}{bf-ag}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*f
- a*g)*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - (4*B*(
-(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d*f - c*g)*(a + b*x)
)/((b*f - a*g)*(c + d*x))])/(d*f - c*g)) - (2*B*PolyLog[2, ((d*f - c*g)*(a
 + b*x))/((b*f - a*g)*(c + d*x))])/(d*f - c*g)))/(b*f - a*g))
```

3.277. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}dx$

3.277.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.277.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^2}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)
```

3.277.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="frica
s")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.277.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**2,x)
```

output
```
Timed out
```

3.277.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="maxim
a")
```

output
```
2*A*B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2)
 + 2*(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(
b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - B^2*(4*log(d*x + c)^2/
(g^2*x + f*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x + c*g
)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((g*log(
e) - 2*g)*d*x + c*g*log(e) - 2*d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x
 + c))/(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g
^2)*x), x)) - A^2/(g^2*x + f*g)
```

3.277.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^2, x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2, x)
```

3.278 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx$

3.278.1 Optimal result

Integrand size = 31, antiderivative size = 381

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = \frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)} + \frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(bf-ag)^2} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{4B^2(bc-ad)^2g\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2} + \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2} + \frac{4B^2(bc-ad)(2bdf-bcg-adg)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)^2(df-cg)^2}$$

output
```
2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*
g+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^2-1/
2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^2+4*B^2*(-a*d+b*c)^2*g*ln((g
*x+f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*
d*f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d
*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*p
olylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

3.278.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B(f+gx)\left((bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\log\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

output
```
-1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(f + g*x)*((b*c -
a*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) -
b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]) + d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f
+ g*x)*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*
c - a*d)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)]) - B*d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(
-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)]) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Lo
g[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f
 + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))
/(d*f - c*g)])))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

3.278.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)^3}\,dx$$

$$\Big\downarrow \ 2954$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2798$$

$$(bc-ad)\left(\frac{2B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^2}\right)$$

$$\Big\downarrow \ 2804$$

$$(bc-ad)\left(\frac{2B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^2}{(bf-ag)^2(a+bx)}+\frac{(bc-ad)g(-2bdf+bcg+adg)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^2(df-cg)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+b}{(c+d}\right.\right.}{(bf-ag)(df-cg)\left(bf-ag-\frac{(df-cg}{c-}\right.}\right.}{g(bc-ad)}\right.$$

$$\Big\downarrow \ 2009$$

$$(bc-ad)\left(\frac{2B\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4B(bf-ag)^2}+\frac{g^2(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(c+dx)(bf-ag)^2(df-cg)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)}+\frac{g(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)}{(c+dx)}\right.}{(bf-ag)^2(df-}\right.}{g}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

3.278. $\displaystyle\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx$

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(
c + d*x))^2) + (2*B*(((b*c - a*d)^2*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2]))/((b*f - a*g)^2*(d*f - c*g)*(c + d*x)*(b*f - a*g - ((d*f
- c*g)*(a + b*x))/(c + d*x))) + (b^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2])^2)/(4*B*(b*f - a*g)^2) + (2*B*(b*c - a*d)^2*g^2*Log[b*f - a*g - ((d*f
- c*g)*(a + b*x))/(c + d*x)])/((b*f - a*g)^2*(d*f - c*g)^2) + ((b*c - a*d)
*g*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[
1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f
- c*g)^2) + (2*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, ((d*f
- c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2))
)/((b*c - a*d)*g))
```

3.278.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.278. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx$

3.278.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^3}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

3.278.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="frica
s")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

3.278.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**3,x)
```

output
```
Timed out
```

3.278.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="maxim
a")
```

output
```
(2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x +
c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b^2
*c^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d +
a*b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^
2*c*d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (
b*d*f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c
*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d
*x + c^2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*A*B - B^2*(2*log(d*x + c)^2/(g^3
*x^2 + 2*f*g^2*x + f^2*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*
(d*g*x + c*g)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a)
- 4*((g*log(e) - g)*d*x + c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))
*log(d*x + c))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g
^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 + 2
*f*g^2*x + f^2*g)
```

3.278.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^3, x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^3}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3, x)
```

3.279 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$

3.279.1 Optimal result

Integrand size = 31, antiderivative size = 724

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$$
$$= \frac{4B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)} - \frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}$$
$$+ \frac{4B(bc-ad)g(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^3(df-cg)^2(f+gx)}$$
$$+ \frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(bf-ag)^3} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{4B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$- \frac{4B^2(bc-ad)^3g^2\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3} + \frac{8B^2(bc-ad)^2g(3bdf-bcg-2adg)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$
$$+ \frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(d}{(b}\right.}{3(bf-ag)^3(df-cg)^3}$$
$$+ \frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(3d^2f^2-3cdfg+c^2g^2)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bf-ag)^3(df-cg)^3}$$

output
```
4/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-2/3*B*(
-a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(-c*g+d
*f)^3/(g*x+f)^2+4/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*l
n(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*l
n(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)^2/(d*x+c)
^2))^2/g/(g*x+f)^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)
^3/(-c*g+d*f)^3-4/3*B^2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/
(-c*g+d*f)^3+8/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d
*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c
*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2
))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+8
/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g
+3*d^2*f^2))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3
/(-c*g+d*f)^3
```

3.279.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 909, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx =$$

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B(f+gx)\left((bc-ad)g(bf-ag)^2(df-cg)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2(bc-ad)g(bf-ag)(-df+cg)(-2bdf}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

output
```
-1/3*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*((b*c -
a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
) + 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*
(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*b^3*(d*f - c*g)^3*(
f + g*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(
b*f - a*g)^3*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c +
d*x] - 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*
f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2])*Log[f + g*x] - 4*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x
)^2*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c -
 a*d)*g*Log[f + g*x]) + 2*B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f -
a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a
*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f
 + g*x)*Log[f + g*x]) + 2*b^3*B*(d*f - c*g)^3*(f + g*x)^2*(Log[a + b*x]*(L
og[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) - 2*B*d^3*(b*f - a*g)^3*(f + g*x)^2*((2*Log[(d*(a + b*
x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)]) + 4*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g
) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*((Log[(g*(a + b*x))
/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + Po...
```

3.279.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)^4}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{4B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^3}\right)$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{4B\int \left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^3}{(bf-ag)^3(a+bx)}+\frac{(bc-ad)g(-((3d^2f^2-3cdgf+c^2g^2)b^2)+adg(3df-cg)b-a^2d^2g^2)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^3(df-cg)^2\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{3g(bc-ad)}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{4B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2b^3}{4B(bf-ag)^3}+\frac{(bc-ad)^2g^2(3bdf-bcg-2adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^3(df-cg)^2(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}-\frac{(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+\right.\right.}{2(bf-ag)(df-cg)^3\left(bf-ag-\frac{(df-\right.}}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

3.279. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$

output
```
(b*c - a*d)*(-1/3*((b - (d*(a + b*x))/(c + d*x))^3*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(
c + d*x))^3) + (4*B*((B*(b*c - a*d)^3*g^3)/((b*f - a*g)^2*(d*f - c*g)^3*(b
*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) - ((b*c - a*d)^3*g^3*(A + B
*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*(b*f - a*g)*(d*f - c*g)^3*(b*f - a*
g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*g^2*(3*b*d*f -
b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f
 - a*g)^3*(d*f - c*g)^2*(c + d*x)*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
+ d*x))) + (b^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*B*(b*f - a*
g)^3) + (B*(b*c - a*d)^3*g^3*Log[(a + b*x)/(c + d*x)])/((b*f - a*g)^3*(d*f
 - c*g)^3) - (B*(b*c - a*d)^3*g^3*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/
(c + d*x)])/((b*f - a*g)^3*(d*f - c*g)^3) + (2*B*(b*c - a*d)^2*g^2*(3*b*d*
f - b*c*g - 2*a*d*g)*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])/(
(b*f - a*g)^3*(d*f - c*g)^3) + ((b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*
f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])
/((b*f - a*g)^3*(d*f - c*g)^3) + (2*B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g
*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f -
 c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^3*(d*f - c*g)^3)))
/(3*(b*c - a*d)*g))
```

3.279.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.279. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.279.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^4}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

3.279.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="frica
s")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

3.279.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**4,x)
```

output
```
Timed out
```

3.279.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="maxim
a")
```

output
```
2/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3
*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 -
c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*
g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(
b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f
^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(
a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^
2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)
*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^
2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^
2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^
5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*e*x^2/(d
^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2
*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B
- 1/3*B^2*(4*log(d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
+ 3*integrate(-1/3*(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 12*(d*g*x + c*g)*l
og(b*x + a)^2 + 12*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((3*g*l...
```

3.279.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^4, x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^4}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4, x)
```

3.280 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx$

3.280.1 Optimal result

Integrand size = 31, antiderivative size = 1154

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = -\frac{B^2(bc-ad)^2g^3(c+dx)^2}{3(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$-\frac{2B^2(bc-ad)^3g^3(c+dx)}{3(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c+dx)}{(bf-ag)^3(df-cg)^4(f+gx)}$$
$$+\frac{B(bc-ad)g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^4(f+gx)^3}$$
$$-\frac{B(bc-ad)g^2(4bdf-bcg-3adg)(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^4(f+gx)^2}$$
$$+\frac{B(bc-ad)g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^4(df-cg)^3(f+gx)}$$
$$+\frac{b^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4}$$
$$-\frac{2B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4}+\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$+\frac{2B^2(bc-ad)^4g^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^4(df-cg)^4}-\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$+\frac{2B^2(bc-ad)^2g(3a^2d^2g^2-2abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^4(df-cg)^4}$$
$$-\frac{B(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^4(df-cg)^4}$$
$$-\frac{2B^2(bc-ad)(2bdf-bcg-adg)\left(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+}{(bf-ag)(c+}\right.}{(bf-ag)^4(df-cg)^4}$$

output
```
-1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-2/
3*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+B^2*(-a*d
+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*
x+f)+1/3*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+
b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/2*B*(-a*d+b*c)*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*
(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f
)^2+B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*
d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^4/(-c
*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^4
-1/4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^4-2/3*B^2*(-a*d+b*c)^4*g^
3*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+B^2*(-a*d+b*c)^3*g^2*(-3*a
*d*g-b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+2/3*B^2*
(-a*d+b*c)^4*g^3*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B^2*(-a*d+b
*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+
d*f)^4+2*B^2*(-a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2
*g^2-4*c*d*f*g+6*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B
*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2
-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*
x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-2*B^2*(-a*d+b*c)*(-a*d*
g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^...
```

3.280.2 Mathematica [A] (verified)

Time = 3.70 (sec) , antiderivative size = 1317, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx =$$

$$-\frac{3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B(f+gx)\left(2(bc-ad)g(bf-ag)^3(df-cg)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-3(bc-ad)g(bf-ag)^2(df-cg)^2(-2}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

output
```
-1/12*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*(2*(b
*c - a*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]) - 3*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a
*d*g)*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*(b*c - a*d)*g
*(b*f - a*g)*(d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^
2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2]) - 6*b^4*(d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g +
 a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g
^2))*(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 1
2*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 -
 3*c*d*f*g + c^2*g^2))*(f + g*x)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f)
 + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + 6*B*(b*c - a*d)*g*(
2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g
) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)
*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f +
 g*x]) + 2*B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c
*g)^2 + 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d
*g)*(f + g*x) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b...
```

3.280.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 1400, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2954, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)^5}dx$$

$$\Big\downarrow \text{2954}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{B\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g(bc-ad)\left(-\frac{(a+bx)(df-cg)}{c+dx}-ag+bf\right)^4}\right)$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{B\int\left(\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^4}{(bf-ag)^4(a+bx)}+\frac{(bc-ad)g(2bdf-bcg-adg)(-2d^2f^2b^2-c^2g^2b^2+2cdfgb^2+2ad^2fgb-a^2d^2g^2)\left(A+B\log\left(\frac{e(a}{(c+}\right.\right.}{(bf-ag)^4(df-cg)^3\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2b^4}{4B(bf-ag)^4}+\frac{(bc-ad)^2g^2\left((6d^2f^2-4cdgf+c^2g^2)b^2-2adg(4df-cg)b+3a^2d^2g^2\right)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^4(df-cg)^3(c+dx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}-\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

3.280. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx$

output
```
(b*c - a*d)*(-1/4*((b - (d*(a + b*x))/(c + d*x))^4*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2])^2)/((b*c - a*d)*g*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(
c + d*x))^4) + (B*(-1/3*(B*(b*c - a*d)^4*g^4)/((b*f - a*g)^2*(d*f - c*g)^4
*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^2) - (2*B*(b*c - a*d)^4*g
^4)/(3*(b*f - a*g)^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
 + d*x))) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g))/((b*f - a*g)
^3*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))) + ((b*c
- a*d)^4*g^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f
 - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x))^3) - ((b*c - a*d
)^3*g^3*(4*b*d*f - b*c*g - 3*a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(b*f - a*g - ((d*f - c*g)*(a + b*x))/(c
 + d*x))^2) + ((b*c - a*d)^2*g^2*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g)
+ b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2]))/((b*f - a*g)^4*(d*f - c*g)^3*(c + d*x)*(b*f - a*g - ((d
*f - c*g)*(a + b*x))/(c + d*x))) + (b^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2])^2)/(4*B*(b*f - a*g)^4) - (2*B*(b*c - a*d)^4*g^4*Log[(a + b*x)/(c +
d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B*(b*c - a*d)^3*g^3*(4*b*d*f - b
*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) +
(2*B*(b*c - a*d)^4*g^4*Log[b*f - a*g - ((d*f - c*g)*(a + b*x))/(c + d*x)])
/(3*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)^3*g^3*(4*b*d*f - b*c*...
```

3.280.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

3.280. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx$

rule 2954
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)
 Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m
 + 2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B
, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m
] && IGtQ[p, 0]
```

3.280.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^5}dx$$

input
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)
```

output
```
int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)
```

3.280.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="frica
s")
```

output
```
integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*f^4*
g*x + f^5), x)
```

3.280.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**5,x)
```

output
```
Timed out
```

3.280.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="maxim
a")
```

output
```
1/6*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^
2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3
*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4
)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8
 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*
a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 +
a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a
^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c
^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2
*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*
d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d
 - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3
+ 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3
*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3
*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^
3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*
d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^
8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b
^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^...
```

3.280.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^5, x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{(f+g\,x)^5}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5,x)
```

output
```
int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5, x)
```

3.281 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

3.281.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.281.2 Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

3.281.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
$Aborted
```

3.281.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.281.4 Maple [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.281.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

3.281.6 Sympy [N/A]

Not integrable

Time = 21.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(f+gx)^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral((f + g*x)**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a
*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x
**2))), x)
```

3.281.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.281.8 Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(gx+f)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.281.9 Mupad [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{(f+g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.282 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

3.282.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \text{Int}\left(\frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.282.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

3.282.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$
$$\Big\downarrow \text{ 2956}$$
$$\int \frac{f+gx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

output
```
$Aborted
```

3.282.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.282.4 Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

input
```
int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.282.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

output
```
integral((g*x + f)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d
*x + c^2)) + A), x)
```

3.282.6 Sympy [N/A]

Not integrable

Time = 6.77 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.86

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{f+gx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

input
```
integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral((f + g*x)/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*
e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2
))), x)
```

3.282.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.282.8 Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.282.9 Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.283 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

3.283.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \text{Int}\left(\frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

output
```
Unintegrable(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.283.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x]
```

3.283.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2938}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A} \, dx$$

$$\Big\downarrow \text{2938}$$

$$\int \frac{1}{B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}dx$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
```

output
```
$Aborted
```

3.283.3.1 Defintions of rubi rules used

rule 2938
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x] && EqQ[n + mn, 0]
```

3.283.4 Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{A+B \ln \left(\frac{e (b x+a)^{2}}{(d x+c)^{2}}\right)}dx$$

input
```
int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.283.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

output
```
integral(1/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
)) + A), x)
```

3.283.6 Sympy [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

input
```
integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Integral(1/(A + B*log(e*(a + b*x)**2/(c + d*x)**2)), x)
```

3.283.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

output
```
integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.283.8 Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

3.283.9 Mupad [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

input
```
int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

output
```
int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.284 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

3.284.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.284.2 Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

3.284.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
$Aborted
```

3.284.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.284.4 Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.284.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas"
)
```

output
```
integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e
)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.284.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Timed out
```

3.284.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima"
)
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.284.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.284.9 Mupad [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.285 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

3.285.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.285.2 Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

3.285.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$
$$\Big\downarrow \text{2956}$$
$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
$Aborted
```

3.285.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.285.4 Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.285.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="frica
s")
```

output
```
integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2
)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.285.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Timed out
```

3.285.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.285.8 Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.285.9 Mupad [N/A]

Not integrable

Time = 7.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.286 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

3.286.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

output
```
Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.286.2 Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

3.286.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

$$\Big\downarrow \; 2956$$

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

output
```
$Aborted
```

3.286.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.286.4 Maple [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{3} \left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

3.286.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="frica
s")
```

output
```
integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.286.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

output
```
Timed out
```

3.286.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxim
a")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.286.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

3.286.9 Mupad [N/A]

Not integrable

Time = 12.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.287 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.287.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.287.2 Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

3.287.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
$Aborted
```

3.287.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.287.4 Maple [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.287.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.71

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="frica
s")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e
)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

3.287.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.287.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 10.35

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxim
a")
```

output
```
-1/2*(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*
f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a
)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b
*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b*d*g^2*
x^3 + b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*
a + 2*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/(2*(b*c - a*d)*B^2*lo
g(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e
) - a*d*log(e))*B^2), x)
```

3.287.8 Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac"
)
```

output
```
integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.287.9 Mupad [N/A]

Not integrable

Time = 5.85 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

input
```
int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.288 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.288.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.288.2 Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

3.288.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{f+gx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

output
```
$Aborted
```

3.288.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.288.4 Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{gx+f}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.288.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.59

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas"
)
```

output
```
integral((g*x + f)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c
*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2)) + A^2), x)
```

3.288.6 Sympy [N/A]

Not integrable

Time = 26.30 (sec) , antiderivative size = 729, normalized size of antiderivative = 25.14

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$
$$=\frac{acf+acgx+adfx+adgx^2+bcfx+bcgx^2+bdfx^2+bdgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}$$
$$-\frac{\int \frac{acg}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{adf}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+}{}$$

input
```
integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**
2 + b*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(
a + b*x)**2/(c + d*x)**2)) - (Integral(a*c*g/(A + B*log(a**2*e/(c**2 + 2*c
*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(a*d*f/(A + B*log(a**2*e/(c**2
 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*
x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c*f/(A + B*log(a**2*e
/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b
**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*d*g*x/(A + B*
log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2
*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*c*g
*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*
d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integr
al(2*b*d*f*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c
**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x
) + Integral(3*b*d*g*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) +
 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d*
*2*x**2))), x))/(2*B*(a*d - b*c))
```

3.288.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 7.86

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima"
)
```

output
```
-1/2*(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*
g)*a)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c)
+ (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b*d*
g*x^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/(2*(b*c - a*d
)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b
*c*log(e) - a*d*log(e))*B^2), x)
```

3.288.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

3.288.9 Mupad [N/A]

Not integrable

Time = 6.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{f+g\,x}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.289 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.289.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.289.2 Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x]
```

3.289.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2938}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2} \, dx$$
$$\Big\downarrow \ 2938$$
$$\int \frac{1}{\left(B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]
```

output
```
$Aborted
```

3.289.3.1 Defintions of rubi rules used

rule 2938
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_), x_Symbol] :> Unintegrable[(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])^p, x] /; FreeQ[{a, b, c, d, e, A, B, n, p}, x] && EqQ[n + mn, 0]
```

3.289.4 Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(A+B \ln \left(\frac{e (b x+a)^{2}}{(d x+c)^{2}}\right)\right)^{2}}d x$$

input
```
int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.289.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.26

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

output
```
integral(1/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c
^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^
2)) + A^2), x)
```

3.289.6 Sympy [N/A]

Not integrable

Time = 8.94 (sec) , antiderivative size = 333, normalized size of antiderivative = 14.48

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\frac{ac+adx+bcx+bdx^2}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}$$
$$-\frac{\int \frac{ad}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{bc}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+}{2B(ad-bc)}$$

input
```
integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
(a*c + a*d*x + b*c*x + b*d*x**2)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*
B**2*b*c)*log(e*(a + b*x)**2/(c + d*x)**2)) - (Integral(a*d/(A + B*log(a**
2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2)
+ b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c/(A + B*log
(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x*
*2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*x/(A
 + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x +
 d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(2*B*(a*d -
b*c))
```

3.289.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 7.57

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

output
```
-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(
b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*
B^2) + integrate(1/2*(2*b*d*x + b*c + a*d)/(2*(b*c - a*d)*B^2*log(b*x + a)
 - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*lo
g(e))*B^2), x)
```

3.289.8 Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^(-2), x)
```

3.289.9 Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

output
```
int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.289. $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.290 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.290.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.290.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

3.290.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
$Aborted
```

3.290.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.290.4 Maple [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.290.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.97

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="frica
s")
```

output
```
integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*e*x^2 + 2*a*b*e*x
 + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*e*x^
2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.290.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.290.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 455, normalized size of antiderivative = 14.68

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxim
a")
```

output
```
-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*log(e)
- a*d*f*log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))
*B^2)*x + 2*((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - 2
*((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(d*x + c)) + integrate(1
/2*(b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*
B + (b*c*f^2*log(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*
c*g^2*log(e) - a*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*
c*f*g*log(e) - a*d*f*g*log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2
*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b
*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f
^2)*B^2)*log(d*x + c)), x)
```

3.290.8 Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

3.290.9 Mupad [N/A]

Not integrable

Time = 8.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.290. $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.291 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.291.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.291.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

3.291.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2956}$$

$$\int \frac{1}{(f+gx)^2 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
$Aborted
```

3.291.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.291.4 Maple [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}dx$$

input
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.291.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.29

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fri
cas")
```

output
```
integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g
*x + B^2*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2
))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x
+ a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.291.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 690, normalized size of antiderivative = 22.26

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="max
ima")
```

output
```
-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*l
og(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) -
 a*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) -
 a*d*f*g*log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*
d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b*c*g^2 - a*d*g
^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d
*x + c)) - integrate(-1/2*(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g
)*b)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(e) - a*d*g^3*log(e))*B^2)
*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e) - a*d*f^3*log(e))*B^2 + 3
*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e) - a*d*f*g^2*log(e))*B^2)
*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*log(e) - a*d*f^2*g*log(
e))*B^2)*x + 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^
2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x
 + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2
 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)
), x)
```

3.291.8 Giac [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

3.291.9 Mupad [N/A]

Not integrable

Time = 32.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

input
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.292 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

3.292.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.292.2 Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

3.292.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

$$\Big\downarrow \ 2956$$

$$\int \frac{1}{(f+gx)^3 \left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

output
```
$Aborted
```

3.292.3.1 Defintions of rubi rules used

rule 2956
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(f +
g*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p, x] /; FreeQ[{a, b, c, d,
 e, f, g, A, B, m, n, p}, x] && EqQ[n + mn, 0] && IntegerQ[n]
```

3.292.4 Maple [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(gx+f\right)^{3} \left(A+B\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)\right)^{2}}dx$$

input
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

output
```
int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

3.292.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 6.61

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fri
cas")
```

output
```
integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2
*g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*e*x^2 + 2*a
*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^
2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*
x^2 + 2*c*d*x + c^2))), x)
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

output
```
Timed out
```

3.292.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 924, normalized size of antiderivative = 29.81

$$\int \frac{1}{(f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(gx+f)^3 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="max
ima")
```

output
```
-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*
log(e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log
(e) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*lo
g(e) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*
f^2*g*log(e) - a*d*f^2*g*log(e))*B^2)*x + 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 +
 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*
c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b
*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3
 - a*d*f^3)*B^2)*log(d*x + c)) - integrate(1/2*(b*d*g*x^2 - b*c*f - (d*f -
 3*c*g)*a + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*
g^4*log(e) - a*d*g^4*log(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (
b*c*f*g^3*log(e) - a*d*f*g^3*log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B +
(b*c*f^4*log(e) - a*d*f^4*log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B
 + (b*c*f^2*g^2*log(e) - a*d*f^2*g^2*log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*
d*f^3*g)*A*B + (b*c*f^3*g*log(e) - a*d*f^3*g*log(e))*B^2)*x + 2*((b*c*g^4
- a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 -
a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^
4)*B^2)*log(b*x + a) - 2*((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d
*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a
*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(d*x + c)), x)
```

3.292.8 Giac [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gia
c")
```

output
```
integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

3.292.9 Mupad [N/A]

Not integrable

Time = 48.71 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

input
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

output
```
int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.293 $\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

3.293.1 Optimal result

Integrand size = 31, antiderivative size = 365

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{B(bc-ad)h(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(10d^2g^2-5cdgh+c^2h^2)-b^3(10d^3g^3-10cd^2g^2h+5c^2}{5b^4d^4}$$
$$-\frac{B(bc-ad)h^2(a^2d^2h^2-abdh(5dg-ch)+b^2(10d^2g^2-5cdgh+c^2h^2))\,nx^2}{10b^3d^3}$$
$$-\frac{B(bc-ad)h^3(5bdg-bch-adh)nx^3}{15b^2d^2}-\frac{B(bc-ad)h^4nx^4}{20bd}-\frac{B(bg-ah)^5n\log(a+bx)}{5b^5h}$$
$$+\frac{B(dg-ch)^5n\log(c+dx)}{5d^5h}+\frac{(g+hx)^5\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{5h}$$

output
```
1/5*B*(-a*d+b*c)*h*(a^3*d^3*h^3-a^2*b*d^2*h^2*(-c*h+5*d*g)+a*b^2*d*h*(c^2*
h^2-5*c*d*g*h+10*d^2*g^2)-b^3*(-c^3*h^3+5*c^2*d*g*h^2-10*c*d^2*g^2*h+10*d^
3*g^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*h^2*(a^2*d^2*h^2-a*b*d*h*(-c*h+5*d*g
)+b^2*(c^2*h^2-5*c*d*g*h+10*d^2*g^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*h^3*
(-a*d*h-b*c*h+5*b*d*g)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*h^4*n*x^4/b/d-1/5*B
*(-a*h+b*g)^5*n*ln(b*x+a)/b^5/h+1/5*B*(-c*h+d*g)^5*n*ln(d*x+c)/d^5/h+1/5*(
h*x+g)^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

3.293.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.27

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{bdx(12Ab^4d^4(5g^4+10g^3hx+10g^2h^2x^2+5gh^3x^3+h^4x^4)+B(bc-ad)hn(12a^3d^3h^3-6a^2bd^2h^2(10dg-}{}$$

input
```
Integrate[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(12*A*b^4*d^4*(5*g^4 + 10*g^3*h*x + 10*g^2*h^2*x^2 + 5*g*h^3*x^3 +
h^4*x^4) + B*(b*c - a*d)*h*n*(12*a^3*d^3*h^3 - 6*a^2*b*d^2*h^2*(10*d*g - 2
*c*h + d*h*x) + 2*a*b^2*d*h*(6*c^2*h^2 - 3*c*d*h*(10*g + h*x) + d^2*(60*g^
2 + 15*g*h*x + 2*h^2*x^2)) - b^3*(-12*c^3*h^3 + 6*c^2*d*h^2*(10*g + h*x) -
 2*c*d^2*h*(60*g^2 + 15*g*h*x + 2*h^2*x^2) + d^3*(120*g^3 + 60*g^2*h*x + 2
0*g*h^2*x^2 + 3*h^3*x^3)))) + 12*a^2*B*d^5*h*(-10*b^3*g^3 + 10*a*b^2*g^2*h
 - 5*a^2*b*g*h^2 + a^3*h^3)*n*Log[a + b*x] - 12*b^4*B*(-5*a*d^5*g^4 + b*c*
(5*d^4*g^4 - 10*c*d^3*g^3*h + 10*c^2*d^2*g^2*h^2 - 5*c^3*d*g*h^3 + c^4*h^4
))*n*Log[c + d*x] + 12*b^4*B*d^5*(5*a*g^4 + b*x*(5*g^4 + 10*g^3*h*x + 10*g
^2*h^2*x^2 + 5*g*h^3*x^3 + h^4*x^4))*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(60
*b^5*d^5)
```

3.293.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^4 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{(g+hx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5h}-\frac{Bn(bc-ad)\int \frac{(g+hx)^5}{(a+bx)(c+dx)}dx}{5h}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{(g+hx)^5 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5h} - \frac{Bn(bc-ad)\int \left(\frac{x^3h^5}{bd} + \frac{(5bdg-bch-adh)x^2h^4}{b^2d^2} + \frac{((10d^2g^2-5cdhg+c^2h^2)b^2-adh(5dg-ch)b+a^2d^2h^2)xh^3}{b^3d^3} + \frac{((10d^3g^3-10cd^2hg^2+5c^2dl}{}\right.}{5h}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^5 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5h} - \frac{Bn(bc-ad)\left(\frac{h^3x^2\left(a^2d^2h^2-abdh(5dg-ch)+b^2(c^2h^2-5cdgh+10d^2g^2)\right)}{2b^3d^3} - \frac{h^2x\left(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(c^2h^2-5cdgh+10d^2g^2)-\right.}{b^4d^4}\right.}{5h}$$

input
```
Int[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/5*(B*(b*c - a*d)*n*(-((h^2*(a^3*d^3*h^3 - a^2*b*d^2*h^2*(5*d*g - c*h) +
 a*b^2*d*h*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2) - b^3*(10*d^3*g^3 - 10*c*d^2
*g^2*h + 5*c^2*d*g*h^2 - c^3*h^3))*x)/(b^4*d^4)) + (h^3*(a^2*d^2*h^2 - a*b
*d*h*(5*d*g - c*h) + b^2*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2))*x^2)/(2*b^3*d
^3) + (h^4*(5*b*d*g - b*c*h - a*d*h)*x^3)/(3*b^2*d^2) + (h^5*x^4)/(4*b*d)
+ ((b*g - a*h)^5*Log[a + b*x])/(b^5*(b*c - a*d)) - ((d*g - c*h)^5*Log[c +
d*x])/(d^5*(b*c - a*d))))/h + ((g + h*x)^5*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]))/(5*h)
```

3.293.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.293.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. 2(351) = 702.

Time = 85.50 (sec) , antiderivative size = 1170, normalized size of antiderivative = 3.21

method	result	size
parallelrisch	Expression too large to display	1170
risch	Expression too large to display	2612

input
```
int((h*x+g)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/60*(12*B*ln(b*x+a)*a^6*c*d^5*h^4*n^2-12*B*ln(b*x+a)*a*b^5*c^6*h^4*n^2+12
*A*x^5*a*b^5*c*d^5*h^4*n+12*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^6*h^4*n+
12*B*x^5*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c*d^5*h^4*n+60*A*x^4*a*b^5*c*d^
5*g*h^3*n+20*B*x^3*a^2*b^4*c*d^5*g*h^3*n^2-20*B*x^3*a*b^5*c^2*d^4*g*h^3*n^
2+120*A*x^3*a*b^5*c*d^5*g^2*h^2*n-30*B*x^2*a^3*b^3*c*d^5*g*h^3*n^2+60*B*x^
2*a^2*b^4*c*d^5*g^2*h^2*n^2+30*B*x^2*a*b^5*c^3*d^3*g*h^3*n^2-60*B*x^2*a*b^
5*c^2*d^4*g^2*h^2*n^2+120*A*x^2*a*b^5*c*d^5*g^3*h*n+60*B*x*ln(e*(b*x+a)^n/
((d*x+c)^n))*a*b^5*c*d^5*g^4*n+60*B*x*a^4*b^2*c*d^5*g*h^3*n^2-120*B*x*a^3*
b^3*c*d^5*g^2*h^2*n^2+120*B*x*a^2*b^4*c*d^5*g^3*h*n^2-60*B*x*a*b^5*c^4*d^2
*g*h^3*n^2+120*B*x*a*b^5*c^3*d^3*g^2*h^2*n^2-120*B*x*a*b^5*c^2*d^4*g^3*h*n
^2-60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^5*d*g*h^3*n+120*B*ln(e*(b*x+a)
^n/((d*x+c)^n))*a*b^5*c^4*d^2*g^2*h^2*n-120*B*ln(e*(b*x+a)^n/((d*x+c)^n))*
a*b^5*c^3*d^3*g^3*h*n-60*B*ln(b*x+a)*a^5*b*c*d^5*g*h^3*n^2+120*B*ln(b*x+a)
*a^4*b^2*c*d^5*g^2*h^2*n^2-120*B*ln(b*x+a)*a^3*b^3*c*d^5*g^3*h*n^2+60*B*ln
(b*x+a)*a*b^5*c^5*d*g*h^3*n^2-120*B*ln(b*x+a)*a*b^5*c^4*d^2*g^2*h^2*n^2+12
0*B*ln(b*x+a)*a*b^5*c^3*d^3*g^3*h*n^2+3*B*x^4*a^2*b^4*c*d^5*h^4*n^2-3*B*x^
4*a*b^5*c^2*d^4*h^4*n^2-4*B*x^3*a^3*b^3*c*d^5*h^4*n^2+4*B*x^3*a*b^5*c^3*d^
3*h^4*n^2+6*B*x^2*a^4*b^2*c*d^5*h^4*n^2-6*B*x^2*a*b^5*c^4*d^2*h^4*n^2-12*B
*x*a^5*b*c*d^5*h^4*n^2+12*B*x*a*b^5*c^5*d*h^4*n^2+60*A*x*a*b^5*c*d^5*g^4*n
+60*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^5*c^2*d^4*g^4*n+60*B*ln(b*x+a)*a^...
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(351) = 702$.

Time = 0.34 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.21

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5h^4x^5 + 3\,(20\,Ab^5d^5gh^3 - (Bb^5cd^4 - Bab^4d^5)h^4n)x^4 + 4\,(30\,Ab^5d^5g^2h^2 - (5\,(Bb^5cd^4 - Bab^4d^5)gh}{}$$

input
```
integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/60*(12*A*b^5*d^5*h^4*x^5 + 3*(20*A*b^5*d^5*g*h^3 - (B*b^5*c*d^4 - B*a*b^
4*d^5)*h^4*n)*x^4 + 4*(30*A*b^5*d^5*g^2*h^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^
5)*g*h^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*h^4)*n)*x^3 + 6*(20*A*b^5*d^5*g
^3*h - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^2*h^2 - 5*(B*b^5*c^2*d^3 - B*a^2*
b^3*d^5)*g*h^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*h^4)*n)*x^2 + 12*(5*A*b^5
*d^5*g^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^3*h - 10*(B*b^5*c^2*d^3 - B*a
^2*b^3*d^5)*g^2*h^2 + 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g*h^3 - (B*b^5*c^4
*d - B*a^4*b*d^5)*h^4)*n)*x + 12*(B*b^5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*
n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^
5*g^4*n*x + (5*B*a*b^4*d^5*g^4 - 10*B*a^2*b^3*d^5*g^3*h + 10*B*a^3*b^2*d^5
*g^2*h^2 - 5*B*a^4*b*d^5*g*h^3 + B*a^5*d^5*h^4)*n)*log(b*x + a) - 12*(B*b^
5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 1
0*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x + (5*B*b^5*c*d^4*g^4 - 10*B*
b^5*c^2*d^3*g^3*h + 10*B*b^5*c^3*d^2*g^2*h^2 - 5*B*b^5*c^4*d*g*h^3 + B*b^5
*c^5*h^4)*n)*log(d*x + c) + 12*(B*b^5*d^5*h^4*x^5 + 5*B*b^5*d^5*g*h^3*x^4
+ 10*B*b^5*d^5*g^2*h^2*x^3 + 10*B*b^5*d^5*g^3*h*x^2 + 5*B*b^5*d^5*g^4*x)*l
og(e))/(b^5*d^5)
```

3.293.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.84

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{1}{5}\,Bh^4x^5\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)$$
$$+\frac{1}{5}\,Ah^4x^5+Bgh^3x^4\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Agh^3x^4+2\,Bg^2h^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)$$
$$+2\,Ag^2h^2x^3+2\,Bg^3hx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+2\,Ag^3hx^2+Bg^4x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Ag^4x$$
$$+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Bg^4}{e}-\frac{2\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bg^3h}{e}$$
$$+\frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3}-\frac{2\,c^3en\log(dx+c)}{d^3}-\frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bg^2h^2}{e}$$
$$-\frac{\left(\frac{6\,a^4en\log(bx+a)}{b^4}-\frac{6\,c^4en\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2en-ab^2d^3en)x^3-3\,(b^3c^2den-a^2bd^3en)x^2+6\,(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bgh^3}{6\,e}$$
$$+\frac{\left(\frac{12\,a^5en\log(bx+a)}{b^5}-\frac{12\,c^5en\log(dx+c)}{d^5}-\frac{3\,(b^4cd^3en-ab^3d^4en)x^4-4\,(b^4c^2d^2en-a^2b^2d^4en)x^3+6\,(b^4c^3den-a^3bd^4en)x^2-12\,(b^4c^4en-}{b^4d^4}\right.}{60\,e}$$

input

```
integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output

```
1/5*B*h^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*h^4*x^5 + B*g*h^3*x^4
*log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^3*x^4 + 2*B*g^2*h^2*x^3*log((b*x +
 a)^n*e/(d*x + c)^n) + 2*A*g^2*h^2*x^3 + 2*B*g^3*h*x^2*log((b*x + a)^n*e/(
d*x + c)^n) + 2*A*g^3*h*x^2 + B*g^4*x*log((b*x + a)^n*e/(d*x + c)^n) + A*g
^4*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^4/e - 2*(a^2*e*n*
log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))
*B*g^3*h/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((
b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2
))*B*g^2*h^2/e - 1/6*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/
d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^
3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*g*h^3/e + 1/60*
(12*a^5*e*n*log(b*x + a)/b^5 - 12*c^5*e*n*log(d*x + c)/d^5 - (3*(b^4*c*d^3
*e*n - a*b^3*d^4*e*n)*x^4 - 4*(b^4*c^2*d^2*e*n - a^2*b^2*d^4*e*n)*x^3 + 6*
(b^4*c^3*d*e*n - a^3*b*d^4*e*n)*x^2 - 12*(b^4*c^4*e*n - a^4*d^4*e*n)*x)/(b
^4*d^4))*B*h^4/e
```

3.293.8 Giac [F(-1)]

Timed out.

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
Timed out
```

3.293.9 Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 1434, normalized size of antiderivative = 3.93

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx=\text{Too large to display}$$

input
```
int((g + h*x)^4*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

output
```
x*((5*A*b*d*g^4 + 20*A*a*d*g^3*h + 20*A*b*c*g^3*h + 30*A*a*c*g^2*h^2 + 10*
B*a*d*g^3*h*n - 10*B*b*c*g^3*h*n)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*g*
h^3 + 20*A*b*d*g^3*h + 30*A*a*d*g^2*h^2 + 30*A*b*c*g^2*h^2 + 10*B*a*d*g^2*
h^2*n - 10*B*b*c*g^2*h^2*n)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*h^4 + 5
*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*
(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a
*d*g*h^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g
*h^3*n)/(5*b*d) + (A*a*c*h^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*h^4 + 5*A*b
*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a
*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*h^4 + 5*A*b*c*h^
4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d +
5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3
+ 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(
5*b*d) + (A*a*c*h^4)/(b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B
*h^4*x^5)/5 + B*g^4*x + 2*B*g^2*h^2*x^3 + 2*B*g^3*h*x^2 + B*g*h^3*x^4) + x
^4*((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*
n)/(20*b*d) - (A*h^4*(5*a*d + 5*b*c))/(20*b*d)) - x^3*((((5*A*a*d*h^4 + 5*
A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(
5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*h^4 + 20*A*a
*d*g*h^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*...
```

3.294 $\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

3.294.1 Optimal result

Integrand size = 31, antiderivative size = 236

$$\begin{aligned}\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx &= -\frac{B(bc-ad)h(a^2d^2h^2-abdh(4dg-ch)+b^2(6d^2g^2-4cdgh+c^2h^2))\,nx}{4b^3d^3} \\ &\quad -\frac{B(bc-ad)h^2(4bdg-bch-adh)nx^2}{8b^2d^2}-\frac{B(bc-ad)h^3nx^3}{12bd}-\frac{B(bg-ah)^4n\log(a+bx)}{4b^4h} \\ &\quad +\frac{B(dg-ch)^4n\log(c+dx)}{4d^4h}+\frac{(g+hx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4h}\end{aligned}$$

output
```
-1/4*B*(-a*d+b*c)*h*(a^2*d^2*h^2-a*b*d*h*(-c*h+4*d*g)+b^2*(c^2*h^2-4*c*d*g
*h+6*d^2*g^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*h^2*(-a*d*h-b*c*h+4*b*d*g)*n*x
^2/b^2/d^2-1/12*B*(-a*d+b*c)*h^3*n*x^3/b/d-1/4*B*(-a*h+b*g)^4*n*ln(b*x+a)/
b^4/h+1/4*B*(-c*h+d*g)^4*n*ln(d*x+c)/d^4/h+1/4*(h*x+g)^4*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))/h
```

3.294.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.33

$$\int (g+hx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$$
$$= \frac{bdx(6Ab^3d^3(4g^3+6g^2hx+4gh^2x^2+h^3x^3)-B(bc-ad)hn(6a^2d^2h^2-3abdh(8dg-2ch+dhx)+b^2(6c^2\cdots}{\cdots}$$

input
```
Integrate[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(b*d*x*(6*A*b^3*d^3*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) - B*(b*c -
 a*d)*h*n*(6*a^2*d^2*h^2 - 3*a*b*d*h*(8*d*g - 2*c*h + d*h*x) + b^2*(6*c^2*
h^2 - 3*c*d*h*(8*g + h*x) + 2*d^2*(18*g^2 + 6*g*h*x + h^2*x^2)))) - 6*a^2*
B*d^4*h*(6*b^2*g^2 - 4*a*b*g*h + a^2*h^2)*n*Log[a + b*x] + 6*b^3*B*(4*a*d^
4*g^3 + b*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*n*Log[
c + d*x] + 6*b^3*B*d^4*(4*a*g^3 + b*x*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h
^3*x^3))*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(24*b^4*d^4)
```

3.294.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3 \left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(g+hx)^4 \left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4h}-\frac{Bn(bc-ad)\int \frac{(g+hx)^4}{(a+bx)(c+dx)}dx}{4h}$$

$$\Big\downarrow \text{93}$$

$$\frac{(g+hx)^4 \left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4h}-$$
$$\frac{Bn(bc-ad)\int \left(\frac{x^2h^4}{bd}+\frac{(4bdg-bch-adh)xh^3}{b^2d^2}+\frac{((6d^2g^2-4cdhg+c^2h^2)b^2-adh(4dg-ch)b+a^2d^2h^2)h^2}{b^3d^3}+\frac{(bg-ah)^4}{b^3(bc-ad)(a+bx)}+\frac{(dg-\cdots}{d^3(ad-b\cdots}\cdots\right.}{4h}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4h} - \frac{Bn(bc-ad)\left(\frac{h^2x(a^2d^2h^2-abdh(4dg-ch)+b^2(c^2h^2-4cdgh+6d^2g^2))}{b^3d^3}+\frac{(bg-ah)^4\log(a+bx)}{b^4(bc-ad)}+\frac{h^3x^2(-adh-bch+4bdg)}{2b^2d^2}-\frac{(dg-ch)^4\log(}{d^4(bc-ad}\right.}{4h}$$

input
```
Int[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/4*(B*(b*c - a*d)*n*((h^2*(a^2*d^2*h^2 - a*b*d*h*(4*d*g - c*h) + b^2*(6*
d^2*g^2 - 4*c*d*g*h + c^2*h^2))*x)/(b^3*d^3) + (h^3*(4*b*d*g - b*c*h - a*d
*h)*x^2)/(2*b^2*d^2) + (h^4*x^3)/(3*b*d) + ((b*g - a*h)^4*Log[a + b*x])/(b
^4*(b*c - a*d)) - ((d*g - c*h)^4*Log[c + d*x])/(d^4*(b*c - a*d))))/h + ((g
 + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*h)
```

3.294.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.294.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(224) = 448$.

Time = 33.09 (sec) , antiderivative size = 984, normalized size of antiderivative = 4.17

method	result
parallelrisch	$\frac{12B\,a^2b^2c\,d^3g\,h^2n^2-12Ba\,b^3c^2d^2g\,h^2n^2-36Aa\,b^3c\,d^3g^2hn-6B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b^4c^4h^3n+24B\ln\left(e(bx+a)^n(dx+c)^{-}\right.}{}$
risch	Expression too large to display

input
```
int((h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/24*(12*B*a^2*b^2*c*d^3*g*h^2*n^2-12*B*a*b^3*c^2*d^2*g*h^2*n^2-36*A*a*b^3
*c*d^3*g^2*h*n-12*B*x^2*b^4*c*d^3*g*h^2*n^2+6*A*x^4*b^4*d^4*h^3*n+24*A*x*b
^4*d^4*g^3*n-6*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c^4*h^3*n-6*B*ln(b*x+a)*a
^4*d^4*h^3*n^2+6*B*ln(b*x+a)*b^4*c^4*h^3*n^2-24*A*a*b^3*d^4*g^3*n-24*A*b^4
*c*d^3*g^3*n-24*B*x*a^2*b^2*d^4*g*h^2*n^2+36*B*x*a*b^3*d^4*g^2*h*n^2+24*B*
x*b^4*c^2*d^2*g*h^2*n^2-36*B*x*b^4*c*d^3*g^2*h*n^2+24*B*ln(e*(b*x+a)^n/((d
*x+c)^n))*b^4*c^3*d*g*h^2*n-36*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c^2*d^2*g
^2*h*n+24*B*ln(b*x+a)*a^3*b*d^4*g*h^2*n^2-36*B*ln(b*x+a)*a^2*b^2*d^4*g^2*h
*n^2-24*B*ln(b*x+a)*b^4*c^3*d*g*h^2*n^2+36*B*ln(b*x+a)*b^4*c^2*d^2*g^2*h*n
^2+24*B*x^3*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*g*h^2*n+36*B*x^2*ln(e*(b*x
+a)^n/((d*x+c)^n))*b^4*d^4*g^2*h*n+12*B*x^2*a*b^3*d^4*g*h^2*n^2+6*B*x^4*ln
(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*h^3*n+2*B*x^3*a*b^3*d^4*h^3*n^2-2*B*x^3*
b^4*c*d^3*h^3*n^2+24*A*x^3*b^4*d^4*g*h^2*n-3*B*x^2*a^2*b^2*d^4*h^3*n^2+3*B
*x^2*b^4*c^2*d^2*h^3*n^2+36*A*x^2*b^4*d^4*g^2*h*n+24*B*ln(b*x+a)*a*b^3*d^4
*g^3*n^2+24*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*d^4*g^3*n+6*B*x*a^3*b*d^4*
h^3*n^2-6*B*x*b^4*c^3*d*h^3*n^2+24*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^4*c*d^3
*g^3*n-24*B*ln(b*x+a)*b^4*c*d^3*g^3*n^2-6*B*a^4*d^4*h^3*n^2+6*B*b^4*c^4*h^
3*n^2-3*B*a^3*b*c*d^3*h^3*n^2+24*B*a^3*b*d^4*g*h^2*n^2-36*B*a^2*b^2*d^4*g^
2*h*n^2+3*B*a*b^3*c^3*d*h^3*n^2-24*B*b^4*c^3*d*g*h^2*n^2+36*B*b^4*c^2*d^2*
g^2*h*n^2)/b^4/d^4/n
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(224) = 448$.

Time = 0.32 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.42

$$\int (g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{6\,Ab^4d^4h^3x^4+2\,(12\,Ab^4d^4gh^2-(Bb^4cd^3-Bab^3d^4)h^3n)x^3+3\,(12\,Ab^4d^4g^2h-(4\,(Bb^4cd^3-Bab^3d^4)gh^2}{}$$

input
```
integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/24*(6*A*b^4*d^4*h^3*x^4 + 2*(12*A*b^4*d^4*g*h^2 - (B*b^4*c*d^3 - B*a*b^3
*d^4)*h^3*n)*x^3 + 3*(12*A*b^4*d^4*g^2*h - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*
g*h^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*h^3)*n)*x^2 + 6*(4*A*b^4*d^4*g^3 -
 (6*(B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*h - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*
g*h^2 + (B*b^4*c^3*d - B*a^3*b*d^4)*h^3)*n)*x + 6*(B*b^4*d^4*h^3*n*x^4 + 4
*B*b^4*d^4*g*h^2*n*x^3 + 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (
4*B*a*b^3*d^4*g^3 - 6*B*a^2*b^2*d^4*g^2*h + 4*B*a^3*b*d^4*g*h^2 - B*a^4*d^
4*h^3)*n)*log(b*x + a) - 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^4*d^4*g*h^2*n*x^3
+ 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*b^4*c*d^3*g^3 - 6*B
*b^4*c^2*d^2*g^2*h + 4*B*b^4*c^3*d*g*h^2 - B*b^4*c^4*h^3)*n)*log(d*x + c)
+ 6*(B*b^4*d^4*h^3*x^4 + 4*B*b^4*d^4*g*h^2*x^3 + 6*B*b^4*d^4*g^2*h*x^2 + 4
*B*b^4*d^4*g^3*x)*log(e))/(b^4*d^4)
```

3.294.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.294.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(224) = 448$.

Time = 0.22 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.98

$$\int (g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{1}{4}Bh^3x^4\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+\frac{1}{4}Ah^3x^4+Bgh^2x^3\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+Agh^2x^3$$
$$+\frac{3}{2}Bg^2hx^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+\frac{3}{2}Ag^2hx^2+Bg^3x\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+Ag^3x$$
$$+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Bg^3}{e}-\frac{3\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)Bg^2h}{2e}$$
$$+\frac{\left(\frac{2a^3en\log(bx+a)}{b^3}-\frac{2c^3en\log(dx+c)}{d^3}-\frac{(b^2cden-abd^2en)x^2-2(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bgh^2}{2e}$$
$$-\frac{\left(\frac{6a^4en\log(bx+a)}{b^4}-\frac{6c^4en\log(dx+c)}{d^4}+\frac{2(b^3cd^2en-ab^2d^3en)x^3-3(b^3c^2den-a^2bd^3en)x^2+6(b^3c^3en-a^3d^3en)x}{b^3d^3}\right)Bh^3}{24e}$$

input
```
integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/4*B*h^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*h^3*x^4 + B*g*h^2*x^3
*log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^2*x^3 + 3/2*B*g^2*h*x^2*log((b*x +
 a)^n*e/(d*x + c)^n) + 3/2*A*g^2*h*x^2 + B*g^3*x*log((b*x + a)^n*e/(d*x +
c)^n) + A*g^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^3/e -
3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*
e*n)*x/(b*d))*B*g^2*h/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(
d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2
*e*n)*x)/(b^2*d^2))*B*g*h^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e
*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*
d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B
*h^3/e
```

3.294.8 Giac [F(-1)]

Timed out.

$$\int (g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
Timed out
```

3.294.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.25

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= x\left(\frac{4\,A\,b\,d\,g^3+12\,A\,a\,c\,g\,h^2+12\,A\,a\,d\,g^2\,h+12\,A\,b\,c\,g^2\,h+6\,B\,a\,d\,g^2\,h\,n-6\,B\,b\,c\,g^2\,h\,n}{4\,b\,d}\right.$$

$$+\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{4\,A\,a\,c\,h^3+12\,A\,a\,d\,g\,h^2+12\,A}{}\right.}{4\,b\,d}$$

$$\left.-\frac{a\,c\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)}{b\,d}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(B\,g^3\,x+\frac{3\,B\,g^2\,h\,x^2}{2}+B\,g\,h^2\,x^3+\frac{B\,h^3\,x^4}{4}\right)$$

$$-x^2\left(\frac{\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{4\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{4\,b\,d}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}\right.$$

$$-\frac{4\,A\,a\,c\,h^3+12\,A\,a\,d\,g\,h^2+12\,A\,b\,c\,g\,h^2+12\,A\,b\,d\,g^2\,h+4\,B\,a\,d\,g\,h^2\,n-4\,B\,b\,c\,g\,h^2\,n}{8\,b\,d}$$

$$\left.+\frac{A\,a\,c\,h^3}{2\,b\,d}\right)$$

$$+x^3\left(\frac{4\,A\,a\,d\,h^3+4\,A\,b\,c\,h^3+12\,A\,b\,d\,g\,h^2+B\,a\,d\,h^3\,n-B\,b\,c\,h^3\,n}{12\,b\,d}-\frac{A\,h^3\,(4\,a\,d+4\,b\,c)}{12\,b\,d}\right)$$

$$+\frac{A\,h^3\,x^4}{4}-\frac{\ln(a+b\,x)\,\left(B\,n\,a^4\,h^3-4\,B\,n\,a^3\,b\,g\,h^2+6\,B\,n\,a^2\,b^2\,g^2\,h-4\,B\,n\,a\,b^3\,g^3\right)}{4\,b^4}$$

$$+\frac{\ln(c+d\,x)\,\left(B\,n\,c^4\,h^3-4\,B\,n\,c^3\,d\,g\,h^2+6\,B\,n\,c^2\,d^2\,g^2\,h-4\,B\,n\,c\,d^3\,g^3\right)}{4\,d^4}$$

input
```
int((g + h*x)^3*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

output
```
x*((4*A*b*d*g^3 + 12*A*a*c*g*h^2 + 12*A*a*d*g^2*h + 12*A*b*c*g^2*h + 6*B*a
*d*g^2*h*n - 6*B*b*c*g^2*h*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*h^3 +
 4*A*b*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^
3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*h^3 + 12*A
*a*d*g*h^2 + 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g
*h^2*n)/(4*b*d) + (A*a*c*h^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*h^3 + 4*A*b
*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(4*a
*d + 4*b*c))/(4*b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*h^3*x
^4)/4 + B*g^3*x + (3*B*g^2*h*x^2)/2 + B*g*h^2*x^3) - x^2*((((4*A*a*d*h^3 +
 4*A*b*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^
3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*h^3 + 12*A
*a*d*g*h^2 + 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g
*h^2*n)/(8*b*d) + (A*a*c*h^3)/(2*b*d)) + x^3*((4*A*a*d*h^3 + 4*A*b*c*h^3 +
 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(12*b*d) - (A*h^3*(4*a*d + 4*
b*c))/(12*b*d)) + (A*h^3*x^4)/4 - (log(a + b*x)*(B*a^4*h^3*n - 4*B*a*b^3*g
^3*n - 4*B*a^3*b*g*h^2*n + 6*B*a^2*b^2*g^2*h*n))/(4*b^4) + (log(c + d*x)*(
B*c^4*h^3*n - 4*B*c*d^3*g^3*n - 4*B*c^3*d*g*h^2*n + 6*B*c^2*d^2*g^2*h*n))/
(4*d^4)
```

3.294. $\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

3.295 $\int (g+hx)^2 (A+B\log (e(a+bx)^n(c+dx)^{-n})) \, dx$

3.295.1 Optimal result

Integrand size = 31, antiderivative size = 158

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = -\frac{B(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} - \frac{B(bc-ad)h^2nx^2}{6bd} - \frac{B(bg-ah)^3n\log(a+bx)}{3b^3h} + \frac{B(dg-ch)^3n\log(c+dx)}{3d^3h} + \frac{(g+hx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3h}$$

output
```
-1/3*B*(-a*d+b*c)*h*(-a*d*h-b*c*h+3*b*d*g)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*h^
2*n*x^2/b/d-1/3*B*(-a*h+b*g)^3*n*ln(b*x+a)/b^3/h+1/3*B*(-c*h+d*g)^3*n*ln(d
*x+c)/d^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

3.295.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.29

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{2a^2Bd^3h(-3bg+ah)n\log(a+bx)+b(dx(B(bc-ad)hn(-6bdg+2bch+2adh-bdhx)+2Ab^2d^2(3g^2+}{}$$

input
```
Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(2*a^2*B*d^3*h*(-3*b*g + a*h)*n*Log[a + b*x] + b*(d*x*(B*(b*c - a*d)*h*n*(
-6*b*d*g + 2*b*c*h + 2*a*d*h - b*d*h*x) + 2*A*b^2*d^2*(3*g^2 + 3*g*h*x + h
^2*x^2)) - 2*b*B*(-3*a*d^3*g^2 + b*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*
Log[c + d*x] + 2*b*B*d^3*(3*a*g^2 + b*x*(3*g^2 + 3*g*h*x + h^2*x^2))*Log[(
e*(a + b*x)^n)/(c + d*x)^n]))/(6*b^3*d^3)
```

3.295.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{(g+hx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h}-\frac{Bn(bc-ad)\int \frac{(g+hx)^3}{(a+bx)(c+dx)}dx}{3h}$$

$$\Big\downarrow \text{93}$$

$$\frac{(g+hx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h}-\frac{Bn(bc-ad)\int \left(\frac{xh^3}{bd}+\frac{(3bdg-bch-adh)h^2}{b^2d^2}+\frac{(bg-ah)^3}{b^2(bc-ad)(a+bx)}+\frac{(dg-ch)^3}{d^2(ad-bc)(c+dx)}\right)dx}{3h}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(g+hx)^3 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h}-\frac{Bn(bc-ad)\left(\frac{(bg-ah)^3\log(a+bx)}{b^3(bc-ad)}+\frac{h^2x(-adh-bch+3bdg)}{b^2d^2}-\frac{(dg-ch)^3\log(c+dx)}{d^3(bc-ad)}+\frac{h^3x^2}{2bd}\right)}{3h}$$

input
```
Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/3*(B*(b*c - a*d)*n*((h^2*(3*b*d*g - b*c*h - a*d*h)*x)/(b^2*d^2) + (h^3*
x^2)/(2*b*d) + ((b*g - a*h)^3*Log[a + b*x])/(b^3*(b*c - a*d)) - ((d*g - c*
h)^3*Log[c + d*x])/(d^3*(b*c - a*d))))/h + ((g + h*x)^3*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n]))/(3*h)
```

3.295.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.295.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(148) = 296$.

Time = 11.35 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.94

method	result
parallelrisch	$\frac{-2B\,b^3c^3h^2n^2+2B\,a^3d^3h^2n^2+6Bxa\,b^2d^3gh\,n^2-6Bx\,b^3c\,d^2gh\,n^2-6B\ln(bx+a)a^2b\,d^3gh\,n^2+6B\,x^2\ln\left(e(bx+a)^n(dx+c)^{-n}\right)b}{}$
risch	Expression too large to display

input
```
int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*B*b^3*c^3*h^2*n^2+2*B*a^3*d^3*h^2*n^2-6*B*ln(e*(b*x+a)^n/((d*x+c)^
n))*b^3*c^2*d*g*h*n+6*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*g*h*n+6*B*
x*a*b^2*d^3*g*h*n^2-6*B*x*b^3*c*d^2*g*h*n^2-6*B*ln(b*x+a)*a^2*b*d^3*g*h*n^
2+6*B*ln(b*x+a)*b^3*c^2*d*g*h*n^2-6*A*a*b^2*c*d^2*g*h*n+B*a^2*b*c*d^2*h^2*
n^2-6*B*a^2*b*d^3*g*h*n^2-B*a*b^2*c^2*d*h^2*n^2+6*B*b^3*c^2*d*g*h*n^2+6*B*
ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*c*d^2*g^2*n+2*B*x^3*ln(e*(b*x+a)^n/((d*x+c
)^n))*b^3*d^3*h^2*n+B*x^2*a*b^2*d^3*h^2*n^2-B*x^2*b^3*c*d^2*h^2*n^2+6*A*x^
2*b^3*d^3*g*h*n+6*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*g^2*n-2*B*x*a^2*
b*d^3*h^2*n^2+2*B*x*b^3*c^2*d*h^2*n^2+6*B*ln(b*x+a)*a*b^2*d^3*g^2*n^2-6*B*
ln(b*x+a)*b^3*c*d^2*g^2*n^2-6*A*a*b^2*d^3*g^2*n-6*A*b^3*c*d^2*g^2*n+2*B*ln
(e*(b*x+a)^n/((d*x+c)^n))*b^3*c^3*h^2*n+2*A*x^3*b^3*d^3*h^2*n+6*A*x*b^3*d^
3*g^2*n+2*B*ln(b*x+a)*a^3*d^3*h^2*n^2-2*B*ln(b*x+a)*b^3*c^3*h^2*n^2)/b^3/d
^3/n
```

3.295.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(148) = 296$.

Time = 0.32 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.31

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \frac{2\,Ab^3d^3h^2x^3+(6\,Ab^3d^3gh-(Bb^3cd^2-Bab^2d^3)h^2n)x^2+2\,(3\,Ab^3d^3g^2-(3\,(Bb^3cd^2-Bab^2d^3)gh-(Bb^3}{}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="frica
s")
```

output
```
1/6*(2*A*b^3*d^3*h^2*x^3 + (6*A*b^3*d^3*g*h - (B*b^3*c*d^2 - B*a*b^2*d^3)*
h^2*n)*x^2 + 2*(3*A*b^3*d^3*g^2 - (3*(B*b^3*c*d^2 - B*a*b^2*d^3)*g*h - (B*
b^3*c^2*d - B*a^2*b*d^3)*h^2)*n)*x + 2*(B*b^3*d^3*h^2*n*x^3 + 3*B*b^3*d^3*
g*h*n*x^2 + 3*B*b^3*d^3*g^2*n*x + (3*B*a*b^2*d^3*g^2 - 3*B*a^2*b*d^3*g*h +
 B*a^3*d^3*h^2)*n)*log(b*x + a) - 2*(B*b^3*d^3*h^2*n*x^3 + 3*B*b^3*d^3*g*h
*n*x^2 + 3*B*b^3*d^3*g^2*n*x + (3*B*b^3*c*d^2*g^2 - 3*B*b^3*c^2*d*g*h + B*
b^3*c^3*h^2)*n)*log(d*x + c) + 2*(B*b^3*d^3*h^2*x^3 + 3*B*b^3*d^3*g*h*x^2
+ 3*B*b^3*d^3*g^2*x)*log(e))/(b^3*d^3)
```

3.295.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.86

$$\begin{aligned}
&\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx \\
&= \frac{1}{3}Bh^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{3}Ah^2x^3 + Bghx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aghx^2 \\
&\quad + Bg^2x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Ag^2x + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Bg^2}{e} \\
&\quad - \frac{\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Bgh}{e} \\
&\quad + \frac{\left(\frac{2\,a^3en\log(bx+a)}{b^3} - \frac{2\,c^3en\log(dx+c)}{d^3} - \frac{(b^2cden-abd^2en)x^2-2\,(b^2c^2en-a^2d^2en)x}{b^2d^2}\right)Bh^2}{6\,e}
\end{aligned}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxim
a")
```

output
```
1/3*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*h^2*x^3 + B*g*h*x^2*l
og((b*x + a)^n*e/(d*x + c)^n) + A*g*h*x^2 + B*g^2*x*log((b*x + a)^n*e/(d*x
 + c)^n) + A*g^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^2/e
 - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e
*n)*x/(b*d))*B*g*h/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x
 + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*
n)*x)/(b^2*d^2))*B*h^2/e
```

3.295.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(148) = 296$.

Time = 46.92 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.92

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{1}{3}\left(Bh^2\log(e)+Ah^2\right)x^3 + \frac{1}{3}\left(Bh^2nx^3+3\,Bghnx^2+3\,Bg^2nx\right)\log(bx+a) - \frac{1}{3}\left(Bh^2nx^3+3\,Bghnx^2+3\,Bg^2nx\right)\log(dx+c) - \frac{(Bbch^2n-Badh^2n-6\,Bbdgh\log(e)-6\,Abdgh)x^2}{6\,bd} + \frac{(3\,Bab^2g^2n-3\,Ba^2bghn+Ba^3h^2n)\log(bx+a)}{3\,b^3} - \frac{(3\,Bcd^2g^2n-3\,Bc^2dghn+Bc^3h^2n)\log(-dx-c)}{3\,d^3} - \frac{(3\,Bb^2cdghn-3\,Babd^2ghn-Bb^2c^2h^2n+Ba^2d^2h^2n-3\,Bb^2d^2g^2\log(e)-3\,Ab^2d^2g^2)x}{3\,b^2d^2}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac"
)
```

output
```
1/3*(B*h^2*log(e) + A*h^2)*x^3 + 1/3*(B*h^2*n*x^3 + 3*B*g*h*n*x^2 + 3*B*g^
2*n*x)*log(b*x + a) - 1/3*(B*h^2*n*x^3 + 3*B*g*h*n*x^2 + 3*B*g^2*n*x)*log(
d*x + c) - 1/6*(B*b*c*h^2*n - B*a*d*h^2*n - 6*B*b*d*g*h*log(e) - 6*A*b*d*g
*h)*x^2/(b*d) + 1/3*(3*B*a*b^2*g^2*n - 3*B*a^2*b*g*h*n + B*a^3*h^2*n)*log(
b*x + a)/b^3 - 1/3*(3*B*c*d^2*g^2*n - 3*B*c^2*d*g*h*n + B*c^3*h^2*n)*log(-
d*x - c)/d^3 - 1/3*(3*B*b^2*c*d*g*h*n - 3*B*a*b*d^2*g*h*n - B*b^2*c^2*h^2*
n + B*a^2*d^2*h^2*n - 3*B*b^2*d^2*g^2*log(e) - 3*A*b^2*d^2*g^2)*x/(b^2*d^2
)
```

3.295.9 Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.35

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= x^2\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{6\,b\,d}-\frac{A\,h^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(B\,g^2\,x+B\,g\,h\,x^2+\frac{B\,h^2\,x^3}{3}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{3\,b\,d}-\frac{A\,h^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)}{3\,b\,d}\right.$$
$$-\frac{3\,A\,a\,c\,h^2+3\,A\,b\,d\,g^2+6\,A\,a\,d\,g\,h+6\,A\,b\,c\,g\,h+3\,B\,a\,d\,g\,h\,n-3\,B\,b\,c\,g\,h\,n}{3\,b\,d}$$
$$\left.+\frac{A\,a\,c\,h^2}{b\,d}\right)+\frac{A\,h^2\,x^3}{3}+\frac{\ln(a+b\,x)\,\left(B\,n\,a^3\,h^2-3\,B\,n\,a^2\,b\,g\,h+3\,B\,n\,a\,b^2\,g^2\right)}{3\,b^3}$$
$$-\frac{\ln(c+d\,x)\,\left(B\,n\,c^3\,h^2-3\,B\,n\,c^2\,d\,g\,h+3\,B\,n\,c\,d^2\,g^2\right)}{3\,d^3}$$

input
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

output
```
x^2*((3*A*a*d*h^2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)
/(6*b*d) - (A*h^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x)^n)/(c + d*x
)^n)*((B*h^2*x^3)/3 + B*g^2*x + B*g*h*x^2) - x*(((3*a*d + 3*b*c)*((3*A*a*d
*h^2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)/(3*b*d) - (A
*h^2*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*h^2 + 3*A*b*d*g^2 + 6*A
*a*d*g*h + 6*A*b*c*g*h + 3*B*a*d*g*h*n - 3*B*b*c*g*h*n)/(3*b*d) + (A*a*c*h
^2)/(b*d)) + (A*h^2*x^3)/3 + (log(a + b*x)*(B*a^3*h^2*n + 3*B*a*b^2*g^2*n
- 3*B*a^2*b*g*h*n))/(3*b^3) - (log(c + d*x)*(B*c^3*h^2*n + 3*B*c*d^2*g^2*n
 - 3*B*c^2*d*g*h*n))/(3*d^3)
```

3.296 $\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

3.296.1 Optimal result

Integrand size = 29, antiderivative size = 116

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = -\frac{B(bc-ad)hnx}{2bd}-\frac{B(bg-ah)^2n\log(a+bx)}{2b^2h}+\frac{B(dg-ch)^2n\log(c+dx)}{2d^2h}+\frac{(g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{2h}$$

output
```
-1/2*B*(-a*d+b*c)*h*n*x/b/d-1/2*B*(-a*h+b*g)^2*n*ln(b*x+a)/b^2/h+1/2*B*(-c
*h+d*g)^2*n*ln(d*x+c)/d^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))
)/h
```

3.296.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx = \frac{-a^2Bd^2hn\log(a+bx)+bB(2ad^2g+bc(-2dg+ch))\,n\log(c+dx)+bd(x(B(-bc+ad)hn+Abd(2g+}{2b^2d^2}$$

input
```
Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
(-(a^2*B*d^2*h*n*Log[a + b*x]) + b*B*(2*a*d^2*g + b*c*(-2*d*g + c*h))*n*Lo
g[c + d*x] + b*d*(x*(B*(-(b*c) + a*d)*h*n + A*b*d*(2*g + h*x)) + B*d*(2*a*
g + b*x*(2*g + h*x))*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b^2*d^2)
```

3.296.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx) \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$

$$\Big\downarrow \; 2948$$

$$\frac{(g+hx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bc-ad)\int \frac{(g+hx)^2}{(a+bx)(c+dx)}dx}{2h}$$

$$\Big\downarrow \; 93$$

$$\frac{(g+hx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bc-ad)\int \left(\frac{h^2}{bd}+\frac{(bg-ah)^2}{b(bc-ad)(a+bx)}+\frac{(dg-ch)^2}{d(ad-bc)(c+dx)}\right)dx}{2h}$$

$$\Big\downarrow \; 2009$$

$$\frac{(g+hx)^2 \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bc-ad)\left(\frac{(bg-ah)^2 \log(a+bx)}{b^2(bc-ad)}-\frac{(dg-ch)^2 \log(c+dx)}{d^2(bc-ad)}+\frac{h^2 x}{bd}\right)}{2h}$$

input
```
Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*n*((h^2*x)/(b*d) + ((b*g - a*h)^2*Log[a + b*x])/(b^2*(
b*c - a*d)) - ((d*g - c*h)^2*Log[c + d*x])/(d^2*(b*c - a*d))))/h + ((g + h
*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*h)
```

3.296.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.296.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(108) = 216$.

Time = 3.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.20

method	result
parallelrisch	$\frac{-B\ln(bx+a)a^2d^2hn+B\ln(bx+a)abcdhn-2B\ln(dx+c)ab\,d^2gn-4B\ln(dx+c)b^2cdgn-B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)abcdh+2B\ln}{}$
risch	Expression too large to display

input
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-B*ln(b*x+a)*a^2*d^2*h*n+B*ln(b*x+a)*a*b*c*d*h*n-2*B*ln(d*x+c)*a*b*d^
2*g*n-4*B*ln(d*x+c)*b^2*c*d*g*n-B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b*c*d*h+2*
B*ln(b*x+a)*b^2*c*d*g*n-B*a^2*d^2*h*n+B*b^2*c^2*h*n-A*a*b*c*d*h+A*b^2*d^2*
h*x^2+4*B*ln(b*x+a)*a*b*d^2*g*n+B*a*b*d^2*h*n*x-B*b^2*c*d*h*n*x-2*B*ln(e*(
b*x+a)^n/((d*x+c)^n))*a*b*d^2*g-2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*c*d*g+
B*ln(d*x+c)*b^2*c^2*h*n+B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*d^2*h+2*B*x*
ln(e*(b*x+a)^n/((d*x+c)^n))*b^2*d^2*g+2*A*x*b^2*d^2*g-B*ln(d*x+c)*a*b*c*d*
h*n-2*A*a*b*d^2*g-2*A*b^2*c*d*g)/b^2/d^2
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.66

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{Ab^2d^2hx^2 + (2\,Ab^2d^2g - (Bb^2cd - Babd^2)hn)x + (Bb^2d^2hnx^2 + 2\,Bb^2d^2gnx + (2\,Babd^2g - Ba^2d^2h)n)\,\mathrm{lo}}{}$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas"
)
```

output
```
1/2*(A*b^2*d^2*h*x^2 + (2*A*b^2*d^2*g - (B*b^2*c*d - B*a*b*d^2)*h*n)*x + (
B*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*a*b*d^2*g - B*a^2*d^2*h)*n)*l
og(b*x + a) - (B*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*b^2*c*d*g - B*
b^2*c^2*h)*n)*log(d*x + c) + (B*b^2*d^2*h*x^2 + 2*B*b^2*d^2*g*x)*log(e))/(
b^2*d^2)
```

3.296.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$= \frac{1}{2}\,Bhx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{2}\,Ahx^2 + Bgx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Agx + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)Bg}{e} - \frac{\left(\frac{a^2en\log(bx+a)}{b^2} - \frac{c^2en\log(dx+c)}{d^2} + \frac{(bcen-aden)x}{bd}\right)Bh}{2\,e}$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima"
)
```

output
```
1/2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*h*x^2 + B*g*x*log((b*x
+ a)^n*e/(d*x + c)^n) + A*g*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)
/d)*B*g/e - 1/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*
c*e*n - a*d*e*n)*x/(b*d))*B*h/e
```

3.296.8 Giac [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{1}{2}\,(Bh\log(e)+Ah)x^2+\frac{1}{2}\left(Bhnx^2+2\,Bgnx\right)\log(bx+a)$$
$$-\frac{1}{2}\left(Bhnx^2+2\,Bgnx\right)\log(dx+c)-\frac{(Bbchn-Badhn-2\,Bbdg\log(e)-2\,Abdg)x}{2\,bd}$$
$$+\frac{(2\,Babgn-Ba^2hn)\log(bx+a)}{2\,b^2}-\frac{(2\,Bcdgn-Bc^2hn)\log(-dx-c)}{2\,d^2}$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

output
```
1/2*(B*h*log(e) + A*h)*x^2 + 1/2*(B*h*n*x^2 + 2*B*g*n*x)*log(b*x + a) - 1/
2*(B*h*n*x^2 + 2*B*g*n*x)*log(d*x + c) - 1/2*(B*b*c*h*n - B*a*d*h*n - 2*B*
b*d*g*log(e) - 2*A*b*d*g)*x/(b*d) + 1/2*(2*B*a*b*g*n - B*a^2*h*n)*log(b*x
+ a)/b^2 - 1/2*(2*B*c*d*g*n - B*c^2*h*n)*log(-d*x - c)/d^2
```

3.296.9 Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{B\,h\,x^2}{2}+B\,g\,x\right)$$
$$+x\left(\frac{2\,A\,a\,d\,h+2\,A\,b\,c\,h+2\,A\,b\,d\,g+B\,a\,d\,h\,n-B\,b\,c\,h\,n}{2\,b\,d}-\frac{A\,h\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)$$
$$-\frac{\ln(a+b\,x)\,\left(B\,a^2\,h\,n-2\,B\,a\,b\,g\,n\right)}{2\,b^2}+\frac{\ln(c+d\,x)\,\left(B\,c^2\,h\,n-2\,B\,c\,d\,g\,n\right)}{2\,d^2}+\frac{A\,h\,x^2}{2}$$

input
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

output
```
log((e*(a + b*x)^n)/(c + d*x)^n)*(B*g*x + (B*h*x^2)/2) + x*((2*A*a*d*h + 2
*A*b*c*h + 2*A*b*d*g + B*a*d*h*n - B*b*c*h*n)/(2*b*d) - (A*h*(2*a*d + 2*b*
c))/(2*b*d)) - (log(a + b*x)*(B*a^2*h*n - 2*B*a*b*g*n))/(2*b^2) + (log(c +
 d*x)*(B*c^2*h*n - 2*B*c*d*g*n))/(2*d^2) + (A*h*x^2)/2
```

3.296. $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

3.297 $\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))\,dx$

3.297.1 Optimal result

Integrand size = 23, antiderivative size = 57

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))\,dx = Ax-\frac{B(bc-ad)n\log(c+dx)}{bd}+\frac{B(a+bx)\log (e(a+bx)^n(c+dx)^{-n})}{b}$$

output
```
A*x-B*(-a*d+b*c)*n*ln(d*x+c)/b/d+B*(b*x+a)*ln(e*(b*x+a)^n/((d*x+c)^n))/b
```

3.297.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))\,dx = Ax-\frac{B(bc-ad)n\log(c+dx)}{bd}+\frac{B(a+bx)\log (e(a+bx)^n(c+dx)^{-n})}{b}$$

input
```
Integrate[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

output
```
A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/b
```

3.297.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B(a+bx)\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{Bn(bc-ad)\log (c+dx)}{bd}+Ax$$

input
```
Int[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

output
```
A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/b
```

3.297.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.297.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

method	result
default	$Ax+B\left(\ln \left(e (bx+a)^{n} (dx+c)^{-n}\right) x+n(ad-cb) \left(-\frac{c\ln (dx+c)}{(ad-cb)d}+\frac{a\ln (bx+a)}{(ad-cb)b}\right)\right)$
parts	$Ax+B\left(\ln \left(e (bx+a)^{n} (dx+c)^{-n}\right) x+n(ad-cb) \left(-\frac{c\ln (dx+c)}{(ad-cb)d}+\frac{a\ln (bx+a)}{(ad-cb)b}\right)\right)$
parallelrisch	$\frac{B\left(\ln (bx+a)ad\,n^{2}-\ln (bx+a)bc\,n^{2}+x\ln \left(e (bx+a)^{n} (dx+c)^{-n}\right)bdn+\ln \left(e (bx+a)^{n} (dx+c)^{-n}\right)bcn\right)}{bdn}+Ax$
risch	$Ax-Bx\ln \left((dx+c)^{n}\right)-\frac{iB\pi x\,\mathrm{csgn}\left(i (bx+a)^{n} (dx+c)^{-n}\right)^{3}}{2}+\frac{iB\pi x\,\,\mathrm{csgn}\left(i (bx+a)^{n}\right)\mathrm{csgn}\left(i (bx+a)^{n} (dx+c)^{-n}\right)^{2}}{2}$

input
```
int(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)),x,method=_RETURNVERBOSE)
```

output
```
A*x+B*(ln(e*(b*x+a)^n/((d*x+c)^n))*x+n*(a*d-b*c)*(-c/(a*d-b*c)/d*ln(d*x+c)
+a/(a*d-b*c)/b*ln(b*x+a)))
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int (A + B\log(e(a+bx)^n(c+dx)^{-n}))\,dx = \frac{Bbdx\log(e) + Abdx + (Bbdnx + Badn)\log(bx+a) - (Bbdnx + Bbcn)\log(dx+c)}{bd}$$

input
```
integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="fricas")
```

output
```
(B*b*d*x*log(e) + A*b*d*x + (B*b*d*n*x + B*a*d*n)*log(b*x + a) - (B*b*d*n*
x + B*b*c*n)*log(d*x + c))/(b*d)
```

3.297.6 Sympy [F(-2)]

Exception generated.

$$\int (A + B\log(e(a+bx)^n(c+dx)^{-n}))\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int (A + B\log(e(a+bx)^n(c+dx)^{-n}))\,dx = Bx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Ax + \frac{\left(\frac{aen\log(bx+a)}{b} - \frac{cen\log(dx+c)}{d}\right)B}{e}$$

input
```
integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="maxima")
```

output
```
B*x*log((b*x + a)^n*e/(d*x + c)^n) + A*x + (a*e*n*log(b*x + a)/b - c*e*n*l
og(d*x + c)/d)*B/e
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))\ dx = \left(nx\log (bx+a)-nx\log (dx+c)+\frac{an\log (bx+a)}{b}-\frac{cn\log (-dx-c)}{d}+x\log (e)\right)B + Ax$$

input
```
integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="giac")
```

output
```
(n*x*log(b*x + a) - n*x*log(d*x + c) + a*n*log(b*x + a)/b - c*n*log(-d*x -
 c)/d + x*log(e))*B + A*x
```

3.297.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))\ dx = A\,x+B\,x\,\ln \left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)+\frac{B\,a\,n\,\ln \left(a+b\,x\right)}{b}-\frac{B\,c\,n\,\ln \left(c+d\,x\right)}{d}$$

input
```
int(A + B*log((e*(a + b*x)^n)/(c + d*x)^n),x)
```

output
```
A*x + B*x*log((e*(a + b*x)^n)/(c + d*x)^n) + (B*a*n*log(a + b*x))/b - (B*c
*n*log(c + d*x))/d
```

3.298 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$

3.298.1 Optimal result

Integrand size = 31, antiderivative size = 148

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx = -\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} + \frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log(g+hx)}{h} - \frac{Bn\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h} + \frac{Bn\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}$$

output

```
-B*n*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h+B*n*ln(-h*(d*x+c)/(-c*h+d*g))*l
n(h*x+g)/h+(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(h*x+g)/h-B*n*polylog(2,b*(
h*x+g)/(-a*h+b*g))/h+B*n*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

3.298.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$$
$$= \frac{(A+B(-n\log(a+bx)+n\log(c+dx)+\log\left(e(a+bx)^n(c+dx)^{-n}\right)))\log(g+hx)+Bn\Big(\log(a+bx)\log}{h}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

output
```
((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x
)^n]))*Log[g + h*x] + B*n*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - B*n*(Log[c + d*x]*Log[(d*(g + h
*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]))/h
```

3.298.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2946, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{g+hx}\,dx$$
$$\Big\downarrow \text{2946}$$
$$-\frac{bBn\int \frac{\log(g+hx)}{a+bx}dx}{h}+\frac{Bdn\int \frac{\log(g+hx)}{c+dx}dx}{h}+\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}$$
$$\Big\downarrow \text{2841}$$

$$-\frac{bBn\left(\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{h\int\frac{\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{g+hx}dx}{b}\right)}{h}+\frac{Bdn\left(\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{h\int\frac{\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{g+hx}dx}{d}\right)}{h}+\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}$$

$$\Big\downarrow \; 2840$$

$$-\frac{bBn\left(\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}-\frac{\int\frac{\log\left(1-\frac{b(g+hx)}{bg-ah}\right)}{g+hx}d(g+hx)}{b}\right)}{h}+\frac{Bdn\left(\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}-\frac{\int\frac{\log\left(1-\frac{d(g+hx)}{dg-ch}\right)}{g+hx}d(g+hx)}{d}\right)}{h}+\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}$$

$$\Big\downarrow \; 2838$$

$$\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}-\frac{bBn\left(\frac{\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{b}+\frac{\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{b}\right)}{h}+\frac{Bdn\left(\frac{\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{d}+\frac{\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{d}\right)}{h}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

output
```
((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[g + h*x])/h - (b*B*n*((Log[-
((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/b + PolyLog[2, (b*(g + h*x))/(b
*g - a*h)]/b))/h + (B*d*n*((Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x]
)/d + PolyLog[2, (d*(g + h*x))/(d*g - c*h)]/d))/h
```

3.298. $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$

3.298.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2946
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Simp[b*B*(n/g)   Int[Log[f + g*x]/
(a + b*x), x], x] + Simp[B*d*(n/g)   Int[Log[f + g*x]/(c + d*x), x], x]) /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*
d, 0]
```

3.298.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.52 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.52

method	result
risch	$\frac{\left(-iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(i(bx+a)^{n}(dx+c)^{-n}\right)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^{n}\right)+iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^{n}\right)^{2}-iB\pi\ \mathrm{csgn}(i(b\cdots}{\cdots}$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-I*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*
(b*x+a)^n)+I*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(
I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*csgn
(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*csgn(I/((d*x+c)^n))*c
sgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*
Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)+2*A)*ln(h*x+g)/h+B*ln((b*x+a)^
n)*ln(h*x+g)/h-B/h*n*dilog(((h*x+g)*b+a*h-b*g)/(a*h-b*g))-B/h*n*ln(h*x+g)*
ln(((h*x+g)*b+a*h-b*g)/(a*h-b*g))-B*ln((d*x+c)^n)*ln(h*x+g)/h+B/h*n*dilog(
(d*(h*x+g)+c*h-d*g)/(c*h-d*g))+B/h*n*ln(h*x+g)*ln((d*(h*x+g)+c*h-d*g)/(c*h
-d*g))
```

3.298.5 Fricas [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx=\int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="fricas"
)
```

output
```
integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

3.298.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.298.7 Maxima [F]

$$\int \frac{A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)}{g + hx}\, dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{hx + g}\, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="maxima"
)
```

output
```
-B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(h*x + g), x)
 + A*log(h*x + g)/h
```

3.298.8 Giac [F]

$$\int \frac{A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)}{g + hx}\, dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{hx + g}\, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)}{g + hx}\, dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{g + h\,x}\, dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x), x)
```

3.299 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx$

3.299.1 Optimal result

Integrand size = 31, antiderivative size = 120

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{bBn\log(a+bx)}{h(bg-ah)}-\frac{Bdn\log(c+dx)}{h(dg-ch)}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)}+\frac{B(bc-ad)n\log(g+hx)}{(bg-ah)(dg-ch)}$$

output
```
b*B*n*ln(b*x+a)/h/(-a*h+b*g)-B*d*n*ln(d*x+c)/h/(-c*h+d*g)+(-A-B*ln(e*(b*x+
a)^n/((d*x+c)^n)))/h/(h*x+g)+B*(-a*d+b*c)*n*ln(h*x+g)/(-a*h+b*g)/(-c*h+d*g
)
```

3.299.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{-\frac{A}{g+hx}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}+\frac{Bn(b(dg-ch)\log(a+bx)+(-bdg+adh)\log(c+dx)+(bc-ad)h\log(g+hx))}{(bg-ah)(dg-ch)}}{h}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

output
```
(-(A/(g + h*x)) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x) + (B*n*(b
*(d*g - c*h)*Log[a + b*x] + (-(b*d*g) + a*d*h)*Log[c + d*x] + (b*c - a*d)*
h*Log[g + h*x]))/((b*g - a*h)*(d*g - c*h)))/h
```

3.299.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(g+hx)}dx}{h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)}$$

$$\Big\downarrow \text{93}$$

$$\frac{Bn(bc-ad)\int \left(\frac{b^2}{(bc-ad)(bg-ah)(a+bx)}+\frac{d^2}{(bc-ad)(ch-dg)(c+dx)}+\frac{h^2}{(bg-ah)(dg-ch)(g+hx)}\right)dx}{h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{Bn(bc-ad)\left(\frac{b\log(a+bx)}{(bc-ad)(bg-ah)}-\frac{d\log(c+dx)}{(bc-ad)(dg-ch)}+\frac{h\log(g+hx)}{(bg-ah)(dg-ch)}\right)}{h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

output
```
-((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x))) + (B*(b*c - a*d)
*n*((b*Log[a + b*x])/((b*c - a*d)*(b*g - a*h)) - (d*Log[c + d*x])/((b*c -
a*d)*(d*g - c*h)) + (h*Log[g + h*x])/((b*g - a*h)*(d*g - c*h))))/h
```

3.299.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.299.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(122) = 244$.

Time = 8.52 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.07

method	result
parallelrisch	$\frac{Axabcd\,g^2n - Ax\,a^2cdghn + B\ln(bx+a)a^2cd\,g^2n^2 - B\ln(bx+a)ab\,c^2g^2n^2 - B\ln(hx+g)a^2cd\,g^2n^2 + B\ln(hx+g)ab\,c^2g^2n^2 - B\ln}{}$
risch	Expression too large to display

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
(A*x*a*b*c*d*g^2*n-A*x*a^2*c*d*g*h*n+B*ln(b*x+a)*a^2*c*d*g^2*n^2-B*ln(b*x+
a)*a*b*c^2*g^2*n^2-B*ln(h*x+g)*a^2*c*d*g^2*n^2+B*ln(h*x+g)*a*b*c^2*g^2*n^2
-B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*c^2*g*h*n+B*ln(e*(b*x+a)^n/((d*x+c)^n))
*a*b*c^2*g^2*n-B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*c*d*g*h*n+B*x*ln(e*(b*x
+a)^n/((d*x+c)^n))*a*b*c*d*g^2*n+B*ln(b*x+a)*x*a^2*c*d*g*h*n^2-B*ln(b*x+a)
*x*a*b*c^2*g*h*n^2-B*ln(h*x+g)*x*a^2*c*d*g*h*n^2+B*ln(h*x+g)*x*a*b*c^2*g*h
*n^2-A*x*a*b*c^2*g*h*n+A*x*a^2*c^2*h^2*n)/(a*h-b*g)/(h*x+g)/n/(c*h-d*g)/a/
c/g
```

3.299.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(120) = 240$.

Time = 3.58 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.08

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = -\frac{Abdg^2 + Aach^2 - (Abc + Aad)gh - ((Bbdgh - Bbch^2)nx + (Badgh - Bach^2)n)\log(bx+a) + ((Bbd}{bdg^3h +}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="frica
s")
```

output
```
-(A*b*d*g^2 + A*a*c*h^2 - (A*b*c + A*a*d)*g*h - ((B*b*d*g*h - B*b*c*h^2)*n
*x + (B*a*d*g*h - B*a*c*h^2)*n)*log(b*x + a) + ((B*b*d*g*h - B*a*d*h^2)*n*
x + (B*b*c*g*h - B*a*c*h^2)*n)*log(d*x + c) - ((B*b*c - B*a*d)*h^2*n*x + (
B*b*c - B*a*d)*g*h*n)*log(h*x + g) + (B*b*d*g^2 + B*a*c*h^2 - (B*b*c + B*a
*d)*g*h)*log(e))/(b*d*g^3*h + a*c*g*h^3 - (b*c + a*d)*g^2*h^2 + (b*d*g^2*h
^2 + a*c*h^4 - (b*c + a*d)*g*h^3)*x)
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**2,x)
```

output
```
Timed out
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int \frac{A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)}{(g + hx)^2}\,dx$$
$$= \frac{\left(\frac{ben\log(bx+a)}{bgh-ah^2} - \frac{den\log(dx+c)}{dgh-ch^2} - \frac{(bcen-aden)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)B}{e} - \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{h^2x + gh} - \frac{A}{h^2x + gh}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="maxim
a")
```

output
```
(b*e*n*log(b*x + a)/(b*g*h - a*h^2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) -
 (b*c*e*n - a*d*e*n)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))
*B/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A/(h^2*x + g*h)
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.41

$$\int \frac{A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)}{(g + hx)^2}\,dx = \frac{Bb^2n\log\left(|bx + a|\right)}{b^2gh - abh^2} - \frac{Bd^2n\log\left(|-dx - c|\right)}{d^2gh - cdh^2} - \frac{Bn\log\left(bx + a\right)}{h^2x + gh} + \frac{Bn\log\left(dx + c\right)}{h^2x + gh} + \frac{(Bbcn - Badn)\log\left(hx + g\right)}{bdg^2 - bcgh - adgh + ach^2} - \frac{B\log\left(e\right) + A}{h^2x + gh}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="giac"
)
```

output
```
B*b^2*n*log(abs(b*x + a))/(b^2*g*h - a*b*h^2) - B*d^2*n*log(abs(-d*x - c))
/(d^2*g*h - c*d*h^2) - B*n*log(b*x + a)/(h^2*x + g*h) + B*n*log(d*x + c)/(
h^2*x + g*h) + (B*b*c*n - B*a*d*n)*log(h*x + g)/(b*d*g^2 - b*c*g*h - a*d*g
*h + a*c*h^2) - (B*log(e) + A)/(h^2*x + g*h)
```

3.299.9 Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \frac{B\,d\,n\,\ln\left(c+d\,x\right)}{c\,h^2-d\,g\,h} - \frac{\ln\left(g+h\,x\right)\,\left(B\,a\,d\,n-B\,b\,c\,n\right)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h} - \frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{h\,\left(g+h\,x\right)} - \frac{B\,b\,n\,\ln\left(a+b\,x\right)}{a\,h^2-b\,g\,h} - \frac{A}{x\,h^2+g\,h}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^2,x)
```

output
```
(B*d*n*log(c + d*x))/(c*h^2 - d*g*h) - (log(g + h*x)*(B*a*d*n - B*b*c*n))/
(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (B*log((e*(a + b*x)^n)/(c + d*x)
^n))/(h*(g + h*x)) - (B*b*n*log(a + b*x))/(a*h^2 - b*g*h) - A/(g*h + h^2*x
)
```

3.300 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$

3.300.1 Optimal result

Integrand size = 31, antiderivative size = 191

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = &-\frac{B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} \\ &+\frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2}-\frac{Bd^2n\log(c+dx)}{2h(dg-ch)^2} \\ &-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2} \\ &+\frac{B(bc-ad)(2bdg-bch-adh)n\log(g+hx)}{2(bg-ah)^2(dg-ch)^2}\end{aligned}$$

output
```
-1/2*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)+1/2*b^2*B*n*ln(b*x+a)/h/
(-a*h+b*g)^2-1/2*B*d^2*n*ln(d*x+c)/h/(-c*h+d*g)^2+1/2*(-A-B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/h/(h*x+g)^2+1/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln(h*
x+g)/(-a*h+b*g)^2/(-c*h+d*g)^2
```

3.300.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.93

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx =$$
$$-\frac{\frac{A}{(g+hx)^2}+\frac{B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2}+B(bc-ad)n\left(-\frac{b^2\log(a+bx)}{(bc-ad)(bg-ah)^2}+\frac{\frac{d^2\log(c+dx)}{bc-ad}+\frac{h\left(\frac{(bg-ah)(dg-ch)}{g+hx}+(-2bdg+bch+adh\right.}{(bg-ah)^2}}{(dg-ch)^2}\right.}{2h}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

output
```
-1/2*(A/(g + h*x)^2 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2 + B
*(b*c - a*d)*n*(-((b^2*Log[a + b*x])/((b*c - a*d)*(b*g - a*h)^2)) + ((d^2*
Log[c + d*x])/(b*c - a*d) + (h*(((b*g - a*h)*(d*g - c*h))/(g + h*x) + (-2*
b*d*g + b*c*h + a*d*h)*Log[g + h*x]))/(b*g - a*h)^2)/(d*g - c*h)^2))/h
```

3.300.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(g+hx)^3}\,dx$$
$$\Big\downarrow \ 2948$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(g+hx)^2}dx}{2h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2}$$
$$\Big\downarrow \ 93$$
$$\frac{Bn(bc-ad)\int \left(\frac{b^3}{(bc-ad)(bg-ah)^2(a+bx)}-\frac{d^3}{(bc-ad)(ch-dg)^2(c+dx)}-\frac{h^2(-2bdg+bch+adh)}{(bg-ah)^2(dg-ch)^2(g+hx)}+\frac{h^2}{(bg-ah)(dg-ch)(g+hx)^2}\right)dx}{2h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bg-ah)^2}-\frac{d^2\log(c+dx)}{(bc-ad)(dg-ch)^2}-\frac{h}{(g+hx)(bg-ah)(dg-ch)}+\frac{h\log(g+hx)(-adh-bch+2bdg)}{(bg-ah)^2(dg-ch)^2}\right)}{2h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)^2) + (B*(b*c -
a*d)*n*(-(h/((b*g - a*h)*(d*g - c*h)*(g + h*x))) + (b^2*Log[a + b*x])/((b*
c - a*d)*(b*g - a*h)^2) - (d^2*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^2) +
 (h*(2*b*d*g - b*c*h - a*d*h)*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2))
)/(2*h)
```

3.300.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.300.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. 2(184) = 368.

Time = 28.81 (sec) , antiderivative size = 1385, normalized size of antiderivative = 7.25

method	result	size
parallelrisch	Expression too large to display	1385
risch	Expression too large to display	4925

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*B*ln(h*x+g)*b^3*c*d^2*g^3*h^2*n-B*ln(b*x+a)*x^2*b^3*c^2*d*h^5*n-B
*ln(b*x+a)*x^2*b^3*d^3*g^2*h^3*n+B*ln(d*x+c)*x^2*a^2*b*d^3*h^5*n-B*ln(b*x+
a)*b^3*d^3*g^4*h*n+B*ln(d*x+c)*b^3*d^3*g^4*h*n+B*x*a^2*b*d^3*g*h^4*n+B*x*a
*b^2*c^2*d*h^5*n-B*x*a*b^2*d^3*g^2*h^3*n-B*x*b^3*c^2*d*g*h^4*n+B*x*b^3*c*d
^2*g^2*h^3*n-2*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b*c*d^2*g*h^4-2*B*ln(e*(b
*x+a)^n/((d*x+c)^n))*a*b^2*c^2*d*g*h^4+4*B*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b
^2*c*d^2*g^2*h^3+B*ln(d*x+c)*x^2*b^3*d^3*g^2*h^3*n-B*ln(h*x+g)*x^2*a^2*b*d
^3*h^5*n+B*ln(h*x+g)*x^2*b^3*c^2*d*h^5*n-2*B*ln(b*x+a)*x*b^3*d^3*g^3*h^2*n
+2*B*ln(d*x+c)*x*b^3*d^3*g^3*h^2*n-B*ln(b*x+a)*b^3*c^2*d*g^2*h^3*n+2*B*ln(
b*x+a)*b^3*c*d^2*g^3*h^2*n+B*ln(d*x+c)*a^2*b*d^3*g^2*h^3*n-2*B*ln(d*x+c)*a
*b^2*d^3*g^3*h^2*n-B*ln(h*x+g)*a^2*b*d^3*g^2*h^3*n+2*B*ln(h*x+g)*a*b^2*d^3
*g^3*h^2*n+B*ln(h*x+g)*b^3*c^2*d*g^2*h^3*n+2*B*ln(h*x+g)*x^2*a*b^2*d^3*g*h
^4*n-2*B*ln(h*x+g)*x^2*b^3*c*d^2*g*h^4*n-2*B*ln(b*x+a)*x*b^3*c^2*d*g*h^4*n
+4*B*ln(b*x+a)*x*b^3*c*d^2*g^2*h^3*n+2*B*ln(d*x+c)*x*a^2*b*d^3*g*h^4*n-4*B
*ln(d*x+c)*x*a*b^2*d^3*g^2*h^3*n-2*B*ln(h*x+g)*x*a^2*b*d^3*g*h^4*n+4*B*ln(
h*x+g)*x*a*b^2*d^3*g^2*h^3*n+2*B*ln(h*x+g)*x*b^3*c^2*d*g*h^4*n-4*B*ln(h*x+
g)*x*b^3*c*d^2*g^2*h^3*n+B*ln(e*(b*x+a)^n/((d*x+c)^n))*b^3*d^3*g^4*h+A*b^3
*d^3*g^4*h+B*a^2*b*d^3*g^2*h^3*n-B*a*b^2*d^3*g^3*h^2*n-B*b^3*c^2*d*g^2*h^3
*n+B*b^3*c*d^2*g^3*h^2*n-2*A*a^2*b*c*d^2*g*h^4-2*A*a*b^2*c^2*d*g*h^4+4*A*a
*b^2*c*d^2*g^2*h^3-B*x*a^2*b*c*d^2*h^5*n+2*B*ln(b*x+a)*x^2*b^3*c*d^2*g*...
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(181) = 362$.

Time = 52.00 (sec) , antiderivative size = 1127, normalized size of antiderivative = 5.90

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx =$$
$$-\frac{Ab^2d^2g^4 + Aa^2c^2h^4 - 2\,(Ab^2cd + Aabd^2)g^3h + (Ab^2c^2 + 4\,Aabcd + Aa^2d^2)g^2h^2 - 2\,(Aabc^2 + Aa^2cd)gh}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="frica
s")
```

output
```
-1/2*(A*b^2*d^2*g^4 + A*a^2*c^2*h^4 - 2*(A*b^2*c*d + A*a*b*d^2)*g^3*h + (A
*b^2*c^2 + 4*A*a*b*c*d + A*a^2*d^2)*g^2*h^2 - 2*(A*a*b*c^2 + A*a^2*c*d)*g*
h^3 + ((B*b^2*c*d - B*a*b*d^2)*g^2*h^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3 + (
B*a*b*c^2 - B*a^2*c*d)*h^4)*n*x + ((B*b^2*c*d - B*a*b*d^2)*g^3*h - (B*b^2*
c^2 - B*a^2*d^2)*g^2*h^2 + (B*a*b*c^2 - B*a^2*c*d)*g*h^3)*n - ((B*b^2*d^2*
g^2*h^2 - 2*B*b^2*c*d*g*h^3 + B*b^2*c^2*h^4)*n*x^2 + 2*(B*b^2*d^2*g^3*h -
2*B*b^2*c*d*g^2*h^2 + B*b^2*c^2*g*h^3)*n*x + (2*B*a*b*d^2*g^3*h - B*a^2*c^
2*h^4 - (4*B*a*b*c*d + B*a^2*d^2)*g^2*h^2 + 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^
3)*n)*log(b*x + a) + ((B*b^2*d^2*g^2*h^2 - 2*B*a*b*d^2*g*h^3 + B*a^2*d^2*h
^4)*n*x^2 + 2*(B*b^2*d^2*g^3*h - 2*B*a*b*d^2*g^2*h^2 + B*a^2*d^2*g*h^3)*n*
x + (2*B*b^2*c*d*g^3*h - B*a^2*c^2*h^4 - (B*b^2*c^2 + 4*B*a*b*c*d)*g^2*h^2
 + 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*n)*log(d*x + c) - ((2*(B*b^2*c*d - B*a
*b*d^2)*g*h^3 - (B*b^2*c^2 - B*a^2*d^2)*h^4)*n*x^2 + 2*(2*(B*b^2*c*d - B*a
*b*d^2)*g^2*h^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3)*n*x + (2*(B*b^2*c*d - B*a
*b*d^2)*g^3*h - (B*b^2*c^2 - B*a^2*d^2)*g^2*h^2)*n)*log(h*x + g) + (B*b^2*
d^2*g^4 + B*a^2*c^2*h^4 - 2*(B*b^2*c*d + B*a*b*d^2)*g^3*h + (B*b^2*c^2 + 4
*B*a*b*c*d + B*a^2*d^2)*g^2*h^2 - 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*log(e))
/(b^2*d^2*g^6*h + a^2*c^2*g^2*h^5 - 2*(b^2*c*d + a*b*d^2)*g^5*h^2 + (b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*g^4*h^3 - 2*(a*b*c^2 + a^2*c*d)*g^3*h^4 + (b^2*d
^2*g^4*h^3 + a^2*c^2*h^7 - 2*(b^2*c*d + a*b*d^2)*g^3*h^4 + (b^2*c^2 + 4...
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**3,x)
```

output
```
Timed out
```

3.300.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(181)=362$.

Time = 0.22 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.00

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$$
$$=\frac{\left(\frac{b^2en\log(bx+a)}{b^2g^2h-2\,abgh^2+a^2h^3}-\frac{d^2en\log(dx+c)}{d^2g^2h-2\,cdgh^2+c^2h^3}-\frac{(2\,abd^2egn-a^2d^2ehn-(2\,cdegn-c^2ehn)b^2)\log(hx+g)}{(d^2g^2h^2-2\,cdgh^3+c^2h^4)a^2-2\,(d^2g^3h-2\,cdg^2h^2+c^2gh^3)ab+(d^2g^4-2\,cdg^3h+c^2g^2h^2)b^2}\right.}{2\,e}$$
$$-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\,(h^3x^2+2\,gh^2x+g^2h)}-\frac{A}{2\,(h^3x^2+2\,gh^2x+g^2h)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="maxim
a")
```

output
```
1/2*(b^2*e*n*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*e*n*lo
g(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*e*g*n - a^2*d^
2*e*h*n - (2*c*d*e*g*n - c^2*e*h*n)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*
d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (
d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*e*n - a*d*e*n)/((d*g^2*h
- c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g
*h^2)*b)*x))*B/e - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2
*x + g^2*h) - 1/2*A/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

3.300.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(181) = 362$.

Time = 0.52 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = \frac{Bb^3n\log\left(|bx+a|\right)}{2\left(b^3g^2h - 2\,ab^2gh^2 + a^2bh^3\right)}$$
$$- \frac{Bd^3n\log\left(|dx+c|\right)}{2\left(d^3g^2h - 2\,cd^2gh^2 + c^2dh^3\right)} - \frac{Bn\log\left(bx+a\right)}{2\left(h^3x^2 + 2\,gh^2x + g^2h\right)} + \frac{Bn\log\left(dx+c\right)}{2\left(h^3x^2 + 2\,gh^2x + g^2h\right)}$$
$$+ \frac{\left(2\,Bb^2cdgn - 2\,Babd^2gn - Bb^2c^2hn + Ba^2d^2hn\right)\log\left(hx+g\right)}{2\left(b^2d^2g^4 - 2\,b^2cdg^3h - 2\,abd^2g^3h + b^2c^2g^2h^2 + 4\,abcdg^2h^2 + a^2d^2g^2h^2 - 2\,abc^2gh^3 - 2\,a^2cdgh^3 + a^2c^2h\right.}$$
$$- \frac{Bbch^2nx - Badh^2nx + Bbcghn - Badghn + Bbdg^2\log\left(e\right) - Bbcgh\log\left(e\right) - Badgh\log\left(e\right) + Bach^2\,\mathrm{l}}{2\left(bdg^2h^3x^2 - bcgh^4x^2 - adgh^4x^2 + ach^5x^2 + 2\,bdg^3h^2x - 2\,bcg^2h^3x - 2\,adg^2h^3x + 2\,acgh^4x +\right.}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="giac"
)
```

output
```
1/2*B*b^3*n*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) - 1/
2*B*d^3*n*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*
B*n*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*B*n*log(d*x + c)/(h^3
*x^2 + 2*g*h^2*x + g^2*h) + 1/2*(2*B*b^2*c*d*g*n - 2*B*a*b*d^2*g*n - B*b^2
*c^2*h*n + B*a^2*d^2*h*n)*log(h*x + g)/(b^2*d^2*g^4 - 2*b^2*c*d*g^3*h - 2*
a*b*d^2*g^3*h + b^2*c^2*g^2*h^2 + 4*a*b*c*d*g^2*h^2 + a^2*d^2*g^2*h^2 - 2*
a*b*c^2*g*h^3 - 2*a^2*c*d*g*h^3 + a^2*c^2*h^4) - 1/2*(B*b*c*h^2*n*x - B*a*
d*h^2*n*x + B*b*c*g*h*n - B*a*d*g*h*n + B*b*d*g^2*log(e) - B*b*c*g*h*log(e
) - B*a*d*g*h*log(e) + B*a*c*h^2*log(e) + A*b*d*g^2 - A*b*c*g*h - A*a*d*g*
h + A*a*c*h^2)/(b*d*g^2*h^3*x^2 - b*c*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*
x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g^2*h^3*x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x
+ b*d*g^4*h - b*c*g^3*h^2 - a*d*g^3*h^2 + a*c*g^2*h^3)
```

3.300.9 Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$$
$$= \frac{\ln(g+h\,x)\,\left(h\,(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n)-2\,B\,a\,b\,d^2\,g\,n+2\,B\,b^2\,c\,d\,g\,n\right)}{2\,a^2\,c^2\,h^4-4\,a^2\,c\,d\,g\,h^3+2\,a^2\,d^2\,g^2\,h^2-4\,a\,b\,c^2\,g\,h^3+8\,a\,b\,c\,d\,g^2\,h^2-4\,a\,b\,d^2\,g^3\,h+2\,b^2\,c^2\,g^2\,h^2-4\,b^2\,c\,d}$$
$$-\frac{\frac{A\,a\,c\,h^2+A\,b\,d\,g^2-A\,a\,d\,g\,h-A\,b\,c\,g\,h-B\,a\,d\,g\,h\,n+B\,b\,c\,g\,h\,n}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}-\frac{x\,(B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}}{2\,g^2\,h+4\,g\,h^2\,x+2\,h^3\,x^2}$$
$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{2\,h\,(g^2+2\,g\,h\,x+h^2\,x^2)}+\frac{B\,b^2\,n\,\ln(a+b\,x)}{2\,a^2\,h^3-4\,a\,b\,g\,h^2+2\,b^2\,g^2\,h}-\frac{B\,d^2\,n\,\ln(c+d\,x)}{2\,c^2\,h^3-4\,c\,d\,g\,h^2+2\,d^2\,g^2\,h}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^3,x)
```

output
```
(log(g + h*x)*(h*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*g*n + 2*B*b^2*c
*d*g*n))/(2*a^2*c^2*h^4 + 2*b^2*d^2*g^4 + 2*a^2*d^2*g^2*h^2 + 2*b^2*c^2*g^
2*h^2 - 4*a*b*c^2*g*h^3 - 4*a*b*d^2*g^3*h - 4*a^2*c*d*g*h^3 - 4*b^2*c*d*g^
3*h + 8*a*b*c*d*g^2*h^2) - ((A*a*c*h^2 + A*b*d*g^2 - A*a*d*g*h - A*b*c*g*h
 - B*a*d*g*h*n + B*b*c*g*h*n)/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (x
*(B*a*d*h^2*n - B*b*c*h^2*n))/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h))/(2*
g^2*h + 2*h^3*x^2 + 4*g*h^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(2*h
*(g^2 + h^2*x^2 + 2*g*h*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^
2*h - 4*a*b*g*h^2) - (B*d^2*n*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4*c
*d*g*h^2)
```

3.301 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4} dx$

3.301.1 Optimal result

Integrand size = 31, antiderivative size = 284

$$\begin{aligned}
&\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4} dx \\
&= -\frac{B(bc-ad)n}{6(bg-ah)(dg-ch)(g+hx)^2} - \frac{B(bc-ad)(2bdg-bch-adh)n}{3(bg-ah)^2(dg-ch)^2(g+hx)} \\
&\quad + \frac{b^3Bn\log(a+bx)}{3h(bg-ah)^3} - \frac{Bd^3n\log(c+dx)}{3h(dg-ch)^3} - \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3} \\
&\quad + \frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log(g+hx)}{3(bg-ah)^3(dg-ch)^3}
\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^2-1/3*B*(-a*d+b*c)*(-a*d
*h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*B*n*ln(b*x+a
)/h/(-a*h+b*g)^3-1/3*B*d^3*n*ln(d*x+c)/h/(-c*h+d*g)^3+1/3*(-A-B*ln(e*(b*x+
a)^n/((d*x+c)^n)))/h/(h*x+g)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h
+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^3/(-c*h+
d*g)^3
```

3.301.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.96

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx =$$

$$-\frac{\frac{2A}{(g+hx)^3}+\frac{2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}+B(bc-ad)n\left(\frac{h}{(bg-ah)(dg-ch)(g+hx)^2}-\frac{2h(-2bdg+bch+adh)}{(bg-ah)^2(dg-ch)^2(g+hx)}-\frac{2b^3\log(a+\cdots}{(bc-ad)(bg-\cdots}\right.}{6h}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

output
```
-1/6*((2*A)/(g + h*x)^3 + (2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)
^3 + B*(b*c - a*d)*n*(h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (2*h*(-2*b
*d*g + b*c*h + a*d*h))/((b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) - (2*b^3*Lo
g[a + b*x])/((b*c - a*d)*(b*g - a*h)^3) + (2*d^3*Log[c + d*x])/((b*c - a*d
)*(d*g - c*h)^3) - (2*h*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2
*g^2 - 3*c*d*g*h + c^2*h^2))*Log[g + h*x])/((b*g - a*h)^3*(d*g - c*h)^3)))
/h
```

3.301.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(g+hx)^4}dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(g+hx)^3}dx}{3h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3h(g+hx)^3}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{Bn(bc-ad)\int \left(\frac{b^4}{(bc-ad)(bg-ah)^3(a+bx)}+\frac{d^4}{(bc-ad)(ch-dg)^3(c+dx)}+\frac{h^2((3d^2g^2-3cdhg+c^2h^2)b^2-adh(3dg-ch)b+a^2d^2h^2)}{(bg-ah)^3(dg-ch)^3(g+hx)}-\frac{h^2(\cdots}{(bg-ah\cdots}\right.}{3h}$$

$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3h(g+hx)^3}$$

$\downarrow$ 2009

$$\frac{Bn(bc-ad)\left(\frac{h\log(g+hx)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{(bg-ah)^3(dg-ch)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bg-ah)^3}-\frac{d^3\log(c+dx)}{(bc-ad)(dg-ch)^3}-\frac{h(-adh-b\cdots}{(g+hx)(bg-ah\cdots}\right.}{3h}$$

$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3h(g+hx)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

output
```
-1/3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)^3) + (B*(b*c -
a*d)*n*(-1/2*h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (h*(2*b*d*g - b*c*h
 - a*d*h))/((b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) + (b^3*Log[a + b*x])/((
b*c - a*d)*(b*g - a*h)^3) - (d^3*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^3)
 + (h*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h +
c^2*h^2))*Log[g + h*x])/((b*g - a*h)^3*(d*g - c*h)^3)))/(3*h)
```

3.301.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.301. $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx$

3.301.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3285 vs. 2(275) = 550.

Time = 88.44 (sec) , antiderivative size = 3286, normalized size of antiderivative = 11.57

method	result	size
parallelrisch	Expression too large to display	3286
risch	Expression too large to display	9645

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-6*B*ln(b*x+a)*x*a^3*b*d^4*g^2*h^6*n^2+18*B*ln(b*x+a)*x*a^2*b^2*d^4*
g^3*h^5*n^2+15*B*x*a*b^3*c^2*d^2*g^2*h^6*n^2-6*B*x^2*a^2*b^2*c*d^3*g*h^7*n
^2+6*B*x^2*a*b^3*c^2*d^2*g*h^7*n^2+6*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^3*b
*d^4*g^2*h^6*n-18*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*d^4*g^3*h^5*n+18
*B*x*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*d^4*g^4*h^4*n+6*B*x*a^3*b*c*d^3*g*h
^7*n^2-15*B*x*a^2*b^2*c*d^3*g^2*h^6*n^2-6*B*x*a*b^3*c^3*d*g*h^7*n^2-6*B*x^
3*ln(e*(b*x+a)^n/((d*x+c)^n))*a^2*b^2*d^4*g*h^7*n+6*B*x^3*ln(e*(b*x+a)^n/(
(d*x+c)^n))*a*b^3*d^4*g^2*h^6*n-6*B*ln(b*x+a)*x^3*a*b^3*d^4*g^2*h^6*n^2-6*
B*ln(b*x+a)*x^3*b^4*c^2*d^2*g*h^7*n^2+6*B*ln(b*x+a)*x^3*b^4*c*d^3*g^2*h^6*
n^2-18*B*ln(b*x+a)*x*a*b^3*d^4*g^4*h^4*n^2+6*B*ln(b*x+a)*x*b^4*c^3*d*g^2*h
^6*n^2-18*B*ln(b*x+a)*x*b^4*c^2*d^2*g^3*h^5*n^2+18*B*ln(b*x+a)*x*b^4*c*d^3
*g^4*h^4*n^2+6*B*ln(h*x+g)*x*a^3*b*d^4*g^2*h^6*n^2-18*B*ln(h*x+g)*x*a^2*b^
2*d^4*g^3*h^5*n^2+18*B*ln(h*x+g)*x*a*b^3*d^4*g^4*h^4*n^2-6*B*ln(h*x+g)*x*b
^4*c^3*d*g^2*h^6*n^2+18*B*ln(h*x+g)*x*b^4*c^2*d^2*g^3*h^5*n^2+6*B*x^2*ln(e
*(b*x+a)^n/((d*x+c)^n))*a^3*b*d^4*g*h^7*n-18*B*x^2*ln(e*(b*x+a)^n/((d*x+c)
^n))*a^2*b^2*d^4*g^2*h^6*n+18*B*x^2*ln(e*(b*x+a)^n/((d*x+c)^n))*a*b^3*d^4*
g^3*h^5*n-2*B*ln(b*x+a)*a^3*b*d^4*g^3*h^5*n^2+6*B*ln(b*x+a)*x^3*a^2*b^2*d^
4*g*h^7*n^2-18*B*ln(h*x+g)*x*b^4*c*d^3*g^4*h^4*n^2-6*B*ln(b*x+a)*x^2*a^3*b
*d^4*g*h^7*n^2+18*B*ln(b*x+a)*x^2*a^2*b^2*d^4*g^2*h^6*n^2-18*B*ln(b*x+a)*x
^2*a*b^3*d^4*g^3*h^5*n^2+6*B*ln(b*x+a)*x^2*b^4*c^3*d*g*h^7*n^2-18*B*ln(...
```

3.301.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="frica
s")
```

output
```
Timed out
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**4,x)
```

output
```
Timed out
```

3.301.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(272) = 544$.

Time = 0.26 (sec) , antiderivative size = 920, normalized size of antiderivative = 3.24

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx$$
$$=\frac{\left(\frac{2\,b^3en\log(bx+a)}{b^3g^3h-3\,ab^2g^2h^2+3\,a^2bgh^3-a^3h^4}-\frac{2\,d^3en\log(dx+c)}{d^3g^3h-3\,cd^2g^2h^2+3\,c^2dgh^3-c^3h^4}+\frac{2\,(3\,ab^2d^3eg^2n-3\,a^2}{(d^3g^3h^3-3\,cd^2g^2h^4+3\,c^2dgh^5-c^3h^6)a^3-3\,(d^3g^4h^2-3\,cd^2g^3h^3}\right.}{}$$
$$-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{3\,(h^4x^3+3\,gh^3x^2+3\,g^2h^2x+g^3h)}-\frac{A}{3\,(h^4x^3+3\,gh^3x^2+3\,g^2h^2x+g^3h)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="maxim
a")
```

output
```
1/6*(2*b^3*e*n*log(b*x + a)/(b^3*g^3*h - 3*a*b^2*g^2*h^2 + 3*a^2*b*g*h^3 -
 a^3*h^4) - 2*d^3*e*n*log(d*x + c)/(d^3*g^3*h - 3*c*d^2*g^2*h^2 + 3*c^2*d*
g*h^3 - c^3*h^4) + 2*(3*a*b^2*d^3*e*g^2*n - 3*a^2*b*d^3*e*g*h*n + a^3*d^3*
e*h^2*n - (3*c*d^2*e*g^2*n - 3*c^2*d*e*g*h*n + c^3*e*h^2*n)*b^3)*log(h*x +
 g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^
3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*
g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6
- 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - ((3*d^2*e*g*h*n -
c*d*e*h^2*n)*a^2 - (5*d^2*e*g^2*n - c^2*e*h^2*n)*a*b + (5*c*d*e*g^2*n - 3*
c^2*e*g*h*n)*b^2 - 2*(2*a*b*d^2*e*g*h*n - a^2*d^2*e*h^2*n - (2*c*d*e*g*h*n
 - c^2*e*h^2*n)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a^2 -
 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b + (d^2*g^6 - 2*c*d*g^5*h
+ c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 + c^2*h^6)*a^2 - 2*(d^2*g
^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c
^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a^2 -
2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (d^2*g^5*h - 2*c*d*g^4
*h^2 + c^2*g^3*h^3)*b^2)*x))*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(h
^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*A/(h^4*x^3 + 3*g*h^3*x^2
 + 3*g^2*h^2*x + g^3*h)
```

3.301.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. $2(272) = 544$.

Time = 1.14 (sec) , antiderivative size = 1530, normalized size of antiderivative = 5.39

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="giac"
)
```

output
```
1/3*B*b^4*n*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h
^3 - a^3*b*h^4) - 1/3*B*d^4*n*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h
^2 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*B*n*log(b*x + a)/(h^4*x^3 + 3*g*h^
3*x^2 + 3*g^2*h^2*x + g^3*h) + 1/3*B*n*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2
 + 3*g^2*h^2*x + g^3*h) + 1/3*(3*B*b^3*c*d^2*g^2*n - 3*B*a*b^2*d^3*g^2*n -
 3*B*b^3*c^2*d*g*h*n + 3*B*a^2*b*d^3*g*h*n + B*b^3*c^3*h^2*n - B*a^3*d^3*h
^2*n)*log(h*x + g)/(b^3*d^3*g^6 - 3*b^3*c*d^2*g^5*h - 3*a*b^2*d^3*g^5*h +
3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 + 3*a^2*b*d^3*g^4*h^2 - b^3*c^
3*g^3*h^3 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 - a^3*d^3*g^3*h^
3 + 3*a*b^2*c^3*g^2*h^4 + 9*a^2*b*c^2*d*g^2*h^4 + 3*a^3*c*d^2*g^2*h^4 - 3*
a^2*b*c^3*g*h^5 - 3*a^3*c^2*d*g*h^5 + a^3*c^3*h^6) - 1/6*(4*B*b^2*c*d*g*h^
3*n*x^2 - 4*B*a*b*d^2*g*h^3*n*x^2 - 2*B*b^2*c^2*h^4*n*x^2 + 2*B*a^2*d^2*h^
4*n*x^2 + 9*B*b^2*c*d*g^2*h^2*n*x - 9*B*a*b*d^2*g^2*h^2*n*x - 5*B*b^2*c^2*
g*h^3*n*x + 5*B*a^2*d^2*g*h^3*n*x + B*a*b*c^2*h^4*n*x - B*a^2*c*d*h^4*n*x
+ 5*B*b^2*c*d*g^3*h*n - 5*B*a*b*d^2*g^3*h*n - 3*B*b^2*c^2*g^2*h^2*n + 3*B*
a^2*d^2*g^2*h^2*n + B*a*b*c^2*g*h^3*n - B*a^2*c*d*g*h^3*n + 2*B*b^2*d^2*g^
4*log(e) - 4*B*b^2*c*d*g^3*h*log(e) - 4*B*a*b*d^2*g^3*h*log(e) + 2*B*b^2*c
^2*g^2*h^2*log(e) + 8*B*a*b*c*d*g^2*h^2*log(e) + 2*B*a^2*d^2*g^2*h^2*log(e
) - 4*B*a*b*c^2*g*h^3*log(e) - 4*B*a^2*c*d*g*h^3*log(e) + 2*B*a^2*c^2*h^4*
log(e) + 2*A*b^2*d^2*g^4 - 4*A*b^2*c*d*g^3*h - 4*A*a*b*d^2*g^3*h + 2*A*...
```

3.301.9 Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 1183, normalized size of antiderivative = 4.17

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx = \frac{B\,d^3\,n\,\ln\left(c+d\,x\right)}{3\,c^3\,h^4-9\,c^2\,d\,g\,h^3+9\,c\,d^2\,g^2\,h^2-3\,d^3\,g^3\,h}$$
$$-\frac{\ln\left(g+h\,x\right)\,\left(h^2\,\left(B\,a^3\,d^3\,n-B\,b^3\,c\right.\right.}{3\,a^3\,c^3\,h^6-9\,a^3\,c^2\,d\,g\,h^5+9\,a^3\,c\,d^2\,g^2\,h^4-3\,a^3\,d^3\,g^3\,h^3-9\,a^2\,b\,c^3\,g\,h^5+27\,a^2\,b\,c^2\,d\,g^2\,h^4-27\,a^2\,b\,c\,d^2\,g}$$
$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{3\,h\,\left(g^3+3\,g^2\,h\,x+3\,g\,h^2\,x^2+h^3\,x^3\right)}-\frac{B\,b^3\,n\,\ln\left(a+b\,x\right)}{3\,a^3\,h^4-9\,a^2\,b\,g\,h^3+9\,a\,b^2\,g^2\,h^2-3\,b^3\,g^3\,h}$$
$$-\frac{\frac{2\,A\,a^2\,c^2\,h^4+2\,A\,b^2\,d^2\,g^4+2\,A\,a^2\,d^2\,g^2\,h^2+2\,A\,b^2\,c^2\,g^2\,h^2+3\,B\,a^2\,d^2\,g^2\,h^2\,n-3\,B\,b^2\,c^2\,g^2\,h^2\,n-4\,A\,a\,b\,c^2\,g\,h^3-4\,A\,a\,b\,d^2\,g^3\,h-4\,A\,a^2\,c\,d\,g\,h^3-}{2\,\left(a^2\,c^2\,h^4-2\,a^2\,c\,d\,g\,h^3+a^2\,d^2\,g^2\,h^2-2\,a\,b\,c^2\,g\,h^3+4\,a\,b\,c\,d\,g^2\,h^2-2\,a\,b\,d^2\,g^3\,h+\right.}}{}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^4,x)
```

output
```
(B*d^3*n*log(c + d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - 9*c^2*
d*g*h^3) - (log(g + h*x)*(h^2*(B*a^3*d^3*n - B*b^3*c^3*n) - h*(3*B*a^2*b*d
^3*g*n - 3*B*b^3*c^2*d*g*n) + 3*B*a*b^2*d^3*g^2*n - 3*B*b^3*c*d^2*g^2*n))/
(3*a^3*c^3*h^6 + 3*b^3*d^3*g^6 - 3*a^3*d^3*g^3*h^3 - 3*b^3*c^3*g^3*h^3 - 9
*a^2*b*c^3*g*h^5 - 9*a*b^2*d^3*g^5*h - 9*a^3*c^2*d*g*h^5 - 9*b^3*c*d^2*g^5
*h + 9*a*b^2*c^3*g^2*h^4 + 9*a^2*b*d^3*g^4*h^2 + 9*a^3*c*d^2*g^2*h^4 + 9*b
^3*c^2*d*g^4*h^2 + 27*a*b^2*c*d^2*g^4*h^2 - 27*a*b^2*c^2*d*g^3*h^3 - 27*a^
2*b*c*d^2*g^3*h^3 + 27*a^2*b*c^2*d*g^2*h^4) - (B*log((e*(a + b*x)^n)/(c +
d*x)^n))/(3*h*(g^3 + h^3*x^3 + 3*g^2*h*x + 3*g*h^2*x^2)) - (B*b^3*n*log(a
+ b*x))/(3*a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3) - ((2*
A*a^2*c^2*h^4 + 2*A*b^2*d^2*g^4 + 2*A*a^2*d^2*g^2*h^2 + 2*A*b^2*c^2*g^2*h^
2 + 3*B*a^2*d^2*g^2*h^2*n - 3*B*b^2*c^2*g^2*h^2*n - 4*A*a*b*c^2*g*h^3 - 4*
A*a*b*d^2*g^3*h - 4*A*a^2*c*d*g*h^3 - 4*A*b^2*c*d*g^3*h + 8*A*a*b*c*d*g^2*
h^2 + B*a*b*c^2*g*h^3*n - 5*B*a*b*d^2*g^3*h*n - B*a^2*c*d*g*h^3*n + 5*B*b^
2*c*d*g^3*h*n)/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g
^2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g
^3*h + 4*a*b*c*d*g^2*h^2)) + (x*(B*a*b*c^2*h^4*n - B*a^2*c*d*h^4*n + 5*B*a
^2*d^2*g*h^3*n - 5*B*b^2*c^2*g*h^3*n - 9*B*a*b*d^2*g^2*h^2*n + 9*B*b^2*c*d
*g^2*h^2*n))/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2
*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*...
```

3.301. $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx$

3.302 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx$

3.302.1 Optimal result

Integrand size = 31, antiderivative size = 389

$$\begin{aligned}
&\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx \\
&= -\frac{B(bc-ad)n}{12(bg-ah)(dg-ch)(g+hx)^3}-\frac{B(bc-ad)(2bdg-bch-adh)n}{8(bg-ah)^2(dg-ch)^2(g+hx)^2} \\
&\quad -\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n}{4(bg-ah)^3(dg-ch)^3(g+hx)} \\
&\quad +\frac{b^4Bn\log(a+bx)}{4h(bg-ah)^4}-\frac{Bd^4n\log(c+dx)}{4h(dg-ch)^4}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4} \\
&\quad -\frac{B(bc-ad)(2bdg-bch-adh)\left(2abd^2gh-a^2d^2h^2-b^2(2d^2g^2-2cdgh+c^2h^2)\right)n\log(g+hx)}{4(bg-ah)^4(dg-ch)^4}
\end{aligned}$$

output
```
-1/12*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^3-1/8*B*(-a*d+b*c)*(-a*
d*h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)^2-1/4*B*(-a*d+b*c)*
(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n/(-a
*h+b*g)^3/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*B*n*ln(b*x+a)/h/(-a*h+b*g)^4-1/4*B*
d^4*n*ln(d*x+c)/h/(-c*h+d*g)^4+1/4*(-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h/(h
*x+g)^4-1/4*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*(2*a*b*d^2*g*h-a^2*d^2*h^2
-b^2*(c^2*h^2-2*c*d*g*h+2*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^4/(-c*h+d*g)^4
```

3.302.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.94

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx =$$
$$-\frac{\frac{A}{(g+hx)^4}+\frac{B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^4}-B(bc-ad)n\left(-\frac{h}{3(bg-ah)(dg-ch)(g+hx)^3}+\frac{h(-2bdg+bch+adh)}{2(bg-ah)^2(dg-ch)^2(g+hx)^2}-\frac{h(a^2d^2h\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

output
```
-1/4*(A/(g + h*x)^4 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4 - B
*(b*c - a*d)*n*(-1/3*h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) + (h*(-2*b*d*
g + b*c*h + a*d*h))/(2*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2) - (h*(a^2*
d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)))
/((b*g - a*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*Log[a + b*x])/((b*c - a*d)
*(b*g - a*h)^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^4) - (h*(-2*
b*d*g + b*c*h + a*d*h)*(-2*a*b*d^2*g*h + a^2*d^2*h^2 + b^2*(2*d^2*g^2 - 2*
c*d*g*h + c^2*h^2))*Log[g + h*x])/((b*g - a*h)^4*(d*g - c*h)^4)))/h
```

3.302.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(g+hx)^5}\,dx$$
$$\Big\downarrow \text{2948}$$
$$\frac{Bn(bc-ad)\int \frac{1}{(a+bx)(c+dx)(g+hx)^4}dx}{4h}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4h(g+hx)^4}$$
$$\Big\downarrow \text{93}$$

$$\frac{Bn(bc-ad)\int\left(\frac{b^5}{(bc-ad)(bg-ah)^4(a+bx)}-\frac{d^5}{(bc-ad)(ch-dg)^4(c+dx)}+\frac{h^2(2bdg-bch-adh)(2d^2g^2b^2+c^2h^2b^2-2cdghb^2-2ad^2ghb+a^2d^2h^2}{(bg-ah)^4(dg-ch)^4(g+hx)}\right.}{4h}$$

$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4h(g+hx)^4}$$

$\downarrow$ 2009

$$\frac{Bn(bc-ad)\left(-\frac{h(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2))}{(g+hx)(bg-ah)^3(dg-ch)^3}-\frac{h\log(g+hx)(-adh-bch+2bdg)(-a^2d^2h^2+2abd^2gh-(b^2(c^2h^2-2c}{(bg-ah)^4(dg-ch)^4}\right.}{4h}$$

$$\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4h(g+hx)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

output
```
-1/4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)^4) + (B*(b*c -
a*d)*n*(-1/3*h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) - (h*(2*b*d*g - b*c*h
 - a*d*h))/(2*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2) - (h*(a^2*d^2*h^2 -
 a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)))/((b*g - a
*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*g - a*
h)^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^4) - (h*(2*b*d*g - b*c
*h - a*d*h)*(2*a*b*d^2*g*h - a^2*d^2*h^2 - b^2*(2*d^2*g^2 - 2*c*d*g*h + c^
2*h^2))*Log[g + h*x])/((b*g - a*h)^4*(d*g - c*h)^4)))/(4*h)
```

3.302.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

3.302.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5230 vs. 2(378) = 756.

Time = 233.55 (sec) , antiderivative size = 5231, normalized size of antiderivative = 13.45

method	result	size
parallelrisch	Expression too large to display	5231
risch	Expression too large to display	16077

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.302.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="frica
s")
```

output
```
Timed out
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**5,x)
```

output
```
Timed out
```

3.302.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. $2(375) = 750$.

Time = 0.36 (sec) , antiderivative size = 1912, normalized size of antiderivative = 4.92

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="maxim
a")
```

output
```
1/24*(6*b^4*e*n*log(b*x + a)/(b^4*g^4*h - 4*a*b^3*g^3*h^2 + 6*a^2*b^2*g^2*
h^3 - 4*a^3*b*g*h^4 + a^4*h^5) - 6*d^4*e*n*log(d*x + c)/(d^4*g^4*h - 4*c*d
^3*g^3*h^2 + 6*c^2*d^2*g^2*h^3 - 4*c^3*d*g*h^4 + c^4*h^5) - 6*(4*a*b^3*d^4
*e*g^3*n - 6*a^2*b^2*d^4*e*g^2*h*n + 4*a^3*b*d^4*e*g*h^2*n - a^4*d^4*e*h^3
*n - (4*c*d^3*e*g^3*n - 6*c^2*d^2*e*g^2*h*n + 4*c^3*d*e*g*h^2*n - c^4*e*h^
3*n)*b^4)*log(h*x + g)/((d^4*g^4*h^4 - 4*c*d^3*g^3*h^5 + 6*c^2*d^2*g^2*h^6
 - 4*c^3*d*g*h^7 + c^4*h^8)*a^4 - 4*(d^4*g^5*h^3 - 4*c*d^3*g^4*h^4 + 6*c^2
*d^2*g^3*h^5 - 4*c^3*d*g^2*h^6 + c^4*g*h^7)*a^3*b + 6*(d^4*g^6*h^2 - 4*c*d
^3*g^5*h^3 + 6*c^2*d^2*g^4*h^4 - 4*c^3*d*g^3*h^5 + c^4*g^2*h^6)*a^2*b^2 -
4*(d^4*g^7*h - 4*c*d^3*g^6*h^2 + 6*c^2*d^2*g^5*h^3 - 4*c^3*d*g^4*h^4 + c^4
*g^3*h^5)*a*b^3 + (d^4*g^8 - 4*c*d^3*g^7*h + 6*c^2*d^2*g^6*h^2 - 4*c^3*d*g
^5*h^3 + c^4*g^4*h^4)*b^4) - ((11*d^3*e*g^2*h^2*n - 7*c*d^2*e*g*h^3*n + 2*
c^2*d*e*h^4*n)*a^3 - (31*d^3*e*g^3*h*n - 15*c*d^2*e*g^2*h^2*n + 2*c^3*e*h^
4*n)*a^2*b + (26*d^3*e*g^4*n - 15*c^2*d*e*g^2*h^2*n + 7*c^3*e*g*h^3*n)*a*b
^2 - (26*c*d^2*e*g^4*n - 31*c^2*d*e*g^3*h*n + 11*c^3*e*g^2*h^2*n)*b^3 + 6*
(3*a*b^2*d^3*e*g^2*h^2*n - 3*a^2*b*d^3*e*g*h^3*n + a^3*d^3*e*h^4*n - (3*c*
d^2*e*g^2*h^2*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*b^3)*x^2 + 3*((5*d^3*e*
g*h^3*n - c*d^2*e*h^4*n)*a^3 - 3*(5*d^3*e*g^2*h^2*n - c*d^2*e*g*h^3*n)*a^2
*b + (14*d^3*e*g^3*h*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*a*b^2 - (14*c*d^
2*e*g^3*h*n - 15*c^2*d*e*g^2*h^2*n + 5*c^3*e*g*h^3*n)*b^3)*x)/((d^3*g^6...
```

3.302.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3325 vs. $2(375) = 750$.

Time = 2.55 (sec) , antiderivative size = 3325, normalized size of antiderivative = 8.55

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="giac"
)
```

output
```
1/4*B*b^5*n*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2
*h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) - 1/4*B*d^5*n*log(abs(d*x + c))/(d^5*g
^4*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5)
- 1/4*B*n*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*
x + g^4*h) + 1/4*B*n*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 +
 4*g^3*h^2*x + g^4*h) + 1/4*(4*B*b^4*c*d^3*g^3*n - 4*B*a*b^3*d^4*g^3*n - 6
*B*b^4*c^2*d^2*g^2*h*n + 6*B*a^2*b^2*d^4*g^2*h*n + 4*B*b^4*c^3*d*g*h^2*n -
 4*B*a^3*b*d^4*g*h^2*n - B*b^4*c^4*h^3*n + B*a^4*d^4*h^3*n)*log(h*x + g)/(
b^4*d^4*g^8 - 4*b^4*c*d^3*g^7*h - 4*a*b^3*d^4*g^7*h + 6*b^4*c^2*d^2*g^6*h^
2 + 16*a*b^3*c*d^3*g^6*h^2 + 6*a^2*b^2*d^4*g^6*h^2 - 4*b^4*c^3*d*g^5*h^3 -
 24*a*b^3*c^2*d^2*g^5*h^3 - 24*a^2*b^2*c*d^3*g^5*h^3 - 4*a^3*b*d^4*g^5*h^3
 + b^4*c^4*g^4*h^4 + 16*a*b^3*c^3*d*g^4*h^4 + 36*a^2*b^2*c^2*d^2*g^4*h^4 +
 16*a^3*b*c*d^3*g^4*h^4 + a^4*d^4*g^4*h^4 - 4*a*b^3*c^4*g^3*h^5 - 24*a^2*b
^2*c^3*d*g^3*h^5 - 24*a^3*b*c^2*d^2*g^3*h^5 - 4*a^4*c*d^3*g^3*h^5 + 6*a^2*
b^2*c^4*g^2*h^6 + 16*a^3*b*c^3*d*g^2*h^6 + 6*a^4*c^2*d^2*g^2*h^6 - 4*a^3*b
*c^4*g*h^7 - 4*a^4*c^3*d*g*h^7 + a^4*c^4*h^8) - 1/24*(18*B*b^3*c*d^2*g^2*h
^4*n*x^3 - 18*B*a*b^2*d^3*g^2*h^4*n*x^3 - 18*B*b^3*c^2*d*g*h^5*n*x^3 + 18*
B*a^2*b*d^3*g*h^5*n*x^3 + 6*B*b^3*c^3*h^6*n*x^3 - 6*B*a^3*d^3*h^6*n*x^3 +
60*B*b^3*c*d^2*g^3*h^3*n*x^2 - 60*B*a*b^2*d^3*g^3*h^3*n*x^2 - 63*B*b^3*c^2
*d*g^2*h^4*n*x^2 + 63*B*a^2*b*d^3*g^2*h^4*n*x^2 + 21*B*b^3*c^3*g*h^5*n*...
```

3.302.9 Mupad [B] (verification not implemented)

Time = 10.63 (sec) , antiderivative size = 2570, normalized size of antiderivative = 6.61

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^5,x)
```

output
```
((x*(13*B*a^3*d^3*g^2*h^4*n - 13*B*b^3*c^3*g^2*h^4*n - B*a^2*b*c^3*h^6*n +
 B*a^3*c^2*d*h^6*n + 5*B*a*b^2*c^3*g*h^5*n - 5*B*a^3*c*d^2*g*h^5*n + 34*B*
a*b^2*d^3*g^4*h^2*n - 38*B*a^2*b*d^3*g^3*h^3*n - 34*B*b^3*c*d^2*g^4*h^2*n
+ 38*B*b^3*c^2*d*g^3*h^3*n - 12*B*a*b^2*c^2*d*g^2*h^4*n + 12*B*a^2*b*c*d^2
*g^2*h^4*n))/(3*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3
*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c
*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h
^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 -
 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (6*A*a^3*c^3*h^6 + 6*A*
b^3*d^3*g^6 - 6*A*a^3*d^3*g^3*h^3 - 6*A*b^3*c^3*g^3*h^3 + 18*A*a*b^2*c^3*g
^2*h^4 + 18*A*a^2*b*d^3*g^4*h^2 + 18*A*a^3*c*d^2*g^2*h^4 + 18*A*b^3*c^2*d*
g^4*h^2 - 11*B*a^3*d^3*g^3*h^3*n + 11*B*b^3*c^3*g^3*h^3*n - 18*A*a^2*b*c^3
*g*h^5 - 18*A*a*b^2*d^3*g^5*h - 18*A*a^3*c^2*d*g*h^5 - 18*A*b^3*c*d^2*g^5*
h + 2*B*a^2*b*c^3*g*h^5*n - 26*B*a*b^2*d^3*g^5*h*n - 2*B*a^3*c^2*d*g*h^5*n
 + 26*B*b^3*c*d^2*g^5*h*n + 54*A*a*b^2*c*d^2*g^4*h^2 - 54*A*a*b^2*c^2*d*g^
3*h^3 - 54*A*a^2*b*c*d^2*g^3*h^3 + 54*A*a^2*b*c^2*d*g^2*h^4 - 7*B*a*b^2*c^
3*g^2*h^4*n + 31*B*a^2*b*d^3*g^4*h^2*n + 7*B*a^3*c*d^2*g^2*h^4*n - 31*B*b^
3*c^2*d*g^4*h^2*n + 15*B*a*b^2*c^2*d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*
n)/(6*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a
^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^...
```

3.302. $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx$

3.303 $\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

3.303.1 Optimal result

Integrand size = 33, antiderivative size = 570

$$\begin{aligned}
&\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx \\
&= \frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2} + \frac{B^2(bc-ad)^3h^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3} + \frac{B^2(bc-ad)^3h^2n^2\log(c+dx)}{3b^3d^3} \\
&\quad + \frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log(c+dx)}{3b^3d^3} \\
&\quad - \frac{2B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3b^3d^2} \\
&\quad - \frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd^3} \\
&\quad + \frac{2B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+b\right.\right.}{3b^3d^3} \\
&\quad - \frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b^3h} \\
&\quad + \frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3h} \\
&\quad + \frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*h^2*n^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln((b*
x+a)/(d*x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B
^2*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^2*ln(d*x+c)/b^3/d^3-2/3*B*(-a
*d+b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^
n)))/b^3/d^2-1/3*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^
2-3*c*d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^
3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+2/3*B^2*(-a*d+b*c)
*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*
polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

3.303.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 906, normalized size of antiderivative = 1.59

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= \frac{-aB^2d^3(3b^2g^2-3abgh+a^2h^2)\,n^2\log^2(a+bx)+Bn\log(a+bx)\left(2b^3Bc(3d^2g^2-3cdgh+c^2h^2)\,n\log(c-\right.}{}$$

input
```
Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(-(a*B^2*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*n^2*Log[a + b*x]^2) + B*n*L
og[a + b*x]*(2*b^3*B*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*n*Log[c + d*x] +
2*B*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 -
3*c*d*g*h + c^2*h^2))*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*A*d^2*(3*b
^2*g^2 - 3*a*b*g*h + a^2*h^2) + B*(-3*a^2*d^2*h^2 + a*b*d*h*(6*d*g + c*h)
+ 2*b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n + 2*B*d^2*(3*b^2*g^2 - 3*a*b*
g*h + a^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n])) + b*(-(b^2*B^2*c*(3*d^2*
g^2 - 3*c*d*g*h + c^2*h^2)*n^2*Log[c + d*x]^2) + B*n*Log[c + d*x]*(-2*A*b^
2*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2) + B*(2*a^2*c*d^2*h^2 - 3*b^2*c^2*h*(
-2*d*g + c*h) + a*b*d*(-6*d^2*g^2 - 6*c*d*g*h + c^2*h^2))*n - 2*b^2*B*c*(3
*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(a^2
*B*d^2*h^2*n*(-2*A + B*n)*x + a*b*B*n*(A*d^2*(-6*g^2 + 6*g*h*x + h^2*x^2)
- 2*B*n*(3*d^2*g^2 + c^2*h^2 + c*d*h*(-3*g + h*x))) + b^2*x*(B^2*c^2*h^2*n
^2 + A^2*d^2*(3*g^2 + 3*g*h*x + h^2*x^2) + A*B*c*h*n*(2*c*h - d*(6*g + h*x
))) + B*(-2*a^2*B*d^2*h^2*n*x + a*b*B*d^2*n*(-6*g^2 + 6*g*h*x + h^2*x^2) +
 b^2*x*(B*c*h*n*(-6*d*g + 2*c*h - d*h*x) + 2*A*d^2*(3*g^2 + 3*g*h*x + h^2*
x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + b^2*B^2*d^2*x*(3*g^2 + 3*g*h*x +
 h^2*x^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)) + 2*B^2*(3*a*b^2*d^3*g^2 -
3*a^2*b*d^3*g*h + a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n
^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b^3*d^3)
```

3.303.3 Rubi [A] (warning: unable to verify)

Time = 1.21 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2973}$$

$$\int (g+hx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$\downarrow$ 2798

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(c+dx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3h(bc-ad)}\right.$$

$\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\left(\frac{(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)h^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad)^2}{}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(-\frac{h(bc-ad)(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh}{b}\right.}{}\right.$$

input

```
Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

3.303. $\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

output
```
(b*c - a*d)*(((b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)*h*(b - (d*(a + b*x))/(c + d
*x))^3) - (2*B*n*(-1/2*(B*(b*c - a*d)^3*h^3*n)/(b^2*d^3*(b - (d*(a + b*x))
/(c + d*x))) + ((b*c - a*d)^3*h^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(2*b*d^3*(b - (d*(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*h^2*(3*b*d*g -
2*b*c*h - a*d*h)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*d^
2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*g - a*h)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])^2)/(2*b^3*B*n) - (B*(b*c - a*d)^3*h^3*n*Log[(a +
b*x)/(c + d*x)])/(2*b^3*d^3) + (B*(b*c - a*d)^3*h^3*n*Log[b - (d*(a + b*x)
)/(c + d*x)])/(2*b^3*d^3) + (B*(b*c - a*d)^2*h^2*(3*b*d*g - 2*b*c*h - a*d*
h)*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^3*d^3) - ((b*c - a*d)*h*(a^2*d^2
*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(
b^3*d^3) - (B*(b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*
d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
/(b^3*d^3)))/(3*(b*c - a*d)*h))
```

3.303.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.303. $\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

3.303.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 90.91 (sec) , antiderivative size = 8443, normalized size of antiderivative = 14.81

method	result	size
risch	Expression too large to display	8443

input
```
int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.303.5 Fricas [F]

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \int (hx+g)^2\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 \, dx$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*h^2*x^2 + 2*A^2*g*h*x + A^2*g^2 + (B^2*h^2*x^2 + 2*B^2*g*h*x
+ B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*h^2*x^2 + 2*A*B*g*h*x
 + A*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.303.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.303.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1671 vs. $2(549) = 1098$.

Time = 0.76 (sec) , antiderivative size = 1671, normalized size of antiderivative = 2.93

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="max
ima")
```

output
```
2/3*A*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*h^2*x^3 + 2*A*B*g
*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*g*h*x^2 + 2*A*B*g^2*x*log((b*x
 + a)^n*e/(d*x + c)^n) + A^2*g^2*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d
*x + c)/d)*A*B*g^2/e - 2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/
d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*g*h/e + 1/3*(2*a^3*e*n*log(b*x + a)
/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(
b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*h^2/e + 1/3*(2*a^2*c*d^2*h^2*
n^2 - (6*c*d^2*g*h*n^2 - c^2*d*h^2*n^2)*a*b - (6*c*d^2*g^2*n*log(e) + (3*h
^2*n^2 + 2*h^2*n*log(e))*c^3 - 6*(g*h*n^2 + g*h*n*log(e))*c^2*d)*b^2)*B^2*
log(d*x + c)/(b^2*d^3) + 2/3*(3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 +
a^3*d^3*h^2*n^2 - (3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*b^3)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*
c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^3*h^2*x^3*log(e)^2 + 2*(3*c*d^2*
g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^2*b^3*log(b*x + a)*log(d*x + c)
 - (3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^2*b^3*log(d*x + c)^
2 + (a*b^2*d^3*h^2*n*log(e) - (c*d^2*h^2*n*log(e) - 3*d^3*g*h*log(e)^2)*b^
3)*B^2*x^2 - (3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^2*n^2)
*B^2*log(b*x + a)^2 + ((h^2*n^2 - 2*h^2*n*log(e))*a^2*b*d^3 - 2*(c*d^2*h^2
*n^2 - 3*d^3*g*h*n*log(e))*a*b^2 - (6*c*d^2*g*h*n*log(e) - 3*d^3*g^2*log(e
)^2 - (h^2*n^2 + 2*h^2*n*log(e))*c^2*d)*b^3)*B^2*x - ((3*h^2*n^2 - 2*h^...
```

3.303.8 Giac [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="gia
c")
```

output
```
Timed out
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$$
$$= \int (g+h\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 dx$$

input
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

output
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.304 $\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$

3.304.1 Optimal result

Integrand size = 31, antiderivative size = 294

$$\begin{aligned}
&\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx \\
&= \frac{B^2(bc-ad)^2hn^2\log(c+dx)}{b^2d^2} \\
&\quad - \frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d} \\
&\quad + \frac{B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2} \\
&\quad - \frac{(bg-ah)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2h} \\
&\quad + \frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h} \\
&\quad + \frac{B^2(bc-ad)(2bdg-bch-adh)n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

output
```
B^2*(-a*d+b*c)^2*h*n^2*ln(d*x+c)/b^2/d^2-B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))/b^2/d+B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((
-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^2/d^2-1/2*(-a*h+b
*g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b
*x+a)^n/((d*x+c)^n)))^2/h+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*polylo
g(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

3.304.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.61

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$=\frac{aB^2d^2(-2bg+ah)n^2\log^2(a+bx)-2Bn\log(a+bx)\left(b^2Bc(-2dg+ch)n\log(c+dx)-B(bc-ad)(-2b\right.}{}$$

input
```
Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(a*B^2*d^2*(-2*b*g + a*h)*n^2*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*(b^2*B*c
*(-2*d*g + c*h)*n*Log[c + d*x] - B*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*
n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(A*(-2*b*d*g + a*d*h) + B*(-2*b*d*g
 + b*c*h - a*d*h)*n + B*d*(-2*b*g + a*h)*Log[(e*(a + b*x)^n)/(c + d*x)^n])
) + b*(b*B^2*c*(-2*d*g + c*h)*n^2*Log[c + d*x]^2 + 2*B*n*Log[c + d*x]*(A*b
*c*(-2*d*g + c*h) + B*(b*c^2*h - a*d*(2*d*g + c*h))*n + b*B*c*(-2*d*g + c*
h)*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(A*b*x*(2*A*d*g - 2*B*c*h*n + A*d
*h*x) + 2*a*B*n*(-2*A*d*g - 2*B*d*g*n + B*c*h*n + A*d*h*x) + 2*B*(a*B*d*n*
(-2*g + h*x) + b*x*(2*A*d*g - B*c*h*n + A*d*h*x))*Log[(e*(a + b*x)^n)/(c +
 d*x)^n] + b*B^2*d*x*(2*g + h*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)) + 2*
B^2*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*n^2*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)])/(2*b^2*d^2)
```

3.304.3 Rubi [A] (warning: unable to verify)

Time = 0.86 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2\,dx$$
$$\Big\downarrow \text{2973}$$
$$\int (g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2dx$$
$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \frac{(c+dx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{h(bc-ad)}\right.$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)h^2}{bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{(bc-ad)(2\ldots}{}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(-\frac{h(bc-ad)(-adh-bch+2bdg)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\ldots\right.}{b^2d^2}\right.}{}\right.$$

input
```
Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(b*c - a*d)*(((b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)*h*(b - (d*(a + b*x))/(c + d
*x))^2) - (B*n*(((b*c - a*d)^2*h^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*g - a*h)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*B*n) + (B*(b*c - a*d)^2
*h^2*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b^2*d^2) - ((b*c - a*d)*h*(2*b*d
*g - b*c*h - a*d*h)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a +
 b*x))/(b*(c + d*x))])/(b^2*d^2) - (B*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d
*h)*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/((b*c - a*d)*h)
)
```

3.304. $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.304.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

3.304.5 Fricas [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2\,dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*h*x + A^2*g + (B^2*h*x + B^2*g)*log((b*x + a)^n*e/(d*x + c)^n
)^2 + 2*(A*B*h*x + A*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.304.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.304.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(289) = 578$.

Time = 0.71 (sec) , antiderivative size = 903, normalized size of antiderivative = 3.07

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$$
$$= ABhx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}A^2hx^2+2\,ABgx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^2gx$$
$$+\frac{2\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)ABg}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2en\log(dx+c)}{d^2}+\frac{(bcen-aden)x}{bd}\right)ABh}{e}$$
$$-\frac{(acdhn^2+(2\,cdgn\log(e)-(hn^2+hn\log(e))c^2)b)B^2\log(dx+c)}{bd^2}$$
$$+\frac{(2\,abd^2gn^2-a^2d^2hn^2-(2\,cdgn^2-c^2hn^2)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2}$$
$$+\frac{B^2b^2d^2hx^2\log(e)^2+2\,(2\,cdgn^2-c^2hn^2)B^2b^2\log(bx+a)\log(dx+c)-(2\,cdgn^2-c^2hn^2)B^2b^2\log(dx}{}$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxim
a")
```

output
```
A*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*h*x^2 + 2*A*B*g*x*log((
b*x + a)^n*e/(d*x + c)^n) + A^2*g*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(
d*x + c)/d)*A*B*g/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2
 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*h/e - (a*c*d*h*n^2 + (2*c*d*g*n*log(e)
 - (h*n^2 + h*n*log(e))*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*g*n^
2 - a^2*d^2*h*n^2 - (2*c*d*g*n^2 - c^2*h*n^2)*b^2)*(log(b*x + a)*log((b*d*
x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^
2) + 1/2*(B^2*b^2*d^2*h*x^2*log(e)^2 + 2*(2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2
*log(b*x + a)*log(d*x + c) - (2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(d*x + c
)^2 - (2*a*b*d^2*g*n^2 - a^2*d^2*h*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*h*
n*log(e) - (c*d*h*n*log(e) - d^2*g*log(e)^2)*b^2)*B^2*x + 2*((h*n^2 - h*n*
log(e))*a^2*d^2 - (c*d*h*n^2 - 2*d^2*g*n*log(e))*a*b)*B^2*log(b*x + a) + (
B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*h
*x^2 + 2*B^2*b^2*d^2*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*h*x^2*log(e)
 - (2*c*d*g*n - c^2*h*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n -
2*d^2*g*log(e))*b^2)*B^2*x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a
))*log((b*x + a)^n) - 2*(B^2*b^2*d^2*h*x^2*log(e) - (2*c*d*g*n - c^2*h*n)*
B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^2*
x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*h*x^2 +
2*B^2*b^2*d^2*g*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

3.304.8 Giac [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac"
)
```

output
```
integrate((h*x + g)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2 dx$$

input
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

output
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.305 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx$

3.305.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx = \frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{bd} + \frac{(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{b} + \frac{2B^2(bc-ad)n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

output
```
2*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))
)/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+2*B^2*(-a*d+b*c)*n^2*p
olylog(2,d*(b*x+a)/b/(d*x+c))/b/d
```

3.305.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.58

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx = \frac{A^2bdx-2AB(bc-ad)n\log(c+dx)+2ABd(a+bx)\log(e(a+bx)^n(c+dx)^{-n})+B^2d(a+bx)\log^2(e(a$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

output
```
(A^2*b*d*x - 2*A*B*(b*c - a*d)*n*Log[c + d*x] + 2*A*B*d*(a + b*x)*Log[(e*(
a + b*x)^n)/(c + d*x)^n] + B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n
]^2 + B^2*(b*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a +
 b*x))/(-(b*c) + a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c -
 a*d)/(b*c + b*d*x)])) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*d)
```

3.305.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2936, 2942, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{ 2936}$$

$$\frac{(a+bx)\left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{c+dx}dx}{b}$$

$$\Big\downarrow \text{ 2942}$$

$$\frac{(a+bx)\left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{(a+bx)\left(B \log \left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$\Big\downarrow$ 25

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(-\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$\Big\downarrow$ 2778

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$\Big\downarrow$ 2005

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{d}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}\right)}{b}$$

$\Big\downarrow$ 2752

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{2Bn(bc-ad)\left(-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{d}-\frac{Bn\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{d}\right)}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

3.305. $\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

output
```
((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/b - (2*B*(b*c - a*d
)*n*(-((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x
)^n]))/d) - (B*n*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/d))/b
```

3.305.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

3.305.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.85 (sec) , antiderivative size = 2240, normalized size of antiderivative = 16.35

method	result	size
risch	Expression too large to display	2240

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*n^2*B^2*c/d+(-2*x*B^2*ln((b*x+a)^n)-B*(-I*B*Pi*b*d*x*csgn(I*e)*csgn(I*(
b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*b*d*x*csgn(I*
e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b*d*x*csgn(I*(b*x+a)^n)*csgn(I
/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*b*d*x*csgn(I*(b*x+a)^n)
*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*d*x*csgn(I/((d*x+c)^n))*csgn(I*(
b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*b*d*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*P
i*b*d*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*
B*Pi*b*d*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)*b*d*x-2*B*ln(d*x+c)
*b*c*n+2*B*a*d*n*ln(b*x+a)+2*A*b*d*x)/b/d)*ln((d*x+c)^n)+B^2/b*ln((b*x+a)^
n)^2*a+x*B^2*ln((d*x+c)^n)^2-2*B^2/b*ln((b*x+a)^n)*a*n+1/4*x*(-I*B*Pi*csgn
(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi
*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*(b*x+a)^n)*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*csgn(I*(b*x+a)^n)*csg
n(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/(
(d*x+c)^n))^2-I*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*csgn(I*(b*x+a)
^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*e/((d*x+c)
^n)*(b*x+a)^n)^3+2*B*ln(e)+2*A)^2+x*B^2*ln((b*x+a)^n)^2+B*(-I*B*Pi*csgn(I*
e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*cs
gn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*csgn(I*(b*x+a)^n)*csg...
```

3.305.5 Fricas [F]

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx=\int \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas")
```

output
```
integral(B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2, x)
```

3.305.6 Sympy [F(-2)]

Exception generated.

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.305.7 Maxima [F]

$$\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx=\int \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima")
```

output
```
2*A*B*x*log((b*x + a)^n*e/(d*x + c)^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x +
 a)*log(d*x + c) - b*c*n^2*log(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d
*x*log((d*x + c)^n)^2 + 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x
*log(e))*log((b*x + a)^n) - 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b
*d*x*log((b*x + a)^n) + b*d*x*log(e))*log((d*x + c)^n))/(b*d) - integrate(
-(b^2*d*x^2*log(e)^2 + a*b*c*log(e)^2 - ((2*n*log(e) - log(e)^2)*b^2*c - (
2*n*log(e) + log(e)^2)*a*b*d)*x - 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2
)*log(b*x + a))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)) + 2*(a*e*n*lo
g(b*x + a)/b - c*e*n*log(d*x + c)/d)*A*B/e
```

3.305.8 Giac [F]

$$\int \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx=\int \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 \, dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx=\int \left(A+B\,\ln \left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2 \,d x$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.306 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{g+hx}\,dx$

3.306.1 Optimal result

Integrand size = 33, antiderivative size = 301

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{h} \\ &\quad+\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\ &\quad-\frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\ &\quad+\frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\ &\quad+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{h}-\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}\end{aligned}$$

output
```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+(A+B*ln(e*
(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-2*
B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/h+2*B
*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/h+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/h-2*B^2*n^2*polylog
(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

3.306.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1082 vs. $2(301) = 602$.

Time = 0.26 (sec) , antiderivative size = 1082, normalized size of antiderivative = 3.59

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx$$
$$= \frac{(A + B(-n\log(a+bx) + n\log(c+dx) + \log(e(a+bx)^n(c+dx)^{-n})))^2\log(g+hx) + 2Bn(A + B(-n\log(a+bx)}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

output
```
((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x
)^n]))^2*Log[g + h*x] + 2*B*n*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] +
 Log[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g -
 a*h)] + PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - 2*A*B*n*(Log[c + d*x]
*Log[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]
) - 2*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c +
 d*x)^n])*(Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c
+ d*x))/(-(d*g) + c*h)]) + B^2*n^2*(Log[a + b*x]^2*Log[(b*(g + h*x))/(b*g
- a*h)] + 2*Log[a + b*x]*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] - 2*Poly
Log[3, (h*(a + b*x))/(-(b*g) + a*h)]) + B^2*n^2*(Log[c + d*x]^2*Log[(d*(g
+ h*x))/(d*g - c*h)] + 2*Log[c + d*x]*PolyLog[2, (h*(c + d*x))/(-(d*g) + c
*h)] - 2*PolyLog[3, (h*(c + d*x))/(-(d*g) + c*h)]) - 2*B^2*n^2*(Log[a + b*
x]*Log[c + d*x]*Log[(b*(g + h*x))/(b*g - a*h)] + (Log[(h*(c + d*x))/(-(d*g
) + c*h)]*(-2*Log[a + b*x] + Log[(h*(c + d*x))/(-(d*g) + c*h)])*(Log[(b*(g
 + h*x))/(b*g - a*h)] - Log[(d*(g + h*x))/(d*g - c*h)]))/2 + Log[(h*(c + d
*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*
(-Log[(b*(g + h*x))/(b*g - a*h)] + Log[(d*(g + h*x))/(d*g - c*h)]) + (Log[
((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]^2*(Log[(-(b*c) + a*d)/(d*
(a + b*x))] + Log[(b*(g + h*x))/(b*g - a*h)] - Log[((-(b*c) + a*d)*(g + h*
x))/((d*g - c*h)*(a + b*x))]))/2 + (Log[c + d*x] - Log[((b*g - a*h)*(c ...
```

3.306.3 Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2973, 2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{g+hx}dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{g+hx}dx$$

$$\Big\downarrow \text{ 2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2804}$$

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)h\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(ch-dg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)h\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2009}$$

$$(bc-ad)\left(\frac{2Bn\,\text{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{h(bc-ad)}+\frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{h(bc-ad)}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/((b*c - a*d)*h)) + ((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*c -
a*d)*h) - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a +
 b*x))/(b*(c + d*x))])/((b*c - a*d)*h) + (2*B*n*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/
((b*c - a*d)*h) + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*
c - a*d)*h) - (2*B^2*n^2*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(
c + d*x))])/((b*c - a*d)*h))
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.306. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{g+hx}\,dx$

3.306.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{hx+g}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

3.306.5 Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{2}}{g+hx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="frica
s")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(h*x + g), x)
```

3.306.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{2}}{g+hx}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.306.7 Maxima [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="maxim
a")
```

output
```
A^2*log(h*x + g)/h + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)
^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n
) - 2*(B^2*log((b*x + a)^n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(h*x + g
), x)
```

3.306.8 Giac [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{g+h\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x), x)
```

3.307 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx$

3.307.1 Optimal result

Integrand size = 33, antiderivative size = 208

$$\begin{aligned}
\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx
&= \frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bg-ah)(g+hx)} \\
&\quad + \frac{2B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} \\
&\quad + \frac{2B^2(bc-ad)n^2\,\mathrm{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}
\end{aligned}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g)/(h*x+g)+2*B*(-a*d+b
*c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)
/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*h+d*g)*
(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)
```

3.307.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3460 vs. $2(208) = 416$.

Time = 0.56 (sec) , antiderivative size = 3460, normalized size of antiderivative = 16.63

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

output
```
(-(A^2*b*d*g^2) + A^2*b*c*g*h + a*A^2*d*g*h - a*A^2*c*h^2 + 2*A*b*B*d*g^2*
n*Log[a + b*x] - 2*A*b*B*c*g*h*n*Log[a + b*x] + 2*A*b*B*d*g*h*n*x*Log[a +
b*x] - 2*A*b*B*c*h^2*n*x*Log[a + b*x] - b*B^2*d*g^2*n^2*Log[a + b*x]^2 + b
*B^2*c*g*h*n^2*Log[a + b*x]^2 - b*B^2*d*g*h*n^2*x*Log[a + b*x]^2 + b*B^2*c
*h^2*n^2*x*Log[a + b*x]^2 - 2*A*b*B*d*g^2*n*Log[c + d*x] + 2*a*A*B*d*g*h*n
*Log[c + d*x] - 2*A*b*B*d*g*h*n*x*Log[c + d*x] + 2*a*A*B*d*h^2*n*x*Log[c +
 d*x] + 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*d*g*h*n^2*Lo
g[a + b*x]*Log[c + d*x] + 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[c + d*x] -
2*a*B^2*d*h^2*n^2*x*Log[a + b*x]*Log[c + d*x] - b*B^2*d*g^2*n^2*Log[c + d*
x]^2 + a*B^2*d*g*h*n^2*Log[c + d*x]^2 - b*B^2*d*g*h*n^2*x*Log[c + d*x]^2 +
 a*B^2*d*h^2*n^2*x*Log[c + d*x]^2 - 2*b*B^2*c*g*h*n^2*Log[a + b*x]*Log[(h*
(c + d*x))/(-(d*g) + c*h)] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*
x))/(-(d*g) + c*h)] - 2*b*B^2*c*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(
-(d*g) + c*h)] + 2*a*B^2*d*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g
) + c*h)] + b*B^2*c*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 - a*B^2*d*
g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + b*B^2*c*h^2*n^2*x*Log[(h*(c
+ d*x))/(-(d*g) + c*h)]^2 - a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) +
c*h)]^2 - 2*b*B^2*c*g*h*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g -
a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*a*B^2*d*g*h*n^2*Log[(-(b*c) +
 a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)...
```

3.307.3 Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2953, 2755, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)^2}dx$$

$$\Big\downarrow \ 2973$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)^2}dx$$

$$\Big\downarrow \ 2953$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2755$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{2Bn\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bg-ah}\right)$$

$$\Big\downarrow \ 2754$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{2Bn\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{dg-ch}-\frac{\log\left(1-\frac{(a+bx)(dg-c}{(c+dx)(bg-a}\right.}{}\right.}{bg-ah}\right.$$

$$\Big\downarrow \ 2838$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{2Bn\left(-\frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg-ch}-\frac{Bn\,\text{PolyLo}}{}\right.}{bg-ah}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

3.307. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}dx$

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*g -
a*h)*(c + d*x)*(b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))) - (2*B*n*(
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*g - c*h)*(a + b*x))/
((b*g - a*h)*(c + d*x))])/(d*g - c*h)) - (B*n*PolyLog[2, ((d*g - c*h)*(a +
 b*x))/((b*g - a*h)*(c + d*x))])/(d*g - c*h)))/(b*g - a*h))
```

3.307.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.307.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{\left(hx+g\right)^{2}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

3.307.5 Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{2}}{(g+hx)^{2}}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="fri
cas")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

3.307.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{2}}{(g+hx)^{2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**2,x)
```

output
```
Timed out
```

3.307.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="max
ima")
```

output
```
-B^2*(log((d*x + c)^n)^2/(h^2*x + g*h) + integrate(-(d*h*x*log(e)^2 + c*h*
log(e)^2 + (d*h*x + c*h)*log((b*x + a)^n)^2 + 2*(d*h*x*log(e) + c*h*log(e)
)*log((b*x + a)^n) + 2*(d*g*n + (h*n - h*log(e))*d*x - c*h*log(e) - (d*h*x
 + c*h)*log((b*x + a)^n))*log((d*x + c)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*
h^2 + c*h^3)*x^2 + (d*g^2*h + 2*c*g*h^2)*x), x)) + 2*(b*e*n*log(b*x + a)/(
b*g*h - a*h^2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*
log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A*B/e - 2*A*B*log((b
*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A^2/(h^2*x + g*h)
```

3.307.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^2, x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(g+h\,x)^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2, x)
```

3.308 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^3}\,dx$

3.308.1 Optimal result

Integrand size = 33, antiderivative size = 393

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^3}\,dx \\
&= \frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bg-ah)^2(dg-ch)(g+hx)} \\
&\quad + \frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(bg-ah)^2} \\
&\quad - \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(g+hx)^2} + \frac{B^2(bc-ad)^2hn^2\log\left(\frac{g+hx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad + \frac{B(bc-ad)(2bdg-bch-adh)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad + \frac{B^2(bc-ad)(2bdg-bch-adh)n^2\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}
\end{aligned}$$

output
```
B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*h+b*g)^2/(-
c*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(-a*h+b*g)^
2-1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(h*x+g)^2+B^2*(-a*d+b*c)^2*h*n
^2*ln((h*x+g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B*(-a*d+b*c)*(-a*d*h-b*c*
h+2*b*d*g)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a
*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(
-c*h+d*g)^2
```

3.308.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 13182 vs. 2(393) = 786.

Time = 5.23 (sec) , antiderivative size = 13182, normalized size of antiderivative = 33.54

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

output
```
Result too large to show
```

3.308.3 Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)^3}\,dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{h(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2h(bc-ad)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2}\right)$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{Bn\int\left(\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{(bg-ah)^2(a+bx)}+\frac{(bc-ad)h(-2bdg+bch+adh)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bg-ah)^2(dg-ch)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2h^2\left(A+B\log\left(e\left(\frac{a\cdots}{c\cdots}\right.\right.\right.}{(bg-ah)(dg-ch)\left(bg-ah-\frac{(dg-\cdots}{}\right.}\right.}{h(bc-ad)}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bn(bg-ah)^2}+\frac{h^2(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(c+dx)(bg-ah)^2(dg-ch)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}+\frac{h(bc-ad)(-adh-bch+2bdg)\log\left(1-\frac{(a+b\cdots}{(c+d\cdots}\right.}{(bg-ah)^2(dg\cdots}\right.}{}\right.$$

input

```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

output

```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/((b*c - a*d)*h*(b*g - a*h - ((d*g - c*h)*(a + b*x))/(c
+ d*x))^2) + (B*n*(((b*c - a*d)^2*h^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/((b*g - a*h)^2*(d*g - c*h)*(c + d*x)*(b*g - a*h - ((d*g - c*
h)*(a + b*x))/(c + d*x))) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)
/(2*B*(b*g - a*h)^2*n) + (B*(b*c - a*d)^2*h^2*n*Log[b*g - a*h - ((d*g - c*
h)*(a + b*x))/(c + d*x)])/((b*g - a*h)^2*(d*g - c*h)^2) + ((b*c - a*d)*h*(
2*b*d*g - b*c*h - a*d*h)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((
d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)
^2) + (B*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*n*PolyLog[2, ((d*g - c*h)
*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)))/((b*
c - a*d)*h))
```

3.308. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^3}\,dx$

3.308.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.308.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{\left(hx+g\right)^{3}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

3.308.5 Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="fri
cas")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**3,x)
```

output
```
Timed out
```

3.308.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="max
ima")
```

output
```
-1/2*B^2*(log((d*x + c)^n)^2/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 2*integrate(-
(d*h*x*log(e)^2 + c*h*log(e)^2 + (d*h*x + c*h)*log((b*x + a)^n)^2 + 2*(d*h
*x*log(e) + c*h*log(e))*log((b*x + a)^n) + (d*g*n + (h*n - 2*h*log(e))*d*x
 - 2*c*h*log(e) - 2*(d*h*x + c*h)*log((b*x + a)^n))*log((d*x + c)^n))/(d*h
^4*x^4 + c*g^3*h + (3*d*g*h^3 + c*h^4)*x^3 + 3*(d*g^2*h^2 + c*g*h^3)*x^2 +
 (d*g^3*h + 3*c*g^2*h^2)*x), x)) + (b^2*e*n*log(b*x + a)/(b^2*g^2*h - 2*a*
b*g*h^2 + a^2*h^3) - d^2*e*n*log(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h
^3) - (2*a*b*d^2*e*g*n - a^2*d^2*e*h*n - (2*c*d*e*g*n - c^2*e*h*n)*b^2)*lo
g(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c
*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) +
 (b*c*e*n - a*d*e*n)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*
h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*A*B/e - A*B*log((b*x + a)^n*e/
(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*A^2/(h^3*x^2 + 2*g*h^2*x
+ g^2*h)
```

3.308.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2}{(hx+g)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^3, x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(g+h\,x)^3}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3, x)
```

3.309 $\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$

3.309.1 Optimal result

Integrand size = 33, antiderivative size = 875

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = -\frac{B^3(bc-ad)^3h^2n^3\log(c+dx)}{b^3d^3}$$
$$+\frac{B^2(bc-ad)^2h^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^2}$$
$$-\frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^3}$$
$$-\frac{B(bc-ad)h(3bdg-2bch-adh)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{b^3d^2}$$
$$-\frac{B(bc-ad)h^2n(c+dx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2bd^3}$$
$$+\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx\right.\right.}{b^3d^3}$$
$$-\frac{(bg-ah)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3b^3h}$$
$$+\frac{(g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3h}$$
$$-\frac{B^2(bc-ad)^3h^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$-\frac{2B^3(bc-ad)^2h(3bdg-2bch-adh)n^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$
$$+\frac{2B^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^2\left(A+B\log\left(e(a+bx)^n(c+dx\right.\right.}{b^3d^3}$$
$$+\frac{B^3(bc-ad)^3h^2n^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3d^3}$$
$$-\frac{2B^3(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n^3\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3d^3}$$

output
```
-B^3*(-a*d+b*c)^3*h^2*n^3*ln(d*x+c)/b^3/d^3+B^2*(-a*d+b*c)^2*h^2*n^2*(b*x+
a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^3/d^2-2*B^2*(-a*d+b*c)^2*h*(-a*d*h-
2*b*c*h+3*b*d*g)*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))/b^3/d^3-B*(-a*d+b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*
(b*x+a)^n/((d*x+c)^n)))^2/b^3/d^2-1/2*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln
(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h
+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))^3/b^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3
/h-B^2*(-a*d+b*c)^3*h^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+
c)/d/(b*x+a))/b^3/d^3-2*B^3*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^3*po
lylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3+2*B^2*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h
*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*(A+B*ln(e*(b*x+a)^n/(
(d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3+B^3*(-a*d+b*c)^3*h^2*n
^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d^3-2*B^3*(-a*d+b*c)*(a^2*d^2*h^2-a*
b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^3*polylog(3,d*(b*x
+a)/b/(d*x+c))/b^3/d^3
```

3.309.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7279 vs. $2(875) = 1750$.

Time = 2.67 (sec) , antiderivative size = 7279, normalized size of antiderivative = 8.32

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
Result too large to show
```

3.309.3 Rubi [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 1020, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3\,dx$$

$\Big\downarrow$ 2973

$$\int (g+hx)^2 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3\,dx$$

$\Big\downarrow$ 2953

$$(bc-ad)\int \frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2798

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\frac{(c+dx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{h(bc-ad)}\right.$$

$\Big\downarrow$ 2804

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2h^3}{bd^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{(bc-ad)^2}{}\right.}{}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)\left(\frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3(bc-ad)h\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2h^3}{2bd^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B(bc-ad)^3n(a}{b^3d}\right.}{}\right.$$

input
```
Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

3.309. $\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

output
```
(b*c - a*d)*(((b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^3)/(3*(b*c - a*d)*h*(b - (d*(a + b*x))/(c + d
*x))^3) - (B*n*(-((B*(b*c - a*d)^3*h^3*n*(a + b*x)*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(b^3*d^2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))) + ((b*
c - a*d)^3*h^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b*d^3*(b - (d*
(a + b*x))/(c + d*x))^2) + ((b*c - a*d)^2*h^2*(3*b*d*g - 2*b*c*h - a*d*h)*
(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^3*d^2*(c + d*x)*(b
- (d*(a + b*x))/(c + d*x))) + ((b*g - a*h)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^3)/(3*b^3*B*n) - (B^2*(b*c - a*d)^3*h^3*n^2*Log[b - (d*(a + b*x)
)/(c + d*x)])/(b^3*d^3) + (2*B*(b*c - a*d)^2*h^2*(3*b*d*g - 2*b*c*h - a*d*
h)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c +
d*x))])/(b^3*d^3) - ((b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) +
b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) + (B*(b*c - a*d)^3*h
^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/(b^3*d^3) + (2*B^2*(b*c - a*d)^2*h^2*(3*b*d*g - 2*b*c*h - a*d*h)*n
^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) - (2*B*(b*c - a*d)*h
*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h
^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/(b^3*d^3) - (B^2*(b*c - a*d)^3*h^3*n^2*PolyLog[2, (b*(c + ...
```

3.309.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.309.4 Maple [F]

$$\int (hx+g)^2 \left(A+B\ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

input
```
int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

3.309.5 Fricas [F]

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int (hx+g)^2 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fri
cas")
```

output
```
integral(A^3*h^2*x^2 + 2*A^3*g*h*x + A^3*g^2 + (B^3*h^2*x^2 + 2*B^3*g*h*x
+ B^3*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*h^2*x^2 + 2*A*B^2*g
*h*x + A*B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*h^2*x^2 + 2*
A^2*B*g*h*x + A^2*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.309.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.309.7 Maxima [F]

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$= \int (hx+g)^2 \left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="max
ima")
```

output
```
A^2*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*h^2*x^3 + 3*A^2*B*g
*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*h*x^2 + 3*A^2*B*g^2*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^3*g^2*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log
(d*x + c)/d)*A^2*B*g^2/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x +
 c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*g*h/e + 1/2*(2*a^3*e*n*log(b*
x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2
 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*h^2/e - 1/6*(2*(B^3*b
^3*d^3*h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((d*x + c
)^n)^3 + 3*(2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^3*b^3*log(d*x
+ c) - 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^3*log(b
*x + a) - 2*(B^3*b^3*d^3*h^2*log(e) + A*B^2*b^3*d^3*h^2)*x^3 - (6*A*B^2*b^
3*d^3*g*h + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h*log(e))*b^3)*B^3)*
x^2 - 2*(3*A*B^2*b^3*d^3*g^2 + (3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c
*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*log(e))*b^3)*B^3)*x - 2*(B^3*b^3*d^3*
h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((b*x + a)^n))*l
og((d*x + c)^n)^2)/(b^3*d^3) - integrate(-(B^3*b^3*c*d^2*g^2*log(e)^3 + 3*
A*B^2*b^3*c*d^2*g^2*log(e)^2 + (B^3*b^3*d^3*h^2*log(e)^3 + 3*A*B^2*b^3*d^3
*h^2*log(e)^2)*x^3 + (B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (2*d^3*g*h
 + c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*log((b*x +
a)^n)^3 + (3*(2*d^3*g*h*log(e)^2 + c*d^2*h^2*log(e)^2)*A*B^2*b^3 + (2*d...
```

3.309.8 Giac [F(-1)]

Timed out.

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="gia
c")
```

output
```
Timed out
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\int (g+h\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,dx$$

input
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

output
```
int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.310 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

3.310.1 Optimal result

Integrand size = 31, antiderivative size = 466

$$\begin{aligned}
&\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx \\
&= -\frac{3B^2(bc-ad)^2hn^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2} \\
&\quad -\frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d} \\
&\quad +\frac{3B(bc-ad)(2bdg-bch-adh)n\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b^2d^2} \\
&\quad -\frac{(bg-ah)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2b^2h} \\
&\quad +\frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h} \\
&\quad -\frac{3B^3(bc-ad)^2hn^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} \\
&\quad +\frac{3B^2(bc-ad)(2bdg-bch-adh)n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} \\
&\quad -\frac{3B^3(bc-ad)(2bdg-bch-adh)n^3\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}
\end{aligned}$$

output
```
-3*B^2*(-a*d+b*c)^2*h*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))/b^2/d^2-3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))^2/b^2/d+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((-a*d+b*c)/b/
(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/d^2-1/2*(-a*h+b*g)^2*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((
d*x+c)^n)))^3/h-3*B^3*(-a*d+b*c)^2*h*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^
2/d^2+3*B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*
x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2-3*B^3*(-a*d+b*c)*(-a*d*h-
b*c*h+2*b*d*g)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

3.310.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3890 vs. $2(466) = 932$.

Time = 0.94 (sec) , antiderivative size = 3890, normalized size of antiderivative = 8.35

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(-12*A*b^2*B^2*c*d*g*n^2 - 12*a*A*b*B^2*d^2*g*n^2 + 12*a*A*b*B^2*c*d*h*n^2
 + 6*a*b*B^3*c*d*h*n^3 - 6*a^2*B^3*d^2*h*n^3 + 2*A^3*b^2*d^2*g*x - 3*A^2*b
^2*B*c*d*h*n*x + 3*a*A^2*b*B*d^2*h*n*x + A^3*b^2*d^2*h*x^2 + 6*a*A^2*b*B*d
^2*g*n*Log[a + b*x] - 3*a^2*A^2*B*d^2*h*n*Log[a + b*x] - 6*a*A*b*B^2*c*d*h
*n^2*Log[a + b*x] + 6*a^2*A*B^2*d^2*h*n^2*Log[a + b*x] + 12*b^2*B^3*c*d*g*
n^3*Log[a + b*x] + 12*a*b*B^3*d^2*g*n^3*Log[a + b*x] - 12*a*b*B^3*c*d*h*n^
3*Log[a + b*x] - 6*a*A*b*B^2*d^2*g*n^2*Log[a + b*x]^2 + 3*a^2*A*B^2*d^2*h*
n^2*Log[a + b*x]^2 + 3*a*b*B^3*c*d*h*n^3*Log[a + b*x]^2 - 3*a^2*B^3*d^2*h*
n^3*Log[a + b*x]^2 + 2*a*b*B^3*d^2*g*n^3*Log[a + b*x]^3 - a^2*B^3*d^2*h*n^
3*Log[a + b*x]^3 - 6*A^2*b^2*B*c*d*g*n*Log[c + d*x] + 3*A^2*b^2*B*c^2*h*n*
Log[c + d*x] + 6*A*b^2*B^2*c^2*h*n^2*Log[c + d*x] - 6*a*A*b*B^2*c*d*h*n^2*
Log[c + d*x] - 12*b^2*B^3*c*d*g*n^3*Log[c + d*x] - 12*a*b*B^3*d^2*g*n^3*Lo
g[c + d*x] + 12*a*b*B^3*c*d*h*n^3*Log[c + d*x] + 12*A*b^2*B^2*c*d*g*n^2*Lo
g[a + b*x]*Log[c + d*x] + 12*a*A*b*B^2*d^2*g*n^2*Log[a + b*x]*Log[c + d*x]
 - 6*A*b^2*B^2*c^2*h*n^2*Log[a + b*x]*Log[c + d*x] - 6*a^2*A*B^2*d^2*h*n^2
*Log[a + b*x]*Log[c + d*x] - 6*b^2*B^3*c^2*h*n^3*Log[a + b*x]*Log[c + d*x]
 + 6*a*b*B^3*c*d*h*n^3*Log[a + b*x]*Log[c + d*x] - 6*b^2*B^3*c*d*g*n^3*Log
[a + b*x]^2*Log[c + d*x] - 12*a*b*B^3*d^2*g*n^3*Log[a + b*x]^2*Log[c + d*x
] + 3*b^2*B^3*c^2*h*n^3*Log[a + b*x]^2*Log[c + d*x] + 6*a^2*B^3*d^2*h*n^3*
Log[a + b*x]^2*Log[c + d*x] - 12*a*A*b*B^2*d^2*g*n^2*Log[(d*(a + b*x))/...
```

3.310.3 Rubi [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3\,dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int (g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3\,dx$$

$$\Big\downarrow \text{ 2953}$$

$$(bc-ad)\int \frac{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2798}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\int\frac{(c+dx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{2h(bc-ad)}\right.$$

$$\Big\downarrow \text{ 2804}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\int\left(\frac{(bg-ah)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2(a+bx)}+\frac{(bc}{}\right.}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$(bc-ad)\left(\frac{\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2h(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{3Bn\left(-\frac{2Bhn(bc-ad)(-adh-bch+2bdg)\operatorname{PolyLog}\left(2,\frac{d(a+b}{b(c+d}\right.}{b^2d^2}\right.}{}\right.$$

input
```
Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(b*c - a*d)*(((b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^3)/(2*(b*c - a*d)*h*(b - (d*(a + b*x))/(c + d
*x))^2) - (3*B*n*(((b*c - a*d)^2*h^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])^2)/(b^2*d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + ((b*g - a
*h)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*b^2*B*n) + (2*B*(b*c -
a*d)^2*h^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/
(b*(c + d*x))])/(b^2*d^2) - ((b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/
(b^2*d^2) + (2*B^2*(b*c - a*d)^2*h^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/(b^2*d^2) - (2*B*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*n*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(
b^2*d^2) + (2*B^2*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[3, (
d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)))/(2*(b*c - a*d)*h))
```

3.310.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.310.4 Maple [F]

$$\int (hx+g)\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}dx$$

input
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

3.310.5 Fricas [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)^{3}\,dx$$
$$=\int (hx+g)\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{3}dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="frica
s")
```

output
```
integral(A^3*h*x + A^3*g + (B^3*h*x + B^3*g)*log((b*x + a)^n*e/(d*x + c)^n
)^3 + 3*(A*B^2*h*x + A*B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*
h*x + A^2*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

3.310.6 Sympy [F(-2)]

Exception generated.

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.310.7 Maxima [F]

$$\int (g+hx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int (hx+g)\left(B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxim
a")
```

output
```
3/2*A^2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*h*x^2 + 3*A^2*B*g
*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*x + 3*(a*e*n*log(b*x + a)/b - c*
e*n*log(d*x + c)/d)*A^2*B*g/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*lo
g(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*h/e - 1/2*((B^3*b^2*d^
2*h*x^2 + 2*B^3*b^2*d^2*g*x)*log((d*x + c)^n)^3 + 3*((2*c*d*g*n - c^2*h*n)
*B^3*b^2*log(d*x + c) - (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^3*log(b*x + a) - (
B^3*b^2*d^2*h*log(e) + A*B^2*b^2*d^2*h)*x^2 - (2*A*B^2*b^2*d^2*g + (a*b*d^
2*h*n - (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^3)*x - (B^3*b^2*d^2*h*x^2 + 2*B^
3*b^2*d^2*g*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b^2*d^2) - integrate
(-(B^3*b^2*c*d*g*log(e)^3 + 3*A*B^2*b^2*c*d*g*log(e)^2 + (B^3*b^2*d^2*h*x^
2 + B^3*b^2*c*d*g + (d^2*g + c*d*h)*B^3*b^2*x)*log((b*x + a)^n)^3 + (B^3*b
^2*d^2*h*log(e)^3 + 3*A*B^2*b^2*d^2*h*log(e)^2)*x^2 + 3*(B^3*b^2*c*d*g*log
(e) + A*B^2*b^2*c*d*g + (B^3*b^2*d^2*h*log(e) + A*B^2*b^2*d^2*h)*x^2 + ((d
^2*g + c*d*h)*A*B^2*b^2 + (d^2*g*log(e) + c*d*h*log(e))*B^3*b^2)*x)*log((b
*x + a)^n)^2 + (3*(d^2*g*log(e)^2 + c*d*h*log(e)^2)*A*B^2*b^2 + (d^2*g*log
(e)^3 + c*d*h*log(e)^3)*B^3*b^2)*x + 3*(B^3*b^2*c*d*g*log(e)^2 + 2*A*B^2*b
^2*c*d*g*log(e) + (B^3*b^2*d^2*h*log(e)^2 + 2*A*B^2*b^2*d^2*h*log(e))*x^2
+ (2*(d^2*g*log(e) + c*d*h*log(e))*A*B^2*b^2 + (d^2*g*log(e)^2 + c*d*h*log
(e)^2)*B^3*b^2)*x)*log((b*x + a)^n) - 3*(B^3*b^2*c*d*g*log(e)^2 + 2*A*B^2*
b^2*c*d*g*log(e) - (2*c*d*g*n^2 - c^2*h*n^2)*B^3*b^2*log(d*x + c) + (2*...
```

3.310.8 Giac [F]

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$=\int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3 dx$$

input
```
integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac"
)
```

output
```
integrate((h*x + g)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$
$$=\int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3 dx$$

input
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

output
```
int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.311 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\,dx$

3.311.1 Optimal result

Integrand size = 25, antiderivative size = 203

$$\begin{aligned}
&\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\,dx \\
&= \frac{3B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{bd} \\
&\quad + \frac{(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{b} \\
&\quad + \frac{6B^2(bc-ad)n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd} \\
&\quad - \frac{6B^3(bc-ad)n^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}
\end{aligned}$$

output
```
3*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))
)^2/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+6*B^2*(-a*d+b*c)*n^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d-6*B^
3*(-a*d+b*c)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d
```

3.311.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.86

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$
$$=\frac{A^3bdx-3A^2B(bc-ad)n\log(c+dx)+3A^2Bd(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3AB^2d(a+bx)\log^2}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
(A^3*b*d*x - 3*A^2*B*(b*c - a*d)*n*Log[c + d*x] + 3*A^2*B*d*(a + b*x)*Log[
(e*(a + b*x)^n)/(c + d*x)^n] + 3*A*B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^2 + B^3*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + 3*A*B^2
*(b*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a + b*x))/(-
(b*c) + a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c - a*d)/(b*
c + b*d*x)])) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B^3*(b*c -
a*d)*n*(Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)]
+ 2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))] - 2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]))/(b*d)
```

3.311.3 Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2936, 2973, 2951, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3\,dx$$

$$\Big\downarrow \text{2936}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{c+dx}dx}{b}$$

$$\Big\downarrow \text{2951}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}$$

$$\Big\downarrow \text{ 2754}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\left(\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\left(\frac{2Bn\left(Bn\int\frac{(c+dx)\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\right.\right.}{d}\right.}{b}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}-\frac{3Bn(bc-ad)\left(\frac{2Bn\left(Bn\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{d}\right.}{b}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

output
```
((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/b - (3*B*(b*c - a*d
)*n*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*
(c + d*x))])/d) + (2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))]) + B*n*PolyLog[3, (d*(a + b*x))/(b*(c + d
*x))]))/d))/b
```

3.311.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2936
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a +
b*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
 B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.311.4 Maple [F]

$$\int \left(A+B\ln \left(e(bx+a)^{n} \left(dx+c\right)^{-n}\right)\right)^{3}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

3.311.5 Fricas [F]

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))^3\,dx=\int \left(B\log \left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas")
```

output
```
integral(B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3, x)
```

3.311.6 Sympy [F(-2)]

Exception generated.

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))^3\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.311.7 Maxima [F]

$$\int (A+B\log (e(a+bx)^n(c+dx)^{-n}))^3\,dx=\int \left(B\log \left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima")
```

output
```
3*A^2*B*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*x + 3*(a*e*n*log(b*x + a)/b
 - c*e*n*log(d*x + c)/d)*A^2*B/e - (B^3*b*d*x*log((d*x + c)^n)^3 - 3*(B^3*
a*d*n*log(b*x + a) - B^3*b*c*n*log(d*x + c) + B^3*b*d*x*log((b*x + a)^n) +
 (B^3*b*d*log(e) + A*B^2*b*d)*x)*log((d*x + c)^n)^2)/(b*d) - integrate(-(B
^3*b*c*log(e)^3 + 3*A*B^2*b*c*log(e)^2 + (B^3*b*d*x + B^3*b*c)*log((b*x +
a)^n)^3 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*log(e) + A*B^2*b*d)*x)*
log((b*x + a)^n)^2 + (B^3*b*d*log(e)^3 + 3*A*B^2*b*d*log(e)^2)*x + 3*(B^3*
b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + (B^3*b*d*log(e)^2 + 2*A*B^2*b*d*log(e)
)*x)*log((b*x + a)^n) - 3*(2*B^3*a*d*n^2*log(b*x + a) - 2*B^3*b*c*n^2*log(
d*x + c) + B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + (B^3*b*d*x + B^3*b*c)*l
og((b*x + a)^n)^2 + ((2*n*log(e) + log(e)^2)*B^3*b*d + 2*A*B^2*b*d*(n + lo
g(e)))*x + 2*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*(n + log(e)) + A*B^2*b
*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d*x + b*c), x)
```

3.311.8 Giac [F]

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\int \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.312 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx$

3.312.1 Optimal result

Integrand size = 33, antiderivative size = 425

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx \\
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{h} \\
&\quad+\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad-\frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\
&\quad+\frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad+\frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{h} \\
&\quad-\frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h} \\
&\quad-\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{h}+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{h}
\end{aligned}$$

output
```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h+(A+B*ln(e*
(b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-3*
B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,d*(b*x+a)/b/(d*x+c))/h+3
*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*
h+b*g)/(d*x+c))/h+6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,d*
(b*x+a)/b/(d*x+c))/h-6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3
,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-6*B^3*n^3*polylog(4,d*(b*x+a)/b/
(d*x+c))/h+6*B^3*n^3*polylog(4,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

3.312.2 Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x), x]
```

3.312.3 Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2973, 2953, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{g+hx}\,dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{g+hx}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\int \left(\frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)h\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(ch-dg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)h\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(-\frac{6B^2n^2\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{h(bc-ad)}+\frac{6B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{h(bc-ad)}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^3*Log[1 - (d*(a + b
*x))/(b*(c + d*x))])/((b*c - a*d)*h)) + ((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^3*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*c -
a*d)*h) - (3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*PolyLog[2, (d*(a
 + b*x))/(b*(c + d*x))])/((b*c - a*d)*h) + (3*B*n*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x)
)])/((b*c - a*d)*h) + (6*B^2*n^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Po
lyLog[3, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*h) - (6*B^2*n^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g
- a*h)*(c + d*x))])/((b*c - a*d)*h) - (6*B^3*n^3*PolyLog[4, (d*(a + b*x))/
(b*(c + d*x))])/((b*c - a*d)*h) + (6*B^3*n^3*PolyLog[4, ((d*g - c*h)*(a +
b*x))/((b*g - a*h)*(c + d*x))])/((b*c - a*d)*h))
```

3.312.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.312.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{hx+g}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

3.312.5 Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="frica
s")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h*x + g),
 x)
```

3.312.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.312.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="maxim
a")
```

output
```
A^3*log(h*x + g)/h - integrate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c
)^n)^3 + B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e)
+ A*B^2)*log((b*x + a)^n)^2 + 3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2
)*log((d*x + c)^n)^2 + 3*(B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x
+ a)^n) - 3*(B^3*log((b*x + a)^n)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*
B + 2*(B^3*log(e) + A*B^2)*log((b*x + a)^n))*log((d*x + c)^n))/(h*x + g),
x)
```

3.312.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{hx+g}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="giac"
)
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g), x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{g+h\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x), x)
```

3.313 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx$

3.313.1 Optimal result

Integrand size = 33, antiderivative size = 302

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx &= \frac{(a+bx)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(bg-ah)(g+hx)} \\ &\quad + \frac{3B(bc-ad)n(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} \\ &\quad + \frac{6B^2(bc-ad)n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\,\mathrm{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)} \\ &\quad - \frac{6B^3(bc-ad)n^3\,\mathrm{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)(dg-ch)}\end{aligned}$$

output
```
(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*h+b*g)/(h*x+g)+3*B*(-a*d+b
*c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)+6*B^2*(-a*d+b*c)*n^2*(A+B*ln(e*(b*x+a)^n
/((d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)
/(-c*h+d*g)-6*B^3*(-a*d+b*c)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(
d*x+c))/(-a*h+b*g)/(-c*h+d*g)
```

3.313.2 Mathematica [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx=\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2, x]
```

3.313.3 Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2973, 2953, 2755, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2755}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{3Bn\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{bg-ah}\right)$$

$$\Big\downarrow \text{2754}$$

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{3Bn\left(\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{dg-ch}\right.}{bg-ah}$$

$\downarrow$ 2821

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{3Bn\left(\frac{2Bn\left(Bn\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(2,\right.\right.}{dg-ch}\right.}{}$$

$\downarrow$ 7143

$$(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{(c+dx)(bg-ah)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}-\frac{3Bn\left(\frac{2Bn\left(Bn\,\text{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)-\text{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\right.}{dg-ch}\right.}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
```

output
```
(b*c - a*d)*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/((b*g -
a*h)*(c + d*x)*(b*g - a*h - ((d*g - c*h)*(a + b*x))/(c + d*x))) - (3*B*n*(
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((d*g - c*h)*(a + b*x)
)/((b*g - a*h)*(c + d*x))])/(d*g - c*h)) + (2*B*n*(-((A + B*Log[e*((a + b*
x)/(c + d*x))^n])*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x
))]) + B*n*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]))/(
d*g - c*h)))/(b*g - a*h))
```

3.313. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx$

3.313.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.313. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx$

3.313.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{\left(hx+g\right)^{2}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

3.313.5 Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^3}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{3}}{\left(hx+g\right)^{2}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="fri
cas")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^2*x^2 +
 2*g*h*x + g^2), x)
```

3.313.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^3}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**2,x)
```

output
```
Timed out
```

3.313.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="max
ima")
```

output
```
B^3*log((d*x + c)^n)^3/(h^2*x + g*h) + 3*(b*e*n*log(b*x + a)/(b*g*h - a*h^
2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*log(h*x + g)
/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A^2*B/e - 3*A^2*B*log((b*x + a)^
n*e/(d*x + c)^n)/(h^2*x + g*h) - A^3/(h^2*x + g*h) + integrate((B^3*c*h*lo
g(e)^3 + 3*A*B^2*c*h*log(e)^2 + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 +
 3*(B^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x
+ a)^n)^2 + 3*(A*B^2*c*h - (d*g*n - c*h*log(e))*B^3 - ((h*n - h*log(e))*B^
3*d - A*B^2*d*h)*x + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*log((d*x + c)
^n)^2 + (B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 3*(B^3*c*h*log(e)^2
+ 2*A*B^2*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x)*log((b*x
 + a)^n) - 3*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*x + B^3*c*h
)*log((b*x + a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x + 2*(B^3*
c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n))
*log((d*x + c)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2 + c*h^3)*x^2 + (d*g^2
*h + 2*c*g*h^2)*x), x)
```

3.313.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^2}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^2, x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(g+h\,x)^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2, x)
```

3.314 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx$

3.314.1 Optimal result

Integrand size = 33, antiderivative size = 629

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx \\
&= \frac{3B(bc-ad)hn(a+bx)\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{2(bg-ah)^2(dg-ch)(g+hx)} \\
&\quad+\frac{b^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{2h(bg-ah)^2}-\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{2h(g+hx)^2} \\
&\quad+\frac{3B^2(bc-ad)^2hn^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad+\frac{3B(bc-ad)(2bdg-bch-adh)n(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{2(bg-ah)^2(dg-ch)^2} \\
&\quad+\frac{3B^3(bc-ad)^2hn^3\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad+\frac{3B^2(bc-ad)(2bdg-bch-adh)n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2} \\
&\quad-\frac{3B^3(bc-ad)(2bdg-bch-adh)n^3\operatorname{PolyLog}\left(3,\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)}{(bg-ah)^2(dg-ch)^2}
\end{aligned}$$

output
```
3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g
)^2/(-c*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(-a*h
+b*g)^2-1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(h*x+g)^2+3*B^2*(-a*d+b*
c)^2*h*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h
+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h
+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^3*(-a*d+b*c)^2*h*n^3*polylog(
2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^2*(
-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*pol
ylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2-3*
B^3*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a
*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2
```

3.314.2 Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx=\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3, x]
```

3.314.3 Rubi [A] (warning: unable to verify)

Time = 1.35 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2973, 2953, 2798, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(g+hx)^3}\,dx$$

$$\Big\downarrow \; 2973$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2953}$$

$$(bc-ad)\int \frac{\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2798}$$

$$(bc-ad)\left(\frac{3Bn\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2h(bc-ad)}-\frac{\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{2h(bc-ad)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)^2}\right)$$

$$\Big\downarrow \text{2804}$$

$$(bc-ad)\left(\frac{3Bn\int \left(\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bg-ah)^2(a+bx)}+\frac{(bc-ad)h(-2bdg+bch+adh)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bg-ah)^2(dg-ch)\left(bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}\right)}+\frac{(bc-ad)^2h^2\left(A+B\log\left(e\ldots\right.\right.}{(bg-ah)(dg-ch)\left(bg-ah-\ldots\right.}\right.}{2h(bc-ad)}\right.$$

$$\Big\downarrow \text{2009}$$

$$(bc-ad)\left(\frac{3Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bn(bg-ah)^2}+\frac{h^2(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(c+dx)(bg-ah)^2(dg-ch)\left(-\frac{(a+bx)(dg-ch)}{c+dx}-ah+bg\right)}+\frac{2Bh^2n(bc-ad)^2\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\ldots}{(bg-ah)^2(dg-ch\ldots}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

3.314. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx$

output
```
(b*c - a*d)*(-1/2*((b - (d*(a + b*x))/(c + d*x))^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3)/((b*c - a*d)*h*(b*g - a*h - ((d*g - c*h)*(a + b*x))/(c
+ d*x))^2) + (3*B*n*(((b*c - a*d)^2*h^2*(a + b*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2)/((b*g - a*h)^2*(d*g - c*h)*(c + d*x)*(b*g - a*h - ((d*g
- c*h)*(a + b*x))/(c + d*x))) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^3)/(3*B*(b*g - a*h)^2*n) + (2*B*(b*c - a*d)^2*h^2*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))
])/((b*g - a*h)^2*(d*g - c*h)^2) + ((b*c - a*d)*h*(2*b*d*g - b*c*h - a*d*h
)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((d*g - c*h)*(a + b*x))
/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (2*B^2*(b*c - a
*d)^2*h^2*n^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])
/((b*g - a*h)^2*(d*g - c*h)^2) + (2*B*(b*c - a*d)*h*(2*b*d*g - b*c*h - a*d
*h)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, ((d*g - c*h)*(a +
b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) - (2*B^2*(b*
c - a*d)*h*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[3, ((d*g - c*h)*(a + b*x)
)/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)))/(2*(b*c - a*d)
*h))
```

3.314.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2798
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Simp[b*n*(p/((q + 1)
*(e*f - d*g)))   Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
 - d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2953
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)   Sub
st[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2
)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}
, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.314. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx$

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.314.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{\left(hx+g\right)^{3}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

3.314.5 Fricas [F]

$$\int \frac{(A+B\log{(e(a+bx)^n(c+dx)^{-n})})^3}{(g+hx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{3}}{\left(hx+g\right)^{3}}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="fri
cas")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^3*x^3 +
 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log{(e(a+bx)^n(c+dx)^{-n})})^3}{(g+hx)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**3,x)
```

output
```
Timed out
```

3.314.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="max
ima")
```

output
```
1/2*B^3*log((d*x + c)^n)^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 3/2*(b^2*e*n*lo
g(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*e*n*log(d*x + c)/(d^2
*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*e*g*n - a^2*d^2*e*h*n - (2*c*
d*e*g*n - c^2*e*h*n)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h
^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d
*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*e*n - a*d*e*n)/((d*g^2*h - c*g*h^2)*a -
(d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*A^
2*B/e - 3/2*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^
2*h) - 1/2*A^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + integrate(1/2*(2*B^3*c*h*lo
g(e)^3 + 6*A*B^2*c*h*log(e)^2 + 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3
 + 6*(B^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*
x + a)^n)^2 + 3*(2*A*B^2*c*h - (d*g*n - 2*c*h*log(e))*B^3 - ((h*n - 2*h*lo
g(e))*B^3*d - 2*A*B^2*d*h)*x + 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*l
og((d*x + c)^n)^2 + 2*(B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 6*(B^3
*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e
))*x)*log((b*x + a)^n) - 6*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d
*h*x + B^3*c*h)*log((b*x + a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e
))*x + 2*(B^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log
((b*x + a)^n))*log((d*x + c)^n))/(d*h^4*x^4 + c*g^3*h + (3*d*g*h^3 + c*h^4
)*x^3 + 3*(d*g^2*h^2 + c*g*h^3)*x^2 + (d*g^3*h + 3*c*g^2*h^2)*x), x)
```

3.314.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="gia
c")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^3, x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(g+h\,x)^3}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3,x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])  #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```